extrapolation from the physical region to the resonance location. In due course, it may be possible to estimate the effective range from the empirical data in $\bar{K}N$ scattering,[29,30] in which case this uncertainty will be greatly reduced.

In the $I=Y=0$ $s_{1/2}$ PB system, another energy range of strong interactions is known, namely at the threshold for $\Lambda\eta$ production in the $\bar{K}N$ system. Whether the sharp rise and fall in this cross section is due to the influence of a resonance state, or whether it is due to a strong $\Lambda\eta$ scattering interaction, is not yet settled.[31] We may remark here that this model does not account for this effect, in either way. The $\Lambda\eta$ scattering amplitude obtained is slowly varying, with the small value $A_{\eta\Lambda} = (-0.12+0.15i)$ F at threshold; the smallness of this value is related to the fact that our model gives zero diagonal potential for the $\Lambda\eta$ system. We conclude that this $\Lambda\eta$ enhancement is generated by forces not included in our model calculation.

[29] Kittel and Otter (Ref. 30) have very recently included a finite effective range R_0 just for the $I=0$ scattering length, to obtain an over-all fit to the low-energy $K^- - p$ scattering available from Refs. 2, 4, and 6, together with the M^* and Γ values observed for $Y_0^*(1405)$. They obtained an acceptable fit for $R_0=0.08\pm0.05$ F, in which case their zero-energy scattering length takes the value $A_0=(-1.54\pm0.02)+(0.53\pm0.03)$ F. When they also included the 400 MeV/c values for A_0 and A_1, they found that the only acceptable fit was for the Watson III solution, with $R_0=0.11 \pm0.03$ F. This analysis now needs to be repeated in view of the K_2^0-p data (cf. Ref. 27) now becoming available in the 400-MeV/c region.

[30] W. Kittel and G. Otter, Phys. Letters **22**, 115 (1966).

ACKNOWLEDGMENTS

The last two authors (G.R. and T.C.W.) wish to acknowledge that their support was from a U.K.A.E.A. grant during the time that most of this work was being carried out.

[31] D. Berley, P. Connolly, E. Hart, D. Rahm, D. Stonehill, B. Thevenet, W. Willis, and S. Yamamoto, Phys. Rev. Letters **15**, 641 (1965).

PHYSICAL REVIEW VOLUME 153, NUMBER 5 25 JANUARY 1967

$SU(6)_W$ Algebra and the Commutators of Electric Dipoles at Infinite Momentum

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(Received 1 August 1966)

The saturation of the $SU(6)_W$ algebra at infinite momentum is discussed. A possible physical interpretation of the tensor generators of $SU(6)_W$ in terms of an assumption of partial conservation is critically analyzed. The implied occurrence of singularities in the tensor amplitudes requires a careful definition of a limiting procedure defining the tensor charges. A collinear limiting procedure, which relates the tensor charges to the total magnetic moments, appears as the most convenient one. The matrix elements of the tensor charges are then compared in the infinite-momentum limit with those of the electric dipoles, and the following implications are exhibited: The charge radii of baryons are pure F; the D/F ratio of axial charges equals the corresponding ratio for the total baryon magnetic moments; a simple relation exists among the isovector total moment of the nucleon, the axial renormalization constant, and the charge radius of the proton; and an extended form of universality holds for tensor and axial currents. We also discuss the saturation of the unitary symmetric part of the commutators, particularly in connection with the possible occurrence of Schwinger terms.

1. INTRODUCTION AND SUMMARY OF RESULTS

A CLASSIFICATION of the charges generating the compact $U(12)$ according to the behavior of their matrix elements between states of infinite momentum has been proposed,[1,2] leading to the distinction between "good" and "bad" charges. The longitudinal and time components of the vector and axial charges and the transverse components of the tensor charges are good charges. In the limit of infinite momentum the set of nonequivalent good charges generates the algebra of $SU(6)_W$.

In the present paper we analyze the consequences of this algebra, avoiding particular hypotheses of approximate saturation. The starting point of the analysis will be a connection between the tensor charges, following from a hypothesis of partial conservation of tensor

[1] R. F. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy,* (W. H. Freeman and Company, San Francisco, California, 1966).

[2] S. Fubini, G. Segré, and J. D. Walecka, Ann. Phys. (N. Y.) 39, 381 (1966).

currents (PCTC).[1-3] Such a hypothesis is strongly suggestive if one wants to assign a physical interpretation to the tensor charges. On the other hand, its mathematical formulation, in the strong form $\partial_\mu J_{\mu\nu}(x)=\Gamma(-\Box^2)J_\nu(x)$, seems necessarily to lead to the occurrence of singular tensor amplitudes between states of equal mass, and great care must be taken in the examination of the most convenient limiting procedure. Our analysis shows the convenience of a collinear limiting procedure, which avoids singularities and provides for a relation between the nucleon tensor amplitudes and the total magnetic moments.

The assumption of definite commutation relations both for dipoles and for tensor charges allows a number of conclusions in this scheme. From the comparison of the unitary-antisymmetric parts of these commutation relations (which are free of Schwinger terms) we obtain: (i) the F character of the charge radii of baryons; (ii) the determination of the proportionality constant occurring in the PCTC relation in terms of the proton charge radius, namely, $\Gamma^{-2}(0)=\frac{1}{3}\langle r_p^2\rangle_{G_E}$. The knowledge of the proportionality constant appearing in the PCTC allows one to derive in a purely algebraic way relations like

$$(D/F)_{\text{axial}}=(D/F)_{\text{total magnetic moments}}$$
$$(\mu_p-\mu_n)/2m=g_A(\tfrac{1}{3}\langle r_p^2\rangle_{G_E})^{1/2}$$
$$\mu^*/2m=\tfrac{1}{3}\sqrt{2}G^*(\tfrac{1}{3}\langle r_p^2\rangle_{G_E})^{1/2}\,.$$

Remarkably, such relations had been derived previously only from saturation of the algebra of electric dipoles, by a quite different approach using dynamical arguments [see Ref. 16]. These relations compare fairly well with experiment and they are of interest as they are derived without assumptions of partial saturation. Among our conclusions, we also point out the formal elegance of the extended universality exhibited by the relations

$$\partial_\mu J_{\mu\nu}^{(\alpha)}=m_V^2(f/\sqrt{2})\phi_\nu^{(\alpha)}\quad \text{(PCTC)}\,,$$
$$\partial_\mu J_{\mu 5}^{(\alpha)}=m_P^2(f/\sqrt{2})\phi^{(\alpha)}\quad \text{(PCAC)}\,.$$

[*Note added in proof.* Recently, Costa, Savoy, and Zimerman (to be published) have rederived the above extended universality using the method of Kawarabayashi and Suzuki. The extended universality has also been used in a recent calculation of the π^0 lifetime by Maiani and Preparata (to be published).]

The comparison of the unitary symmetric parts of the commutators of dipoles and of tensor charges encounters some difficulties because of the possible existence of Schwinger terms. Although their exclusion leads to acceptable and even pleasing results, such as the sum rules of Drell and Hearn,[4] their presence is suggested from formal arguments. Such points do certainly deserve more discussion and various alternatives are presented in the text.

In Sec. 2 we establish our notation and we discuss the PCTC hypothesis. The connection between tensor charges and electric dipoles is derived in Sec. 3, where the limiting procedure for calculating the tensor amplitudes is also discussed. In Sec. 4 we compare the unitary antisymmetric parts of the commutation relations of dipoles and of tensor charges and we also point out the extended form of universality mentioned above. In Sec. 5 we derive the relations between electromagnetic and axial quantities. In Sec. 6 we compare the unitary symmetric parts of the commutators and we discuss the presence of Schwinger terms in the dipole commutator. Finally in Sec. 7 we extend the analysis to the commutation relations between tensor and axial charges. Some details of calculations are reported in the appendices.

2. TENSOR CHARGES AND THE ASSUMPTION OF PARTIAL CONSERVATION OF TENSOR CURRENTS

It has been recently proposed[1-3] that the tensor currents, whose space integrals are among the generators of $U(12)$,[5] may be given a physical content through a partial-conservation hypothesis (PCTC).

In the quark model the tensor currents are[6]

$$J_{\mu\nu}^{(\alpha)}(x)=-\bar{\psi}(x)\sigma_{\mu\nu}(\tfrac{1}{2}\lambda^{(\alpha)})\psi(x)\,,\quad (\alpha=0,\cdots 8)\quad (1)$$

and the vector and axial currents have the form

$$J_\mu^{(\alpha)}(x)=i\bar{\psi}(x)\gamma_\mu(\tfrac{1}{2}\lambda^\alpha)\psi(x)\,,\quad (2)$$
$$J_{\mu 5}^{(\alpha)}(x)=i\bar{\psi}(x)\gamma_\mu\gamma_5(\tfrac{1}{2}\lambda^\alpha)\psi(x)\,.\quad (3)$$

We also define the following charges:

$$T_i^{(\alpha)}=-i\int d^3x\, J_{4i}^{(\alpha)}(x)\,,\quad (4)$$

$$V^{(\alpha)}=-i\int d^3x\, J_4^{(\alpha)}(x)\,,\quad (5)$$

$$A^{(\alpha)}=-i\int d^3x\, J_{45}^{(\alpha)}(x)\,,\quad (6)$$

$$A_i^{(\alpha)}=-\int d^3x\, J_{i5}^{(\alpha)}(x)\,.\quad (7)$$

The commutation relations between the above tensor charges suggested by the quark model are

$$[T_i^{(\alpha)},T_j^{(\beta)}]=i\delta_{ij}f_{\alpha\beta\gamma}V^{(\gamma)}+i\epsilon_{ijk}d_{\alpha\beta\gamma}A_k^{(\gamma)}\,.\quad (8)$$

[3] W. Krolikowski, Nuovo Cimento **42A**, 435 (1966); **44A**, 745 (1966); **46A**, 106 (1966).

[4] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).

[5] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).

[6] We use the metric where $p\cdot q=\mathbf{p}\cdot\mathbf{q}-p_0q_0$. The γ matrices are Hermitian and $\sigma_{\mu\nu}=(1/2i)[\gamma_\mu,\gamma_\nu]$.

The PCTC hypothesis is generally written in the form

$$\partial_\mu J_{\mu\nu}{}^{(\alpha)}(x) = a\phi_\nu{}^{(\alpha)}(x)\,, \qquad (9)$$

where $\phi_\nu{}^{(\alpha)}(x)$ are the vector-meson fields and $a=a(-\Box^2)$ is supposed to have a weak dependence on the momentum transfer. On the hypothesis of vector meson dominance in the vector currents we have

$$J_\nu{}^{(\alpha)}(x) = g\phi_\nu{}^{(\alpha)}(x)\,, \qquad (10)$$

$$\partial_\mu J_{\mu\nu}{}^{(\alpha)} = \Gamma J_\nu{}^{(\alpha)}(x)\,, \qquad (11)$$

where $\Gamma = a/g$ is also supposed to be a weakly momentum-transfer-dependent operator.

We shall always use in the following the PCTC hypothesis in the form of Eq. (11). In a weaker form we can simply state that the divergences of the tensor currents as well as the vector currents are dominated by the vector meson pole, so that Eq. (11) is actually true only for matrix elements at momentum transfer near the meson pole. In this case the extrapolation procedure from the pole to $k^2=0$ is important, and the results may depend on the form factors one is actually extrapolating.[2] Under the stronger assumption of Eq. (11), there is not such an ambiguity and the only point is to know whether $\Gamma(0)$ is or is not zero. Let us consider the application to nucleons. By Eq. (11) we can directly relate[3] the tensor form factors to the electromagnetic form factors, as shown in Appendix 1. This makes it possible to express the amplitudes of tensor charges in terms of electromagnetic quantities only (see Appendix 2). It turns out that the matrix elements of the transverse components of the tensor current (i.e. of J_{31}, J_{32}, J_{41}, and J_{42}) between states of collinear momenta are proportional to $\Gamma(k^2)$ through a finite quantity. So $\Gamma(0)=0$ would imply the vanishing of the good tensor charges in the limit of infinite momentum. For this reason we assume in the following $\Gamma(0)\neq 0$.

An unpleasant feature of the adopted formulation of PCTC is the need of singularities in the amplitudes of tensor charges. This is made immediately evident by taking $\nu=4$ in Eq. (11) and integrating over all space. The right-hand side becomes proportional to the charge operator, while the left-hand side has the form of a space integral of a spatial divergence and would vanish for a nonsingular behavior. For the Fourier transforms we have $\lim_{\mathbf{k}\to 0} k_i T_i{}^{(\alpha)}(\mathbf{k}) = -i\Gamma(0)V^{(\alpha)}$, so that if we take the momentum transfer $\mathbf{k}$ along the z axis the good charges T_1 and T_2 are regular, while the longitudinal charge T_3 presents a kinematic singularity of the order $1/k_z$. We do not consider of physical relevance the occurrence of this singularity; the tensor currents are here introduced only as mathematical intermediaries and only their divergence is supposed to bear a physical significance. Furthermore the above singularity will not affect the present analysis, as we shall only be concerned with the transverse part of Eq. (11).

3. CONNECTION BETWEEN TENSOR CHARGES AND ELECTRIC DIPOLES

In this section we establish a connection between the tensor charges and the electric dipoles, which will be the starting point of the subsequent analysis of the commutation relations.

From the definition of the electric dipoles

$$D_i{}^{(\alpha)} = \int d^3x\, x_i J_0{}^{(\alpha)}(x) \qquad (12)$$

by virtue of Eq. (11) [which also implies conservation of vector current, (CVC)] and (4) we have

$$\dot{D}_i{}^{(\alpha)} = \int d^3x\, J_i{}^{(\alpha)}(x) = \Gamma^{-1}\dot{T}_i{}^{(\alpha)} \qquad (13)$$

from which it follows

$$T_i{}^{(\alpha)} = \Gamma(D_i{}^{(\alpha)} + C_i{}^{(\alpha)})\,, \qquad (14)$$

where $C_i{}^{(\alpha)}$ are conserved charges.

Equation (14) implies that the matrix elements of T_i and D_i between states of different energy are proportional, through some gentle function $\Gamma(k^2)$, k being the momentum transfer. In the limit of infinite momentum k^2 approaches zero, so that it will be possible to determine $\Gamma(0)$ by comparing the commutation relations of the tensor charges with those of the electric dipoles, as we shall see in the next section. To this end we also need the diagonal matrix elements of T_i and D_i. The evaluation of these matrix elements between states of the baryon octet is carried out in Appendix 2. For the matrix elements of D_i we find that, as is well known, they are related to the anomalous magnetic moments, while for the T_i the evaluation needs a little care and may present some ambiguity.[2] In fact the amplitudes of tensor charges between states of equal masses are only defined through a limiting procedure from nonzero momentum transfer $k=p'-p$. The result is not unique. For $\mathbf{k}$ in the z direction we find that the matrix elements of T_i are related to the total magnetic moments, while for $\mathbf{k}$ orthogonal to the z axis we find that they are related to the anomalous moments, apart from an unphysical singular term which may perhaps be eliminated by a suitable regularization procedure.

We adopt here the first procedure (collinear limit) in view of the following arguments:

(a) The matrix elements $\langle p'|Q(\mathbf{k})|p\rangle$, of the Fourier transform $Q(\mathbf{k})$ of the good charge densities, with a finite momentum transfer $\mathbf{k}$ collinear to the external momenta, are independent of $\mathbf{k}$ in the limit $p\to\infty$. The generalized charges $Q(\mathbf{k})$ coincide, in this sense, with the true charges $Q(0)$, whenever these are well defined, and must of course essentially satisfy the same algebra. Furthermore the matrix elements of the charges $Q(\mathbf{k})$, for collinear $\mathbf{k}$, can be directly evaluated giving unambiguous results in all cases we are interested in. We thus propose to define $\langle p|Q(0)|p\rangle$ as identical to $\langle p'|Q(\mathbf{k})|p\rangle$, at infinite momentum and for collinear $\mathbf{k}$, whenever any ambiguity may arise. No similar results hold for transverse $\mathbf{k}$ and it may be seen that a limiting procedure from a purely transverse $\mathbf{k}$ is not in general justified because of possible singular terms, as in the case of tensor charges (see Appendix 2). It may also be seen that no troubles arise if we take $k_z\neq 0$ and infinitesimal transverse components k_x, k_y. This is also true for the electric dipoles.

(b) The collinear limit is the most suitable one, as we are considering the algebra of those charges which are good for collinear momenta. As a confirmation of this point of view, the results of this procedure are in agreement with the $SU(6)_W$ symmetry. In fact in the limit of infinite momentum the transverse tensor charges we are considering are equivalent to those $SU(6)_W$ generators which behave like $\beta\sigma_i\lambda^\alpha$, which implies a vector character under W spin. The same behavior is also exhibited by the Sachs magnetic form factor G_M, while the Pauli form factor F_2 also contains a W-spin scalar.

Another argument in favor of the total magnetic moments is provided by an approximate saturation of the tensor algebra with octet and decuplet intermediate states. In this case we obtain the results of $SU(6)_W$ symmetry if we take the tensor amplitudes as proportional to the total magnetic moments.[2]

4. COMPARISON OF THE COMMUTATION RELATIONS OF TENSOR CHARGES TO THOSE OF ELECTRIC DIPOLES

We want to compare the commutation relations (8) for tensor charges to those for electric dipoles. The latter are obtained from the charge densities commutators

$$[J_0^{(\alpha)}(x),J_0^{(\beta)}(x')]_{x_0=x_0'}=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_0^{(\gamma)}(x)+S^{(\alpha\beta)}(x,x'),\quad(15)$$

where by $S^{(\alpha\beta)}(x,x')$ we have denoted the possible Schwinger terms. We remark that the necessity of such terms has not been proved so far and they are generally omitted.

Equation (15) gives

$$[D_i^{(\alpha)},D_j^{(\beta)}]=if_{\alpha\beta\gamma}R_{ij}^{(\gamma)}+S_{ij}^{(\alpha\beta)},\quad(16)$$

where

$$R_{ij}^{(\gamma)}=\int d^3x\,x_ix_jJ_0^{\gamma)}(x),\quad(17)$$

and $S_{ij}^{(\alpha\beta)}$ is antisymmetric under the interchange $(i,\alpha)\leftrightarrow(j,\beta)$. By requiring an octet behavior of $J_0^{(\alpha)}(x)$ under commutation with the $SU(3)$ generators, we get the condition

$$\int d^3x'S^{(\alpha\beta)}(x,x')=\int d^3x\,S^{(\alpha\beta)}(x,x')=0.\quad(18)$$

If we assume for $S^{(\alpha\beta)}(x,x')$ a form

$$S^{(\alpha\beta)}(x,x')=\frac{1}{2}\left[\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{x}')-\frac{\partial}{\partial x_k'}\delta(\mathbf{x}-\mathbf{x}')\right]R_k^{(\alpha\beta)}(x,x'),\quad(19)$$

we find from Eq. (18)

$$\left(\frac{\partial}{\partial x_k}R_k^{(\alpha\beta)}(x,x')\right)_{x=x'}=\left(\frac{\partial}{\partial x_k'}R_k^{(\alpha\beta)}(x,x')\right)_{x=x'}.\quad(20)$$

This gives for $S_{ij}^{(\alpha\beta)}$ of Eq. (16)

$$S_{ij}^{(\alpha\beta)}=\frac{1}{2}\int d^3x[x_iR_j^{(\alpha\beta)}(x,x)-x_jR_i^{(\alpha\beta)}(x,x)]\quad(21)$$

showing that $S_{ij}^{(\alpha\beta)}$ must be symmetric in $\alpha\leftrightarrow\beta$.

We now consider in detail the commutation relations (8) and (16) that we can write in the spherical $SU(3)$ basis as

$$[T_i^{(\alpha)},T_j^{(\beta)}]=-\delta_{ij}\sqrt{3}\begin{pmatrix}8&8&8_a\\\alpha&\beta&\gamma\end{pmatrix}V^{(\gamma)}+i\epsilon_{ijk}(5/3)^{1/2}\begin{pmatrix}8&8&8_s\\\alpha&\beta&\gamma\end{pmatrix}A_k^{(\gamma)}-i\epsilon_{ijk}\frac{4}{\sqrt{3}}\begin{pmatrix}8&8&1\\\alpha&\beta&0\end{pmatrix}A_k^{(0)},\quad(22)$$

$$[D_i^{(\alpha)},D_j^{(\beta)}]=-\sqrt{3}\begin{pmatrix}8&8&8_a\\\alpha&\beta&\gamma\end{pmatrix}R_{ij}^{(\gamma)}+S_{ij}^{(\alpha\beta)},\quad(23)$$

where we have used the standard Clebsch-Gordan coefficients for $SU(3)$.[7] By virtue of the symmetry in α, β, $S_{ij}^{(\alpha\beta)}$ has only components on the representations **1**, $\mathbf{8}_s$ and **27** of $SU(3)$ according to

$$S_{ij}^{(\alpha\beta)}=\begin{pmatrix}8&8&1\\\alpha&\beta&0\end{pmatrix}S_{ij}^{(1)}+\begin{pmatrix}8&8&8_s\\\alpha&\beta&\gamma\end{pmatrix}S_{ij}^{(8,\gamma)}+\begin{pmatrix}8&8&27\\\alpha&\beta&\gamma\end{pmatrix}S_{ij}^{(27,\gamma)}.\quad(24)$$

[7] J. J. De Swart, Nuovo Cimento **31**, 420 (1964); Rev. Mod. Phys. **35**, 916 (1963).

From Eqs. (22), (23), and (24) we get

$$\Gamma^{-2}[T_i^{(\alpha)},T_j^{(\beta)}]-[D_i^{(\alpha)},D_j^{(\beta)}]$$

$$=-\sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \alpha & \beta & \gamma\end{pmatrix}(\delta_{ij}\Gamma^{-2}V^{(\gamma)}-R_{ij}^{(\gamma)})+\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \beta & \gamma\end{pmatrix}\left[i\epsilon_{ijk}\left(\sqrt{\frac{5}{3}}\right)\Gamma^{-2}A_k^{(\gamma)}-S_{ij}^{(8,\gamma)}\right]$$
$$+\begin{pmatrix}8 & 8 & 1\\ \alpha & \beta & 0\end{pmatrix}\left(-i\epsilon_{ijk}\frac{4}{\sqrt{3}}\Gamma^{-2}A_k^{(0)}-S_{ij}^{(1)}\right)-\begin{pmatrix}8 & 8 & 27\\ \alpha & \beta & \gamma\end{pmatrix}S_{ij}^{(27,\gamma)}. \quad (25)$$

We now take the matrix elements of Eq. (25) between states of the baryon octet with momentum along the z axis, and we insert in the left-hand side a complete set of intermediate states. For i, $j=1$, 2, T_i and D_i are both "good" operators, so that we may be hopeful about the possibility of saturating the commutators with states of finite mass in the limit of infinite momentum. In this limit one has $k^2\rightarrow 0$, independently of the intermediate state, so that in Eq. (25) $\Gamma=\Gamma(0)$ is actually a constant. Furthermore, since C_i in Eq. (14) has nonvanishing matrix elements only between states of the same four-momentum, and hence of the same mass, all the intermediate-state contributions not coming from the octet cancel out in the left-hand side of Eq. (25). We thus obtain the remarkable result that Eq. (25) supplies an exact sum rule by only retaining the pole terms.

We now use the Wigner-Eckart theorem and the orthogonality relations of the Clebsch-Gordan coefficients to obtain the following equation:

$$\xi_1(88N\eta)\xi_2(88N\eta')\xi_3(8N^*8\eta')\sum_{\zeta\zeta'}\xi_1(888\zeta)\xi_1(888\zeta')$$
$$\times(8/N)^{1/2}(8\zeta\zeta'|\beta_{\text{II}}(8888)|N\eta\eta')[A_{ij}^{\zeta\zeta'}(r,r')-\xi_1(88N\eta)A_{ji}^{\zeta\zeta'}(r,r')]$$
$$=-\sqrt{3}\delta_{N8}\delta_{\eta a}[\delta_{ij}\Gamma^{-2}\langle r|V(88\eta')|r'\rangle-\langle r|R_{ij}(88\eta')|r'\rangle]$$
$$+\delta_{N8}\delta_{\eta s}[i\epsilon_{ijk}(\sqrt{\tfrac{5}{3}})\Gamma^{-2}\langle r|A_k(88\eta')|r'\rangle-\langle r|S_{ij}^{(8)}(88\eta')|r'\rangle]$$
$$+\delta_{N1}[-i\epsilon_{ijk}(4/\sqrt{3})\Gamma^{-2}\langle r|A_k^{(0)}(88)|r'\rangle-\langle r|S_{ij}^{(1)}(88)|r'\rangle]-\delta_{N27}\langle r|S_{ij}^{(27)}(88)|r'\rangle, \quad (26)$$

where ξ_1, ξ_2, and ξ_3 are sign factors and $(8\zeta\zeta'|\beta_{\text{II}}(8888)|N\eta\eta')$ are recoupling coefficients as defined in Ref. 7. We have also defined

$$A_{ij}^{\zeta\zeta'}(r,r')=\sum_s[\Gamma^{-2}\langle r|T_i(88\zeta)|s\rangle\langle s|T_j(88\zeta')|r'\rangle-\langle r|D_i(88\zeta)|s\rangle\langle s|D_j(88\zeta')|r'\rangle]. \quad (27)$$

The quantities $V(88\eta)$ and the similar ones appearing in Eqs. (26) and (27) are reduced matrix elements in $SU(3)$ and are explicitly given in Appendix 3. Finally r, r', and s are helicity labels.

Letting N, η, and η' take on all possible values, we obtain from (26) a set of independent equations. Those coming from the $SU(3)$ symmetric parts will be discussed in Sec. 6. We consider here the antisymmetric part which is free from Schwinger terms and gives rise to definite predictions. For $N=10$ and $N=10^*$ we get trivial results. Taking $N=8$, $\eta=a$, $\eta'=s$ we obtain

$$-\mu_n/2m^2=\tfrac{1}{3}\langle r_n^2\rangle_{F_1}, \quad (28)$$

or equivalently[8]

$$\langle r_n^2\rangle_{G_E}=0 \quad (29)$$

G_E being the electric Sachs form factor. The result of Eq. (29) can also be expressed by saying that the charge radii of the baryons are pure F.

We next consider the case $N=8$, $\eta=\eta'=a$ in Eq. (26). By taking into account the preceding result, we get

$$\Gamma^{-2}=\tfrac{1}{3}\langle r_p^2\rangle_{G_E}. \quad (30)$$

Inserting the experimental value for the proton radius[9] we have $\Gamma=0.43\ m_p$. We note that the relation (30) can also be directly obtained by comparing the sum rule of Cabibbo and Radicati[10] with that obtained from the unitary antisymmetric part of the commutation relations of tensor charges.[2]

Equation (30) appears quite interesting as it relates the proportionality constant of the PCTC relation to a physical quantity. As a consequence, it is possible to obtain by use of PCTC a number of relations between measurable objects, some of which are considered in the next section. In addition, the sum rules derived in Ref. 2 from tensor commutation relations acquire a more direct physical meaning.

Another interesting aspect of Eq. (30) is that it im-

[8] The charge radius is defined as

$$\tfrac{1}{6}\langle r^2\rangle_{G_E}=-((d/dq^2)G_E(q^2))_{q^2=0}+G_E(0)/8m^2$$

and

$$G_E(q^2)=F_1(q^2)-(q^2/4m^2)F_2(q^2).$$

[9] We use the value $\langle r_p^2\rangle_{G_E}=0.72$ F^2 deduced from the fit by T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, 922 (1966).

[10] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966).

plies a sort of extended universality, in connection with partial conservation of axial currents (PCAC). In the vector-meson-pole dominance scheme we can use $\frac{1}{6}\langle r_p^2\rangle \simeq m_V^{-2}$ and $g=m_V^2/[2G_{VNN}(0)]$, where m_V is the common mass of the vector meson nonet, g is the constant defined in Eq. (10) and G_{VNN} is the nucleon-vector-meson coupling constant. From (9) and (30) we thus obtain

$$\partial_\mu J_{\mu\nu}{}^{(\alpha)}=\frac{m_V{}^3}{2\sqrt{2}G_{VNN}(0)}\phi_\nu{}^{(\alpha)} \qquad (31)$$

which looks quite analogous to the Goldberger and Treiman relation. If we insert in Eq. (31) the relation $2G_{VNN}=m_V/f$, obtained from the commutators of the $U(3)\otimes U(3)$ chiral algebra,[11] the PCTC relation can be written in the form

$$\partial_\mu J_{\mu\nu}{}^{(\alpha)} \quad m_V{}^2(f/\sqrt{2})\phi_\nu{}^{(\alpha)} \qquad (32)$$

exhibiting a strong analogy[12] to the PCAC relation

$$\partial_\mu J_{\mu 5}{}^{(\alpha)}=m_{P_\alpha}{}^2(f/\sqrt{2})\phi^{(\alpha)}. \qquad (33)$$

We note that the universality expressed by Eqs. (32) and (33) is in agreement with a symmetry $U(2)_W$ in the limit of degenerate meson masses. This can be seen as follows. Let us take the matrix elements of (32) and (33) between one-particle states moving in the z direction. One gets

$$-ik_3\langle\beta|J_{3\nu}{}^{(\alpha)}|\gamma\rangle-ik_4\langle\beta|J_{4\nu}{}^{(\alpha)}|\gamma\rangle =m^2(f/\sqrt{2})\langle\beta|\phi_\nu{}^{(\alpha)}|\gamma\rangle \qquad (34)$$

and

$$-ik_3\langle\beta|J_{35}{}^{(\alpha)}|\gamma\rangle-ik_4\langle\beta|J_{45}{}^{(\alpha)}|\gamma\rangle =m^2(f/\sqrt{2})\langle\beta|\phi^{(\alpha)}|\gamma\rangle. \qquad (35)$$

Since J_{23}, J_{31}, J_{35} transform like the x, y, z components of a W-spin vector, and the same holds for $-J_{42}$, J_{41}, J_{45} and for $-\phi_2$, ϕ_1, ϕ, Eq. (34) follows from Eq. (35) by W-spin invariance for $\nu=1$, 2. A similar connection is contained in a recent paper by Costa and Tonin.[13,14]

5. RELATIONS BETWEEN ELECTROMAGNETIC AND AXIAL QUANTITIES

In this section we consider some relations arising from a comparison of the matrix elements of tensor and axial charges in the limit $p\to\infty$.

Let us first consider the matrix elements of the baryon octet. From the Lorentz behavior of the charges we obtain for instance[15]

$$\langle\beta|T_1{}^{(\alpha)}|\gamma\rangle_{p\to\infty}=-\langle\beta|A(\gamma_4\sigma_2\lambda^\alpha)|\gamma\rangle_{p\to\infty}. \qquad (36)$$

According to the usual models of baryons we suppose that the W-spin generators coincide with the quark spin operators between baryons at rest. So we get

$$\langle\beta|T_1{}^{(\alpha)}|\gamma\rangle_{p\to\infty}=-\langle\beta|A_2{}^{(\alpha)}|\gamma\rangle_{p=0}. \qquad (36')$$

Reducing with respect to the spin and $SU(3)$ variables and using (A15) and (30), we get

$$(D/F)_{\text{axial}}—=(D/F)_{\text{total magnetic moments}}, \qquad (37)$$

$$(\mu_p-\mu_n)/2m=g_A(\tfrac{1}{3}\langle r_p{}^2\rangle_{G_E})^{1/2}. \qquad (38)$$

These relations coincide with those obtained, by a different procedure, by Buccella, Gatto, and Veneziano.[16]

A relation analogous to (38) can be derived for $N-N^*$ transition matrix elements neglecting the mass-difference effects. We obtain in this case

$$\mu^*/2m=\tfrac{1}{3}\sqrt{2}G^*(\tfrac{1}{3}\langle r_p{}^2\rangle_{G_E})^{1/2}, \qquad (39)$$

where μ^* is the $N-N^*$ magnetic transition amplitude ($\mu^*_{\text{exp}}=1.21\mu_p$)[17] and G^* is the $N-N^*$ axial amplitude ($G^*_{\text{exp}}=1.50$).[18] Numerically (38) gives 0.50 F=0.58 F and (39) gives 0.36 F=0.35 F. The agreement is fairly good in view of the approximations introduced in the pole model. One can also consider the ratio of (39) and (38) in order to eliminate the constant Γ obtaining

$$\mu^*/(\mu_p-\mu_n)=\tfrac{1}{3}\sqrt{2}(G^*/g_A), \qquad (40)$$

also in good agreement with experiment. Equation (40) had been deduced, from dynamical arguments, by Gatto and Veneziano.[19]

We observe that the above results can also be achieved under the stronger assumption of W-spin invariance, with definite W-spin assignment of the baryons. In this case the right-hand side of Eq. (36) can be connected, by a W-spin rotation, to the matrix element of the axial charge A_3 always at infinite momentum. Consequently, Eq. (39) and similar ones can be deduced without any assumption of mass degeneracy.

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[11] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); **16**, 384 (1966); M. Ademollo, Nuovo Cimento **46A**, 156 (1966).

[12] If we had limited to $SU(2)$ symmetry in Eq. (32), m_V should have been replaced by m_ρ for $\alpha=1$, 2, 3 and similarly for the other isomultiplets. Therefore we may suppose that in presence of mass breaking in $SU(3)$ we have $m_V\to m_{V_\alpha}$ in Eq. (32).

[13] G. Costa and M. Tonin, Nuovo Cimento **42A**, 1015 (1966).

[14] A relation similar to (11) holds in a $U(6,6)$ scheme (Ref. 5) with a pole-dominance assumption giving $J_R{}^{(\alpha)}=g\phi_R{}^{(\alpha)}$. On the meson pole we have then $\partial_\mu J_{\mu\nu}{}^{(\alpha)}=mJ_\nu{}^{(\alpha)}$ which differs by a factor $\sqrt{2}$ from (11) with $\Gamma=m/\sqrt{2}$.

[15] The definition of $A(\gamma_4\sigma_2\lambda^\alpha)$ is as in Ref. (1). Equation (36) connects the matrix elements of the total magnetic moments to those of the $SU(6)_W$ generators. An interesting application can be done when the physical states $\langle\beta|$ and $|\gamma\rangle$ are supposed to belong to reducible $SU(6)_W$ representations. See R. Gatto, L. Maiani, and G. Preparata, Phys. Letters, **21**, 459 (1966).

[16] F. Buccella, G. Veneziano, and R. Gatto, Nuovo Cimento **42A**, 1019 (1966); **43A**, 768 (1966).

[17] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

[18] C. H. Albright and L. S. Liu, Phys. Rev. **140**, B748 (1965).

[19] R. Gatto and G. Veneziano, Phys. Letters **19**, 512 (1965); **20**, 439 (1966).

6. CONSEQUENCES FROM THE UNITARY SYMMETRIC PARTS OF THE COMMUTATORS

We now go back to Eq. (26) and consider the unitary symmetric projections. For $N=1$ we get

$$\langle r|S_{12}^{(1)}(88)|r'\rangle = +i\left[\frac{3}{4\sqrt{2}m^2}(2\mu_p+\mu_n-1)-\frac{4}{\sqrt{3}}\Gamma^{-2}a\right]\chi_r^\dagger\sigma_3\chi_{r'}. \quad (41)$$

For $N=8$, $\eta=\eta'=s$, we have

$$\langle r|S_{12}^{(8)}(88D)|r'\rangle = -i\left[\frac{3}{4m^2}(2\mu_p+\mu_n-1)-\left(\sqrt{\frac{5}{3}}\right)\Gamma^{-2}d\right]\chi_r^\dagger\sigma_3\chi_{r'}. \quad (42)$$

For $N=8$, $\eta=s$, $\eta'=a$:

$$\langle r|S_{12}^{(8)}(88F)|r'\rangle = +i\left[\frac{3\sqrt{5}}{4m^2}\mu_n+\left(\sqrt{\frac{5}{3}}\right)\Gamma^{-2}f\right]\chi_r^\dagger\sigma_3\chi_{r'}. \quad (43)$$

For $N=27$:

$$\langle r|S_{12}^{(27)}(88)|r'\rangle = -i[2\mu_p+\mu_n-1]\frac{1}{4\sqrt{6}m^2}\chi_r^\dagger\sigma_3\chi_{r'}. \quad (44)$$

Equation (44) requires $S_{12}^{(27)}\neq 0$. Furthermore if $S_{12}^{(8)}$ were zero we would have from (42) and (43) the relations

$$(\sqrt{\tfrac{5}{3}})f=-(3\Gamma^2/4m^2)(\sqrt{5})\mu_n\,, \quad (45)$$

$$(\sqrt{\tfrac{5}{3}})d=(3\Gamma^2/4m^2)(2\mu_p+\mu_n-1)\,. \quad (46)$$

These relations would give $d/f=0.65$ against $(d/f)_{\rm exp}\simeq 1.28$, and $g_A=0.49$. Therefore we are led to conclude that $S_{12}^{(8)}$ also has to be different from zero.

The presence of Schwinger-like terms in the commutators of the charge densities seems to be a consequence of the $SU(6)_W$ algebra and the PCTC hypothesis. Such a result may not be surprising as we are actually comparing the compact $SU(6)_W$ algebra with the algebra of current densities, which necessitates infinite dimensional representations.[20] One can escape the unpleasant situation of having Schwinger terms by one of the following ways:

(a) Reject the PCTC hypothesis. In this case we can no longer relate the electric dipoles to the tensor charges.

(b) Modify the commutation relation of tensor charges [Eq. (8)]. We may actually suppose that only the unitary symmetric part of the commutator has to be changed, maintaining the validity of the results of Secs. 4 and 5. In this connection, we remark that the two parts of the commutator test two different aspects of the quark model, as stressed in Ref. 2. The unitary antisymmetric part is related to the spin-$\frac{1}{2}$ assignment for the fundamental fields while the symmetric part is related to the property of the quark fields to be a unitary triplet. However, we want to emphasize that the commutation relations (8) are equivalent, in the limit of infinite momentum, to those of the $SU(6)_W$ algebra, so that the choice of (b) may amount to invalidation of such algebra.

It may be interesting at this point to examine the consequences of the commutation relation (16) in the absence of Schwinger terms. For the unitary symmetric part we obtain four independent sum rules giving the anomalous magnetic moments as integrals over a certain combination of the total cross sections for γ absorption. These are explicitly written down in Appendix 4. The combinations corresponding to the channels γp and γn give rise to the sum rules recently obtained by Drell and Hearn[4] on the basis of general dispersion arguments. This may be a point in favour of the absence of Schwinger terms in the dipole commutators. From the discussion at the beginning of this section it follows that the above sum rules are not consistent with the analogous rules from the commutators of tensor charges as given in Ref. 2. Apart from the possibilities (a) and (b), mentioned before in order to eliminate this contradiction, a third one could exist, namely: (c) Nonconvergence of the continuum for the unitary symmetric part of the sum rules. This is equivalent to the necessity of subtractions in the dispersion integrals.

We conclude this section by some remarks about the results one would have obtained, using for the tensor pole terms the anomalous (instead of the total) magnetic moments. The main conclusions of the present section would not be modified even in that case: Schwinger terms in the dipole algebra would still be required for consistency; there would still be disagreement between the algebra of tensor charges and the Drell-Hearn sum rule, etc. On the contrary, the positive results of the preceding sections would be ruled out and the consistency with $SU(6)_W$ symmetry would be destroyed. In particular one would obtain $\langle r_n^2\rangle_{F1}=0$ instead of (29), and the D/F ratio of the anomalous magnetic moments would be equal to that of the axial amplitudes. These would be very bad results. Also Eqs. (38), (39), and (40) would have to be accordingly modified, making worse the agreement with experiment. Therefore we are led to the conclusion that, also *a posteriori*, our choice of the collinear limiting procedure is to be preferred.

[20] (a) R. Dashen and M. Gell-Mann, Phys. Letters **17**, 145 (1965); (b) Phys. Rev. Letters **17**, 340 (1966). In our case of dipole algebra and in absence of Schwinger-terms contributions, the necessity of infinite-dimensional representations has been directly shown by (c) G. Veneziano, Nuovo Cimento **44A**, 295 (1966).

7. ANALYSIS OF THE COMMUTATION RELATIONS BETWEEN TENSOR AND AXIAL CHARGES

We now extend the analysis of the preceding sections to the commutation relation

$$[T_i^{(\alpha)},A^{(\beta)}]=id_{\alpha\beta\gamma}T_{i5}^{(\gamma)} \quad (47)$$

that we examine here in connection with

$$[D_i^{(\alpha)},A^{(\beta)}]=if_{\alpha\beta\gamma}D_{i5}^{(\gamma)}+R_i^{(\alpha\beta)}. \quad (48)$$

In Eqs. (47) and (48) we have introduced

$$T_{i5}^{(\gamma)}=-\tfrac{1}{2}\epsilon_{ijk}\int d^3xJ_{jk}^{(\gamma)}(x)\,, \quad (49)$$

$$D_{i5}^{(\gamma)}=-i\int d^3x\, x_iJ_{45}^{(x)}\,, \quad (50)$$

and a term $R_i^{(\alpha\beta)}$ which arises from possible Schwinger terms in the commutators $[J_0,J_{05}]$. Nothing can be said in general about the symmetry in α, β of this term.

Following the method of Sec. 4 we take Eqs. (47) and (48) between states of the baryon octet and go to the limit $p\rightarrow\infty$. By virtue of Eq. (14) the continuum can be eliminated so that we are left with a set of relations involving only matrix elements between the baryon octet. Concerning the right-hand side of Eqs. (47) and (48), we note that the charges T_{i5} are equivalent to the tensor charges and give rise to the total magnetic moments, whereas the operators D_{i5} have vanishing matrix elements in the absence of second-class axial amplitudes.

As a result we obtain that the additional term $R_i^{(\alpha\beta)}$ has no components in the representations **10**, **10***, and **27**. On the other hand, the absence of the $\mathbf{8}_s$ component would imply the vanishing of the total magnetic moments, and the absence of the $\mathbf{8}_a$ component would imply the vanishing of the axial couplings. We conclude that Schwinger terms both symmetric and antisymmetric in α, β are required to save the consistency between Eq. (47) and Eq. (48).

Alternative solutions are offered by following possibilities analogous to (a) and (b) of the preceding section. Concerning the point (b), however, it is now no longer sufficient to modify the unitary symmetric part of the commutation relation, and one would actually have to destroy the whole $SU(6)_W$ algebra. With the alternative (a), of rejecting the PCTC hypothesis, one still encounters a difficulty with the approximate saturation of Eq. (48). Indeed, from the fact that in absence of Schwinger terms the left-hand side of Eq. (48) vanishes in the $p\rightarrow\infty$ limit, it can be shown that if we only take the contributions from the octet and decuplet intermediate states we must have either vanishing anomalous magnetic moments or vanishing axial couplings.[21] A similar unpleasant result is also obtained in the schemes with mixing.[1,15]

It appears from the present work that the $SU(6)_W$ algebra at infinite momentum together with the PCTC hypothesis provides a useful and consistent scheme, giving reasonable results for approximate saturation according to simple mixing schemes. On the contrary the local $U(3)\otimes U(3)$ algebra seems to be in general inadequate in the absence of Schwinger terms. The last is in fact an infinite-dimensional algebra and it leads to representations involving an infinite number of states.[22] The $SU(6)_W$ algebra, supplemented with the PCTC hypothesis, may provide an approximate compact version of the infinite dipole algebra and the present analysis shows that this attempt may in part be successful.

APPENDIX 1: RELATIONS BETWEEN TENSOR AND VECTOR NUCLEON FORM FACTORS

The tensor form factors are defined by[3]

$$\begin{aligned}(2\pi)^3(p_0p_0'/mm')^{1/2}\langle p'|J_{\mu\nu}(0)|p\rangle
&=\bar{u}(\mathbf{p}')[(\gamma_\mu k_\nu-\gamma_\nu k_\mu)G_1(k^2)+\sigma_{\mu\nu}G_2(k^2)\\
&\quad+i(k_\mu P_\nu-k_\nu P_\mu)G_3(k^2)\\
&\quad+(\gamma_\mu P_\nu-\gamma_\nu P_\mu)G_4(k^2)]u(\mathbf{p})\,, \end{aligned} \quad (A1)$$

where $k=p'-p$ and $P=p'+p$; spin and unitary spin indices are omitted for simplicity. Between states of the same particle $G_4(k^2)=0$. The conserved vector current is

$$\begin{aligned}(2\pi)^3(p_0p_0'/mm')^{1/2}\langle p'|J_\nu(0)|p\rangle
&=\bar{u}(\mathbf{p}')\left[\left(i\gamma_\nu+k_\nu\frac{m'-m}{k^2}\right)F_1(k^2)\right.\\
&\quad\left.+i\sigma_{\mu\nu}k_\mu\frac{F_2(k^2)}{m+m'}\right]u(\mathbf{p}). \end{aligned} \quad (A2)$$

Comparing with Eq. (A1) by use of Eq. (11) we obtain the following relations:

$$k^2[G_1(k^2)+(m+m')G_3(k^2)]=\Gamma(k^2)F_1(k^2)\,, \quad (A3)$$

$$\begin{aligned}(m+m')[G_2(k^2)+k^2G_3(k^2)&\\
+(m'-m)G_4(k^2)]&=-\Gamma(k^2)F_2(k^2). \end{aligned} \quad (A3')$$

We observe that the total magnetic moment form factor is

$$\begin{aligned}G_M(k^2)&=F_1(k^2)+F_2(k^2)\\
&=\Gamma^{-1}(k^2)[k^2G_1(k^2)-(m+m')G_2(k^2)\\
&\quad+(m^2-m'^2)G_4(k^2)]. \end{aligned} \quad (A4)$$

We note that for $\Gamma(0)\neq0$ and $F_1(0)\neq0$ (e.g., the

[21] Consequences of the $[A,J_\mu(x)]$ commutators, which are equivalent to the $[A,D]$ for zero momentum transfer, have been derived by S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **43A**, 161 (1966) who however consider only a subset of the possible relations. The extension to the whole set gives the result mentioned in the text.

[22] See Ref. 20 (b).

case of a proton) the tensor form factors G_1 and/or G_3 must have a pole at $k^2=0$. They can be regular for $m'\neq m$ since in this case $F_1(0)=0$. These singularities, however, cannot receive a physical interpretation, as the tensor form factors are not separately connected, at the moment, to measurable quantities. In the absence of a more direct physical meaning of the tensor currents, we may suppose that the invariant functions exhibiting the simplest analytic properties are just the terms k^2G_1, G_2, k^2G_3, and G_4 appearing in Eqs. (A3), (A3′), and (A4) above, none of which is singular for $k^2=0$.

APPENDIX 2: MATRIX ELEMENTS OF D_i AND T_i

A. Electric Dipole Moments of the Nucleon

It is well known[1,10] that the electric dipole moments of the nucleon are proportional to the anomalous magnetic moments in the limit of infinite momentum. For later convenience we shall here report a detailed derivation of this result. We start from the definition

$$D_i=-i\lim_{\mathbf{k}\to 0}\frac{\partial}{\partial k_i}\int \exp(i\mathbf{k}\cdot\mathbf{x})J_0(x)d^3x \tag{A5}$$

and we obtain

$$\begin{aligned}\langle p'|D_i|p\rangle&=-i\lim_{\mathbf{k}\to 0}(\partial/\partial k_i)(2\pi)^3\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k})\langle p'|J_0(0)|p\rangle\\&=-i(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')\lim_{\mathbf{k}\to 0}(\partial/\partial k_i)\langle p+k|J_0(0)|p\rangle,\end{aligned} \tag{A6}$$

where $|p'\rangle$ and $|p\rangle$ are nucleon states with momentum in the direction 3, and $i=1, 2$. The last step in (A6) can be justified by taking wave packets for the nucleon wave functions and then going to the limit of plane waves.[23] We have also

$$\lim_{\mathbf{k}\to 0}\frac{\partial}{\partial k_i}\langle p+k|J_0(0)|p\rangle=(2\pi)^{-3}\frac{m}{p_0}\lim_{\mathbf{k}\to 0}\left[F_1(k^2)\frac{\partial}{\partial k_i}\bar{u}(\mathbf{p}+\mathbf{k})\gamma_4 u(\mathbf{p})+\frac{F_2(k^2)}{2m}\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p})\right]. \tag{A7}$$

The spinor derivative can be calculated by use of the formula[24] (at first order in $\mathbf{k}$)

$$u(\mathbf{p}+\mathbf{k})=[I+i\gamma_4(\boldsymbol{\gamma}\cdot\mathbf{k}/2p_0)]u(\mathbf{p}). \tag{A8}$$

We thus get from (A6)

$$\begin{aligned}\langle p'|D_i|p\rangle&=-i\delta(\mathbf{p}-\mathbf{p}')\frac{m}{p_0}\left[i\frac{F_1(0)}{2p_0}\bar{u}(\mathbf{p})\gamma_i u(\mathbf{p})+\frac{F_2(0)}{2m}\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p})\right]\\&=-i\delta(\mathbf{p}-\mathbf{p}')\frac{F_2(0)}{2m}(m/p_0)\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p}).\end{aligned} \tag{A9}$$

In the limit $p\to\infty$ we have

$$\langle p'|D_i|p\rangle=-\delta(\mathbf{p}-\mathbf{p}')(F_2(0)/2m)\epsilon_{3ij}\chi^\dagger\sigma_j\chi \tag{A10}$$

where χ are Pauli spinors.

An alternative procedure consists in the use of CVC. In this case, to render the procedure unambiguous, we let the nucleon masses be different and go to the limit of equal masses at the end. We thus obtain the same result as before.

B. Nucleon Tensor Charges

Let us now consider the tensor charges. We write

$$T_i=\lim_{\mathbf{k}\to 0}\int d^3x\exp(i\mathbf{k}\cdot\mathbf{x})J_{0i}(x). \tag{A11}$$

Using (A1) we obtain

$$\begin{aligned}\langle p'|T_i|p\rangle=-i\delta(\mathbf{p}-\mathbf{p}')\lim_{\mathbf{k}\to 0}(mm'/p_0p_0')^{1/2}\bar{u}(\mathbf{p}+\mathbf{k})[&(\gamma_4k_i-\gamma_ik_4)G_1(k^2)+\sigma_{4i}G_2(k^2)\\&+i(k_4P_i-k_iP_4)G_3(k^2)+(\gamma_4P_i-\gamma_iP_4)G_4(k^2)]u(\mathbf{p}).\end{aligned} \tag{A12}$$

[23] F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).

[24] Equation (A8) corresponds to an infinitesimal pure Lorentz transformation. In order to maintain the same helicity a spatial rotation is also needed. For $\mathbf{k}\cdot\mathbf{p}=0$ we should have simply

$$u(\mathbf{p}+\mathbf{k})=\left(I-\gamma_3\frac{\boldsymbol{\gamma}\cdot\boldsymbol{\kappa}}{2|\mathbf{p}|}\right)u(\mathbf{p})$$

which in the limit $p\to\infty$ is equivalent to (A8).

The treatment of Eq. (A12) is not unambiguous and may depend on the limiting procedure. To make evident the various situations that may arise we shall consider three different limiting procedures:

(i) We let $\mathbf{k}$ tend to zero parallel to $\mathbf{p}$.
(ii) We let $\mathbf{k}$ tend to zero along the x_i direction (where i is just the index of T_i).
(iii) We keep the initial and final mass different while performing the limit $\mathbf{k}\to 0$ (and no restriction on the direction of $\mathbf{k}$).

The method (i) seems to be the most direct one; it avoids the difficulties of cases (ii) and (iii), as we shall see. In fact in case (ii) a divergent term will appear, which is clearly unphysical; while case (iii) presents some ambiguity because $F_1(0)$ is one for equal masses and zero for different masses.

Following the prescription (i) we find from (A12) and (A4):

$$\langle p'|T_i|p\rangle = -i\delta(\mathbf{p}'-\mathbf{p})(m/p_0)\lim_{\mathbf{k}\to 0,\mathbf{k}\parallel\mathbf{p}}\bar{u}(\mathbf{p}+\mathbf{k})[-\gamma_i k_4 G_1(k^2)+\sigma_{4i}G_2(k^2)]u(\mathbf{p})$$
$$= -i\delta(\mathbf{p}'-\mathbf{p})\Gamma(0)[G_M(0)/2m](m/p_0)\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p}),\quad \text{(A13)}$$

and in the limit $p\to\infty$:

$$\langle p'|T_i|p\rangle = -\delta(\mathbf{p}'-\mathbf{p})\Gamma(0)(G_M(0)/2m)\epsilon_{3ij}\chi^\dagger\sigma_j\chi. \quad \text{(A14)}$$

Such a result is analogous to that for the corresponding matrix elements of the electric dipoles, Eq. (A10), except for the substitution of anomalous magnetic moments with total moments.

The method (ii) gives

$$\langle p'|T_i|p\rangle = -i\delta(\mathbf{p}-\mathbf{p}')\frac{m}{p_0}\lim_{\mathbf{k}\to 0,\mathbf{k}\perp\mathbf{p}}\bar{u}(\mathbf{p}+\mathbf{k})[\gamma_4 k_i G_1(k^2)+\sigma_{4i}G_2(k^2)-ik_iP_4G_3(k^2)]u(\mathbf{p})$$
$$= -i\delta(\mathbf{p}'-\mathbf{p})\Gamma(0)\left[\frac{F_2(0)}{2m}\frac{m}{p_0}\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p})+\lim_{\mathbf{k}\to 0}\left(k_i\frac{F_1(k^2)}{k^2}\right)\right]. \quad \text{(A15)}$$

In this expression the term proportional to F_1 is singular; however this term is clearly unphysical, being a non-spin-flip term. This divergence can formally be avoided by a suitable regularization procedure, as for example by taking a symmetrized limit in $\mathbf{k}$. One thus would obtain the anomalous magnetic moments. However, it seems to us that the singularity is essentially required by PCTC, as noted in Sec. 2.

Finally, the method (iii) gives

$$\langle p'|T_i|p\rangle = -i\delta(\mathbf{p}'-\mathbf{p})\lim_{\Delta m\to 0}(mm'/p_0p_0')\bar{u}(\mathbf{p}')[-\gamma_i k_4 G_1(k^2)+\sigma_{4i}G_2(k^2)-\gamma_i P_4 G_4(k^2)]u(\mathbf{p})$$
$$= -i\delta(\mathbf{p}-\mathbf{p}')\Gamma(0)[G_M(0)/2m](m/p_0)\bar{u}(\mathbf{p})\sigma_{i4}u(\mathbf{p}) \quad \text{(A16)}$$

which is identical to that of (A13). In the last step we have supposed that $F_1(k^2)$ tends to the electric charge as k^2 tends to zero, in spite of the fact that the masses are different.

APPENDIX 3: EXPLICIT FORM OF THE MATRIX ELEMENTS APPEARING IN EQUATIONS (21) AND (21′)

The reduced matrix elements are defined by

$$\langle 8\lambda,r|A^{(N,\alpha)}|8\lambda',r'\rangle = \sum_\eta \begin{pmatrix} 8 & N & 8_\eta \\ \lambda' & \alpha & \lambda \end{pmatrix}\langle r|A^{(N)}(88\eta)|r'\rangle. \quad \text{(A17)}$$

For the operators V, A_k, R_{ij}, T_i, and D_i of Eq. (20) the index N of the unitary representation is always 8 and is omitted, except for the unitary singlet axial charges denoted by $A_k^{(0)}$.

From the definitions of Sec. 2, taking the matrix elements between baryon states and letting $p\to\infty$, we obtain

$$\langle r|V(88\eta)|r'\rangle = \delta_{\eta F}\sqrt{3}\chi_r^\dagger\chi_{r'}, \quad \text{(A18)}$$
$$\langle r|A_k(88F)|r'\rangle = f\chi_r^\dagger\sigma_k\chi_{r'}, \quad \text{(A19)}$$
$$\langle r|A_k(88D)|r'\rangle = d\chi_r^\dagger\sigma_k\chi_{r'}, \quad \text{(A20)}$$
$$\langle r|A_k^{(0)}(88)|r'\rangle = a\chi_r^\dagger\sigma_k\chi_{r'}, \quad \text{(A21)}$$
$$\langle r|R_{ij}(88\eta)|r'\rangle = \tfrac{1}{3}\langle r^2\rangle_{F_1}{}^\eta\delta_{ij}\chi_r^\dagger\chi_{r'}, \quad \text{(A22)}$$
$$\langle r|T_i(88\eta)|r'\rangle = -\Gamma(0)(\mu_{\text{tot}}{}^\eta/2m)\epsilon_{3ij}\chi_r^\dagger\sigma_j\chi_{r'}, \quad \text{(A23)}$$
$$\langle r|D_i(88\eta)|r'\rangle = -(\mu_{\text{an}}{}^\eta/2m)\epsilon_{3ij}\chi_r^\dagger\sigma_j\chi_{r'}, \quad \text{(A24)}$$

where $\mu_{\text{tot}}{}^\eta$ are the total, and $\mu_{\text{an}}{}^\eta$ the anomalous, magnetic moments.

In the right-hand side of the above relations we have omitted the factor $\delta(\mathbf{p}-\mathbf{p}')$. The D and F combinations of the magnetic moments (both anomalous and total) expressed in terms of those of proton and neutron are

$$\mu^D=-\tfrac{1}{2}(\sqrt{15})\mu_n \qquad \text{(A25)}$$

$$\mu^F=\sqrt{3}(\mu_p+\tfrac{1}{2}\mu_n) \qquad \text{(A26)}$$

and exactly the same combinations hold for the charge radii. Finally the axial amplitude g_A of nucleon β decay is given by

$$g_A=(1/\sqrt{3})f+(\sqrt{\tfrac{3}{5}})d. \qquad \text{(A27)}$$

APPENDIX 4: SUM RULES FROM THE UNITARY-SYMMETRIC PART OF DIPOLE COMMUTATORS

We start from the commutation relation

$$[D_i^{(\alpha)},D_j^{(\beta)}]+[D_i^{(\beta)},D_j^{(\alpha)}]=0, \qquad \text{(A28)}$$

where Schwinger terms have been assumed to be absent. We take Eq. (A28) between baryon octet states and insert a complete set of intermediate states. The pole term can be directly evaluated by (A17) and (A24). The continuum can be written as[25]

$$\text{Continuum}=-\frac{2}{\pi}\int\frac{d\nu}{\nu^2}\,\text{Im}C^{(2)}(\nu), \qquad \text{(A29)}$$

where $C^{(2)}$ is the forward Compton amplitude antisymmetric in the γ polarization and ν is the photon laboratory energy. The optical theorem gives

$$4\pi\alpha\,\text{Im}C^{(2)}(\nu)=\tfrac{1}{4}\nu[\sigma_P(\nu)-\sigma_A(\nu)], \qquad \text{(A30)}$$

where $\alpha=e^2/4\pi$ and σ_P and σ_A are the total photon-baryon cross sections for parallel and antiparallel spin states, respectively. By performing the four independent $SU(3)$ projections we get

$$-\frac{1}{\sqrt{8}}\frac{1}{4m^2}[(\mu_{\text{an}}{}^F)^2+(\mu_{\text{an}}{}^D)^2] -\frac{1}{8\pi^2\alpha}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{(1)}=0, \qquad \text{(A31)}$$

$$\frac{1}{4m^2}\mu_{\text{an}}{}^F\mu_{\text{an}}{}^D-\frac{1}{8\pi^2a}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{(8sa)}=0, \qquad \text{(A32)}$$

$$-\frac{3}{10}\frac{1}{4m^2}[(\mu_{\text{an}}{}^D)^2-(5/3)(\mu_{\text{an}}{}^F)^2] -\frac{1}{8\pi^2\alpha}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{(8ss)}=0, \qquad \text{(A33)}$$

$$-\sqrt{\frac{3}{8}}\frac{1}{4m^2}[\tfrac{3}{5}(\mu_{\text{an}}{}^D)^2-(\mu_{\text{an}}{}^F)^2] -\frac{1}{8\pi^2\alpha}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{(27)}=0, \qquad \text{(A34)}$$

where the $SU(3)$ indices indicate the coupling between the $\gamma\gamma$ and the $N\bar{N}$ states, referring, respectively, to the first and second subindices for the **8**. The combination corresponding to the γp channel is

$$-\frac{\sqrt{2}}{3}(\mathbf{1})+\frac{2}{3\sqrt{5}}(\mathbf{8}_{sa})+\frac{2}{15}(\mathbf{8}_{ss})+\frac{2}{5\sqrt{6}}(\mathbf{27}), \qquad \text{(A35)}$$

giving rise to the sum rule

$$\left(\frac{\mu_p-1}{2m}\right)^2=\frac{1}{8\pi^2\alpha}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{\gamma p}. \qquad \text{(A36)}$$

The combination for the γn channel is

$$-(\sqrt{2}/3)(\mathbf{1})-\frac{4}{15}(\mathbf{8}_{ss})-(2/15\sqrt{6})(\mathbf{27}), \qquad \text{(A37)}$$

giving

$$\frac{\mu_n{}^2}{4m^2}=\frac{1}{8\pi^2\alpha}\int\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]_{\gamma n}. \qquad \text{(A38)}$$

The relations (A36) and (A38) have been derived by Drell and Hearn[4] on the general assumptions of non-subtracted dispersion relations and of a low-energy theorem for Compton scattering. The comparison with experiment of Eq. (A36) is not definitive and is also discussed in Ref. 4.

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[25] For the definition of the invariant amplitudes and the dispersion method for deriving the sum rules see, e.g., Ref. (2).