Model Calculation for the $Y_0^*(1405)$ Resonance State

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The $Y_0^*(1405)$ resonance state is considered in terms of a multichannel potential arising from vector meson (V) exchange between the baryon octet (B) and the pseudoscalar meson octet (P). The interactions are assumed to follow $SU(3)$ symmetry; the only symmetry breaking included is that arising from the PB threshold separations, due to the P and B mass splittings. The resonance mass corresponds to a reasonable value ($f^2/4\pi=1.25$) for the VPP coupling; the width then calculated is 27 MeV, comparable with the empirical value. The unitary impurities found in this $Y_0^*(1405)$ state are quite small, the octet admixtures being dominant and only 15% in total intensity. The effective range calculated for the $\bar{K}N$ scattering length is not large, being of order $1/m_V$. The relationship between the various choices possible for the definition of the resonance energy is also discussed explicitly for this case.

1. INTRODUCTION

THE $\pi\Sigma$ resonance $Y_0^*(1405)$ is now generally believed to have spin-parity $(\frac{1}{2}-)$. Although there exists a little direct evidence[1] consistent with this interpretation, from the study of its decay mode, this belief is based mainly on the determination of the $I=0$ $\bar{K}N$ scattering length from the analysis of the $K^- - p$ reaction processes, for K^- interactions at rest and for low momenta,[2–5] in the approximation of zero effective range. This scattering length, $A_0=a_0+ib_0$, has a large real part, with sign appropriate to the presence of a bound state in the $\bar{K}N$ system. Kim[3] obtained the value $A_0=[(-1.67\pm0.04)+i(0.72\pm0.04)]$ F; Kittel *et al.*[5] have given the value $[(-1.57\pm0.04)+i(0.54\pm0.06)]$ F, obtained using their own charge-exchange data and the data of Sakitt *et al.*[4] With the zero-range approximation for the $\bar{K}N$ channel, the $I=0$ $\bar{K}N$ scattering amplitude $A_0/(1-ikA_0)$ has a resonance pole at

$$E^*=M^*-\tfrac{1}{2}i\Gamma \tag{1.1}$$

corresponding to vanishing of the denominator,

$$1-ikA_0=0. \tag{1.2}$$

This Y_0^* virtual bound-state resonance then has the parameters

$$M^*=(m_K+m_N)\left[1-\frac{1}{2m_Km_N}\frac{(a_0^2-b_0^2)}{(a_0^2+b_0^2)^2}\right], \tag{1.3a}$$

$$\Gamma=[(m_K+m_N)/m_Km_N][2a_0b_0/(a_0^2+b_0^2)^2], \tag{1.3b}$$

with the values $(M^*,\Gamma)=(1419.5,\ 26.5)$ MeV for the Kim parameters and $(M^*,\Gamma)=(1414.5,\ 27)$ MeV for the parameters of Kittel *et al.*, not far from the values $M^*=1405$ MeV, $\Gamma=35\pm5$ MeV observed empirically[6] for $Y_0^*(1405)$.

Some doubts have been expressed in the literature about the identification of the observed $Y_0^*(1405)$ with this virtual bound state. For example, Martin and Wali[7] and Martin[8] have argued that, with the $(\frac{1}{2}-)$ assignment, the observed width corresponds to an effective $Y_0^*\Sigma\pi$ coupling constant, $g_Y^2/4\pi=0.065$, which they consider to be unreasonably small for strongly interacting particles; for this reason, Martin has preferred the assignment $(\frac{1}{2}+)$. There is no doubt that a direct determination of the Y_0^* spin-parity would be very desirable, using the Byers-Fenster method based on the decay sequence $Y_0^*\to\Sigma^+\pi^-$, $\Sigma^+\to p\pi^0$, for some production reaction leading to Y_0^* particles with nonzero polarization.

The use of zero-range theory for the discussion of the $\bar{K}N$ interaction between 1405 and 1465 MeV (corresponding to laboratory momentum p_K about 200 MeV/c) may also be criticized. It is certainly true that a more complete analysis of the phenomena in this region should be based[9] on an energy-dependent two-channel reaction matrix $K_{R2}(E)$ for the $I=0$ state,[10,11]

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[1] A. Engler, H. E. Fisk, R. W. Kraemer, C. M. Meltzer, J. B. Westgard, T. C. Bacon, D. G. Hill, H. W. Hopkins, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **15**, 224 (1965).

[2] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

[3] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965); Nevis Laboratory Report No. 149, 1966 (unpublished).

[4] M. Sakitt, T. B. Day, R. G. Glasser, N. Seeman, J. Friedman, W. E. Humphrey, and R. R. Ross, Phys. Rev. **139**, B719 (1965).

[5] W. Kittel, G. Otter, and I. Wacek, Phys. Letters **21**, 349 (1966).

[6] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

[7] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964).

[8] A. W. Martin, Nuovo Cimento **34**, 1809 (1964).

[9] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).

[10] In this paper, we shall denote by $K_{Rn}(E)$ the reduced K matrix (defined in Ref. 11) for the lowest n channels. The properties of $K_{Rn}(E)$, and the relationship between these $K_{Rn}(E)$ for different n, are discussed in Ref. 11. For energy E below the $(n+1)$th threshold, the elements of $K_{Rn}(E)$ are all real.

[11] R. H. Dalitz, Rev. Mod. Phys. **33**, 471 (1961).

with real elements

$$K_{R2}(E)=\begin{pmatrix}\alpha(E) & \beta(E)\\ \beta(E) & \gamma(E)\end{pmatrix} \begin{matrix}\bar{K}N\\ \pi\Sigma\end{matrix}. \qquad (1.4)$$

With momenta k and q for the $\bar{K}N$ and $\pi\Sigma$ channels, respectively, the $I=0$ $\bar{K}N$ scattering amplitude is then given by

$$A_0=\alpha(E)+i\{q\beta^2(E)/[1-iq\gamma(E)]\}. \qquad (1.5)$$

For $E<m_K+m_N$, it is appropriate to use the single-channel reduced K matrix

$$K_{R1}(E)=\gamma(E)-\{|k|\beta^2(E)/[1+|k|\alpha(E)]\}, \qquad (1.6)$$

related to the $\pi\Sigma$ scattering phase shift $\delta_{\Sigma\pi}$ as follows:

$$q\cot\delta_{\Sigma\pi}=K_{R1}^{-1}(E). \qquad (1.7)$$

The $\pi\Sigma$ *resonance energy* ($\delta_{\Sigma\pi}=90°$) is therefore given by the equation[12]

$$1+|k|\alpha(E)=0. \qquad (1.8)$$

From Eq. (1.5), we have the scattering amplitudes

$$b_0=q\beta^2(E)/[1+q^2\gamma^2(E)], \qquad (1.9a)$$

$$a_0=\alpha(E)-q\gamma(E)b_0. \qquad (1.9b)$$

From a_0, b_0 alone, it is not possible to deduce $\alpha(E)$ without some assumption concerning $\gamma(E)$. Hence there may be some appreciable uncertainty in the location M^* assigned to the $\pi\Sigma$ resonance energy associated with a given $\bar{K}N$ scattering amplitude. The expressions (1.3) for $(M^*-\frac{1}{2}i\Gamma)$ give the location of the complex pole associated with this resonance and common to all the scattering amplitudes $T_{\bar{K}N,\bar{K}N}$, $T_{\bar{K}N,\pi\Sigma}$, $T_{\pi\Sigma,\pi\Sigma}$, etc., given by Eq. (1.2). With expression (1.5) for A_0, this equation takes the following form,

$$[1-ik\alpha(E)][1-iq\gamma(E)]+q(E)k(E)\beta^2(E)=0. \qquad (1.10)$$

The energy dependence of $K(E)$ may be parametrized approximately by the effective range expansion of Ross and Shaw,[13]

$$K^{-1}(E)=K^{-1}(E_0)+R(E-E_0), \qquad (1.11)$$

where R is an "effective range" matrix. However, the energy dependence of the reaction matrix K may be quite weak, even in the neighborhood of the $\pi\Sigma$ resonance corresponding to Eq. (1.8). It is true that the *reduced K-matrix* K_{R1} for the $\pi\Sigma$ channel alone, given by Eq. (1.6), necessarily has a pole at the resonance energy, but this does not at all require that the two-channel K matrix K_{R2} nor the scattering length A_0 related with it by Eq. (1.5), should necessarily have any strong energy dependence. This remark will be well illustrated by the model to be discussed below.

In this paper, we wish to discuss a specific model for the $Y_0^*(1405)$ state, viz., that it results from the attraction generated by the exchange of a vector meson (V) between the pseudoscalar octet (P) and the baryon octet (B). We assume that these interactions obey $SU(3)$ symmetry, but we shall take into account the $SU(3)$ symmetry breaking which results from the mass differences within the P and B octets, since this gives rise to the separation of the thresholds $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, and $K\Xi$ appropriate to the $I=0$ PB states. This model represents a generalization of the one-channel model of Arnold and Sakurai,[14] who considered V exchange between $\bar{K}$ and N, and who calculated the scattering amplitude by the unitarized Born approximation method (N/D with N given by the Born approximation). Das and Mahanthappa[15] have extended this N/D calculation to include the $\pi\Sigma$ channel. However, for V exchange, this N/D procedure is notoriously unreliable. The singular nature of V exchange necessitates a subtraction and the solution obtained depends sensitively on the subtraction point assumed. The function $D(E)$ thus obtained is far from constant on the left-hand cut corresponding to the V exchange, so that the N/D function obtained is not at all a reasonable approximation to the T matrix appropriate to V exchange; in fact, it is well known that there exists no exact solution corresponding to this situation, in consequence of the singular nature of V exchange. Hence, we have chosen to use a multichannel potential model.

In Sec. 2, the equations for the multichannel system are set up. They are approximated to provide a Schrödinger equation for convenient solution; it would be desirable to solve the complete equations at some later stage. The one free parameter (the VPP coupling constant) is adjusted to place the $I=0$ s-wave $\pi\Sigma$ resonant state at the mass value observed for $Y_0^*(1405)$. The $I=0$ and $I=1$ $\bar{K}N$ scattering lengths are then calculated as function of the total energy E, and their comparison with the empirical situation is discussed. In Sec. 3, we discuss the $SU(3)$ character found for the calculated $Y_0^*(1405)$ state, the relation between its mass value and the $I=0$ $\bar{K}N$ scattering length, and the validity of the effective range expansion for this scattering length.

To conclude the Introduction, we wish to emphasize that our purpose in discussing this model has been to provide an explicit demonstration that the assumptions which underlie the usual phenomenological analysis of $\bar{K}N$ interactions and the discussion of its relation with the $Y_0^*(1405)$ state are well justified for a quite plausible physical model. In these respects, the properties of this multichannel potential model will be quite typical of any dispersion-theoretic approach to this situation. We do not believe that this V-exchange model necessarily

[12] R. H. Dalitz, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 391.

[13] M. Ross and G. Shaw, Ann. Phys. (N. Y.) 9, 391 (1960).

[14] R. C. Arnold and J. J. Sakurai, Phys. Rev. **128**, 2808 (1962).

[15] T. Das and K. J. Mahanthappa, Nuovo Cimento **39**, 206 (1965).

provides the correct dynamical explanation for the existence of the $Y_0^*(1405)$ state (although the model does correspond to parameters roughly consistent with those determined from quite different phenomena), but this is not the point. It is our intention to discuss these questions further for models of a quite different kind [for example, for the three-quark model[16] for $Y_0^*(1405)$ and other such resonances] in future work.

2. EQUATIONS FOR THE MODEL SYSTEMS

Our model consists of a multichannel system of two-particle PB channels (P=pseudoscalar meson, B=baryon) interacting through exchange of vector mesons (V=vector meson octet). For isospin I, the wave equations for this system take the following form, in momentum space:

$$[(M_\alpha{}^2+k^2)^{1/2}+(m_\alpha{}^2+k^2)^{1/2}-E]\psi_\alpha(\mathbf{k}) = -\sum_\beta \int V_{\alpha\beta}{}^{(I)}(\mathbf{k},\mathbf{k}')\psi_\beta(\mathbf{k}')d_3\mathbf{k}', \quad (2.1)$$

where α and β label the PB channels and $V_{\alpha\beta}{}^{(I)}$ denotes the vector-exchange potential. We shall consider only the static limit for this potential, given by

$$V_{\alpha\beta}{}^{(I)}(\mathbf{k},\mathbf{k}')=C_{\alpha\beta}{}^{(I)}\frac{f^2}{4\pi}\frac{1}{m_V{}^2+(\mathbf{k}-\mathbf{k}')^2}, \quad (2.2)$$

where, for $I=0$ and 1, the matrices $C_{\alpha\beta}{}^{(I)}$ are given by

$$C^{(0)}=\begin{pmatrix} -\frac{3}{2} & \frac{1}{4}\sqrt{6} & -\frac{3}{4}\sqrt{2} & 0 \\ \frac{1}{4}\sqrt{6} & -2 & 0 & -\frac{1}{4}\sqrt{6} \\ -\frac{3}{4}\sqrt{2} & 0 & 0 & \frac{3}{4}\sqrt{2} \\ 0 & -\frac{1}{4}\sqrt{6} & \frac{3}{4}\sqrt{2} & -\frac{3}{2} \end{pmatrix} \begin{matrix} \bar{K}N \\ \pi\Sigma \\ \eta\Lambda \\ K\Xi \end{matrix}$$

$$C^{(1)}=\begin{pmatrix} -\frac{1}{2} & \frac{1}{4}\sqrt{6} & \frac{1}{2} & \frac{1}{4}\sqrt{6} & 0 \\ \frac{1}{4}\sqrt{6} & 0 & 0 & 0 & \frac{1}{4}\sqrt{6} \\ \frac{1}{2} & 0 & -1 & 0 & -\frac{1}{2} \\ \frac{1}{4}\sqrt{6} & 0 & 0 & 0 & \frac{1}{4}\sqrt{6} \\ 0 & \frac{1}{4}\sqrt{6} & -\frac{1}{2} & \frac{1}{4}\sqrt{6} & -\frac{1}{2} \end{pmatrix} \begin{matrix} \bar{K}N \\ \pi\Lambda \\ \pi\Sigma \\ \eta\Sigma \\ K\Xi \end{matrix}$$

where the PB channels appropriate to these states are those listed on the right. The parameter f denotes the $\rho\pi\pi$ coupling constant, which characterizes the VPP coupling. We assume that the ρ meson is coupled universally with the isospin current, so that the VBB coupling is characterized by the same coupling parameter. $SU(3)$ symmetry breaking is neglected for the vector mesons; thus the vector meson octet is assumed degenerate, with mass m_V, there is then no singlet-octet mixing, and no V_1PP coupling for the singlet vector meson V_1. Hence the potentials $V^{(I)}$ are unitary symmetric; the eigenvalues of the coefficient matrices $C^{(I)}$ correspond to pure $SU(3)$ representations, as given in Table I. For $I=0$, the $SU(3)$ representations which contribute are $\{1\}$, $\{8\}_a$, $\{8\}_s$, and $\{27\}$. For $I=1$, the representations which contribute are $\{8\}_a$, $\{8\}_s$, $\{10\}$, $\{\overline{10}\}$, and $\{27\}$. We note that the most attractive potential $V(\{\alpha\})$ is that for the singlet representation. It is therefore not surprising that the $I=0$ attraction turns out to be much greater than the $I=1$ attraction, since $V(\{1\})$ does not contribute to the latter.

TABLE I. The eigenvalues of C appropriate to each of the $SU(3)$ representations possible for the PB systems.

$\{\alpha\}$	$\{1\}$	$\{8\}_s$	$\{8\}_a$	$\{10\}$	$\{\overline{10}\}$	$\{27\}$
$C(\{\alpha\})$	-3	$-\frac{3}{2}$	$-\frac{3}{2}$	0	0	1

$SU(3)$ symmetry breaking is taken into account for the pseudoscalar mesons P and the baryons B, insofar as the physical masses, m_α and M_α, respectively, are used for these octets. Experience has shown that the splitting of the PB thresholds can have a major effect on the $SU(3)$ properties of scattering and resonance states, especially through the constraints of unitarity. It is even possible that an s-wave state which is resonant in the limit of exact unitary symmetry may no longer be resonant when the threshold mass splittings are taken into account.[17,18]

Since the solution of a set of simultaneous integral equations such as (2.1) is a major task, we shall simplify the form of its left-hand side, leading to an approximate set of differential equations, which may be regarded as a model system in its own right. We note that

$$(M^2+k^2)^{1/2}=M+k^2/[M+(M^2+k^2)^{1/2}], \quad (2.3)$$

so that the left-hand side of Eq. (2.1) may be written

$$[-E-M_\alpha-m_\alpha-k^2/2\mu_\alpha(k)]\psi_\alpha(\mathbf{k}), \quad (2.4)$$

where

$$[2\mu_\alpha(k)]^{-1}=[M_\alpha+(M_\alpha{}^2+k^2)^{1/2}]^{-1} + [m_\alpha+(m_\alpha{}^2+k^2)^{1/2}]^{-1}. \quad (2.5)$$

Our approximation consists in replacing the variable k in $\mu_\alpha(k)$ by the constant k_E given by

$$(M_\alpha{}^2+k_E{}^2)^{1/2}+(m_\alpha{}^2+k_E{}^2)^{1/2}=E. \quad (2.6)$$

Explicitly, Eqs. (2.5) and (2.6) lead to the approximation

$$\mu_\alpha(E)=(E+M_\alpha+m_\alpha)[E^2-(M_\alpha-m_\alpha)^2]/8E^2. \quad (2.7)$$

Thus, we replace the model system (2.1) by the model

[16] R. H. Dalitz, in *High Energy Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1966), p. 251.

[17] This remark holds only for resonant states which are generated by effective potentials between the particles appropriate to the channels with the lowest thresholds. It is well illustrated by the model calculation of Deloff and Wyld (Ref. 18) for the s-wave BB system. It would not hold for resonant states which are due to strongly attractive forces between substructure particles (such as the quarks) whose threshold energies lie far above the resonance mass considered; although these states can be shifted in mass value by their coupling with low-lying channels where there is strong mass splitting, they cannot be destroyed by these symmetry-breaking effects.

[18] A. Deloff and H. W. Wyld, Phys. Letters **12**, 245 (1964).

system of differential equations,

$$-(1/2\mu_\alpha(E))\nabla^2\psi_\alpha+\sum_\beta V_{\alpha\beta}^{(I)}(r)\psi_\beta(r)=(E-M_\alpha-m_\alpha)\psi_\alpha(r). \quad (2.8)$$

These differential equations are then integrated, by the standard numerical methods, for an appropriate set of initial conditions on ψ_α and $d\psi_\alpha/dr$ at the origin, to lead to the T-matrix $T_{\alpha\beta}(E)$ for the potential $V^{(I)}$ at energy E, for this multichannel system.

Equations (2.8) depend on two parameters, the mass m_V and the coupling constant f. The calculations have been carried through for the particular choice $m_V=888$ MeV for the mean vector meson mass. The coupling constant f is then chosen to place the $I=0$ resonance in the $\pi\Sigma$ channel (defined by the energy such that the $\pi\Sigma$ phase shift is 90°) at mass value 1405 MeV, corresponding to the observed $Y_0^*(1405)$ state. The value required for the coupling constant is then[19]

$$f^2/4\pi=1.25. \quad (2.9)$$

This appears to be a reasonable value for this parameter. From the observed $\rho\pi\pi$ decay width of 124 MeV, Sakurai[20,21] obtained the value $f^2/4\pi=2.5$. From the isospin dependence of the πN s-wave scattering lengths, i.e., from (a_3-a_1), Sakurai obtained the same value for $f^2/4\pi$; from the energy dependence of (a_3-a_1), he obtained the further estimate $f^2/4\pi=2.1\pm0.3$.

The $I=0$ $\pi\Sigma$ s-wave cross section obtained from the solution of (2.8) for the coupling constant value (2.9) is shown in Fig. 1. The calculated width for this resonance

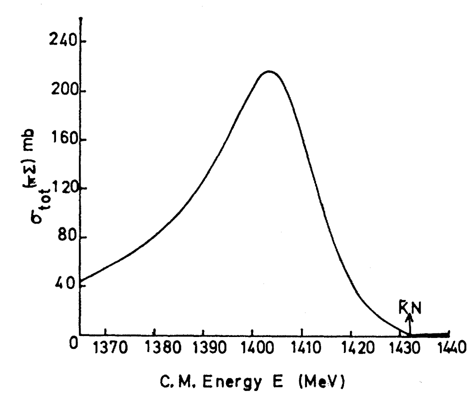

FIG. 1. The total s-wave $\pi\Sigma$ scattering cross section calculated for the multichannel potential model for $Y_0^*(1405)$ is plotted as a function of the total c.m. energy. The cross section becomes very small at the $\bar{K}N$ threshold, where only the term $\gamma(E)$ contributes to the $\pi\Sigma$ scattering.

[19] The general conclusions do not depend sensitively on the value chosen for m_V. For a different choice of m_V, in the same mass region, the corresponding value for f is adequately given by $f^2/4\pi=1.25(m_V/888)^2$.

[20] J. J. Sakurai, *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 227. More recently, Yellin (Ref. 21) has used the width $\Gamma_\rho=106\pm5$ MeV given by Rosenfeld *et al.* (Ref. 6) to obtain the estimate $f^2/4\pi=2.07\pm0.12$.

[21] J. Yellin, Phys. Rev. **147**, 1080 (1966).

peak is

$$\Gamma=27\text{ MeV}, \quad (2.10)$$

comparable with (but smaller than) the observed width $\Gamma=35\pm5$ MeV for $Y_0^*(1405)$. This model calculation shows quite clearly that the observed width is quite compatible with the assignment $s_{1/2}$ for $Y_0^*(1405)$; in fact our model calculation has led to a somewhat smaller value than the empirical width. We note that the calculated resonance shape is quite asymmetric (upper half-width 9 MeV, lower half-width 18 MeV), in consequence of the $\bar{K}N$ threshold lying close above the resonance energy. The complete T matrix, calculated for the resonance energy, is listed in Table II.

TABLE II. The T-matrix elements (unit F) for the $I=0$ PB system at the resonance $Y_0^*(1405)$.

	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\bar{K}N$	$0.87+4.03i$			
$\pi\Sigma$	$0.00-1.93i$	$0.00+0.92i$		
$\eta\Lambda$	$0.68+2.69i$	$0.00-1.29i$	$0.43+1.80i$	
$K\Xi$	$-0.13-2.03i$	$0.00+0.97i$	$-0.40-1.36i$	$0.46+1.02i$

For energies above the $\bar{K}N$ threshold, the $\bar{K}N$ scattering length $A=a+ib$ may be obtained directly from the calculated T-matrix element $(\bar{K}N|T|\bar{K}N)$, since

$$(\bar{K}N|T|\bar{K}N)^{-1}=A^{-1}-ik, \quad (2.11)$$

where k denotes here the c.m. momentum in the $\bar{K}N$ channel. The $I=0$ values obtained at three energies are given in Table III. An effective range fit to the values

TABLE III. The $\bar{K}N$ scattering lengths calculated for the multichannel potential due to vector meson exchange, given as function of the total energy E.

E (MeV)	1434	1455	1520
A_0 (F)	$-1.275+0.424i$	$-1.165+0.453i$	$-0.849+0.460i$
A_0^{-1} (F^{-1})	$-0.707-0.235i$	$-0.746-0.290i$	$-0.911-0.494i$
A_1 (F)	$0.235+0.0435i$	$0.231+0.0466i$	$0.218+0.0557i$
A_1^{-1} (F^{-1})	$4.11\ -0.76i$	$4.16\ -0.84i$	$4.31\ -1.10i$

$A_0(1434)$ and $A_0(1455)$ is given by

$$A_0^{-1}(E)=(-0.703-0.230i)-\left(\frac{E-1432}{1000}\right)(1.88+2.60i), \quad (2.12a)$$

$$=(-1.285+0.420i)^{-1}-\tfrac{1}{2}(0.226+0.313i)k^2, \quad (2.12b)$$

for c.m. total energy E MeV, or c.m. momentum k F^{-1}. The value obtained for the effective range R_0 has a reasonable order of magnitude,

$$R_0=(1.02+1.41i)/m_V, \quad (2.13)$$

TABLE IV. The $\bar{K}N$ scattering amplitudes obtained from data on low-energy and at-rest K^-p interactions. The errors quoted correspond only to the diagonal elements of the error matrix. In fact, there are strong correlations between the uncertainties in the values of these four parameters.

	$A_{0t}=a_0+ib_0$ (F)	$A_{1t}=a_1+ib_1$ (F)
Kim[a]	$(-1.67\pm0.04)+i(0.72\pm0.04)$	$(-0.00\pm0.06)+i(0.69+0.03)$
Kittel *et al.*[b]	$(-1.57\pm0.04)+i(0.54\pm0.06)$	$(-0.24\pm0.05)+i(0.43\pm0.05)$

[a] Reference 3.
[b] Reference 5.

in terms of the range parameter for the interaction potential. The effective range expansion appears to be quite good even up to 1520 MeV; the expression (2.12a) gives $A_0^{-1}(1520)=(-0.87-0.46i)$ F, quite close to the calculated value given in Table IV. The zero-energy $\bar{K}N$ scattering length $A_{0t}=(-1.285+0.420i)$ F is comparable with the empirical value given in Table IV; its interpretation will be considered in Sec. 3 below.

The $I=1$ amplitudes calculated from the parameters obtained from the fit to $Y_0^*(1405)$ are also listed in Table III. The zero-energy $\bar{K}N$ scattering length is $A_{1t}=(0.24+0.04\text{f})$ F, not at all in accord with the empirical value. In the potential $V^{(1)}$, the $\bar{K}N$ potential is attractive but much weaker than the corresponding term in $V^{(0)}$. In fact, the first Born approximation provides a fair approximation for the calculated A_{1t}. In this situation, since the imaginary part of A_{1t} arises first in second Born approximation, it is naturally much smaller than the real part of A_{1t}. It appears that other interactions (for example, those arising from baryon, decuplet, and $D_{3/2}$ singlet exchanges[7]) must contribute dominantly to the absorptive processes observed for the $I=1$ state. Since such interactions would be expected to contribute appreciably also for the absorptive processes in the $I=0$ state, this means that our model calculation can certainly not be taken as a serious representation of the complete dynamics of the $Y_0^*(1405)$ state. Rather, as we have said in the Introduction, our model is intended to provide a concrete illustration that the assumptions made in relating the $Y_0^*(1405)$ state with the $I=0$ low-energy $\bar{K}N$ scattering are well justified for the case of a straightforward model.

3. DISCUSSION AND CONCLUSIONS

The $Y_0^*(1405)$ state has frequently been described[11] as a *virtual bound-state resonance* of the $I=0$ $\bar{K}N$ system. This description corresponds to the way in which the resonance pole arises in the $\pi\Sigma$ reduced K matrix[10] K_{R1}, namely, that the reduced K matrix K_{R2},

$$K_{R2}=\begin{pmatrix}\alpha & \beta\\ \beta & \gamma\end{pmatrix}\quad\begin{matrix}\bar{K}N\\ \pi\Sigma\end{matrix}\tag{3.1}$$

for the $\bar{K}N$ and $\pi\Sigma$ channels has no pole at this energy and the resonance pole arises from the denominator depending on the element α in the expression[11] for K_{R1},

$$K_{R1}=\gamma-\beta^2|k|/(1+\alpha|k|).\tag{3.2}$$

When the interactions coupling these two channels are relatively weak, this situation arises only when the $\bar{K}N$ forces are such as to give a bound state for the uncoupled $\bar{K}N$ channel, since only the $\bar{K}N$ forces then contribute significantly to the value for α. In this way, a resonance state is possible for the s-wave $\pi\Sigma$ system, even though there are no repulsive (or centrifugal) forces present to provide a potential barrier to contain the resonant state.

More generally, the interactions coupling these channels may be strong and all of the interactions within and between these channels may contribute to the element α determining the $\pi\Sigma$ resonant state. In the present model, for the parameters used, the potential interaction in the $\bar{K}N$ channel has well-depth parameter $s\approx0.8$, insufficient for the generation of a bound state in this channel. Hence, the interactions between the $\bar{K}N$ and other channels and within these other channels (especially in the $\pi\Sigma$ channel) provide the significant additional attraction necessary for the $Y_0^*(1405)$ resonance state.

It is not obvious *a priori* whether the $Y_0^*(1405)$ state has any simple character in terms of $SU(3)$ symmetry. Although the potential matrix assumed does follow the $SU(3)$ symmetry, the strongest attraction being that for the singlet representation, the symmetry breaking introduced by the use of the physical masses for the P and B octets could well mix different $SU(3)$ representations to such a degree that the $SU(3)$ classification scheme might have no value here (a situation suggested by the fact that this resonance can be seen only in the $\pi\Sigma$ channel, as well as by the description "virtual $\bar{K}N$ bound-state resonance"). We may settle this question for our model calculation by examining the complete T matrix calculated for the energy E^* corresponding to the complex resonance pole. In the neighborhood of $[M^*-\frac{1}{2}i\Gamma]$, the T-matrix elements are necessarily all of the form[22]

$$(j|T|i)=c_jc_i^*/[E-M^*+\tfrac{1}{2}(i\Gamma)]+\text{finite terms},\tag{3.3}$$

where the labels i, j refer to the various channels. The resonance eigenstate is then given by the channel wave function

$$|\psi_{\text{res}})=N\{\sum_{i=1}^{n}c_i|i)\},\tag{3.4}$$

where N is the appropriate normalization factor.

Since $Y_0^*(1405)$ is quite a narrow resonance, the resonance pole is close to the real axis, and we shall approximate the products $c_jc_i^*$ by their values at the resonance energy, on the real axis. From Table II, we see that the T_{ji} are dominantly imaginary at resonance, as expected. The imaginary parts $\text{Im}T_{ji}$ do have the product structure $c_jc_i^*$ expected, and the coefficients c_i

[22] This is simply the statement that the resonance pole is necessarily a simple pole of $\det(T)$. If double poles (or more complicated poles) ever occur, these are considered to arise from the accidental coincidence of two (or more) simple poles.

found correspond to the following expression for the resonant state:

$$|Y_0^*(1405)) = [0.72|\bar{K}N) - 0.345|\pi\Sigma) + 0.48|\eta\Lambda) - 0.36|K\Xi)]. \quad (3.5)$$

We see that the resonant state is dominantly in the $\bar{K}N$ channel, although not overwhelmingly so. This state is to be compared with the $SU(3)$ singlet state,

$$|\{1\}, I = Y = 0) = [\tfrac{1}{2}|\bar{K}N) - \tfrac{1}{4}\sqrt{6}|\pi\Sigma) + \tfrac{1}{4}\sqrt{2}|\eta\Lambda) - \tfrac{1}{2}|K\Xi)]. \quad (3.6)$$

Alternatively, we may express the state (3.5) in terms of $SU(3)$ eigenfunctions, as follows:

$$|Y_0^*(1405)) = [0.92|\{1\}) + 0.29|\{8\}_s) - 0.25|\{8\}_a) - 0.03|\{27\})].$$

We see that $Y_0^*(1405)$ is dominantly in the singlet configuration but that it includes octet admixtures of total intensity 15%. It is remarkable that the $Y_0^*(1405)$ state retains such a high degree of unitary purity, even though it is an s-wave resonance. As remarked by Rajasekaran,[23,24] this can be understood simply if $|ka| \ll 1$ holds for the c.m. momenta in all the channels contributing significantly to the resonant state, where a is the effective radius of interaction. Here, a is typically $1/m_V \approx 0.22$ F, and $|ka| \approx 0.5$ for the most extreme channel ($K\Xi$), which contributes only 12% to the Y_0^* state.

As remarked in Sec. 2, the calculated width for this state is quite comparable with the empirical value. We may note that only the $\pi\Sigma$ channel is available for $Y_0^*(1405)$ decay, and this channel contributes only 12% in intensity to the $Y_0^*(1405)$ state. This factor reduces the $Y_0^*\Sigma\pi$ effective coupling constant by an order of magnitude: following Martin,[8] allowance for this factor leads to the estimate $g(Y_0^*BP)^2/4\pi \approx 0.5$ for the over-all Y_0^*BP coupling, a rather large coupling constant for an s-wave interaction.

The effective ranges R_0 and R_1 for the $\bar{K}N$ interaction are not large in this model. R_0 is given by Eq. (2.13); R_1 has the value $(0.8-2.0i)/m_V$. Hence, this model gives no reason to believe that these effective ranges should be particularly large; the energy dependences of A_0^{-1} and A_1^{-1} are such that the effective range term gives an adequate representation for them, certainly as far as 1520 MeV. From a study of $\bar{K}N$ scattering and reaction processes in the vicinity of the $Y_0^*(1520)$ resonance, Watson *et al.*[25] have obtained values of the $\bar{K}N$ s-wave elastic scattering amplitudes at 1520 MeV. Their result is that T_0 and T_1 are essentially pure imaginary (cf. Fig. 32 of Ref. 26); the value obtained here for kT_0 is $(-0.29+0.52i)$, rather far from the value $kT_0 = (0.8i)$ found by Watson *et al.* Their empirical value requires a rather strong energy dependence for A_0 over this energy range.[27] Since the value calculated for A_{1t} did not agree even qualitatively with the empirical value, we shall not make a corresponding comparison for kT_1 at 1520 MeV.

There is always some uncertainty involved in the assignment of a mass value to a resonant state. The resonance mass may be defined in several ways:

(a) By the resonance pole E^*, which is given by the Eq. (1.3). However, it must be remembered that a_0 and b_0 vary with energy, in general. For example, in our model calculation, $A_{0t} = (-1.275+0.42i)$ F holds at the $\bar{K}N$ threshold (1432 MeV); even with the small effective range calculated, A_0 changes to $(-1.45+0.36i)$ F as the total energy falls to 1405 MeV.

(b) By the condition $\delta_{\pi\Sigma} = 90°$ for the $\pi\Sigma$ phase shift. In view of the relation (1.6), this mass M_R is defined by Eq. (1.8). The relation between α and a_0 is given explicitly by Eq. (1.9b); this depends on a knowledge of $\gamma(E)$. From Eqs. (1.6) and (1.7), it is apparent that $\gamma(E)$ corresponds to the nonresonant background scattering in the $\pi\Sigma$ s-wave system. For a fixed resonance pole, the existence of background scattering can cause the peak in the $\pi\Sigma$ cross section as function of energy to shift quite appreciably from the pole position M^*. Even when $\gamma(E)$ is small, the nonresonant term of (1.6) gives rise to an unsymmetrical resonance shape. In any case, the peak in the $\pi\Sigma$ cross section always lies a little below the mass M_R for which $\delta_{\pi\Sigma}$ passes through 90°, since the factor $1/q^2$ decreases monotonically with increasing energy. However, the mass value M_R always lies close to the peak in the cross section, whereas the mass M^* can lie far from the peak.

In our model calculation, it turns out that $\gamma(E_t)$ is very small (cf. Table V). When we can take $\gamma \equiv 0$, then a_0 and α are identical, and the resonance energy defined by procedure (b) is at $-(2\mu_{KN}a_0^2)^{-1}$ below the $\bar{K}N$ threshold.[28] Of course, it may still be necessary to take into account the energy dependence of a_0 and b_0. These remarks all go to emphasize the uncertainty in any

TABLE V. Elements of the reduced K matrix K_{R2} (unit F) as function of the total energy E.

E (MeV)	$(\bar{K}N\|K_{R2}\|\bar{K}N)$	$(\bar{K}N\|K_{R2}\|\pi\Sigma)$	$(\pi\Sigma\|K_{R2}\|\pi\Sigma)$
1432 (threshold)	−1.303	−0.479	−0.035
1455	−1.185	−0.467	−0.033
1520	−0.861	−0.412	−0.014

[23] G. Rajasekaran, Nuovo Cimento **37**, 1004 (1965).

[24] R. H. Dalitz, Proc. Roy. Soc. (London) **A288**, 183 (1965).

[25] M. Watson, M. Ferro-Luzzi, and R. D. Tripp, Phys. Rev. **131**, 2248 (1963).

[26] R. D. Tripp, *Italian Physical Society, Course 33* (Academic Press Inc., New York, 1966), p. 70.

[27] Since completion of this work, we have learned from Professor G. Trilling that the study of low-energy K_2^0-p processes by Dr. J. Kadyk and collaborators at the Lawrence Radiation Laboratory has led to values for the ratio $(K_1^0p)/(\Lambda\pi+\Sigma\pi)$ in the region of 400 MeV/c which are in good agreement with the value calculated using A_1 from the zero-range $\bar{K}N$ analysis, but which are quite far from the value calculated using $A_1(1520)$ from Ref. 25. This result casts considerable doubt on the accuracy of the values $A_0(1520)$ and $A_1(1520)$ given by Watson *et al.* [*Note added in proof.* This work has now been published: J. A. Kadyk, Y. Oren, G. Goldhaber, S. Goldhaber, and G. Trilling, Phys. Rev. Letters **17**, 599 (1966).]

[28] We note that this is the quantity which has actually been computed to obtain the resonance energies quoted in Refs. 2–5.

extrapolation from the physical region to the resonance location. In due course, it may be possible to estimate the effective range from the empirical data in $\bar{K}N$ scattering,[29,30] in which case this uncertainty will be greatly reduced.

In the $I=Y=0$ $s_{1/2}$ PB system, another energy range of strong interactions is known, namely at the threshold for $\Lambda\eta$ production in the $\bar{K}N$ system. Whether the sharp rise and fall in this cross section is due to the influence of a resonance state, or whether it is due to a strong $\Lambda\eta$ scattering interaction, is not yet settled.[31] We may remark here that this model does not account for this effect, in either way. The $\Lambda\eta$ scattering amplitude obtained is slowly varying, with the small value $A_{\eta\Lambda} = (-0.12+0.15i)$ F at threshold; the smallness of this value is related to the fact that our model gives zero diagonal potential for the $\Lambda\eta$ system. We conclude that this $\Lambda\eta$ enhancement is generated by forces not included in our model calculation.

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[29] Kittel and Otter (Ref. 30) have very recently included a finite effective range R_0 just for the $I=0$ scattering length, to obtain an over-all fit to the low-energy $K^- - p$ scattering available from Refs. 2, 4, and 6, together with the M^* and Γ values observed for $Y_0^*(1405)$. They obtained an acceptable fit for $R_0=0.08\pm0.05$ F, in which case their zero-energy scattering length takes the value $A_0=(-1.54\pm0.02)+(0.53\pm0.03)$ F. When they also included the 400 MeV/c values for A_0 and A_1, they found that the only acceptable fit was for the Watson III solution, with $R_0=0.11\pm0.03$ F. This analysis now needs to be repeated in view of the K_2^0-p data (cf. Ref. 27) now becoming available in the 400-MeV/c region.

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$SU(6)_W$ Algebra and the Commutators of Electric Dipoles at Infinite Momentum

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The saturation of the $SU(6)_W$ algebra at infinite momentum is discussed. A possible physical interpretation of the tensor generators of $SU(6)_W$ in terms of an assumption of partial conservation is critically analyzed. The implied occurrence of singularities in the tensor amplitudes requires a careful definition of a limiting procedure defining the tensor charges. A collinear limiting procedure, which relates the tensor charges to the total magnetic moments, appears as the most convenient one. The matrix elements of the tensor charges are then compared in the infinite-momentum limit with those of the electric dipoles, and the following implications are exhibited: The charge radii of baryons are pure F; the D/F ratio of axial charges equals the corresponding ratio for the total baryon magnetic moments; a simple relation exists among the isovector total moment of the nucleon, the axial renormalization constant, and the charge radius of the proton; and an extended form of universality holds for tensor and axial currents. We also discuss the saturation of the unitary symmetric part of the commutators, particularly in connection with the possible occurrence of Schwinger terms.

1. INTRODUCTION AND SUMMARY OF RESULTS

A CLASSIFICATION of the charges generating the compact $U(12)$ according to the behavior of their matrix elements between states of infinite momentum has been proposed,[1,2] leading to the distinction between "good" and "bad" charges. The longitudinal and time components of the vector and axial charges and the transverse components of the tensor charges are good charges. In the limit of infinite momentum the set of nonequivalent good charges generates the algebra of $SU(6)_W$.

In the present paper we analyze the consequences of this algebra, avoiding particular hypotheses of approximate saturation. The starting point of the analysis will be a connection between the tensor charges, following from a hypothesis of partial conservation of tensor

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[2] S. Fubini, G. Segré, and J. D. Walecka, Ann. Phys. (N. Y.) 39, 381 (1966).