Regge-Pole Model for High-Energy Backward $\pi^{\pm}p$ Scattering*

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It is well known by now that both the π^+p and π^-p elastic-scattering reactions show peaks near the backward direction at high energy. The most striking difference between the two distributions in the backward region is the dip in $d\sigma/du$ which appears in π^+p but not π^-p. We show in this paper that the present experimental situation in high-energy $\pi^{\pm}p$ scattering can be readily understood in terms of the exchange of the N and Δ Regge trajectories. The interpretation of the dip is that N trajectory exchange, which contributes only to π^+p, becomes numerically very small when the N pole moves near $J=-\frac{1}{2}$.

IN recent years a number of experiments have been carried out on high-energy backward $\pi^{\pm}p$ scattering, with both reactions showing peaks in the backward direction which fall away rather rapidly with increasing energy.[1–4] The most striking difference between the two is the dip in $d\sigma/du$ near the backward direction which appears in π^+p but not in π^-p, as was first noted by Brody *et al.*[4] In the present work we have attempted to understand the experimental situation in terms of the exchange of the N and Δ Regge trajectories which are thought to be the dominant trajectories communicating with the πN system. We find that the present experimental situation can be readily understood in terms of these two trajectories. The interpretation of the dip is that N trajectory exchange, which contributes only to π^+p, becomes numerically very small when the N pole moves near $J=-\frac{1}{2}$. This effect, which depends strongly on the even signature of the N trajectory, is explained in more detail below.

We may write the contributions to $\pi^{\pm}p$ scattering of the N and Δ trajectories in the crossed channel by writing the amplitude for $\pi^{\pm}p$ scattering as follows[5]:

$$f^{\pm}(\sqrt{s},u)=f_1^{\pm}(\sqrt{s},u)-\cos\theta\, f_1(-\sqrt{s},u)+i\sin\theta\,\boldsymbol{\sigma}\cdot\hat{n}f_1^{\pm}(-\sqrt{s},u). \quad (1)$$

At large s and fixed u, the contributions of the crossed-channel Regge poles dominate $f_1^{\pm}(\sqrt{s},u)$. In this limit $f_1^{\pm}(\sqrt{s},u)$ can be written

$$f_1^{\pm}(\sqrt{s},u) \underset{\substack{s\to\infty,\\ u\text{ fixed}}}{\longrightarrow} \sum_{i=N,\Delta}\Bigg\{\frac{[(\sqrt{s}+M)^2-\mu^2][\sqrt{u}-\sqrt{s}+2M]}{4s\sqrt{u}}\frac{C_i^{\pm}\beta_i(\sqrt{u})}{\cos\pi\alpha_i(\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha_i(\sqrt{u})-1/2}\{1+\eta_i\exp[-i\pi(\alpha_i(\sqrt{u})-\tfrac{1}{2})]\}$$
$$-\frac{[(\sqrt{s}+M)^2-\mu^2][-\sqrt{u}-\sqrt{s}+2M]}{4s\sqrt{u}}\frac{C_i^{\pm}\beta_i(-\sqrt{u})}{\cos\pi\alpha_i(-\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha_i(-\sqrt{u})-1/2}\{1+\eta_i\exp[-i\pi(\alpha_i(-\sqrt{u})-\tfrac{1}{2})]\}\Bigg\}, \quad (2)$$

where α_i is the pole position, β_i is a modified reduced residue, η_i is the signature of the trajectory, and $C_i^{\pm}$ is the isospin crossing coefficient. These expressions represent the leading asymptotic terms in s from each pole. Correction terms are $O(1/s)$ compared to these leading terms. This asymptotic form is used throughout the backward direction, including the region near $u=0$. The justification of this use for the present case of unequal-mass scattering is more elaborate than for the equal-mass case and has recently been supplied in detail in a paper by Freedman and Wang.[6] A further point about Eqs. (2) is that as written they refer to poles in the $l=J-\frac{1}{2}$ amplitudes in the u channel. The partial-wave amplitudes in the u channel obey the MacDowell symmetry,[7]

$$T_{J-1/2}{}^{J}(\sqrt{u})=T_{J+1/2}{}^{J}(-\sqrt{u}), \quad (3)$$

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[1] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Letters **10**, 248 (1964).

[2] W. R. Frisken, A. L. Read, H. Ruderman, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and D. H. White, Phys. Rev. Letters **15**, 313 (1965); Phys. Rev. (to be published).

[3] C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, Phys. Rev. Letters **15**, 838 (1965).

[4] H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Shochet, and R. Van Berg, Phys. Rev. Letters **16**, 828 (1966).

[5] Here $\sqrt{s}$ and θ are c.m. energy and scattering angle, respectively; u is the square of the c.m. energy in the crossed baryon channel. In the direct-channel physical region, u is given by $u=2(M^2+\mu^2)-s+2q^2(1-\cos\theta)$, where q is the c.m. momentum, and M and μ are the masses of the nucleon and the pion, respectively. The quantity $\hat{n}$ is a unit vector given by $\hat{n}=(\mathbf{q}_i\times\mathbf{q}_f)/|\mathbf{q}_i\times\mathbf{q}_f|$, where $\mathbf{q}_i$ and $\mathbf{q}_f$ are the initial and final c.m. proton momenta. The quantity $f_1(\sqrt{s},u)$ is defined in the paper by V. Singh, Phys. Rev. **129**, 1889 (1963). The differential cross section and the polarization are given by

$$\frac{d\sigma}{du}=\frac{\pi}{q^2}|f(\sqrt{s},u)|^2$$

and

$$\mathbf{P}=\frac{2\,\mathrm{Im}f_1(\sqrt{s},u)f_1^*(-\sqrt{s},u)\hat{n}}{|f(\sqrt{s},u)|^2}\sin\theta.$$

[6] D. Z. Freedman and Jiunn-Ming Wang, Phys. Rev. Letters **17**, 569 (1966); see also Phys. Rev. (to be published).

[7] S. W. MacDowell, Phys. Rev. **116**, 774 (1959).

where

$$T_l^J(\sqrt{u})=\frac{\sqrt{u}}{q(\sqrt{u})}\exp[i\delta_l^J(\sqrt{u})]\sin\delta_l^J(\sqrt{u}).$$

We make the convention of always dealing with the $l=J-\frac{1}{2}$ amplitude and eliminating the $l=J+\frac{1}{2}$ amplitude by the use of Eq. (3). With this convention $\alpha_\Delta(1238)=\frac{3}{2}, \alpha_\Delta(1924)=\frac{7}{2}$, but $\alpha_N(-939)=\frac{1}{2}, \alpha_N(-1688)=\frac{5}{2}$, etc., since the N trajectory goes through the nucleon and the $\frac{5}{2}^+$ in the $l=J+\frac{1}{2}$ amplitude, which is the continuation to negative energies of the $l=J-\frac{1}{2}$ amplitude.

The functions $\alpha(\sqrt{u})$ and $\beta(\sqrt{u})$ are real analytic in the cut $\sqrt{u}$ plane with cuts $[-\infty,-(M+\mu)][M+\mu,\infty]$. The precise definition of $\beta(\sqrt{u})$ is

$$\beta(\sqrt{u})=\frac{(\sqrt{\pi})\gamma(\sqrt{u})\Gamma(\alpha+1)}{\Gamma(\alpha+\frac{1}{2})},$$

where

$$\gamma(\sqrt{u})=\left\{\frac{T_{J-1/2}{}^J(\sqrt{u})[J-\alpha(\sqrt{u})]}{(q^2)^{J-1/2}(E+M)}\right\}_{J=\alpha(\sqrt{u})}. \quad (4)$$

The quantity E is the energy of the nucleon in the c.m. system. The factor $\Gamma(\alpha+1)$ in $\beta(\sqrt{u})$ at first sight would cause poles to appear in $\beta(\sqrt{u})$ and therefore the amplitude at $\alpha=-1, -2$, etc. However, the Mandelstam symmetry[8]

$$T_{J-1/2}{}^J(\sqrt{u})=T_{-(J+1/2)}{}^{-(J+1)}(\sqrt{u}),\quad \text{for } J=\text{integer}, \quad (5)$$

holds. Therefore if a pole moves through $J=-1$, say, either its residue must vanish or another pole must move through $J=0$. Since in the present case we are by assumption dealing with the leading trajectories, the first alternative must hold, so the functions $\gamma(\sqrt{u})$ vanish at $\alpha=-1, -2$, etc. and therefore the combination $\Gamma(\alpha+1)\gamma(\sqrt{u})$ is a smooth function having no poles.

Now consider the combination which occurs in Eqs. (2):

$$\frac{\gamma(\sqrt{u})\Gamma(\alpha+1)}{\Gamma(\alpha+\frac{1}{2})}\frac{[1+\eta\exp[-i\pi(\alpha-\frac{1}{2})]]}{\cos\pi\alpha}. \quad (6)$$

If α passes through a positive half integer, i.e., a physical value of J, this combination either gives the amplitude a pole in $\sqrt{u}$ or a finite contribution, depending on whether the trajectory in question has "right" or "wrong" signature at this point. On the other hand, if α passes through a negative half integer, the combination of factors above yields either a finite contribution or zero again, depending on whether the trajectory has "right" or "wrong" signature at this point. To take the example which is most relevant for this paper, if $\alpha_N(\sqrt{u})$ goes through $J=-\frac{1}{2}$ the above combination of factors vanishes. In contrast, since the Δ trajectory has opposite signature, the corresponding combination of factors would give a finite contribution if $\alpha_\Delta(\sqrt{u})$ reached $J=-\frac{1}{2}$ and would vanish only if $\alpha_\Delta(\sqrt{u})$ reached $J=-\frac{3}{2}$. Note that in Eq. (2), two terms of the above type appear for each Regge pole, one containing $\alpha(\sqrt{u})$ and $\beta(\sqrt{u})$, and the other $\alpha(-\sqrt{u})$ and $\beta(-\sqrt{u})$. If there is a value of $\sqrt{u}$, for which both $\alpha_N(-\sqrt{u})$ and $\alpha_N(\sqrt{u})$ are near $J=-\frac{1}{2}$, then in the neighborhood of this value the contribution of the N trajectory will be drastically reduced and a dip will appear in $(d\sigma/du)_{\pi^+p}$. This dip will appear at a fixed value of u, for high s. This is the explanation proposed in this paper for the dip observed in $(d\sigma/du)_{\pi^+p}$ near the backward direction. This explanation has some nontrivial consequences for the shape of the N trajectory. To see this, we note that experimentally the dip appears for a negative value of u, $u\approx-0.2$. This means $\sqrt{u}$ is pure imaginary and, therefore, using the real analyticity of $\alpha_N(\sqrt{u})$, we have $\alpha_N(\sqrt{u})=\alpha_N^*(-\sqrt{u})$ for any $u<0$. In the region of the dip we have $\alpha_N(\sqrt{u})\approx-\frac{1}{2}\approx\alpha_N(-\sqrt{u})=\alpha_N^*(\sqrt{u})$. Expanding $\alpha_N(\sqrt{u})$ around the origin in $\sqrt{u}$, we see that this requires that the odd powers of $\sqrt{u}$ make a weak contribution compared with the even powers of $\sqrt{u}$. In other words, any simple parametrization of $\alpha_N(\sqrt{u})$ will require that it be approximately even in $\sqrt{u}$. This means that we may expect to find resonances of orbital angular momentum $l=J-\frac{1}{2}$ appearing on the N trajectory as well as the known cases, which have $l=J+\frac{1}{2}$. This last remark follows from the MacDowell symmetry, Eq. (3), and the evenness of $\alpha_N(\sqrt{u})$. In Fig. 1 the N trajectory coming from the best fit to the data is shown. It was constrained to go through the $1688F_{5/2}$ resonance and the nucleon. It turns out to go through $J=\frac{5}{2}$ at $\sqrt{u}=1600$ also, which corresponds to a $D_{5/2}$ resonance. The fact that such a resonance exists experimentally[9] we take as an additional piece of evidence in favor of our explanation of the dip in high-energy backward π^+p scattering. There is no particle corresponding to the $J=\frac{1}{2}$ intersection at $\sqrt{u}=850$; therefore the nucleon residue has been constrained to vanish at this point, so that no particle appears.[10]

The parametrizations for the Δ and N residue functions β_i and the trajectory functions α_i are

$$\beta_\Delta(\sqrt{u})=(\alpha_\Delta+\tfrac{1}{2})(\alpha_\Delta+\tfrac{3}{2})D_\Delta\exp[a_\Delta\sqrt{u}+b_\Delta u],$$

$$\beta_N(\sqrt{u})=(\alpha_N+\tfrac{1}{2})(\alpha_N+\tfrac{3}{2})\times(\sqrt{u}-\sqrt{u_0})D_N\exp[a_N\sqrt{u}+b_N u],$$

where u_0 is the energy at which $\alpha_N(\sqrt{u_0})=\frac{1}{2}$ and $\alpha_i=\alpha_i^0+\alpha_i^1\sqrt{u}+\alpha_i^2 u$ for both Δ and N. We also

[8] S. Mandelstam, Ann. Phys. 19, 254 (1962).

[9] See for example the phase-shift solutions in the papers by P. Bareyre, C. Bricman, A. V. Stirling, and G. Villet, Phys. Letters 18, 342 (1965); B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, *ibid.* 19, 420 (1965).

[10] It can be shown that when a trajectory is in the range of physical J values for $\sqrt{u}>M+\mu$ and $\sqrt{u}<-(M+\mu)$, $\gamma(\sqrt{u})$ must change sign between $-(M+\mu)$ and $M+\mu$. [See B. R. Desai, Phys. Rev. Letters 17, 498 (1966).] The zero we have put in the nucleon residue accomplishes this sign change.

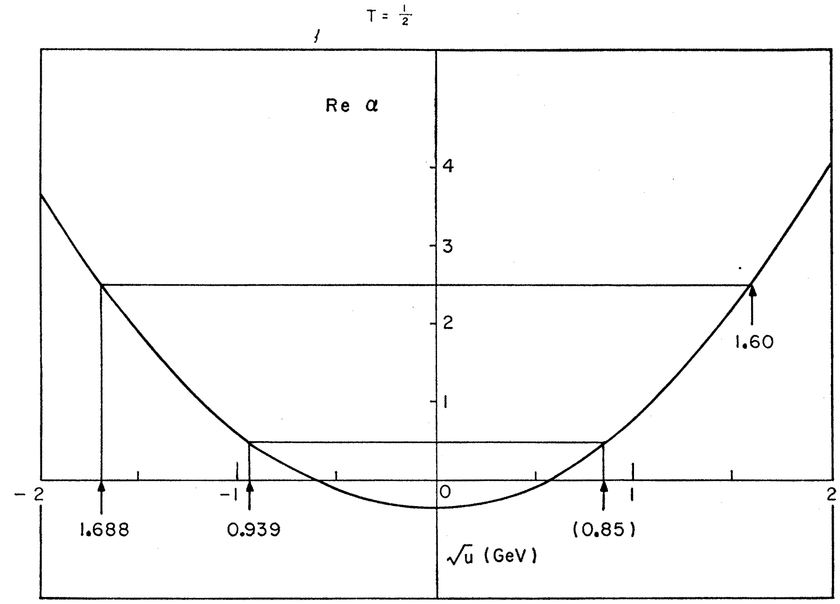

FIG. 1. The nucleon trajectory.

constrained α_Δ to pass through $\Delta(1238)$ and $\Delta(1924)$. The residue functions β_Δ and β_N are constrained to have the known values at Δ and at the nucleon, respectively. More explicitly,

$$\beta_\Delta(1238)=\frac{9\pi}{64}g^2\frac{d\alpha_\Delta}{d\sqrt{u}}\bigg|_{\sqrt{u}=1238}$$

and

$$\beta_N(-939)=\frac{3\pi}{4}g^2\frac{d\alpha_N}{d\sqrt{u}}\bigg|_{\sqrt{u}=-939}, \tag{7}$$

where g^2 is the dimensionless πN coupling constant, which is 14.6. So in actual least-squares fitting to the experimental data there is one free parameter for each α_i, and two free parameters for each residue function β_i.

Both Δ and N trajectories can be exchanged in backward π^+p scattering, whereas for the π^-p case only the Δ trajectory can be exchanged. To study the Regge amplitude for the Δ trajectory, we first looked at the π^-p data. The data[2–4] which are available at 4, 6, and 8 GeV/c show a backward peak which is somewhat broader than the corresponding peak for the π^+p case.[2,4] The differential cross section for π^-p is in general considerably smaller than for π^+p in the region of the backward peak. For example, at 8 GeV/c and $u=0$, the π^-p differential cross section is only one-third as large as that for π^+p. Although the data are quite crude, there is no evidence for any appreciable structure in the π^-p case, the data being consistent with a smooth drop-off in moving away from the backward direction. Because of the crudity of the data, a range of fits is possible; consequently the Δ parameters are not very well determined. However, this does not affect the main result of this paper, namely, the explanation of the dip in the π^+p case. This is true because, as mentioned above, the π^+p cross section is several times the π^-p cross section, which implies that exchange of the N trajectory is the dominant effect in the π^+p case. For example, in Fig. 2, the solid curve at 7.8 GeV/c shows a fit to the π^+p data due to the contribution of the N trajectory alone, whereas the dashed curve indicates the resultant contribution to the cross section after a typical Δ-trajectory contribution is added to the N-trajectory amplitude.

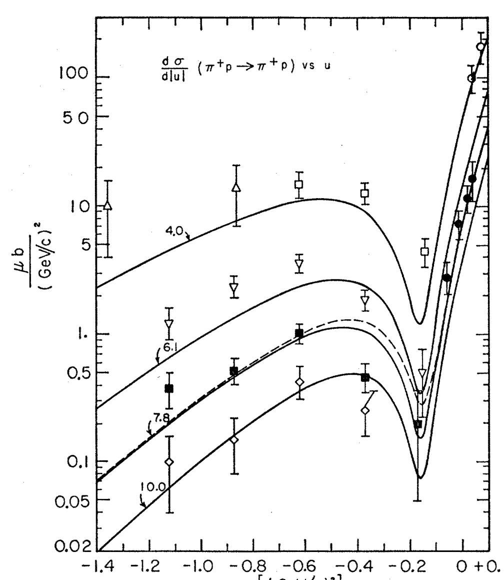

FIG. 2. Backward π^+p differential cross-section data compared with Regge fits. Symbols: Δ is for 4-GeV/c data points of Ref. 1; ○ and ● are for 4- and 8-GeV/c points of Ref. 2; □, ▽, ■ and ◇ are for 4.4-, 6.1-, 7.8-, and 10-GeV/c points of Ref. 4, respectively. The solid curves are the Regge fits due to the N-trajectory exchange contribution alone, and the dashed curve is the differential cross section after the Δ-trajectory contribution is added to the N-trajectory amplitude at 7.8 GeV/c.

Because of the uncertainties in the contribution of the Δ, the rest of the solid curves in Fig. 2, also represent the contribution of the N trajectory alone. The experimental data that are available range over incident lab momenta from 4 to 10 GeV/c. However, in least-squares fits to the data, fits were made only to the 6- to 10-GeV/c data. The reason for this is that near 4 GeV/c, the total cross section for π^+p scattering shows a bump,[11] indicating the possible presence of a resonance in the direct channel at this energy. Rather than attempt to include the possible effects of this resonance in addition to the Regge amplitude, we took the simpler course of fitting only the high-energy data. At 6 GeV/c and above, we are 3 to 4 half-widths above this last resonance and from 8 to 70 half-widths above the lower resonances. Thus we assume that resonance contributions are negligible above 6 GeV/c. For comparison we plot the contribution of the N trajectory at 4 GeV/c also, where the fit is still a qualitatively good one. In general the fits over the whole range of energies represent the qualitative features of the data quite well, where the fact that the solid curves fall somewhat below the experimental points at the lower energies and larger u values may indicate the presence of some relatively small background terms not included in our fits. The parameters for α_N and β_N coming from the best fit to the data are

$$\alpha_N{}^0=-0.340\,,\qquad a_N=-0.123\ \text{GeV}^{-1}\,,$$
$$\alpha_N{}^1=\ 0.093\ \text{GeV}^{-1}\,,\qquad b_N=\ 0.227\ \text{GeV}^{-2}\,,$$
$$\alpha_N{}^2=\ 1.052\ \text{GeV}^{-2}\,,\quad D_N=264.0\quad \mu\text{b}^{1/2}\ \text{GeV}^{-1}\,.$$

One can see from this that the position of the dip corresponds essentially to the point where $\text{Re}(\alpha_N+\frac{1}{2})=0$, as mentioned earlier.

[11] A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontić, R. H. Phillips, and A. Rousset, Phys. Rev. Letters **13**, 205 (1964).

To conclude, our model successfully explains the existing features of backward π^+p scattering. To test these ideas further, it is suggested that differential cross-section measurements of greater accuracy and at higher energies be carried out. Also, measurements of $\pi^\pm p$ polarization would be very useful. In an earlier paper by one of us,[12] it was shown that the sign of the polarization is controlled by the terms in $\alpha_i(\sqrt{u})$ which are odd in $\sqrt{u}$, i.e., $\alpha_i{}^1$ in our parametrization. The Δ-trajectory parameters as mentioned earlier are not well determined. However, if the contribution from direct-channel resonances are small, any Δ trajectory which passes through the $\frac{3}{2}^+(1238)$ and $\frac{7}{2}^+(1924)$ resonances and gives a rough fit to the energy dependence of the π^-p data will have a strong positive $\alpha_\Delta{}^1$. Therefore, the prediction of large positive polarization in π^-p is still maintained. For the π^+p case, the situation is somewhat more complicated. Because of the approximate evenness of $\alpha_N(\sqrt{u})$, $\alpha_N{}^1$ is small. However, preliminary calculations show that the sign of the polarization away from the backward direction is still determined by the sign of $\alpha_N{}^1$, with the magnitude of the polarization showing a bump at the position of the dip in the differential cross section. The polarization is extremely sensitive to small variations in $\alpha_N{}^1$; whereas, as long as $\alpha_N{}^1$ is small, the differential cross-section is rather insensitive to small changes in $\alpha_N{}^1$. Therefore, a measurement of polarization in π^+p would give important further information about the N trajectory.

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[12] J. D. Stack, Phys. Rev. Letters **16**, 286 (1966).