Optical Potential for High-Energy Physics: Theory and Applications*

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A field-theoretic exact two-body optical potential V^0 is defined, corresponding to the exact pseudopotential in nuclear physics. A small-angle, high-energy approximation for scattering amplitudes in the absence of resonances or bound states is suggested, based largely on investigations of Torgerson in a fairly realistic model field theory. This approximation, which corresponds to the eikonal or linearized WKB approximation in a nonrelativistic limit, involves only mass-shell values of V^0 and can be discussed in the framework of dispersion relations and analyticity. The longest range contributions to V^0 are one-pion exchange (when allowed) and the multiperipheral diagrams of Amati, Fubini, and Stanghellini; these contributions are termed "multipheripheral optical potential" (MOP). One possibility for the asymptotic high-energy limit of MOP brings in Regge poles through the multipheripheral diagrams. At energies which are not asymptotic, but are high enough to ensure the usefulness of the eikonal formalism, important nonpole contributions to V^0 are discussed. The difference between $\bar{p}p$ and pp elastic scattering is explained in such an energy region. As a natural consequence of the picture presented, one obtains distorted-wave Born-approximation (DWBA) correction formulas applicable to any small contributions in V_0; e.g., real part, spin flip, and charge exchange. A special case is the absorptive correction to the ρ Regge-pole expression for charge exchange in $\pi^- p \to \pi^0 n$, which has been discussed in a previous paper. The Serber potential which accommodates large-t behavior (although only $-t/s \ll 1$ can be properly described by the eikonal expression) is shown to be a special case of MOP. If MOP is dominated by the Pomeranchuk Regge pole (P) in elastic pp scattering above 10 BeV/c, and if we ignore spin, a real part of the forward scattering amplitude for this case is obtained which agrees in sign, with the observed value, but is too small in magnitude; it has an energy dependence $[\ln(s/s_0)]^{-1}$. The $\bar{p}p$ results should become identical to the pp ones for energies which are asymptotic for $\bar{p}p$ also. Similar results for the real part hold for all two-body reactions. In πp scattering, a formalism incorporating spin properly into the eikonal method is presented, and in the asymptotic limit with no "anomalous-moment" terms in the Born approximation (as suggested by a Pomeranchuk pole), a spin-orbit coupling is obtained corresponding to use of the Dirac equation with a 4-vector static central potential. The resulting πp spin-flip amplitude decreases with increasing energy like $s^{-1/2}$ relative to nonflip terms, but is presumably dominated by effects of secondary Regge Poles such as P'. To describe multichannel reactions, and to obtain absorptive correction formulas including reactive damping, an exact multichannel optical potential V_{ij}^0 is defined, and a matrix eikonal mass-shell approximation is proposed. Such a method is valid only when certain commutation relations are satisfied for the matrix Born approximation; these are satisfied, for example, if the t dependence of all elements of this matrix is the same, which can be true in many cases if mass differences are ignored. Regge poles and the Byers-Yang model are considered in this context. To include resonances in the s channel, possibilities for extending the eikonal formalism are discussed. A method of formulation utilizing dispersion relations for phase shifts allows an alternative, purely S-matrix approach to the eikonal approximation, but is physically more obscure than the field-theoretic and static-potential-theory approaches. Singularities in complex J of the MOP-eikonal approach are explored. It is found that infinite numbers of branch cuts in J correspond to absorptive (DWBA) corrections when Regge poles are used in MOP. An apparent paradox concerning results of Mandelstam on cancellation of cuts is discussed and a possible avenue of resolution, involving treating external particles as Regge poles with signature, is described.

INTRODUCTION

INTERPRETATIONS of high-energy elastic scattering and other two-body reactions have, in the past, been proposed on the basis of various special models. One-elementary-particle exchange,[1] Regge poles in the t channel,[2] phenomenological optical models.[3,4] coherent (semiclassical) droplet models,[5] and statistical (incoherent) interaction models[6] as well as various combinations of these have more or less successfully been fitted to much of the known features of such processes. It may be noted that no one model is universally successful. Thus, one-particle exchange with absorptive correction from empirical optical-model fits has exhibited dramatic agreement with experiment[7] in cases where π exchange in peripheral inelastic reactions is allowed, but such a model does not correctly describe many cases where π exchange is not allowed, and is irrelevant to the description of elastic scattering. Alternatively, phenomenological optical-model fits have shown striking correlations between elastic-scattering polarizations and angular distributions,[8,9] but are irrelevant for inelastic reactions.

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Previous attempts to calculate high-energy elastic scattering in terms of inelastic processes using the K-matrix formalism[10] [or combining it with distorted-wave Born-approximation (DWBA) prescriptions[11]] have not been quantitatively impressive; they give reasonable results[12] only for the highest partial waves or large impact parameters; they do not lead to a semiclassical absorption picture ($e^{2i\delta(b)} \rightarrow 0$) for small impact parameters; and only two-body inelastic channels can enter in an explicit way, which apparently is not adequate for a realistic description of inelastic processes.[13]

The preceding remarks have concerned reactions with small momentum transfer. Quantitatively successful models for large-$(-t)$ reactions have involved incoherent statistical models[6] or purely phenomenological absorptive potentials,[4] with little success in deriving such behavior using either specific field-theory diagrams or S-matrix theory.

A phenomenological statistical approach of Krisch[14] has been reasonably successful in connecting small and large-$(-t)$ behavior of elastic scattering to inelastic processes, at the expense of introducing *ad hoc* Gaussian source functions.

The most ambitious program for understanding high-energy reactions to date has been the multiperipheral model of Amati, Fubini, and Stanhellini (AFS).[15] This is based on summing a class of field-theory diagrams; for elastic scattering it yields Regge-pole behavior as a first approximation, and successive approximations bring in cuts in the complex angular momentum (J) plane.[16] The role of such cuts in this model and in more complex models has been extensively debated; it was concluded[17] that such cuts should not appear in the perturbation diagrams actually summed by AFS, but should in fact be present in models which include a broader topological class of diagrams.[18,19] This fact presented a theoretical barrier to further development of the AFS model.

Subsequent investigations by Gribov, Pomeranchuk, and co-workers using S-matrix techniques[20] have lent support to the conclusion that such cuts are present, are important, and that their properties may be determined essentially by the lowest approximation (Regge-pole) terms in elastic and inelastic amplitudes. Since an S-matrix viewpoint makes it possible to remove the distinction between elementary particles, bound states, and resonances implicit in a Feynman-diagram approach such as used by AFS, it may be possible to avoid the topological difficulty mentioned above if high-energy theory can be formulated directly in terms of S-matrix elements on the mass shell. The K-matrix approach for example does this,[10] but does not exhibit a natural way for obtaining semiclassical optical-model behavior, especially when Regge poles are to be included in a semi-phenomenological theory.

The purpose of this paper is to describe an optical-model formalism which is (in principle) exact, and within its framework discuss useful models and resolution of the above-mentioned difficulties.

In Sec. II, a basis for further discussion is provided by first reviewing the two-body optical (psuedo-) potential idea in static potential theory. The well-known high-energy small-angle (eikonal) approximation[21,22] involving such an optical potential is then characterized in terms which are equally applicable in S-matrix theory.

In Sec. III, the work of Torgerson[23] on high-energy approximations in a field theory is used as a basis for defining an exact field-theoretic optical potential V^0, which is appropriate for utilizing the eikonal approximation, and also in principle to calculate bound states. This potential differs from other previously defined potentials, e.g., the Bethe-Salpeter kernel,[24] the associated equal-time equivalent potential of Logunov and Tavkhelidze,[25] and the mass-shell potential of Chew and Frautschi.[26] The long-range contributions of V^0 are then identified with the AFS multipheripheral graphs. These contributions are referred to as the multiperipheral optical potential (MOP).

In Sec. IV, the dispersion relation in s for the associated eikonal function $\chi(s,b^2)$ is used as a basis for an S-matrix approach to calculation of χ. This permits consideration of dispersion graphs in principle more complicated than the multiperipheral diagrams, and suggests reasonable approximations to be used for χ at moderately high energies where the leading Regge poles (or other leading singularities in the J plane) do not provide an adequate approximation to V^0. A comparison of pp and $p\bar{p}$ elastic scattering is presented,

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ignoring spin; an explanation of the $p\bar{p}$ expanding diffraction peak is given based on diagrams for $p\bar{p}$ annihilation channels analogous to the multiperipheral diagrams of AFS but involving Regge-pole nucleonic state exchanges. Large $-t$ behavior is estimated on the basis of the Pomeranchuk Regge pole (P) dominating V^0, and is seen to be consistent with Serber's results on the potential.[4]

In Sec. V, the eikonal approximation is extended to spin-$\frac{1}{2}$-spin-0 scattering. It is shown that when the dominant term in V^0 does not contain any helicity-flip amplitude, there is an induced spin-orbit coupling effect which yields a nonzero spin-flip term in the scattering amplitude. This is shown to correspond exactly to the result obtained with a central, static potential in the Dirac equation,[27] when the Born approximation is interpreted as the Fourier transform of a static central potential.

In Sec. VI, it is shown that small contributions to V^0 can be treated by the distorted-wave Born approximation (DWBA) formula of Sopkovich,[28] Durand and Chiu,[7] and Jackson and Gottfried.[29] The DWBA formula cannot be applied at this point to inelastic reactions, but applications to estimate the real part of the amplitudes, the spin-flip amplitudes, and charge exchange are given.

In addition to reproducing previous spinless formulas[30] for πp charge exchange, the real part of the forward pp scattering amplitude is estimated assuming P dominates V^0. The correct sign is found, and a logarithmic energy dependence is obtained which is compatible with experiment, but the magnitude is too small. The real part of forward $\bar{p}p$ scattering would be the same as pp at asymptotic energies, and the same sign (and similar energy dependence) for the real part should also hold for all elastic meson-baryon scattering at asymptotic energies if the leading term in V^0 is a Pomeranchuk pole. The πp charge-exchange polarization (due to absorptive corrections applied to the ρ pole) is computed also in this section.

In Sec. VII, an exact two-body multichannel optical potential V_{ij}^0 is introduced, analogous to the single-channel potentials defined in Sec. II. The purpose here is to obtain results for two-body inelastic reactions which reduce in the limit of small $V_{ij}^0(i\neq j)$ to the multichannel DWBA formula.[7] This generalizes the results of Sec. VI. If the $V_{ij}^0(i\neq j)$ are not small, however, new results are obtained which have a more restricted range of validity than in the single-channel case. A certain commutation condition between the V_{ij}^0 matrices must be satisfied in order that an eikonal approximation be possible. The physical content of the commutation condition is briefly discussed, and a simple two-channel model example constructed. Regge poles may be used for V_{ij}^0 at asymptotic energies, and the possibility of a Byers-Yang model[5] is discussed.

In Sec. VIII, the problem of phenomenological inclusion of resonances in the direct channel is considered, returning to one-channel formulations, and some possibilities for extending the eikonal formalism are explored. An alternative expression of the eikonal approximation is obtained by considering a dispersion relation for the complex phase shifts,[31] which avoids the introduction of an off-mass-shell potential, but is physically more obscure than the approach through field theory or static potentials. The relation of this result to classical limits in S-matrix theory is briefly discussed.

In Sec. IX, the singularities in the J plane of the eikonal approximation with P-dominated V^0 are examined, and it is concluded that an infinite number of branch points (accumulating at $J=1$ for $t\rightarrow 0$) are present. This is the singularity structure of the iterated multiperipheral model,[16] but it is argued that the cut discontinuities obtained are not those of Ref. 16. This suggests that Mandelstam's diagrammatic analysis[17] may not contradict the use of Regge poles in MOP. A possible proof of this conjecture is indicated if the MOP can be extended to handle the external particles as Regge poles with signature. This requires (strictly speaking) discussion of 4 particle$\rightarrow$4 particle amplitudes, but a multichannel two-body formalism as described in Sec. VII may be adequate.

A summary of new results is given in Sec. X.

II. STATIC-POTENTIAL MULTICHANNEL SCATTERING AND THE SINGLE-CHANNEL OPTICAL POTENTIAL

1. Definition of the Optical Potential

Consider, in the framework of nonrelativistic static-potential theory, an elastic-scattering process to which many inelastic channels are coupled. In this section only two-body channels are explicitly considered, but the results can be put in a form which does not have this restriction. The system can be formally described then by a multichannel Schrödinger equation. Assuming a central potential matrix $V_{ij}(r)$, the radial wave functions for the ith channel, $U_{li}(k_i,r)$, satisfy:

$$\left[\frac{d^2}{dr^2}+k_i^2-\frac{l(l+1)}{r^2}\right]U_{li}(k_i,r)=\sum_j V_{ij}(r)U_{lj}(k_j,r), \quad (1)$$

where k_i is the center-of-mass momentum in channel i. Conservation of energy allows every k_i to be expressed in terms of $k\equiv k_1$, so the indices on k can be omitted in the argument of the wave functions.

Let $G_{li}^{(+)}(k;r,r')$ be the Green's function with outgoing-wave boundary conditions for the left-hand side

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of Eq. (1), i.e.,

$$\left[\frac{d^2}{dr^2}+k_i^2-\frac{l(l+1)}{r^2}\right]G_{li}^{(+)}(k;r,r')=\delta(r-r'). \quad (2)$$

G can be expressed[32] in terms of spherical Bessel functions, j_l and $h_l^{(1)}$.

Let $U_{l1}^0(k,r)$ be the solution of the homogeneous equation for channel 1 describing a plane wave, proportional to $j_l(kr)$. Then the scattering solution for Eq. (1) satisfies the following set of multichannel Lippman-Schwinger equations for $j\neq 1$:

$$U_{lj}(k,r)=\sum_i\int_0^\infty G_{lj}^{(+)}(k;r,r')V_{ji}(r')U_{li}(k,r')dr'. \quad (3)$$

Separating the $i=1$ term, (3) can be written

$$\sum_{i\neq 1}\int_0^\infty dr'[\delta(r-r')\delta_{ji}-G_{lj}^{(+)}(k;r,r')V_{ji}(r')]U_{li}(k,r)' =\int_0^\infty dr'\, G_{lj}^{(+)}(k;r,r')V_{j1}(r')U_{l1}(k,r'); \quad (4)$$

let $[I-H_l(k)]$ denote the nonlocal operator on the left-hand side of (4), and $[G_j^{(+)}(k)V_{j1}]$ the operator on the right side. Then if the integral operator $[I-H(k)]^{-1}$ exists with outgoing-wave boundary conditions (in all channels except 1) the appropriate symbolic inversion of (4) is

$$U_{lj}(k,r)=\sum_{n\neq 1}[I-H_l(k)]^{-1}{}_{jn}G_{ln}^{(+)}(k)V_{n1}U_{l1}$$

and (1) can then be written

$$\left[\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}\right]U_{l1}(k,r)=V_{11}(r)U_{l1}(r)+\int_0^\infty dr' \times[\sum_{j,n\neq 1}V_{1j}[I-H_l(k)]^{-1}{}_{jn}G_{ln}^{(+)}(k)V_{n1}]_{r,r'}\,U_{l1}(k,r'). \quad (5)$$

Now (5) has the form of a Schrödinger equation with a nonlocal potential operator $V_{11}(r)+W_l$ on the right side, with the following properties of the nonlocal part W:

(A) W is quadratic in the potentials V_{1n} leading to and from channel 1.

(B) If all V_{ij} are energy-independent, W is energy-dependent (even below inelastic thresholds) and l-dependent.

(C) If all V_{ij} are real, and channel 1 is the lowest mass state, W is real below the lowest threshold for real inelastic processes, but complex above this threshold.

This effective potential ($V_{11}+W$) is called the optical potential, or pseudopotential, for channel 1.

The perturbation solution of Eq. (5) provides a physical interpretation. The nonlocal part of the optical potential describes the disappearance of particles from channel 1 (V_{1n}) at point r', propagation with the exact Green's function for other channels ($[I-H]^{-1}G^{(+)}$), and reappearance of these particles at point r in channel 1 (V_{m1}). Such a potential can obviously be defined without restrictions on the number of particles in the other channels, and with relativistic kinematics (e.g., Dirac equation) if desired.

2. The Local Approximation

At high energies, the wavelength of the particles becomes small, while the range of the potentials remains constant; a semiclassical picture is appropriate then for sufficiently high energies.

In the classical limit, the particles can be considered as localized in space, interacting with an immediately adjacent region of the "medium" described by the potential operator; this operator then should be well approximated by a local (i.e., diagonal in r) operator:

$$V_1^{\text{eff}}(r)=V_{11}(r)+\bar{W}_l(r),$$

where

$$\bar{W}_l(r)=\int_0^\infty j_l(kr')W_l(r,r')dr'$$

aside from a normalization factors; the j_l represents an unperturbed plane-wave weight factor.

These ideas have been extensively exploited in treating scattering from a complex nucleus.[33,21] In the formalism presented above, the excited states of the nucleus are represented by particles in the other (eliminated) channels, and the V_{ij} are transition matrix elements (overlap integrals).[21] In actual practice, comparatively little success has been achieved in computing the V_{ij} (and hence W); the emphasis has historically been on fitting data by an empirical V^{eff}, with a few adjustable parameters in a definite functional form.

3. The Eikonal Approximation

A closed-form approximation for the scattering solution of the Schrödinger equation with local (or effective local) potential, good for small angles and large k values, is well known: the eikonal approximation. Various derivations of this form have been given: By summing the Born series using the stationary-phase method in each term[22]; by representing the wave function as a product of a plane wave and a modulating function and retaining the dominant term (as $k^{-1}\to 0$) in the modulating function[21]; and by linearizing a one-dimensional WKB approximation, dropping terms of order k^{-2} in the phase shifts, and assuming that high partial waves dominate the scattering. The final results in each case are equiva-

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lent to the following prescription (for nonrelativistic, spinless problems):

(A) Take the Born approximation for the (complex) phase shifts

$$\delta_l(k)=k\int_0^\infty V(r)j_l^2(kr)r^2dr$$

[even when this is large].

(B) Then at the same time replace the partial-wave sum

$$f(k,\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)(e^{2i\delta_l}-1) \quad (6)$$

by an integral over impact parameter $b\equiv(l+\frac{1}{2})/k$ involving a Bessel function instead of Legendre polynomials; using covariant (Mandelstam) variables s and t instead of k and θ, this is

$$f(s,t)=ik\int_0^\infty bdb\,J_0[b(-t)^{1/2}][1-e^{i\chi(s,b^2)}], \quad (7)$$

where $-t=4k^2\sin^2(\theta/2)$, and $\chi(s,b^2)\equiv\delta_{kb-1/2}(k)$. This continuous function of b is known as the eikonal function. Note that it is not asserted that (A) alone yields a good approximation for δ_l, nor that (B) yields a good approximation to $f(s,t)$ with the exact phase shifts δ_l; only the combination (A+B) is equivalent to the eikonal approximation as derived by the various methods mentioned above.

Alternatively, the eikonal approximation can be characterized in terms of the Born approximation f_B to the amplitude $f(s,t)$, if the *exact* (covariant) impact-parameter representation[34] of $f(s,t)$ is adopted. From this viewpoint, (7) is considered an exact representation, thus defining the function χ. Then the eikonal approximation may be defined by the following prescription:

(C) χ is a linear homogeneous functional of the Born approximation f_B.

Since the Born approximation must be reproduced for small values of f_B, and hence χ, a specific formula for χ can be obtained by expanding the exponential retaining only first order in χ on the right side of (7) and setting the left side equal to $f_B(s,t)$. Applying the inverse Fourier-Bessel transform, the following result is obtained for the approximate eikonal:

$$\chi(s,b^2)\cong\frac{1}{k}\int_0^\infty xdx\,J_0(xb)f_B(s,-x^2); \quad (8)$$

where $x=(-t)^{1/2}=2k\sin(\theta/2)$.

The second way of characterizing the approximation is more convenient for application to S-matrix theory since only the Born approximation to the amplitude ie required. Furthermore, it is easy to generalize to ths case of spin-$\frac{1}{2}$–spin-0 scattering, which will be done later.

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The prescription (C) has a direct physical significance in the semiclassical limit at high energies; the values of Imχ, in a simple optical-ray picture, are proportional to the inverse of the mean free path of particles in the optical medium. This, in turn, should be linear in the imaginary part of the potential, which describes removal of flux from the beam by absorption. Since χ should (in an S-matrix picture) be an analytic functional of the potential, this means χ should be linear in the potential, and hence linear in the Born approximation.

4. DWBA-Type Formulas

In the absence of a complete dynamical theory it is necessary to rely on experimental data for determining χ. This is complicated in general since χ occurs in an exponential form, and if V is a sum of several terms, this will not be reflected in a simple way in the experimental data. However, the bulk of high-energy small-angle elastic-scattering data is consistent with a simple imaginary Gaussian for χ, similar for scattering in various isospin states (indicating isosinglet exchange dominates V). The effect of small terms in V (and χ) can be exhibited then by expanding the exponential in (7) keeping only first order in the small terms.

Let $V=V_0+\delta V$, $\chi=\chi_0+\delta\chi$; then to first order in $\delta\chi$,

$$f(s,t)=ik\int_0^\infty bdb\,J_0[b(-t)^{1/2}]\{1-e^{i\chi_0}[1+i\delta\chi(s,b^2)]\}$$

or

$$f(s,t)=f_0(s,t)+\int_0^\infty bdb\,J_0[b(-t)^{1/2}]e^{i\chi_0(s,b^2)}\times\int_0^\infty xdx\,J_0(xb)\delta f_B(s,-x^2), \quad (9)$$

where

$$f_0(s,t)=ik\int_0^\infty bdb\,J_0[b(-t)^{1/2}][1-e^{i\chi_0(s,b^2)}]$$

and δf_B is the small contribution to f_B associated with δV;

$$\delta\chi(s,b^2)=\frac{1}{k}\int_0^\infty xdx\,J_0(xb)\delta f_B(s,-x^2).$$

The resulting formula (9) has the structure of a distorted-wave Born approximation for the perturbation δV. This can be applied to charge exchange scattering,[30] helicity flip terms, and the real part of the amplitude; these will be discussed later when a specific idea of χ is available in realistic cases.

III. THE FIELD-THEORETIC SINGLE-CHANNEL OPTICAL POTENTIAL

1. Definition of the Potential

The meaning of a potential function is connected with a specific prescription for obtaining the scattering am-

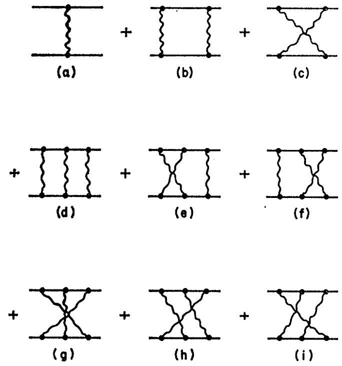

FIG. 1. Generalized ladder graphs in nucleon-nucleon scattering with vector-meson exchanges.

plitude, given the potential. If a one-body wave equation (Dirac or Klein-Gordon) is given with an associated (static) potential, the solution of this equation yields the scattering amplitude. This has been the primary method used in previous applications of the optical-model ideas.[4]

In the relativistic two-body context, two types of exact potential have previously been described at length in the literature. One is the Chew-Frautschi (CF) potential,[26] which is a function defined on the mass shell only; the definition specifies that this potential, when combined with the analytically continued two-body elastic unitarity condition (and analytic properties as determined by the Mandelstam representation) must lead through a certain well-defined iterative procedure[35] to the exact scattering amplitude. The other is the Bethe-Salpeter (BS) potential, which is defined as the irreducible kernel of the BS integral equation,[24] and is a function of four 4-momentum vectors (off the mass shell as well as on).

Neither of these potentials has had an extensive application to high-energy processes. Their original motivation, in both cases, was primarily concerned with bound states and resonances. It was argued that single-particle exchanges should be a reasonable approximation in such problems. In the BS case, this was justified by a space-time picture[24] of a weakly bound electron-positron pair in quantum electrodynamics (QED); in the CF case nearest-singularity arguments were employed.

For a potential to be useful, it is desirable that simple approximations for the potential lead to qualitatively good scattering amplitudes; otherwise nothing has been gained in the introduction of a potential, since a complete dynamical calculation is out of the question. The application of eikonal formulas, with an empirical absorptive potential in the spirit of nuclear physics, has led to a qualitatively good description of high-energy scattering.

Thus one is led to the question of the *field-theoretic significance of the eikonal approximation*. This has been investigated recently by Torgerson,[23] using nucleon-nucleon scattering by exchange of massive neutral vector mesons (with no anomalous moment couplings) as a field-theoretic framework. (The main points of the argument are independent of the presence of nucleon spin.) He has given strong arguments for the following conjecture: When a single-particle-exchange diagram is used as the Born approximation, the eikonal formula (8) is a good high-energy, small-angle approximation for the sum of all generalized ladder diagrams (crossed as well as uncrossed) shown in Fig. 1.

This is explicitly verified by Torgerson for the sum of the 2 two-rung diagrams (the one-rung is trivially reproduced), and the functional form is shown to be correct for the sum of all the three-rung diagrams (after several subtle cancellations are taken into account). Independent of the perturbation expansion, he also exhibits a semiclassical limit for the problem in which the nucleon fields are replaced by classical current distributions describing straight-line trajectories; in this case the eikonal expression appears through a result of Glauber[36] on semiclassical matrix elements. The diagrammatic interpretation of this limit is consistent with the generalized ladder series in Fig. 1.

Now if the eikonal formula is to be regarded as an approximation scheme for the scattering amplitude, given an exact potential, it is apparent that one requires a definition of the potential (and hence what is meant by the Born approximation) appropriately chosen so the series in Fig. 1 will reproduce the scattering amplitude, at least in the high-energy limit. This considera-consideration motivates the following definition of a field-theoretic exact optical potential (EOP). Let $F(k_1,k_2;k_1',k_2')$ [any four-point function] be represented by (a) in Fig. 2; consider the infinite series of Feynman diagrams constructed from F and free nucleon propagators, indicated in Fig. 2, topologically identical to the generalized ladder series of Fig. 1 with F replacing the meson propagator.

Let $A(k_1,k_2;\, k_1',k_2')$ be the sum of this series, defined by analytic continuation in the strength of F if necessary. Then the EOP, $V^0(k_1,k_2;\, k_1',k_2')$ is defined by the statement that when $F=V^0$, A becomes the exact scattering amplitude. This obviously guarantees the eikonal formula (if Torgerson's results are not misleading) and

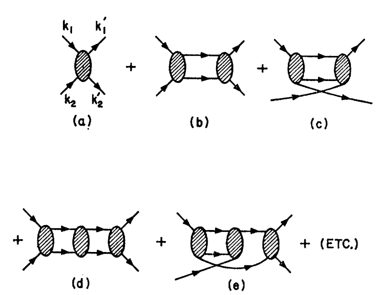

FIG. 2. Graphs used in definition of optical potential V^0.

[35] S. Mandelstam, Phys. Rev. 121, 1344 (1958).

[36] R. J. Glauber, Phys. Rev. 84, 394 (1951).

also provides an exact formalism, valid in principle for computing low-energy processes.

To explicate the meaning of V^0, one may enumerate the lowest order perturbation graphs which contribute to V^0 in QED. The graphs of order α^3 and lower are shown in Fig. 3 for particle-particle scattering, and in Fig. 4 are given the additional contributions for particle-antiparticle scattering. In addition one has, in QED, a one-photon pole in particle-antiparticle scattering. Note that in a classical limit for the electron-positron field, which ignores vertex renormalizations, self- interaction, annihilations, and closed loops, only the one-photon term (a) survives. This shows the great economy of V^0 in such cases compared to the BS definition of a potential. (A more realistic approximation might include annihilations in particle-antiparticle scattering, e.g., Figs. 4(a) and 4(b), but without renormalization, self-interaction, and loop graphs.)

The principal drawback of this definition is the absence of a closed-form integral equation which would allow investigation of bound states and resonances. Thus, we cannot use V^0 in practice, unless an approximation is made such as the retention of only uncrossed diagrams in the series of Fig. 2. The BS equation sums these, if V^0 is used as an approximate BS kernel, and one obtains essentially the ladder approximation, augmented (in principle) with vertex and propagator modifications and some multiparticle intermediate states as indicated in Fig. 3. In other words, V^0 offers no advantages over the BS definition if it is desired to compute A using an integral equation, unless a symbolic closed-form representation of the series in Fig. 2 can be conconstructed. The latter is not of primary interest at the moment and will not be discussed further.

In a similar spirit, the potential V^0 can be considered as an approximation to the CF potential. In this case, only mass-shell values of V^0 are required, and the dispersion-relation approach described in the next section will yield useful models for the CF potential as well as being useful in the eikonal formalism. Further remarks on this point will follow later.

Historically, the general idea of an exact optical potential for high-energy physics was proposed by

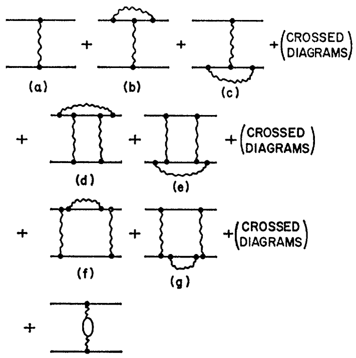

FIG. 3. Contributions of order α^3 and lower to V^0 in quantum electrodynamics for particle-particle scattering.

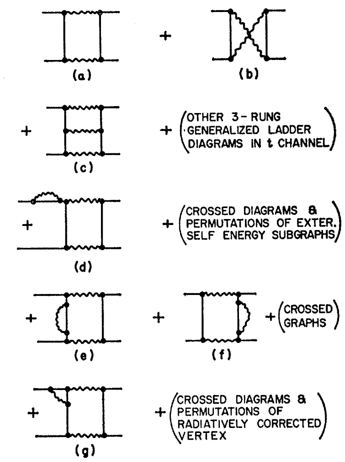

FIG. 4. Additonal contributions of order α^3 and lower to V^0 in QED for particle-antiparticle scattering.

Blokhintsev, Barashenkov, and Barbashov,[37] but without any detailed dynamical models this could not be further exploited. A phenomenological formal eikonal development was proposed by Blokhinstev,[38] but since the potential was phrased in terms of Heisenberg field operators the calculation of such a potential was very obscure.

To obtain useful results, the high-energy small-angle approximation must be adopted, and this will allow utilization of dispersion relations in the calculation of χ.

2. Dispersion Relation for the Eikonal and the Multiperipheral Contribution

For sufficiently large k values the eikonal approximation (7) can be used, assuming no resonances or bound states in the s channel are important. Assuming $A(s,t)\equiv s^{1/2}f(s,t)$ satisfies a Mandelstam representation with normal thresholds and adopting (7) as an exact representation, for fixed b^2 the exact $\chi(s,b^2)$ has a branch cut in s on the real axis starting from the lowest inelastic threshold and running to $+\infty$, a kinematic branch cut from $s=0$ to the elastic threshold $k=0$, and no other singularities, if (as in potential scattering) only one double spectral function $\rho(s,t)$ is present, corresponding to peripheral reactions. At high energies the kinematic branch cuts (and any complications with S-matrix zeros, resonances, and anomalous thresholds) are presumably unimportant, and only the inelastic branch cut need be considered. A dispersion relation in s keeping only this cut takes the form:

$$\chi(s,b^2)=\frac{1}{\pi}\int_{s_1}^{\infty}\frac{ds'}{s'-s}\,\text{Im}\chi(s',b^2)\,. \tag{10}$$

Now χ in the eikonal approximation is the Fourier-Bessel (FB) transform of the Born approximation, A_B.

[37] D. I. Blokhintsev, V. S. Barashenkov, and B. M. Barbashov, Usp. Fiz. Nauk **68**, 417 (1959) [English transl.: Soviet Phys.-Uspekhi **2**, 505 (1959)].

[38] D. I. Blokhintsev, Nuovo Cimento **30**, 1094 (1963).

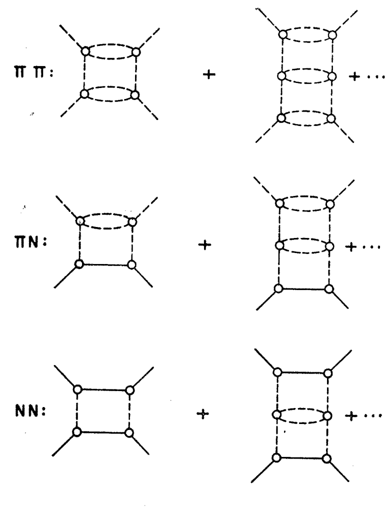

FIG. 5. Multiperipheral graphs in $\pi\pi$, πN, and NN elastic scattering.

The optical-potential Born approximation (at least in model field theories) has (with the above-mentioned restrictions) analytic properties similar to the amplitude, except the elastic branch point in the amplitude is not present. Thus, one may write a dispersion relation in t for $A_B(s,t)$:

$$A_B(s,t)=\frac{1}{2\pi i}\int_{t_0}^{\infty}\frac{dt'}{t'-t}\,\mathrm{disc}_{t'}A_B(s,t')\,.$$

The large-b^2 components of χ will presumably be dominated by the smallest t' values in the integral, i.e., the smallest mass states in the t channel. Except when one-pion exchange is present, these will be two-pion states. In this way the peripheral contributions to χ are identified.

If it is assumed that these longest range contributions dominate disc(A_B), a fairly definite model emerges. This assumption is essentially the same as the assumption that all inelastic processes are highly peripheral at the energies of interest. Other models of disc(A_B) are, of course, possible; but such attempts, when not directly related to consideration of two-particle states in the t channel, have employed either purely phenomenological considerations[14] or embody statistical arguments[39,40] which are not related to specific dynamical schemes.

What is suggested, then, is a kind of strip approximation[41] for the optical potential on the mass shell. If this V^0 is applied at low energies as a potential in the CF equation, the bound-state N/D calculations are identical to those of the strip approximation.

To go further, a more detailed model for peripheral many-particle inelastic processes is required. The only such model not restricted to two-body (or resonance) final states for inelastic processes is the multiperipheral model of Amati, Fubini, and Stanghellini (AFS).[15] The arguments of Bialas and Van Hove[13] indicate that multiparticle states are essential for obtaining results which agree with experimental elastic-scattering data. The discussion of AFS concerning the relevance of multiperipheral graphs in the unitarity condition for elastic scattering applies to V^0 as readily as to the scattering amplitude. These contributions to V^0 will be called the *multiperipheral optical potential* (MOP). The relevant dispersion graphs for $\pi\pi$, πN, and NN scattering are shown in Fig. 5.

The treatment of elastic scattering presented by Amati, Cini, and Stanghellini[16] employing s-channel unitarity was a form of iteration involving the multiperipheral graphs as input. The idea of the MOP is similar, but the details are quite different; the authors of Ref. 16 did not use an eikonal formula, and their iteration procedure apparently will generate only uncrossed s-channel ladder graphs (where the multiperipheral chains are rungs).

The multiperipheral idea implies a model of inelastic processes in which inelasticity increases slowly with energy. It is inapplicable, therefore, (in its original form) to $\bar{K}N$ and $\bar{p}p$ reactions which have strong inelasticity near threshold. In particular, for the $\bar{p}p$ case, the multiplicity of particles in inelastic reactions is already large (4–5) at low energies[42] and does not increas logarithmically, as implied by MOP. In such cases, the optical potential model must be extended[40] at least to include annihilation graphs, such as Figs. 4(a) and 4(b) in QED, since they apparently contribute a large amount of the inelastic cross section.

IV. PRACTICAL MODELS FOR THE POTENTIAL

1. Low- and High-Energy Approximations

The starting point for the MOP is the assumption that inelastic processes (which determine ImV^0) are dominated by production of pion pairs through the one-pion-exchange mechanism.[15] In some reactions, in some energy regions, this seems to be a reasonable approximation, especially if the pion pairs form a ρ meson. An example is the case of πN reactions from 1–5 BeV/c, where ρ production seems to be the most important inelastic process. In other cases one often finds other channels are more important; in NN collisions, N^* production through one-pion exchange seems to account for the majority of the inelastic cross section in the same energy region. This suggests that a reasonable model for ImV^0 should include contributions such as in Fig. 6, at energies which are such that single or double resonance production dominates.

[39] L. Van Hove, Rev. Mod. Phys. **36**, 655 (1965).

[40] J. J. J. Kokkedee, Nuovo Cimento **43A**, 919 (1966).

[41] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **5**, 580 (1960).

[42] T. Ferbel, A. Firestone, J. Sandweiss, H. D. Taft, M. Gilloud, T. W. Morris, W. J. Willis, A. Bachman, P. Baumel, and R. M. Lea, Phys. Rev. **143**, 1096 (1966); T. Ferbel, A. Firestone, J. Johnson, J. Sandweiss, and H. D. Taft, Nuovo Cimento **38**, 12 (1965); G. R. Lynch, R. E. Foulks, G. R. Kalbfleisch, S. Limentani, J. B. Shafer, M. L. Stevenson, and N. Xuong, Phys. Rev. **131**, 1276 (1963).

At sufficiently high energies, such that multiple-production processes do dominate over single or double isobar production, the sum of all t-channel ladder graphs (Fig. 5) is, in the multiperipheral model (with s-wave $\pi\pi$ vertices), the appropriate approximation to $\mathrm{Im}V^0$. This sum has[15] Regge-pole asymptotic behavior, $s^{\alpha(t)}$. Thus, the asymptotic contributions to $\mathrm{Im}V^0$ in the multiperipheral picture are the Regge poles previously used in phenomenological analysis[43,44] of high-energy reactions; only these poles represent here the Born approximation to be used in an eikonal formalism. (This was previously conjectured[30] on the basis of the Chew-Frautschi potential, but that potential is not the appropriate one to invoke, as is now clear from Torgerson's work.[23]) Quantitative estimates show that the Born approximation gives, for the scattering amplitude, the correct order of magnitude at sufficiently small momentum transfer,[30] which accounts for the success of phenomenological Regge pole-fits in cases other than K^-p and $\bar{p}p$ scattering which do not fit the MOP picture.

In principle, it is possible to incorporate the resonance production diagrams in Fig. 6 in a sum over all possible ladder graphs; the sum over all graphs might be represented by sufficiently many Regge poles, some of which are below $J=0$ in the complex J plane. It is not known whether this is possible in an S-matrix approach, but ladder diagrams in some model theories[45] can be represented as an infinite sum over such poles.

In a pragmatic approach, it seems reasonable to use instead a combination of (A) Regge poles with $\alpha\geqslant 0$ which are associated with known particles and resonances in the t channel, and (B) explicitly computed single- and double-resonance production diagrams, as long as they have over-all energy dependence less dominant than that of the Regge poles in (A). The latter requirement is necessary to minimize duplication of important contributions in both (A) and (B). The assumption that (A) is independent of (B) is essentially equivalent to assuming that the (B) diagrams are a negligible part of the structure of the composite particles described by the (A) poles.

Note that the discussion above is concerned with $\mathrm{Im}V^0$, and thus determines $\mathrm{Im}\chi$. The real part of χ (except at asymptotic energies) must then be calculated from the disperison relation (10). The Regge-pole contributions to $\mathrm{Im}\chi$ (above a transition energy s_1) yield for each pole (see for example Appendix of Ref. 23) a form for $\mathrm{Re}\chi$

$$\frac{1}{\pi}\int_{s_1}^{\infty}\frac{ds'}{s'-s}\beta(t)(s'/s_0)^{\alpha(t)}=\beta(t)\{-\pi\cot[\pi\alpha(t)](s/s_0)^{\alpha(t)} +(s/s_1)^{-1-\alpha(t)}\,{}_1F_1[1,1+\alpha(t);2+\alpha(t);(s/s_1)^{-1}]\} \tag{11}$$

[43] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).
[44] T. Binford and B. R. Desai, Phys. Rev. **138**, B1167 (1965).
[45] G. Tiktopolous, Phys. Rev. **133**, B1231 (1964).

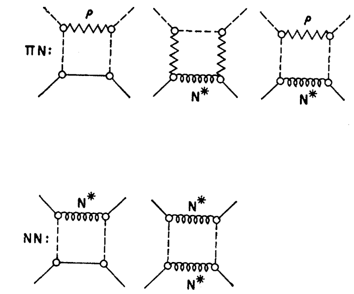

FIG. 6. Typical single- and double-resonance contributions to V^0.

which [when combined with crossing symmetry and the charge-conjugation properties of the pole] reproduces for $\alpha>0$ and $(s/s_1)\to\infty$ the usual signature factor times $(s/s_0)^{\alpha(t)}$, with the t-dependent residues β appearing as an over-all factor. However, at nonasymptotic energies, the phase of χ will be determined not only by (11) but also by the single- and double-resonance contributions; and in any case, χ becomes real below the first inelastic threshold.

In practice, one expects $\mathrm{Im}\chi\gg\mathrm{Re}\chi$, and that the box diagrams of Fig. 6, or similar inelastic diagram estimates as calculated by Amaldi and co-workers[46] will be a reasonable model for the moderate-energy region, augmented by Regge poles.

It is instructive to consider again at this point the possibility of using V^0 on the mass shell, $V^0(s,t)$, as an approximation to the CF potential. If only Regge poles are retained in $\mathrm{Im}V^0$, with a "strip boundary" s_1 chosen very high, then each Regge-pole term V_n in $V^0(s,t)$ evaluated at low energies ($s\ll s_1$) [cf. Eq. (11)] has the form

$$V_n{}^0(s,t)=\beta_n(t)\{-\pi\cot[\pi\alpha_n(t)](s/s_0)^{\alpha_n(t)} +(s/s_1)^{-1-\alpha_n(t)}\}\pm(\text{crossed term}). \tag{12}$$

Now if $\alpha_n(t)$ is well approximated by a straight line with a small slope for $-4k^2<t<\mu^2$, where μ is the physical mass of the lowest physical particle or resonance lying on the trajectory [$\alpha_n(\mu^2)=m\pi$ with $m=0$, 1, or 2] for odd C such as the pion one obtains

$$V_n{}^0(s,t)\cong\frac{\beta_n(t)/\alpha_n'(0)}{\mu^2-t}\left(\frac{s}{s_0}\right)^{\alpha_n(0)} +(\text{terms nonsingular at } t=\mu^2). \tag{13}$$

Then if $\beta_n(t)$ is slowly varying and μ^2 is small, only the first term is important and $V_n{}^0(s,t)$ is of the pole form used in analyzing low-energy scattering and bound-state data in terms of one-boson-exchange potentials.[47,48] The energy dependence does not correspond to elementary-particle exchange unless $\alpha_n(t)$ is a very

[46] U. Amaldi and F. Selleri, Phys. Rev. **128**, 2772 (1962); U. Amaldi, R. Biancastelli, and S. Francariglia, report at the Oxford International Conference on Elementary Particles, 1965 (unpublished).
[47] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965).
[48] R. A. Bryan and B. Scott, Phys. Rev. **135**, B434 (1964).

flat trajectory so that $\alpha(0)=\alpha(\mu^2)$, but since such applications[47,48] ignore inelastic channels they cannot give a decisive test of energy dependence of the potential.

It can be roughly said, then, that the same pole terms in V^0 can be applied both to sufficiently high energies (using the eikonal method) and to sufficiently low energies (which do not probe the energy dependence) with qualitative success.

2. Spinless Discussion of High-Energy pp Scattering

Since the forward diffraction peak in pp scattering appears to shrink with increasing energy, at least between 3 and 15 BeV/c, this process is a good candidate for a direct fit with a few Regge poles. From an *a priori* viewpoint, poles belonging to all known nonstrange mesons can contribute to pp (and $\bar{p}p$) scattering. The Pomeranchuk pole (P) alone is found to be not adequate to account for the energy dependence of the pp total cross section, and some secondary poles (at least P' and ω) are required to give a satisfactory description.[44] However, for a qualitative discussion only P is considered here, with very small shrinkage, which presumably is consistent with data above 10 BeV/c, if only $(-t)<0.30$ BeV^2/c^2 is considered.[44]

With this assumption concerning the high-energy form of V^0, the pp Born approximation for $f(s,t)$ is

$$f_p(s,t)=s^{-1/2}\beta(t)\frac{1+e^{-i\pi\alpha(t)}}{2\sin[\pi\alpha(t)]}\left(\frac{s}{s_0}\right)^{\alpha(t)}. \qquad (14)$$

With the usual assumption for pole trajectories and residues at small $|t|$, $\alpha(t)\cong 1+t\alpha'$ and $\beta(t)\cong\beta(0)$; s_0 is taken as a free parameter to fit the data. Now f_p for small $|t|$ can be roughly approximated by an imaginary exponential in t, and χ_p (the FB transform of f_p) is an imaginary Gaussian in b to the same rough approximation, with amplitude independent of energy in the high-energy limit. The real part of the amplitude vanishes if the eikonal is purely imaginary. Experimentally, the real part is known to be small compared to the imaginary part of the amplitude in the forward direction.[49] It is consistent, then, to assume that the most important part of χ is purely imaginary. Later the real part at $t=0$ due to V_p will be estimated by a DWBA formula.

The large-$|t|$ scattering amplitude will be determined by the small-b behavior of χ, which in turn is essentially determined by the large-$|t|$ behavior of f_B. To estimate this, assume $\alpha(t)$ approaches a definite limit $\alpha(\infty)$ as $t\to-\infty$. Then the energy dependence of the small-b part of the eikonal will be $s^{\alpha(\infty)-1}$ and the corresponding phase will be $(\pi/2)\alpha(\infty)$; if $\alpha(\infty)$ is not far from $+1$, this part of the eikonal will be slowly varying with energy and predominantly imaginary, as in the large-b contribution.

In this discussion we assume that the important contributions to the integral over t [yielding $\chi(b)$] come from a limited range of t, for any s. Thus we are extracting essentially a fixed-t, $s\to\infty$ limit of the potential, corresponding to contributions such that $-t/s\to 0$. If χ were (contrary to such assumptions) to depend essentially on the behavior of V_p for $-t/s$ nonzero, there might be serious doubts concerning the association of Regge-pole terms with the asymptotic behavior of the multipheripheral diagrams.

The small-b dependence of χ then depends on the large-$|t|$ behavior of $\beta(t)$. It is believed[50] that $\beta(t)$ satisfies (at least for the leading trajectories) a dispersion relation with singularities only for positive t:

$$\beta(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{dt'}{t'-t}\,\mathrm{Im}\beta(t'). \qquad (15)$$

A plausible behavior of β for large $(-t)$ then would be

$$\beta(t)\cong a(t_0-t)^{-1}, \qquad (16)$$

where a and t_0 can, in principle, be determined from the dispersion relation (15).

Given (16) and the above assumptions on $\alpha(\infty)$, the Born approximation f_B is the same as for an imaginary Yukawa potential in the Klein-Gordon equation, as proposed by Serber as a good phenomenological description of large-momentum-transfer pp scattering.[4] (Recall that the eikonal approximation is a small-angle method, but allows large $|t|$ if s is large.)

The Pomeranchuk pole (or other dominant singularity at $J=1$) is therefore in the present scheme a replacement for the phenomenological potential of Serber which fits both small- and large-momentum-transfer, high-energy pp scattering. Note however, that the large-momentum-transfer scattering amplitudes with a pole-dominated eikonal will have a behavor $(-t)^{-\rho(s)}$; in Serber's model,[4] ρ was a fixed power, but the present scheme gives a slowly varying $\rho(s)$ which varies as $s^{\alpha(\infty)-1}$. As long as $\alpha(\infty)$ is chosen close enough to $+1$, however, Serber's fits can be reproduced. In fact, energy dependence of ρ is necessary to obtain a better fit than Serber's, as pointed out by Krisch.[14]

In such a picture, it is apparent that $\rho(s)$ is not universal; the power law is different for different reactions. Thus it is not surprising that πp large-$(-t)$ scattering[51] follows a different power law than pp scattering. (Large $-t$ means here center-of-mass scattering angles of 20°–30° at high energies, not the backward scattering, which in πp reactions must utilize a different model such as baryon exchange.)

3. A Model for High- and Moderate-Energy $\bar{p}p$ Scattering

The $p\bar{p}$ problem at nonasymptotic energies is typical of the cases not dominated by poles. As in the qualita-

[49] K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 14, 862 (1965).

[50] H. Cheng and D. Sharp, Phys. Rev. 132, 1854 (1963).

[51] J. Orear, R. Rubinstein, D. B. Scarl, D. H. White, A. D. Krisch, W. Frisken, A. L. Read, and H. Ruderman, Phys. Rev. Letters 15, 309 (1965).

tive pp treatment, assume a spinless formalism. The energy range from 2 to 10 BeV/c will be considered. The annihilation cross sections here are equal in importance to those of production without annihilation,[42] and the latter are comparable to the corresponding pp reactions. A model for $V_{\bar{p}}$ should, therefore, include terms corresponding to the pp potential (V_p) and in addition comparable terms (V_A) obtained from a model of the annihilation reactions. Since strange-particle annihilation cross sections are smaller than those for only pions, only the latter will be considered.

In terms of the eikonals, the elastic-scattering amplitudes will be written

$$A(s,t)=ikW\int_0^\infty bdb\, J_0[b(-t)^{1/2}][1-e^{i\chi_p(s,b^2)}] \quad (17)$$

for pp, and

$$\bar{A}(s,t)=ikW\int_0^\infty bdb\, J_0[b(-t)^{1/2}] \times[1-e^{i\chi_p(s,b^2)+i\chi_A(s,b^2)}] \quad (18)$$

for $\bar{p}p$, where $W^2=s$, and covariant normalization is implied:

$$d\sigma/d\Omega=s^{-1}|A(s,t)|^2.$$

If a Born approximation (using Regge poles for A) is reasonable for pp scattering with only even-signature poles[52] (P only, as indicated above, as a special case), the poles in pp and $\bar{p}p$ will be the same, but in $\bar{A}$ there will be "absorptive corrections" from the extra factor $\exp(i\chi_A)$. If χ_A dominates over χ_p, the poles will be concealed, and the main features of $p\bar{p}$ scattering in this energy region will be determined by χ_A.

For simplicity, assume that χ_A is purely imaginary, and hence can be computed directly from annihilation amplitudes at the same energy as elastic scattering is measured (no dispersion integral required). Then χ_A is

$$\chi_A(s,b^2)\cong\frac{i}{kW}\sum_\gamma H_\gamma(s,b^2)\rho_\gamma(s)H_\gamma{}^*(s,b^2), \quad (19)$$

where $H_\gamma(s,b^2)$ is the FB component (b) of the amplitude $T_\gamma(s)$ for $p\bar{p}$ annihilation into multipion state γ (here γ refers to all nonexplicit variables which describe such a state) and $\rho_\gamma(s)$ is the phase-space factor for the pion state. In (19) we have used the "b-diagonal" approximation to the unitarity condition [following Blankenbecler and Goldberger,[34] and Baker and Blankenbecler[53]; see also Ref. 10] where $l+\frac{1}{2}$ is identified with kb.

To estimate χ_A, then, a model for the most important H's is necessary. Experimental data indicate that the pion directions in annihilations are far from isotropically distributed in the center-of-mass system; a purely statistical model thus seems inappropriate. At the other extreme, it would be convenient if most of the 4- and 5-pion states (which are the most important of annihilations) were accounted for by double-resonance producduction (e.g., ρ,ω) but this is not true.[42] The only familiar kind of dynamical model consistent with the data would seem to be a form of multiperipheral model with baryonic states forming the legs of the ladder, as indicated in Fig. 7.

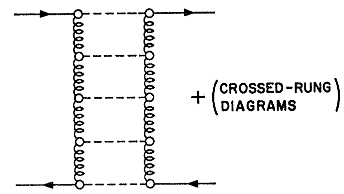

FIG. 7. Baryonic multiperipheral model for $p\bar{p}$ annihilation contributions to V^0.

Before proceeding, it is necessary again to raise the question of duplicating graphs. The graphs of Fig. 7 might, in principle, be included in Regge poles of χ_p since any quasi-two-body, baryon-number-zero states in the t channel will contribute to structure of exchanged-meson states (at least in a bootstrap[54] picture). (The πp, pp, etc. moderate-energy inelasticities do not raise this question since they are dominated by single- or double-resonance production.) In fact, however, the graphs of Fig. 7 cannot contribute to χ_p, and hence cannot contribute to any (even-signature) poles in χ_p. Thus no duplication is involved if χ_A is computed from the graphs of Fig. 7.

Now the model represented by Fig. 7 must be made more precise. Since multiparticle phase space is difficult to handle, the order of magnitude of these diagrams will be estimated by replacing them with a diagram such as Fig. 8, in which all pions are grouped into two "fireballs," Again for simplicity, these groups are considered to have zero angular momentum in their individual center-of-mass systems. Then the box diagrams indicated can be calculated, if the natures of the baryonic exchanges are specified.

Elementary-particle perturbation theory would suggest nucleon exchanges, with a t dependence given by $(M^2-t)^{-1}$. However, this would yield an effective radius for $p\bar{p}$ scattering of order $(2M)^{-1}$, which is much too small. Further, such a weak t dependence would not explain the degree of charged-pion angular anistropy found[42] in the annihilations above 3 BeV/c.

In accordance with the general picture of composite particles, however, the required t dependence can be achieved simply by considering the baryonic exchanges in Fig. 8 as Regge poles, with trajectories and residues that drop off with increasing $(-t)$. The order of magnitude of these t variations may be estimated theoretically by assigning an effective radius of order $(2\mu_\pi)^{-1}$ to the

[52] D. H. Sharp and W. G. Wagner, Phys. Rev. **131**, 2226 (1963); I. J. Muzinich, *ibid.* **130**, 1571 (1963).
[53] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).
[54] R. C. Arnold, Nuovo Cimento **37**, 589 (1965); J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966); Y. Hara, *ibid.* **133**, B1565 (1964).

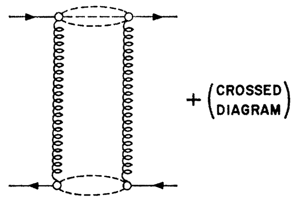

FIG. 8. Rough approximation for graphs of Fig. 7.

composite state (baryon or baryonic resonance), or empirically by observing the backward peak width in high-energy πp scattering[55] (presumably dominated by such baryon Regge poles). Both considerations lead to the expectation that the t dependence of each baryonic exchange will be quite similar to that observed in forward (meson-exchange) processes, such as small-angle elastic scattering, when mesonic Regge poles dominate (excluding the one-pion-exchange case).

To a rough approximation consider then an exponential behavior for the t dependence of each half (T_n) of the diagrams in Fig. 8 (t is the 4-momentum transfer between an incoming nucleon and the associated "fireball"):

$$T_n(s,t)\cong T_n(s,0)e^{R_n^2t/2}, \qquad (20)$$

where each R_n is comparable to the value for forward elastic scattering; R_n may vary slowly with s, but this can be ignored in the rough approximation. Here n refers to the pion population of any particular "fireball." The s dependence of T_n, extracted above as $T_n(s,0)$, will be $(s/s_0)^{\alpha_m(0)}$ where $\alpha_m(\alpha_{\max})$ is the leading baryonic trajectory at $t=0$; the s_0 is an appropriate scale factor.

The FB transform of (20) then is

$$H_n(s,b^2)\cong T_n(s_0,0)(s/s_0)^{\alpha_m(0)}e^{-b^2/2R^2_n}. \qquad (21)$$

With a two-particle phase-space factor $(2/k_nW)$ appropriate for the given normalization and an impact-parameter representation, χ_A then becomes

$$\chi_A(s,b^2)\cong\frac{2i(s/s_0)^{2\alpha_m(0)}}{kW^2}\sum_n k_n^{-1}e^{-b^2R^2_n}|T_n(s_0,0)|^2. \qquad (22)$$

For simplicity again, assume that a single (n) contribution dominates, and make the rough approximation $k\approx k_n$. One obtains the Guassian form

$$\chi_A(s,b^2)\cong i\lambda k^{-2}(s/s_0)^{2\alpha_m(0)-1}e^{-b^2/R^2}, \qquad (23)$$

where λ is a free (positive) parameter; R, s_0, and $\alpha_m(0)$ are also undetermined, but the former two must be of the same order of magnitude as seen in all forward elastic-scattering reactions at high energy, and the latter can in principle be determined from observation of the energy dependence of backward πp scattering. It is estimated[56] that $\alpha_{\max}(0)\approx 0.15$ in backward $\pi^- p$ scattering, which presumably is dominated by a pole associated with $N_{3/2}^*(1238)$. (Since the nucleon pole presumably has a smaller $\alpha(0)$, the former pole is expected to dominate at sufficiently high energy whenever baryonic exchange is considered.) The exact value of $\alpha_{\max}(0)$ does not affect the qualitative conclusions below.

With $\alpha_m(0)\cong 0.15$, χ_A from (23) is a Gaussian in b, purely imaginary (by hypothesis), with asymptotic energy dependence $k^{-3.4}$. Thus χ_A will be dominated by χ_p for sufficiently large s, but may be very large in magnitude in the low end of the energy range under consideration. From (18), the $p\bar{p}$ scattering amplitude in the region where χ_A dominates χ_p is given now by

$$\bar{A}(s,t)\cong ikW\int_0^\infty bdb\, J_0[b(-t)^{1/2}] \times\{1-\exp[-\lambda k^{-2}(s/s_0)^{+\eta}e^{-b^2/R^2}]\}, \qquad (24)$$

where $\eta=2\alpha_m(0)-1\cong -0.7$.

Now the two most striking qualitative features of $p\bar{p}$ elastic scattering[57] can be obtained from the result (24) applied between 2 and 10 BeV/c:

(A) Drop of $(\sigma_{\rm el}/\sigma_{\rm tot})$ from near 0.50 at low energies to a small value (approaching the pp value) above 10 BeV/c; and (B) expansion of diffraction peak width (decrease in effective radius) over the same interval by an appreciable factor (again approaching pp value above 10 BeV/c). [The integrals can, of course, be done numerically but it is sufficient for the purposes of this paper to give a qualitative discussion, especially since (23) is to be considered only a rough guide to the actual physical situation.]

For large k, expansion of the integrand to first order in λ (or k^{-2}) yields for $\bar{A}$ an exponential in t with characteristic radius $R/2$,with amplitude decreasing with energy and proportional to λ. Such a radius is smaller than the pp case (because it is assumed R is given roughly by the pp result). The appropriate eikonal for asymptotically high energies is χ_p, however; so (if λ is not too large) the characteristic radius will actually become the pp radius, as will also $(\sigma_{\rm el}/\sigma_{\rm tot})$.

At the other extreme, consider small k. The value of the factor in braces { } in Eq. (24) will be essentially unity for $b\lesssim R$, and will drop rapidly to zero (with a Gaussian tail) for $b\gg R$. The thickness of the transition region will become small compared to R as k decreases, and at the same time the half-maximum point will expand to larger b values as k decreases. The distribution of optical opacity then resembles a black disk for small k, with a radius$>R$. In such a case one obtains $\sigma_{\rm el}/\sigma_{\rm tot}$

[55] ABBBHLM Collaboration, Phys. Letters **10**, 248 (1964); W. Frisken, A. L. Read, H. Ruderman, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and D. H. White, Phys. Rev. Letters **15**, 313 (1965); C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, *ibid.* **15**, 838 (1965). H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Shochet, and R. VanBerg, *ibid.* **16**, 828 (1966).

[56] G. F. Chew and J. D. Stack, University of California Radiation Laboratory Report No. UCRL-16293 (unpublished); see also J. D. Stack, Phys. Rev. Letters **16**, 286 (1966); V. Barger and D. Cline, *ibid.* **16**, 913 (1966).

[57] O. Czyzewski, B. Escoubes, Y. Goldschmidt-Clermont, M. Guinea-Moorhead, D. R. O. Morrison, and S. de Unamuno-Escoubes, Phys. Letters **15**, 188 (1965).

=0.50, and an effective radius (larger than the pp case) which decerases with increasing energy. Explicit calculations confirm that reasonable fits to the data[57] can be achieved with λ of order 5–10 $(\mathrm{BeV}/c)^2$, with R taken from pp scattering.

V. SPIN-$\frac{1}{2}$–SPIN-0 SCATTERING

1. Eikonal Representation for Helicity Amplitudes

For a description of meson-nucleon scattering the nucleon spin must be taken into account. It might be thought that spin effects are unimportant at high energies, but appreciable polarization has been observed in πp scattering[58] in the range of energies presumably capable of an eikonal description. Thus a successful theory must be capable of predicting this polarization, even if the contribution of the spin-flip amplitudes to the cross section is small, as it can be, and still yield appreciable polarization.[59]

The nonrelativistic treatment of problems with pins with an eikonal formalism has been discussed by Glauber,[21] but a treatment of the relativistic case has not been presented (to the author's knowledge) in the literature. For spin-$\frac{1}{2}$–spin-0 scattering, there seems to be no obvious difficulty in generalizing the spinless formulation presented in Sec. II. However, when higher spin cases are considered there are difficulties in principle connected with coupling different l values. These have been mentioned by Glauber[21] and apparently have been sidered by Amaldi and co-workers[46] in connection with nucleon-nucleon absorptive correction formulas. These problems will not be considered here.

The formulation of the eikonal approximation in Sec. II can be generalized to meson-baryon (πN) scattering because there is still a single phase shift for each J value, and the eikonal is (roughly speaking) obtained by using the Born approximation to the phase shift when many partial waves are important and there are no important resonances. The specific formulation in this case begins with the relativisitic πN scattering amplitude representation in terms of partial-wave amplitudes[60]:

$$f_1(s,t)=\sum_{l=0}^{\infty} f_{l+}(s)P_{l+1}'(z)-\sum_{l=2}^{\infty} f_{l-}(s)P_{l-1}'(z)\,, \quad (25a)$$

$$f_2(s,t)=\sum_{l=1}^{\infty} (f_{l-}-f_{l+})P_l'(z)\,, \quad (25b)$$

where $z=\cos\theta$, $t=-2k^2(1-\cos\theta)$; the normalization is defined by

$$\frac{d\sigma}{d\Omega}=|f_1+f_2\cos\theta|^2+|f_2|^2\sin^2\theta.$$

The partial-wave amplitudes $f_{l\pm}$ are related to the phase shifts $\delta_{l\pm}$ by

$$f_{l\pm}=(e^{2i\delta_{l\pm}}-1)/2ik. \quad (26)$$

Assuming that a large number of partial waves contribute, an impact-parameter respresentation is desired. The choice is not unique,[61] but there is one choice for which the helicity nonflip amplitude is simplest, and this amplitude should be most important at high energies. It is most convenient to express the expansions (25) in terms of rotation matrices $d_{\mu\nu}{}^J(\theta)$ to obtain such a representation. This, in turn, is easiset to express by introducing helicity nonflip and helicity flip amplitudes[62] G_+ and G_-; the normalization is chosen such that

$$\frac{d\sigma}{d\Omega}=s^{-1}[|G_+|^2+|G_-|^2], \quad (27)$$

and the nucleon polarization $P(\theta)$ is given by

$$P(\theta)=2\,\mathrm{Im}(G_+{}^*G_-)/[|G_+|^2+|G_-|^2]. \quad (28)$$

The angular momentum expansions[62] of these amplitudes can be written

$$G_+(s,t)=\sum_J (J+\tfrac{1}{2})g_+{}^J(s)d_{1/2,1/2}{}^J(z)\,, \quad (29a)$$

$$G_-(s,t)=\sum_J (J+\tfrac{1}{2})g_-{}^J(s)d_{1/2,-1/2}{}^J(z)\,. \quad (29b)$$

The explicit relationship between the $G_\pm$ and f_1, f_2 is

$$G_+=W(f_1+f_2)\cos(\theta/2)\,, \quad (30a)$$

$$G_-=W(f_1-f_2)\sin(\theta/2)\,. \quad (30b)$$

Comparing (29), (30), (25), and (26) one obtains expressions for the $g_\pm{}^J$ in terms of phase shifts:

$$g_+{}^J=W[f_{l+}+f_{(l+1)-}]=(i\rho)^{-1}[e^{2i\delta_{l+}}+e^{2i\delta_{(l+1)-}}-2]\,, \quad (31a)$$

$$g_-{}^J=W[f_{l+}-f_{(l+1)-}]=(i\rho)^{-1}[e^{2i\delta_{l+}}-e^{2i\delta_{(l+1)-}}]\,, \quad (31b)$$

where $\rho=2k/W$, and $l\equiv J-\frac{1}{2}$ here.

The expansions (29) now are replaced by integrals over $b\equiv J/k$ and the d^J functions are replaced by Bessel functions to which they correspond in the limit $J\rightarrow\infty$ (see Appendix of Durand and Chiu, Ref. 7). At the same time, the phase shifts $\delta_{l\pm}$ are replaced by continuous functions of b^2, $\chi_\pm(s,b^2)$, such that for large l, $\chi_\pm[s,[(l+\frac{1}{2})/k]^2]\rightarrow\delta_{l\pm}(s)$.

It is convenient to define "flip" and "nonflip" χ functions by

$$\chi_f=(\chi_+-\chi_-)/2\,, \quad (32a)$$

$$\chi_0=(\chi_++\chi_-)/2. \quad (32b)$$

[58] S. Suwa, A. Yokosawa, N. E. Booth, R. J. Esterling, and R. E. Hill, Phys. Rev. Letters **15**, 560 (1965); Enrico Fermi Institute Report (unpublished).

[59] S. Fernbach, W. Heckrotte, and J. V. Lepore, Phys. Rev. **97**, 1059 (1955).

[60] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

[61] E. Predazzi, Ann. Phys. (N.Y.) **36**, 228 (1965); **36**, 250 (1965).

[62] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959); see also M. Jacob, in *Strong Interaction Processes*, edited by M. Jacob and G. F. Chew (W. A. Benjamin, Inc., New York, 1965).

In terms of χ_0 and χ_f, the resulting integral representations are

$$G_+(s,t)=ik^2\cos(\theta/2)\int_0^\infty bdb\,J_0[b(-t)^{1/2}]\times[1-e^{i\chi_0}\cos\chi_f],\quad(33a)$$

$$G_-(s,t)=k^2\int_0^\infty bdb\,J_1[b(-t)^{1/2}][e^{i\chi_0}\sin\chi_f].\quad(33b)$$

The expressions (33) give an exact representation, defining the χ's. The eikonal approximation, by analogy with the spinless case, now can be obtained by equating the Born approximations for G_+ and G_- to the first-order terms in the expansion of the integrands in powers of $\chi_\pm$ (or χ_0 and χ_f). The result is

$$G_+^B(s,t)\cong k^2\cos(\theta/2)\int_0^\infty bdb\,J_0[b(-t)^{1/2}]\chi_0(s,b^2),\quad(34a)$$

$$G_-^B(s,t)\cong k^2\int_0^\infty bdb\,J_1[b(-t)^{1/2}]\chi_f(s,b^2).\quad(34b)$$

The inverse Fourier-Bessel transforms then yield χ_0 and χ_f, which can be written in terms of the Born approximations to f_1 and f_2 as

$$\chi_0(s,b^2)=\left(\frac{W}{k^2}\right)\int_0^\infty xdx\,J_0(xb)\times[f_1^B(s,-x^2)+f_2^B(s,-x^2)]\quad(35a)$$

and

$$\chi_f(s,b^2)=\left(\frac{W}{k^2}\right)\int_0^\infty xdx\,J_1(xb)\sin(\theta/2)\times[f_1^B(s,-x^2)-f_2^B(s,-x^2)],$$

where $x=(-t)^{1/2}=2k\sin(\theta/2)$; the second equation can also be written

$$\chi_f(s,b^2)=\left(\frac{W}{2k^3}\right)\int_0^\infty x^2dx\,J_1(xb)\times[f_1^B(s,-x^2)-f_2^B(s,-x^2)].\quad(35b)$$

These equations define then the eikonal approximation for πN scattering. Further discussion of qualitative features will require specific assumptions regarding the potentials.

2. Covariant Born Approximations, Helicity Flip Amplitudes, and Spin-Orbit Coupling

A description of the Born approximations in terms of the Mandelstam (singularity-free) amplitudes A and B is convenient for a discussion of specific physical models. From Ref. 60,

$$f_1(s,t)=\left(\frac{E+M}{8\pi W}\right)[A(s,t)+(W-M)B(s,t)],\quad(36a)$$

$$f_2(s,t)=\left(\frac{E-M}{8\pi W}\right)[-A(s,t)+(W+M)B(s,t)],\quad(36b)$$

where E is the center-of-mass energy of the nucleon, and M and μ are the nucleon and meson masses.

The corresponding representation of the πN scattering amplitude in terms of the Dirac matrices is[60]

$$T=-A+\tfrac{1}{2}\gamma\cdot(q_1+q_2)B,\quad(37)$$

where q_1, q_2 are the 4-momentum vectors of the incoming and outgoing nucleons. In the Born approximation for field-theoretic models including only nucleon poles, or exchanged vector mesons without direct anomalous-moment couplings, only B is nonzero. In an S-matrix approach, crossing relations for A and B are utilized to express their Born-approximation values in terms of poles associated with particles and resonances in crossed channels.

In particular, when only a single Regge pole is included, the amplitudes A and B have asymptotic energy dependence[63]

$$A(s,t)\approx F_A(t)(s/s_0)^{\alpha(t)},\quad(38a)$$

$$B(s,t)\approx F_B(t)(s/s_0)^{\alpha(t)-1},\quad(38b)$$

where F_A and F_B are products of signature and residue functions. As a consequence, the Born approximations for helicity amplitudes have asymptotic energy dependence

$$G_+^B\approx[F_A^{(1)}(t)(s/s_0)^{\alpha(t)}+F_B^{(1)}(t)(s/s_0)^{\alpha(t)}]\cos(\theta/2),\quad(39a)$$

and

$$G_-^B\approx[F_A^{(2)}(t)(s/s_0)^{\alpha(t)+1/2}+F_B^{(2)}(t)(s/s_0)^{\alpha(t)-1/2}]\sin(\theta/2),$$

where $F_A^{(1)}$, $F_A^{(2)}$, $(F_B^{(1)}, F_B^{(2)})$ are linear in $F_A(F_B)$, and contain $\alpha(t)$ and constants. The second expression can be written

$$G_-^B\approx[F_A^{(3)}(t)(s/s_0)^{\alpha(t)}+F_B^{(3)}(t)(s/s_0)^{\alpha(t)-1}](-t)^{1/2}\quad(39b)$$

using $(-t)^{1/2}=2k\sin(\theta/2)$, where $F_A^{(3)}$ $(F_B^{(3)})$ is again linear in F_A (F_B) and contains $\alpha(t)$ and constants. The factor $\cos(\theta/2)$ in (39a) is quantitatively unimportant since only small values of θ are to be considered.

Now from (27), if $F_A(t)\neq0$, the cross section in Born approximation has a contribution to its asymptotic en-

[63] V. Singh, Phys. Rev. **129**, 1889 (1963).

ergy dependence contributed by G_- for $t\neq 0$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{G_-}\approx(-t)s^{-1}|F_A{}^{(3)}(t)|^2(s/s_0)^{2\alpha(t)} \tag{40}$$

while if $F_A(t)=0$, or at $t=0$, one obtains only the G_+ contribution:

$$\left(\frac{d\sigma}{d\Omega}\right)_{G_+}\approx s^{-1}\{|F_A{}^{(1)}(t)+F_B{}^{(1)}(t)|^2\}(s/s_0)^{2\alpha(t)}. \tag{41}$$

Assume, for the following argument, that these asymptotic Born terms are a good qualitative guide. From general restrictions based on unitarity, $d\sigma/d\Omega$ at $t=0$ should not grow faster than s^1 (aside from logarithmic factors). This limit is satisfied by the Pomeranchuk pole, $\alpha(0)=1$; other poles have smaller intercepts.

Experimental elastic-scattering data, at least on high-energy πp elastic scattering, show that the asymptotic cross section for $t\neq 0$ (compared to the cross section at $t=0$) gives no indication of a rise away from the forward direction as indicated by (40), after the region of energy containing important resonance contributions is excluded. This is a strong indication that for each pole, either $F_A=0$, or $\alpha(0)<\frac{1}{2}$.

In πp scattering, P' and ρ poles are required for a good fit to the data.[43,44] (These three are the only trajectories known to exhibit physical resonances in the t channel whose quantum numbers are consistent with their exchange in πp forward scattering.) The ρ pole residues can be determined by analysis of charge exchange; assuming an uncorrected ρ pole, it is found[64] that F_A is quite large. This is consistent with the above conclusions if $\alpha_\rho(0)<0.50$. Detailed analyses[64] indicate $\alpha_\rho(0)\cong 0.60$, but this may be affected if absorptive corrections are included; in any case the use of $\alpha_\rho(0)=0.50$ still provides a reasonably good fit.[65]

In other reactions (e.g., KN scattering) the ω trajectory (as well as others) is allowed, but it is believed that $F_A\cong 0$ in this case on the basis of isoscalar nucleon form factor analysis. Thus $\alpha_\omega(0)$ is not restricted to be less than $\frac{1}{2}$.

Returning to πp scattering at high energies, consider the P contribution to χ_0 and χ_f. Since $F_A=0$, there is a relation determining χ_f in terms of χ_0. It will be shown below that this relation is exactly the same as in the case of a static central potential in the Dirac equation,[27] in the high-energy approximation $W\gg M$.

Let $B(s,t)$ now represent the Born approximation for B, and take the Born approximation for A to be zero. From (36),

$$(f_1{}^B+f_2{}^B)=(E-M^2/W)(B/4\pi), \tag{42a}$$

$$(f_1{}^B-f_2{}^B)=M(1-E/W)(B/4\pi). \tag{42b}$$

[64] G. Höhler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966).
[65] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965).

This yields, from (35), in the eikonal approximation

$$4\pi\chi_0(s,b^2)=k^{-2}(EW-M^2)\int_0^\infty xdx\,J_0(xb)B(s,-x^2), \tag{43a}$$

$$4\pi\chi_f(s,b^2)=(2k)^{-2}M(W-E)\int_0^\infty x^2dx\,J_1(xb)B(s,-x^2).$$

In the limit $W\gg M$, these become

$$4\pi\chi_0(s,b^2)\cong 2\int_0^\infty xdx\,J_0(xb)B(s,-x^2), \tag{44a}$$

$$4\pi\chi_f(s,b^2)\cong\left(\frac{MW}{4k^2}\right)\int_0^\infty x^2dx\,J_1(xb)B(s,-x^2). \tag{44b}$$

Now consider the static-potential theory problem where the nonflip eikonal χ_0 is derived from a central potential $V_c(r)$ through the expression[21]

$$\chi_0(b^2)=\int_{-\infty}^{+\infty}dz\,V_c[(z^2+b^2)^{1/2}]. \tag{45}$$

The explicit energy dependence of the potentials (and eikonals) is suppressed. Define the Fourier transform $\tilde{V}_c$ by

$$\tilde{V}_c(\Delta)=2\int_0^\infty r^2dr\,V_c(r)\left[\frac{\sin(\Delta r)}{\Delta r}\right], \tag{46}$$

where $\Delta=(-t)^{1/2}$. This is proportional to the Born approximation for non-spin flip amplitude. Then χ_0 can be obtained from $\tilde{V}_c$ by inverting (46) and substituting into (45):

$$\chi_0(b^2)=\int_0^\infty \Delta d\Delta\,J_0(b\Delta)\tilde{V}_c(\Delta). \tag{47}$$

Thus

$$\tilde{V}_c(x)=B(-x^2)/2\pi. \tag{48}$$

Equation (44b) then becomes

$$\chi_f(b^2)=\left(\frac{MW}{4k^2}\right)\int_0^\infty xdx\,\tilde{V}_c(x)[xJ_1(bx)]. \tag{49}$$

Using

$$\frac{d}{dz}[zJ_1(z)]=zJ_0(z),$$

this can be written

$$\chi_f(b^2)=-\left(\frac{MW}{4k^2}\right)b\int_0^\infty xdx\,\tilde{V}_c(x)\left[\int_0^x dy\,yJ_0(by)\right]. \tag{50}$$

Integrating by parts, and assuming $\tilde{V}_c$ drops off rapidly as $x\rightarrow\infty$, one obtains

$$\chi_f(b^2)=-\left(\frac{MW}{4k^2}\right)b\int_0^\infty xdx\,J_0(bx)\int_0^x ydy\,\tilde{V}_c(y). \tag{51}$$

Now

$$\int_0^x ydy\ \tilde{V}_c(y)=2\int_0^\infty rdr\ V_c(r)\int_0^x dy\ \sin(yr)$$

$$=2\int_0^\infty dr\ V_c(r)[1-\cos(xr)]$$

$$=2\int_0^\infty dr\ V_c(r)\frac{d}{dr}\left[\frac{-\sin(xr)}{x}\right]$$

$$+(\text{terms independent of } x)$$

$$=2\int_0^\infty r^2dr\ \frac{\sin(xr)}{xr}\left[\frac{1}{r}\frac{d}{dr}V_c(r)\right]$$

$$+(\text{terms independent of } x).$$

As a consequence, (51) for $b\neq 0$ can be written

$$\chi_f(b^2)=\left(\frac{MW}{4k^2}\right)b\int_0^\infty xdx\ J_0(xb)\tilde{V}_s(x)\,, \qquad (52)$$

where $\tilde{V}_s(x)$ can be considered the Fourier transform of an effective spin-orbit potential $V_s(r)$, with

$$V_s(r)\equiv -\frac{1}{r}\frac{d}{dr}V_c(r)\,. \qquad (53)$$

Note that for a Gaussian V_c (as indicated by cross-section data), V_s has the same Gaussian shape.

The corresponding phase shifts, in a discrete angular momentum representation, will be given then [using (46)] by

$$\delta_{l+}-\delta_{l-}=\left[\frac{MW(l+\frac{1}{2})}{4k^3}\right]\int_{-\infty}^{+\infty} dz\ V_s[(z^2+b^2)^{1/2}]$$

$$\cong\frac{M(l+\frac{1}{2})}{2k^2}\int_{-\infty}^{+\infty} dz\ V_s[(z^2+b^2)^{1/2}] \qquad (54)$$

and

$$(\delta_{l+}+\delta_{l-})/2=\frac{1}{2}\int_{-\infty}^{+\infty} dz\ V_c[(z^2+b^2)^{1/2}]. \qquad (55)$$

These are, as stated above, just the linearized WKB approximations at high energies for the Dirac equation[27] with a potential $V_c(r)$, which again shows the physical content of the eikonal approximation for spin-$\frac{1}{2}$–spin-0 scattering.

It may be noted that the formula (52) is used in the Byers-Yang model[5] by assuming the relevance of an effective static potential as in nuclear physics. The essential results of that model concerning polarization in elastic scattering are then comprehensible within the framework of a B-type Born approximation, such as a Pomeranchuk-pole contribution, giving a Gaussian V_c and hence similar V_s. Phase conditions may "accidently" yield zero polarization in such a case, if only P is present.

A point which should be emphasized, however, is that since χ_f drops off relative to χ_0 like $s^{-1/2}$ at large s when $F_A=0$ as in the above work, secondary Regge poles (e.g., P', ρ) with $F_A\neq 0$ are equally important contributions to χ_f. A numerical estimate using fitted parameters for these poles[43] in fact indicates that their F_A terms dominate the induced helicity-flip contribution from P.

VI. ABSORPTIVE CORRECTION FORMULAS IN ELASTIC SCATTERING

As previously remarked, the helicity flip, charge exchange, and real parts of nonflip eikonal contributions are expected to be small compared to the imaginary part of the nonflip eikonal at high energies. Even for the latter, the nonlinear powers in the expansion of the exponential (higher Born approximations) contribute reasonably small corrections[30] for small $|t|$. Thus it should be a good approximation to retain only first-order terms in helicity flip, charge exchange, and real part contributions to eikonals, at least for small $|t|$. By charge exchange here is meant the difference between eikonals for scattering in different isospin states. A treatment of this in the spinless case has been sketched in a previous paper[30] and will not be repeated here; but the helicity flip formula will be derived, an application of the real part formula will be presented, and the polarization in charge exchange estimated, in what follows.

For the helicity flip case, πN scattering will be considered with χ_f as given by (35b), where $(f_1{}^B-f_2{}^B)$ is presumably given by the Pomeranchon spin-orbit term (see Sec. V) and ρ and P' poles (anomalous-moment coupling contributions). Then, to first order in χ_f, we have from (33)

$$G_+(s,t)\cong ik^2\rho^{-1}\cos(\theta/2)\int_0^\infty bdb\ J_0[b(-t)^{1/2}]$$

$$\times[1-e^{i\chi_0(s,b^2)}], \quad (56)$$

$$G_-(s,t)\cong k^2\rho^{-1}\int_0^\infty bdb\ J_1[b(-t)^{1/2}]e^{i\chi_0(s,b^2)}\chi_f(s,b^2)\,. \quad (57)$$

With the assumption that χ_f and $\mathrm{Re}\chi_0$ are small, $\mathrm{Im}G_+$ should dominate the cross section, and $\mathrm{Im}\chi_0$ thus can be deduced from the inverse Fourier-Bessel transform of the square root of $d\sigma/d\Omega$. With χ_0 estimated from experiment, the formula (57) takes the DWBA form, simplifying the comparison of pole models with experiment. Application of this formula will require comparison of polarization in π^+p and π^-p elastic scattering (or charge-exchange reactions) to separate P' and ρ contributions. An application of such a formula using elementary boson exchange has been given by Dosch and Fridman.[66]

[66] H. G. Dosch and A. Fridman, Nuovo Cimento 42, 1 (1966).

As an application of the real-part "absorptive correction" formula, consider pp scattering in a spinless model at asymptotic energies such that χ is dominated by the Pomeranchuk pole, formula (14). As in Sec. IV 2, assume that the small $|t|$ behavior is most important, and for an order-of-magnitude estimate take

$$\begin{aligned} f_p(s,t) &\cong f_p(s,0)(s/s_0)^{t\alpha'} e^{-i\pi t\alpha'/2} \\ &\cong ik\Gamma \exp\{t\alpha'[\ln(s/s_0)-i\pi/2]\}, \end{aligned} \quad (58)$$

where Γ is real and potitive. Let

$$\Lambda^2=[2\ln(s/s_0)-i\pi]\alpha'.$$

From (8),

$$\chi(s,b^2)\cong i\Gamma\int_0^\infty xdx\, J_0(xb)e^{-x^2\Lambda^2/2}=\frac{i\Gamma e^{-b^2/2\Lambda^2}}{\Lambda^2}. \quad (59)$$

For $\ln(s/s_0)\gg\pi/2$, $\mathrm{Im}\chi\gg|\mathrm{Re}\chi|$, and in (17) only first order in $\mathrm{Re}\chi$ need be retained. Then (17) becomes

$$A(s,t)=A_p(s,t)+k^2\int_0^\infty bdb\, J_0[b(-t)^{1/2}] \times e^{-\mathrm{Im}\chi(s,b^2)}\,\mathrm{Re}\chi(s,b^2) \quad (60)$$

with $\mathrm{Re}A_p(s,t)=0$;

$$A_p(s,t)=ik^2\int_0^\infty bdb\, J_0[b(-t)^{1/2}][1-e^{-\mathrm{Im}\chi(s,b^2)}]. \quad (61)$$

Now

$$\mathrm{Re}\chi(s,b^2)\cong-\Gamma\,\mathrm{Im}[\Lambda^{-2}\exp(-b^2/2\Lambda^2)].$$

Let $R^2=2\alpha'\ln(s/s_0)$; to first order in the imaginary part,

$$\Lambda^{-2}\cong R^{-2}[1+i\kappa],$$

where

$$\kappa=\pi/[2\ln(s/s_0)].$$

So

$$\mathrm{Re}\chi(s,b^2)\cong-\kappa[(b^2/2R^2)-1]\Gamma R^{-2}e^{-b^2/2R^2}.$$

From (60),

$$\mathrm{Re}A(s,t)=-\kappa\Gamma R^{-2}k^2\int_0^\infty bdb\, e^{-\mathrm{Im}\chi(s,b)^2}J_0[b(-t)^{1/2}] \times[(b^2/2R^2)-1]e^{-b^2/2R^2}. \quad (62)$$

Since the experimental $(d\sigma/d\Omega)$ is reasonably well fit by an exponential in t, with effective radius R corresponding to the R defined (63) above,

$$A_p(s,t)\cong ik^2C^2R^2e^{tR^2/2}$$

and

$$1-\exp[-\mathrm{Im}\chi(s,b^2)]\cong Ce^{-b^2/2R^2}, \quad (63)$$

where C is of order 0.90 in pp scattering. Using (63) instead of the *a priori* formula (59) implies a model only for the real part of χ, not the imaginary part. Substituting (63) into (62), the result is

$$\mathrm{Re}A(s,t)=-\kappa\Gamma R^{-2}\int_0^\infty bdb\,[1-Ce^{-b^2/2R^2}] \times J_0[b(-t)^{1/2}][(b^2/2R^2)-1]e^{-b^2/2R^2}. \quad (64)$$

In particular,

$$\mathrm{Re}A(s,0)=-C\kappa\Gamma R^{-2}k^2\int_0^\infty bdb\, e^{-b^2/2R^2}(1-b^2/2R^2). \quad (65)$$

The ratio of (65) to (61) at $t=0$ yields

$$\alpha\equiv\mathrm{Re}A(s,0)/\mathrm{Im}A(s,0)=-\kappa\Gamma/4. \quad (66)$$

Now

$$\Gamma\cong C \quad (67)$$

[expanding (61) to first order in Γ and comparing (63)]. With $C=0.9$, this yields

$$\alpha\cong-0.36/\ln(s/s_0). \quad (68)$$

A one-pole fit to the high-energy pp scattering gives an estimate for s_0 which is rather small since the peak does not shrink very rapidly. A reasonable choice is $s_0=0.50$ BeV^2; at 30 BeV/c the predicted value of α is then

$$\alpha(30)\cong-0.09.$$

This value is to be compared with the experimental value[49] of -0.33 ± 0.03 at 30 BeV/c. Clearly the magnitude is much too small. The slow energy dependence given by (68) is to be compared on the one hand with the experimental constancy[49] of α between 20 and 30 BeV/c, and on the other hand with power-law behavior expected from secondary poles in Born approximation.[67] The inadequacy of the latter has been discussed by Sakurai.[68]

Note that $\bar{p}p$ scattering at asymptotic energies would exhibit the same value of α, in contradistinction to models[68] wherein vector-meson exchanges are responsible for the real part; in such a case α changes sign. The $\pi^\pm p$ and $K^\pm p$ values of α would be comparable to the pp value at asymptotic energies. However, it should be noted that $\bar{p}p$ scattering up to 12 BeV/c cannot be fit by the Pomeranchon alone, and the conclusions regarding α should not be drawn until such energies are achieved that the $\bar{p}p$ diffraction peak and total cross section agree with pp. Similar precautions apply to πp and Kp values for α.

It appears, however, that the small magnitude of α predicted here rules out a simple (pole+absorption) picture of the real part coming from P.

The question of polarization in the reaction $\pi^-p\to\pi^0n$ may be investigated using the absorptive correction formulas with f_1^B, f_2^B given by the ρ Regge-pole terms. The polarization vanishes when no absorptive

[67] V. Barger and M. Olsson, Phys. Letters **16**, 545 (1966).
[68] J. J. Sakurai, Phys. Rev. Letters **16**, 1181 (1966).

TABLE I. Neutron-polarization prediction for small-angle $\pi^- p$ charge exchange.

$-t$ (BeV^2/c^2)	$P(\theta)$
0.00	0.00
0.05	+0.044
0.10	+0.048
0.20	+0.054
0.30	0.00
0.40	(−)

corrections are applied because the helicity flip and helicity nonflip amplitudes have the same phase for all t; but the absorptive corrections disturb this phase relationship. If the calculation is done in the same spirit of approximation as in the estimate of the real part of the amplitude as above, the integrals can be carried out analytically and a closed form obtained for the estimated polarization.

Assuming the elastic πN scattering can be well described with an imaginary nonflip eikonal $\bar{\chi}$, which is chosen to yield the empirical forward πN diffraction-peak shape

$$\exp[i\bar{\chi}(s,b)]\cong 1-Ce^{-b^2/2R^2}$$

(where R^2 is energy-independent), the "correction" formulas for G_+^{CE} and G_-^{CE} (retaining only first order in χ_0^ρ and χ_f^ρ) are

$$G_+=-\cos(\theta/2)\int_0^\infty bdb\, J_0[b(-t)^{1/2}] \times[1-Ce^{-b^2/2R^2}]k^2\chi_0^\rho(s,b^2),$$

$$G_-=\int_0^\infty bdb\, J_1[b(-t)^{1/2}][1-Ce^{-b^2/2R^2}]k^2\chi_f^\rho(s,b^2)$$

with

$$k^2\chi_0^\rho=W\int_0^\infty xdx\, J_0(bx)[f_1^\rho(s,-x^2)+f_2^\rho(s,-x^2)],$$

$$k^2\chi_f^\rho=(W/2k)\int_0^\infty x^2dx\, J_1(bx) \times[f_1^\rho(s,-x^2)-f_2^\rho(s,-x^2)].$$

For large s, assuming the dominant contributions to these integrals come from small $(-t)$, and that the residue functions are slowly varying for small $(-t)$, the ρ-pole expressions can be written

$$W(f_1^\rho+f_2^\rho)\cong d_+(s)(s/s_0)^{t\alpha'}\exp(-i\pi t\alpha'/2)$$

and

$$2M(f_1^\rho-f_2^\rho)\cong[d_+(s)-\alpha(t)d_-(s)] \times(s/s_0)^{t\alpha'}\exp(-i\pi t\alpha'/2),$$

where $d_\pm$ are proportional to the residues for $\pi\pi\to N\bar{N}$ (t channel) helicity parallel and antiparallel states,[63] have energy dependence $(s/s_0)^{\alpha(0)}$, and phase $[-\frac{1}{2}i\pi\alpha(0)]$. Here $\alpha(t)$ is the ρ trajectory function, and a linear form has been assumed:

$$\alpha(t)\cong\alpha(0)+t\alpha'.$$

As in the real-part calculation, the factors involving α' can be written

$$(s/s_0)^{t\alpha'}\exp(-i\pi t\alpha'/2)=\exp[t\alpha'(\Lambda-i\pi/2)] \equiv\exp(tR_1^2/2),$$

where $\Lambda=\ln(s/s_0)$, and R_1^2 is complex. Then

$$k^2\chi_0^\rho(s,b)^2\cong d_+\int_0^\infty xdx\, J_0(bx)\exp(-x^2R_1^2/2)$$

and

$$2Mk^2\chi_f^\rho(s,b^2)\cong\int_0^\infty x^2dx\, J_1(bx) \times\{d_+-[\alpha(0)-x^2\alpha']d_-\}\exp(-x^2R_1^2/2).$$

These integrals can be evaluated analytically, to give

$$k^2\chi_0^\rho(s,b^2)\cong d_+(s)R_1^{-2}\exp(-b^2/2R_1^2),$$

$$2Mk^2\chi_f^\rho(s,b^2)\cong R_1^{-4}b\exp(-b^2/2R_1^2)\{d_+(s)-d_-(s) \times[\alpha(0)-4\alpha'R_1^{-2}-2\alpha'R_1^{-2}(b^2/2R_1^2)]\}.$$

The integrals for G_+ and G_-, in turn, can be analytically evaluated. Let $\eta=R^2/(R^2+R_1^2)$ and $z=R_1(-t)^{1/2}$. Then

$$G_+=-d_+\cos(\theta/2)[e^{-z^2/2}-C\eta e^{-\eta z^2/2}]$$

and

$$G_-=[(-t)^{1/2}/2M][\{d_+-\alpha(t)d_-\}e^{-z^2/2}-C\eta^2 \times\{d_+-d_-[\alpha(t)-\Delta(t)]\}e^{-\eta z^2/2}],$$

where

$$\Delta(t)=[(1-\eta^2)t+2(1-\eta)R_1^{-2}]\alpha'.$$

Since η and R_1^2 are complex, the phases of G_+ and G_- are now different, provided $C\neq 0$ and $\eta\neq 0$. Empirically, $C\cong 0.7$ and $|\eta|\cong 0.5$. The corrections to the differential cross section are relatively small, and to estimate polarization, one may use parameters $d_\pm$, α', $\alpha(0)$ as determined by Höhler *et al.*[64] through a fit to the differential charge-exchange cross section. The predicted polarization then has no free parameters, aside from an over-all undetermined sign, and a possible logarithmic uncertainty in Λ because s_0 is now well determined.

For energies such that $\Lambda=4$, and using the parameters of Höhler *et al.*,[64] the polarization was evaluated and values are given in Table I. Note these qualitative polarization features: (1) rapid rise to maximum value tween $t=0$ and $t=-0.05$ BeV^2/c^2; (2) zero around $t=-0.30$; (3) logarithmic energy dependence of magnitude; (4) maximum value about 5%.

VII. MULTICHANNEL OPTICAL POTENTIALS

1. Multichannel Formalisms and the Quasiclassical Condition

Within the context of single-channel reaction theory, using a model for an exact one-channel optical potential, it is not possible to treat genuinely inelastic reactions. However, most of the successful applications of single-particle-exchange models with absorptive corrections are concerned with two-body inelastic reactions such as $\pi N \rightarrow \rho N$ and $NN \rightarrow N^*N^*$.[69] In order to develop models for this class of reactions it is convenient to introduce a more general type of potential,[70] and a multichannel generalization of the eikonal approximation.

An n-channel generalization of the optical potential is quite simple in the context of a nonrelativistic many-channel problem as described in Sec. II. One simply eliminates all but n of the N channels from the N original coupled Schrödinger equations, yielding n coupled integro-differential equations involving n^2 nonlocal potential operators. In the high-energy limit, the $n\times n$ nonlocal (optical) potential matrix can be replaced with an effective approximate local optical-potential matrix which is complex above the lowest inelastic threshold not explicitly included among the n channels.

The eikonal form of solution in the multichannel case is, however, not always possible. The essential condition[21] is the commutation at different z values [see Eq. 71 below] of the potential operator occurring in the exponential development of the wave function; this refers to the optical-potential matrix (local approximation) at different points along any classical trajectory,[22] or straight line.[21] This requires all n-channel momenta to be the same,[71] and

$$[\mathbf{V}(r),\mathbf{V}(r')]=0 \tag{69}$$

for all r, r' (both conditions understood to the degree of approximation desired). The superscript O on V (denoting optical potential) is dropped in this section.

If (69) is satisfied, a path-ordered matrix exponential can be replaced by a simple matrix exponential, and the eikonal expression for the n-channel scattering amplitude at high energies and small angles is

$$\mathbf{f}(s,t)=ik\int_0^\infty bdb\, J_0[b(-t)^{1/2}][\mathbf{I}-\exp(i\boldsymbol{\chi}(s,b^2))], \tag{70}$$

where

$$\boldsymbol{\chi}(b^2)=\frac{1}{k}\int_{-\infty}^{+\infty} dz\, \mathbf{V}[(z^2+b^2)^{1/2}]. \tag{71}$$

The energy dependence of $\mathbf{V}$ and $\boldsymbol{\chi}$ is not explicitly noted. These results can be easily obtained from Eq. (139) of Ref. (21), where the wave function is an n-component vector in channel space.

In terms of the n-channel Born approximation

$$\mathbf{V}(x)=2\int_0^\infty r^2dr\, \mathbf{V}(r)\left[\frac{\sin(xr)}{xr}\right], \tag{72}$$

where $x=(-t)^{1/2}$, the eikonal matrix is

$$\boldsymbol{\chi}(b^2)=\frac{1}{k}\int_0^\infty xdx\, J_0(xb)\mathbf{V}(x). \tag{73}$$

The condition (69) applied to $\mathbf{V}$, or in more general notation to the Born approximation to the scattering matrix is,

$$[\mathbf{f}_B(s,t),\mathbf{f}_B(s,t')]=0 \text{ for all } t,t'. \tag{74}$$

This condition will be referred to as the "quasiclassical multichannel condition."

From an S-matrix viewpoint, (74) can be adopted as an initial requirement that the eikonal method be applicable. Then $\boldsymbol{\chi}$ can be defined by

$$\boldsymbol{\chi}(s,b^2)=\frac{1}{k}\int_0^\infty xdx\, J_0(bx)\mathbf{f}_B(s,-x^2) \tag{75}$$

and as a consequence of (74), one finds

$$[\boldsymbol{\chi}(s,b^2),\boldsymbol{\chi}(s,b'^2)]=0 \quad \text{for all} \quad b,b'. \tag{76}$$

Since (76) is satisfied, $\boldsymbol{\chi}$ can be diagonalized at each s for all b by a (b-independent but s-dependent) similarity transform in channel indices:

$$\boldsymbol{\chi}_D(s,b^2)=\mathbf{S}^{-1}\boldsymbol{\chi}(s,b^2)\mathbf{S}.$$

The same $\mathbf{S}$ also diagonalizes $\mathbf{f}_B$ since $\mathbf{f}_B$ is a matrix functional of $\boldsymbol{\chi}$, and vice versa. Thus one obtains n uncoupled effective one-channel scattering problems, with complex eigenphases $\delta_l(s)$. To each of these uncoupled problems one may apply the eikonal approximation as defined in terms of prescription (c), and Eq. (8), of Sec. II. After transforming with $\mathbf{S}^{-1}$ back to the physical n-channel problem, (70) is obtained, with $\boldsymbol{\chi}$ as defined by (75). It remains to be decided what significance $\mathbf{f}_B$ has, outside the nonrelativistic framework.

The definition of a two-body multichannel optical potential in field-theoretic context following the procedure of Sec. III is straightforward in terms of a specified ladder set of Feynman graphs, and an $n\times n$ matrix 4-point function $V_{ij}(k_1,k_2;\, k_1',k_2')$ which reproduces the n^2 exact S-matrix elements of the exact field theory when used in this set. If the quasiclassical condition (74) is satisfied, where $\mathbf{f}_B$ is given by the mass-shell values $V_{ij}(s,t)$, then the semiclassical results of Torgerson[23] can be generalized to the multichannel case. A simple instance is the NN scattering problem with spin treated by Torgerson, which in general is intractable by eikonal method because of coupled spin states, but the special case of vector-meson exchange without anomalous-moment coupling gives helicity conservation and hence (74) is satisfied.

One class of models in which (74) can always be satisfied is the class in which all t dependence of Born-approximation matrix elements is the same, reflecting a similar radial dependence of the equivalent static-optical-potential matrix elements. At first thought,

[69] B. Margolis and A. Rotsstein, Nuovo Cimento **42**, 180 (1966) have given an application of such a formula to $NN \rightarrow NN^*$.

[70] This was suggested to the author by Y. Nambu. Such potentials for collective excitation states have been used in nuclear physics; cf. T. Tamura, Rev. Mod. Phys. **37**, 679 (1965); D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. **110**, 1080 (1958).

[71] This is similar to the coherence requirement of Byers and Yang (Ref. 5).

these seem to be very restrictive, since elementary one-particle-exchange potentials give very different ranges for different quantum numbers. However, the Byers-Yang droplet model[5] implies exactly such an assumption. The degree of success of this model for inelastic reactions is open to question, but it is conceivable that such a circumstance is present at least for non-strangeness-transfer reactions within the framework of a multi-peripheral optical potential, as follows:

The multiperipheral idea applied to such inelastic two-body reactions suggests that ladder diagrams with pion ladders will be the important Born approximations (V_{ij}) at high energy. The ladders for different V_{ij}, however, for non-strangeness-exchange processes, can be obtained from each other by changing the first or last rungs only.[15] This idea has been exploited by Berman and Drell in constructing models for photoproduction of vector mesons,[72] and their models illustrate the similar behavior as a function of t for all V_{ij}; this t dependence depends essentially on the existence of large ladders, and not on their end diagrams, at high energies. A similar viewpoint can be compatible with Regge poles if all residues and trajectories drop off at a similar rate with increasing $(-t)$; such pole characteristics are, of course, implied if the above multiperipheral ladder diagrams are represented by Regge poles; i.e., that their contributions dominate the poles of interest in the t channel. These arguments only raise the possibility that a model of equal radial distributions might be comprehensible; comparison with experiment, however, is the real test.

It may be noted that a multichannel formalism, when possible, is able to include a more complete set of graphs, if the analogy between QED(w) and Regge-pole Born approximation is pursued. Diagrams analogous to those Fig. 3(d), 3(e), 3(f), 3(g) [as well as those analogous to the single exchange, Fig. 3(a)] appear in the multichannel case in a certain approximation, where the "photon" and "electron" on the top (or bottom) of the diagram resonate to form an isobar.

2. Absorptive Correction Formulas and Reaction Damping

In the case where off-diagonal elements of $\mathbf{f}_B$ are small (of order ϵ) and the diagonal (elastic scattering) elements identical for all channels (to order ϵ^2), retention of first order in off-diagonal χ yields the DWBA formulas of the absorptive correction model. The equality of the elastic-scattering Born approximation in all channels satisfies the quasiclassical condition (to order ϵ^2). Since the results in this case are well known,[7,28,29] they will not be reproduced here.

A more interesting question is the opposite limit, where the off-diagonal Born-approximation terms are large. For simplicity the elastic-scattering (diagonal) Born terms will be taken to be zero, and a simple two-channel application considered. This example will illustrate the reaction damping mechanism implied by formulas (70) and (75).

[72] S. M. Berman and S. D. Drell, Phys. Rev. 133, B791 (1964).

Let $(f_B)_{11}=(f_B)_{22}=0$, and $(f_B)_{21}=(f_B)_{12}=f^B$. Then

$$\chi_{11}=\chi_{22}=0\,,$$

and

$$\chi_{21}=\chi_{21}=\Phi(s,b^2)\,,$$

where Φ is k^{-1} times the Fourier-Bessel transform of f^B. The formula (70) now yields (for real Φ):

$$f_{11}(s,t)=f_{22}(s,t)=ik\int_0^\infty bdb\, J_0[b(-t)^{1/2}]\times[1-\cos\Phi(s,b^2)] \quad (77a)$$

and

$$f_{12}(s,t)=f_{21}{}^*(s,t)=ik\int_0^\infty bdb\, J_0[b(-t)^{1/2}]\sin\Phi(s,b^2)\,. \quad (77b)$$

Unitarity bounds on the f_{ij} are clearly satisfied, at least if Φ is real. (For complex Φ one also obtains a damped result.) This shows that (70) is useful in circumstances where the DWBA formula is not, and at least gives a prescription for unitarizing a model of inelastic reactions which is in better correspondence to a semiclassical picture than, for example, K-matrix approximations.[10] Note that for real Φ, if $|\Phi|$ and $|\partial\Phi/\partial b|$ are large for small b, then the integrand in (77a) at small b will be rapidly oscillating as a function of b; for small $(-t)$ then the contributions from $\cos\Phi$ will average to zero over the periods of the trigonometric function, and the *effective* "central" opacity will be unity, as in a black disk model. In this way a semiclassical picture is obtained when the inealstic Born terms are large.

VIII. RESONANCES IN THE s CHANNEL AND ALTERNATIVE EIKONAL FORMULATIONS

1. Motivation

In applying the eikonal formalism for either elastic scattering or inelastic multichannel problems, it is necessary to assume the absence of important resonances at the energies of interst. This seems to be reasonable in pp and $\bar{p}p$ reactions at all energies such that $k\gg 2\mu_\pi$, i.e., k^{-1} much less than the range of the optical potential, a necessary condition that the eikonal form be reasonable.

However, in πp (and possibly for K^-p) scattering cross sections there is evidence[73] for resonant structure at momenta as high as 3 BeV/c. The utility of the optical-potential–eikonal outlook can be extended, possibly for momenta down to 1 BeV/c, if a means for phenomenologically including resonances in given partial-wave amplitudes can be found. Such methods could be used in the determination of spin and parity of reso-

[73] S. W. Kormanyos, A. D. Krisch, J. R. O'Fallon, K. Ruddick, and L. G. Ratner; Phys. Rev. Letters 16, 709 (1966).

nances, extending the utility of phase-shift analyses at high energies.

The simplest and most conservative approach consists in assuming all partial-wave amplitudes except the resonant one to be given by the eikonal formula; and for the resonant amplitude, assuming arbitrary phase shift and inelasticity at each energy. A parametrized phase-shift formula, for example a Breit-Wigner form with constant inelasticity chosen to match the eikonal prediction for that partial wave above and below the resonance, would presumably be a reasonable approximation for a narrow resonance. However, for wide resonances, the inelasticity may change appreciably during passage through the resonance, and more parameters must be introduced at least to describe the energy variation of the inelasticity in the resonant partial wave.

A more efficient approach should retain nonresonant contributions, both real and imaginary, to the resonant partial wave given in terms of the optical potential, and superpose on them "pure" resonance terms corresponding to a resonance in many channels coupled with a partial coupling to the elastic-scattering channel. Purely *ad hoc* methods of this kind have been used,[74] but there are many possible ways of accomplishing the purpose. It is desired to have some theoretical framework within which to make approximations to ensure consistency.

A dynamical theory of multichannel reactions with absorption has been proposed by Warnock,[75] involving multichannel N/D equations with arbitrary matrix inelasticity input terms. If such a dynamical theory were used to obtain resonances through introduction of models for the left-hand cuts (or potentials), and if the inelasticity were taken from an eikonal formula, it would be possible to obtain a completely consistent description of resonances with absorptive background. Such a theory is more detailed than is desired for the purposes outlined above, although it might be possible to introduce effective-range type approximations to obtain phenomenological formulas.[76]

2. Modified Cheng Representation

A more practical alternative is the utilization of a representation, proved exact in the nonrelativistic potential scattering context, for the partial-wave scattering amplitudes, developed by Abbe, Kaus, Nath, and Srivastava.[77,78] This representation (modified Cheng) utilizes s-channel Regge trajectory functions and the Born approximation for the given potential to obtain an expression including both resonances and an eikonal-type "background" term in each partial wave. The representation is valid for absorptive, energy-dependent potentials,[78] and should therefore be adequate for high-energy physics applications.

Specifically, let the partial-wave Born approximation be given in the form of an integral representation

$$B_l(s)=\frac{1}{k}\int_{4\mu^2}^{\infty} dm^2\, \sigma(s,m^2)Q_l(1+m^2/2k^2)\,. \qquad (78)$$

Let $\alpha_n(n=0, 1, 2, \cdots, \infty)$ be the s-channel Regge trajectory functions, and let $\cosh\xi(s)=1+8\mu^2/k^2$. Then the modified Cheng representation, in terms of usual phase shifts, can be written as

$$\delta_l(s)=B_l(s)+\frac{1}{2i}\sum_{n=0}^{\infty}\left\{\int_{\alpha_n(s)}^{\alpha_n^*(s)}\frac{d\lambda}{\lambda-l}\exp[(\lambda-l)\xi(s)] - k^{-1}\frac{\exp[-(l+n)\xi(s)]}{l+n} \times\int_{4\mu^2}^{\infty} dm^2\, \sigma(s,m^2)P_{n-1}(1+m^2/2k^2)\right\}\,. \qquad (79)$$

The α_n must be indexed by n in the order in which their asymptotic values lie in the l plane as $s\to+\infty$; $n=0$ must be highest. It was assumed in writing the last term in (79) that the α_n retreat to the negative integers as $s\to\infty$ as with Yukawa-type energy-independent potentials.[77] This property is known to be true also for potentials with energy dependence[79]; this suggests that the representation (79) will be valid in S-matrix theory.[78] The identification of the Born approximation with the optical potential defined in Sec. III above must, however, be considered conjecture, based on the identification of the correspondence of (79) to the eikonal approximation in the high-energy limit.

Observe that (79) has the form of a background-plus-resonance contribution in the neighborhood of a resonance, when one of the complex α_n passes close to the integer l. However, the last term is an energy-dependent correction which would not be expected from the intuitive or *ad hoc* point of view. It is chosen such that the second and third terms cancel at high energies, away from resonances.

A representation in terms of Regge trajectory functions is economical in the sense that two or more resonances which share the same trajectory can in principle be included with fewer additional parameters.[80] In practice, trajectory functions may be approximated with an effective-range type expression, with real part linear with energy, and with imaginary part given by two-body phase space.

The modified Cheng representation has not been written down for spin-$\frac{1}{2}$–spin-0 problems, but the generalization should be straightforward following lines of the original derivation.[77]

[74] W. Johnson, F. C. Smith, and P. C. DeCelles, Phys. Rev. **138**, B938 (1965).
[75] R. L. Warnock, Phys. Rev. **146**, 1109 (1966).
[76] R. L. Warnock (private communication).
[77] W. J. Abbe, P. Kaus, P. Nath, and Y. N. Srivastava, Phys. Rev. **140**, B1595 (1965).
[78] W. J. Abbe, P. Kaus, P. Nath, and Y. N. Srivastava, Phys. Rev. **141**, 1513 (1966).
[79] H. Bethe and T. Kinoshita, Phys. Rev. **128**, 1418 (1963).
[80] See for example Barger and Cline, Ref. 56.

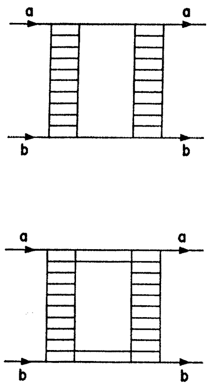

FIG. 9. Planar diagrams: without J-plane cuts.

3. Dispersion Relations in s for Phase Shifts

Another approach to a more complete representation for the partial-wave scattering amplitude may be obtained from analyticity properties in energy of the (complex) phase shifts. If no resonances or bound states are present in the lth partial wave, the phase shift $\delta_l(s)$ (in a spinless problem) is an analytic function of s with branch cuts on the negative real axis and on the positive real axis beginning at the threshold for inelastic processes.[81] If the optical-potential Born approximation is used for $\ln S_l = \delta_l$, the result is one of the eikonal approximation characterizations given in Sec. II, provided a large number of partial waves (each contributing a comparable amount) make up the complete scattering amplitude.

If this formulation is adopted, it is possible to include the effect of resonances simply by explicitly incorporating complex conjugate poles in S_l. A reduced phase shift $\tilde{\delta}_l$ may be defined, in case there is one resonance in the lth partial wave, by[81]

$$\delta_l = \tilde{\delta}_l + \ln\frac{[(W-W_0)^{1/2}+\gamma][(W-W_0)^{1/2}-\gamma]}{[(W-W_0)^{1/2}+\gamma^*][(W-W_0)^{1/2}-\gamma^*]}, \quad (80)$$

where W_0 is the threshold for elastic scattering and γ determines the position width of the resonance. The reduced phase shift can then be adopted as the function approximated by the optical Born approximation at high energies. This procedure, compared to the modified Cheng representation, has the advantage that the extra parameters (a complex γ for each resonance) are constants unambigously defined by the formula (80) when $\tilde{\delta}_l$ is given by the optical Born approximation. However, the dynamical origin of resonances in this formulation is completely ignored. Representations such as (80) have been discussed to some extent by Ball and Frazer.[31]

The expression (80), with $\tilde{\delta}_l = B_l$, represents in some sense a separation into semiclassical contributions (eikonal) and purely quantum effects (resonances) in elastic scattering. At the same time, the relevant equations of motion are completely concealed, thus making it difficult to assess the accuracy of the eikonal approximation for $\tilde{\delta}_l$. The S-matrix significance of a semiclassical high-energy contribution may be related to the accumulation of singularities near the physical region corresponding to classically accessible processes.[82]

[81] N. G. Van Kampen, Phys. Rev. **91**, 1267 (1953).

IX. SINGULARITIES IN THE ANGULAR MOMENTUM PLANE

1. Presence of Cuts with Pomeranchon in Born Approximation

It is generally true that the asymptotic behavior in s of the elastic-scattering amplitude is determined by the singularities with largest real part in complex angular momentum of the t channel amplitude; this can be exhibited through the Sommerfeld-Watson transform.[83] Regge poles give a particular case, in which the dominant poles in the J plane yield a power-law behavior $s^{\alpha(t)}$. Such simple power laws are, conversely, associated with poles in J; asymptotic behaviors involving in addition nonzero powers of logarithms of s are associated with branch points in J, at points $J=\alpha(t)$.

If a Regge pole is used in the optical-potential Born approximation and the eikonal approximation for the scattering amplitude calculated, it is found that its asymptotic behavior is quite complicated, and is certainly not a simple power law or superposition of power laws. Thus the singularities in the J plane are not simple poles. The character of the leading singularities may be investigated by using the simple exponential approximation (58) for the Pomeranchuk pole at small momentum transfer, which gives an eikonal such as (59).

Expanding the integrand of (17) in powers of the strength (Γ) of the Born term, and integrating term-by-term, a series of terms all of order $[\ln(s/s_0)]^{-1}$ compared to the first (Born) term is obtained. This shows that (17) yields an infinite sequence of branch points in the J plane, in addition to the pole term, which are all of comparable asymptotic importance (when the Pomeranchuk pole is considered). Since all terms have the same dominant power of s for $t=0$, the branch points must all be coincident at the point $J=1$ when $t\to 0$, forming an essential singularity at this point when $t\to 0$.

Such a sequence of branch points, generated by an iteration formula for the leading pole, was originally obtained in the development of the multiperipheral model.[15,16] It was originally believed that such branch points were general consequences of unitarity in the s channel.[16] However, Mandelstam showed[17] this was not true, by exhibiting a class of once-iterated Feynman diagrams (containing two exchanged ladders, each having Regge-pole behavior) in which the dominant asymptotic behavior expected was not present. Such diagrams are shown in Fig. 9.

[82] S. Coleman and R. E. Norton, Nuovo Cimento **38**, 438 (1965).
[83] R. Oehme, in *Strong Interactions and High Energy Physics*, edited by R. G. Moorhouse (Oliver and Boyd, Edinburgh, 1963).

Subsequent investigations,[18–20] taking into account a wider class of diagrams with multiple Regge-pole exchange but incorporating multiparticle intermediate states, have shown that cuts of the above character are present in the complete field-theoretic scattering amplitude. The structure of the simplest class of these diagrams is shown in Fig. 10, there X_1 and X_2 are nonplanar subgraphs, and in Fig. 11 is shown a concrete example of this class in a trilinear coupling (ϕ^3) field theory.

Thus, if a Regge-pole Born approximation (represented by a ladder in Figs. 9, 10, and 11) is iterated in such a way as to include only planar graphs such as in Fig. 9, a cut should not be obtained; while if graphs including nonplanar segments (X_1 and X_2 in Fig. 10) are generated during iteration, cuts should be obtained.[84] In either case, branch points are generated having the same location, but the discontinuity across the associated branch cuts should vanish in the former case.

In a nondiagrammatic approximation method based on an S-matrix approach it is not clear, in general, which dispersion graphs are included in the iterative process. In the perturbation development of the eikonal approximation developed by Torgerson,[23] the reproduction of the fourth-order generalized ladder diagrams could be verified directly, but even sixth-order generalized ladder diagrams were not quantitatively checked because of the computation difficulties. The self-energy diagrams, in particular, were not included in Torgerson's analysis; if it is believed that the eikonal approximation is a good one at high energies, using one-meson exchange as the Born approximation, one must simultaneously believe that self-energy, closed-loop, and renormalization graphs are unimportant in some sense for elastic scattering at sufficiently high energies and sufficiently small scattering angles. This was observed in Ref. 23. This point will be referred to again below.

At first glance, diagrams such as in Fig. 9(a) would seem to be the only class included (at second order) in the eikonal approximation with a Regge-pole Born term. If this were true, the Mandelstam results[17] would preclude any contribution from an associated J-plane branch cut (with strength of second order in the Born-

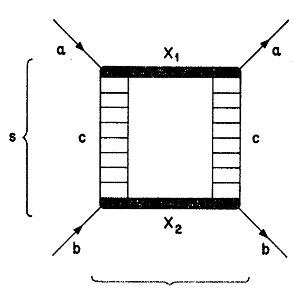

FIG. 10. Nonplanar diagrams: with J-plane cuts.

[84] A model using such an approach has been investigated by E. Abers, H. Burkhardt, V. L. Telplitz, and C. Wilkin, Nuovo Cimento **42**, 365 (1966).

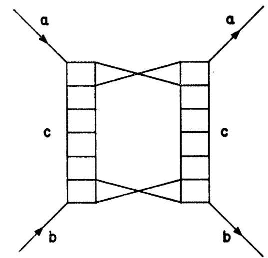

FIG. 11. Concrete example of nonplanar diagram with J-plane cuts.

term strength) as given by the eikonal formula (expanded to second order); this would mean the eikonal expression is completely misleading when a special Born approximation, i.e., (17), is used.

The resolution of this difficulty was conjectured in a previous note.[30] The following discussion will be concerned with a more explicit formulation of this possible means of resolution.

2. External Particles as Regge Poles with Signature

As a means of discussing diagrams with external particles as composite entities in S-matrix language, consider an amplitude for four particles in and four particles out.[85] If pairs of incoming and outgoing particles can exist in states with quantum numbers of pairs coincident with quantum numbers of stable particles a and b, then a graph such as in Fig. 12 will contribute to the 4-particle–4-particle S-matrix element a multiple pole which is a product of simple poles in each of the subenergies s_1, s_2, s_1', s_2'. The residue of each such multiple pole is the $ab \to ab$ scattering amplitude. Further, assuming a and b lie on Regge trajectories, one may continue the four-particle scattering amplitude away from this multiple pole and find other such poles corresponding to Regge recurrences associated with a and b; we can consider such multiple pole residues as defining pole-pole scattering amplitudes.

A graphical analysis of this pole-pole scattering amplitude would include diagrams such as in Fig. 10, where now X_1 and X_2 indicate the amplitudes T_{ac} for a pole-pole scattering $a+c \to a+c$, where c is the exchanged pole, in an unphysical region. The graph represented in Fig. 10 then could be calculated using unitarity in the t channel, given T_{ac} and T_{bc}.

If the pole c has the Pomeranchon quantum numbers, one of the important intermediate states in T_{ac} will be the pole a. Similarly considering X_2 as a pole-pole scat-

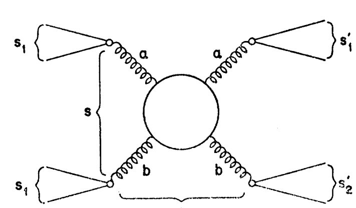

FIG. 12. Regge-pole "scattering amplitude" as a pole contribution in 4-particle scattering amplitude.

[85] This device has been used in other connections; cf. R. Hwa, Phys. Rev. **134**, B1086 (1964).

tering amplitude T_{bc}, there will be a pole in T_{bc} contributed by b. The graph of Fig. 10 including these poles only may be considered as a generalization of Fig. 1(b), but with *all* lines in the diagram representing Regge poles.

Now if T_{ac} and T_{bc} contain all three nonvanishing double spectral functions, as they will in any realistic model, one obtains in Fig. 10 nonplanar diagrams of the type required (when a and b are particles) to generate cuts.[20] The presence of the third double spectral function in T_{ac} and T_{bc} is guaranteed if there are nontrivi alsignature factors associated with the poles a and b, as intermediate states of T_{ac} and T_{bc}. Thus if a and b are considered as Regge poles with signature, and the correspondence between pole-pole diagrams and dispersion graphs involving particles is not deceptive, the iteration of such pole-pole diagrams (including the pole "box") will generate cuts. If the eikonal approximation includes a description of this fact there is no contradiction with previous results.[17,18] The latter conjecture cannot be proved at the present state of the art, but is connected in field-theoretic language with the question of the role of self-energy diagrams, and in S-matrix language with the possibility of constructing multichannel eikonals to take into account Regge recurrences of the external particles.

The conclusion of this discussion is essentially that one should not, on the basis of the presence of cuts, hesitate to use the eikonal method of iterating Regge poles; the method may give a better idea of reality than some special Feynman diagrams such as Fig. 9(a). The composite nature of *all* particles, external as well as exchanged, is presumably important in assessing any high-energy approximation method in S-matrix theory.

X. SUMMARY OF RESULTS

Aside from introducing the dynamical framework of the optical potentials, several specific results have been obtained in the body of this paper. They are gathered here for convenient reference, not necessarily in the order they appeared.

(1) S-matrix characterizations of the eikonal approximation have been given, but are not throughly justified.

(2) Criteria for validity of Regge-pole and DWBA approximations for amplitudes have been implicitly obtained.

(3) A semispecific model for $p\bar{p}$ annihilations is described, and qualitative elastic-scattering features correctly obtained.

(4) The real part of pp and $p\bar{p}$ forward scattering amplitudes at asymptotically high energies has been calculated; the energy dependence was found to be in qualitative agreement with pp data, but the magnitude too small.

(5) An eikonal formalism for relativistic spin-$\frac{1}{2}$–spin-0 scattering has been given, and it was shown that an effective spin-orbit potential is present when the Pomeranchon pole [or any term with no anomalous-moment (A) Born term] dominates the eikonal.

(6) An estimate of polarization expected in πp charge exchange (absorptively corrected ρ pole) was given.

(7) A multichannel eikonal formlaism was proposed which yields a reactive damping effect when inelastic channels are important.

(8) Phenomenological expressions were given to include s-channel resonances.

(9) The problem of cuts in angular momentum was discussed with reference to effects of structure in external particles.

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