Utilizing the orthogonality properties of the d^J functions, we obtain the uncorrected two-pole prediction for the g's through an inversion of these relations:

$$(J+\tfrac{1}{2})(g_J{}^{\pm})_{\rm pole}=\int_{-1}^{+1} d(\cos\theta)G_{\pm{\rm pole}}(s,\cos\theta)d_{\pm\frac{1}{2},\frac{1}{2}}{}^{J}(\theta)$$

where $G_{\pm\rm pole}$ is given by (14) in the text. In terms of Legendre polynomials, putting $z=\cos\theta$, $\bar{G}_+=G_+\cos(\theta/2)$ and $\bar{G}_-=G_-\sin(\theta/2)$, we have

$$(g_J{}^{\pm})_{\rm pole}=\int_{-1}^{+1} dz\, \bar{G}_{\pm{\rm pole}}(s,z)[P_{J+\frac{1}{2}}{}'(z)\mp P_{J-\frac{1}{2}}{}'(z)]. \quad \text{(A3)}$$

The corrected amplitudes G_+, G_- then are expressed as the sums (A2), where the g's are S_J from (A1) times $g_{\rm pole}$ from (A3). Our point of view is that these corrections are essential from an *a priori* standpoint.

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Strangeness-Changing Decay Processes†

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The effective coupling constant $G(\sin\theta\sin\delta)/\sqrt{2}$ of $\Delta S=1$, $\Delta Q=0$ processes and $G(\sin\theta\cos\delta)/\sqrt{2}$ of $\Delta S=\Delta Q=1$ processes is studied on the basis of the decay rates of the leptonic and photonic decay modes of hardons, where S is the strangeness quantum number, Q is the charge, and θ is the Cabibbo angle. The decay rates give information on $\sin\delta$ together with arbitrary mass factors. When the mass factors are eliminated, one finds both for the vector current and the axial-vector current a very small angle δ, which expresses a drastic reduction of the $\Delta S=1$, $\Delta Q=0$ processes. The decay modes connected with the weak vertex $(\Sigma^-\to n)$; $\Sigma^-\to n+\pi^-$, $\Sigma^-\to n+e^-+\bar{\nu}$, and $\Sigma^-\to n+\pi^-+\gamma$, are examined together and it is found that the vector-current dominance of $\Sigma^-\to n+e^-+\bar{\nu}$ decay is consistent with the S-wave decay of $\Sigma^-\to n+\pi^-$ and parity-conserving decay of $\Sigma^+\to p+\gamma$. The decay rate of $\Sigma^-\to n+\pi^-+\gamma$ is estimated and found to be in agreement with a previous estimate and experiments.

I. INTRODUCTION

THE octet-current hypothesis[1] that the weak currents $(j_\mu)_i{}^j$ of strongly interacting particles transform according to an eight representation of $SU(3)$ and that the semi-leptonic effective Lagrangian transforms like members of these weak-current operators, has played a fundamental role in our understanding of elementary-particle weak interactions.

In particular, the vector and axial-vector currents written in the combination[1]

$$J_\mu=(\cos\theta)(j_\mu)_1{}^2+(\sin\theta)(j_\mu)_1{}^3,$$
$$(j_\mu)_i{}^j=(j_\mu{}^V)_i{}^j+(j_\mu{}^A)_i{}^j,$$

have been found to be very useful in relating leptonic decay processes of hadrons with $\Delta S=0$ to processes with $\Delta S=\pm1$, where S is the strangeness quantum number, and the $SU(3)$ content of the currents is given by

$$j_i{}^j\to\begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\eta}{\sqrt{6}} \end{pmatrix}.$$

The Cabibbo angle $\theta=0.26$ has the effect of decreasing the effective coupling constant $G(\sin\theta)/\sqrt{2}$ of the $\Delta S=1$ processes compared to the coupling constant $G(\cos\theta)/\sqrt{2}$ of the $\Delta S=0$ processes, so as to agree with the experimental results on hadron decays.[2]

It is also known that among the $\Delta S=1$ processes the decays with $\Delta Q=0$ are suppressed compared to decays with $\Delta Q=1$. We therefore make an attempt to de-

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[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[2] H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, G. A. Snow, and W. Willis, Phys. Rev. **136**, B1791 (1964).

termine the corresponding angle δ (analogously to the Cabibbo angle θ) that yields the ratio of the coupling constants. It is emphasized here that because of ambiguities in the mass factors the effective coupling constants are not strictly amenable to a geometrical discription in terms of angles but we do so for convenience and $\sin\delta$ can be interpreted as a reduction factor.

The angle δ_V for the vector current is determined by considering the hadronic parts of the processes $\Sigma^+ \to p+\gamma$ and $\Sigma^- \to n+e^-+\bar{\nu}$, and also $K^+ \to \pi^+ + e^+ + e^-$ and $K^+ \to \pi^0 + e^+ + \nu$ in Sec. 2. Similarly, the angle δ_A for the axial-vector current is determined in Sec. 3 by considering the processes $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ and $K^+ \to \pi^+ + \pi^0 + \gamma$. The strength of the vector part of the weak vertex ($\Sigma^- \to n$) is estimated from the S-wave part of the nonleptonic amplitude $\Sigma^- \to n+\pi^-$, and the branching ratio of $\Sigma^- \to n+\pi^- + \gamma$ is obtained in Sec. 4. Finally some remarks are made in Sec. 5.

2. VECTOR CURRENT

The effective coupling constant for the $\Delta S=1$, $\Delta Q=0$ current $(j_\mu{}^V)_2{}^3$ is given by $G(\sin\theta \sin\delta_V)/\sqrt{2}$ and that of the $\Delta S=\Delta Q=1$ current $(j_\mu{}^V)_1{}^3$ is given by $G(\sin\theta \cos\delta_V)/\sqrt{2}$. The reduction of the decay rate of $\Delta S=\Delta Q=\pm1$ processes compared to shose of $\Delta S=0$, $\Delta Q=\pm1$ is expressible by θ.

The parity-conserving part of the amplitude $\Sigma^+(p) \to p(q)+\gamma(k)$ that occurs via $(j_\mu{}^V)_2{}^3$ can be written from invariance arguments as[3]

$$\mathfrak{M}=e\frac{G}{\sqrt{2}}M_1{}^2(\sin\theta \sin\delta_V)\bar{u}_p(q)\left[\left(\frac{i\gamma_\mu k^2}{k^2}+\frac{k_\mu \Delta}{k^2}\right)F_1(k^2)\right.$$
$$\left.+i\sigma_{\mu\nu}\frac{k_\nu}{M}F_2(k^2)\right]u_\Sigma(p)\epsilon_\mu/(2|k|)^{1/2}, \quad (1)$$

where

$$\hbar=c=1\,,\; 2M=M_\Sigma+M_p\,,\; \Delta=M_\Sigma-M_p\,,\; G=10^{-5}M_p{}^{-2}\,,$$

and $\sin\delta_V$ is the constant that characterizes the reduction in strength of the $\Delta S=\pm1$, $\Delta Q=0$ decay modes due to the strong interactions. The M_1 is an arbitrary mass that was taken to be equal to M in Ref. 3. From the experimental decay rate $\Gamma=2.33\times10^7$ sec^{-1}, we obtain

$$(\sin\delta_V)(M_1/M)^2=0.097\,. \quad (2)$$

The $\Sigma^+ \to p+\gamma$ decay occurs via the vector current because we will assume that the CP-invariant interaction is constructed from the $\Delta S=1$, $\Delta Q=0$ component of the unitary spin current.

Another estimate of the angle δ_V can be made from the decay rate of $K^+(p) \to \pi^+(q)+e^+(s)+e^-(r)$. The amplitude may be written as

$$\mathfrak{M}=\sqrt{2}e^2G(\sin\theta \sin\delta_V)p_\mu\bar{u}(r)\gamma_\mu v(s)/(4\omega_K\omega_\pi)^{1/2}\,, \quad (3)$$

where ω_K and ω_π are the energies of the kaon and pion, respectively. From the upper limit of the decay rate $\Gamma\leq 90$ sec^{-1}, we obtain

$$\sin\delta_V \leq 0.05\,. \quad (4)$$

Equations (2) and (4) then yield $M_1 \geq 1.2M$.

The decay $\Sigma^-(p) \to n(q)+e^-(r)+\bar{\nu}(s)$ has contributions from the vector current and axial-vector current. The contribution to the decay rate from the axial-vector current has been found to be 12% of that of the vector current.[2] The decay rates of $\Sigma^+ \to p+\gamma$ and $\Sigma^- \to n+e^-+\bar{\nu}$ receive most of their contribution from the vector current. The amplitude for $\Sigma^- \to n+e^-+\bar{\nu}$ due to the vector current can be written from invariance arguments as[4]

$$\mathfrak{M}=\frac{G}{\sqrt{2}}(\sin\theta \cos\delta_V)\bar{u}_n\left[\gamma_\mu F_1(k^2)+\sigma_{\mu\nu}\frac{k_\nu}{M}F_2(k^2)\right.$$
$$\left.+i\frac{k_\mu}{M}F_3(k^2)\right]u_\Sigma(\bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu)\,, \quad (5)$$

where $k=p-q=r+s$. In the Cabibbo theory[1] $F_1(k^2)=1$, and $F_2(k^2)=F_3(k^2)=0$.

The weak vertices ($\Sigma^+ \to p$) and ($\Sigma^- \to n$) are expected to be closely related since in both vertices a component of she Σ triplet is transformed into a component of the N doublet. When there are no other strongly interacting particles in the final decay products of Σ one can obtain information on the strength of the effective coupling constants of the weak vertex which appear in its decays. The decay rate of $\Sigma^- \to n+e^-+\bar{\nu}$ computed from Eq. (5), leads to $\cos\delta_V \sim 1$. The angle δ_V is very small consistent with our previous estimates.

The decay rate of K_{e3}; $K^+(p) \to \pi^0(q)+e^+(s)+\nu(r)$, can be obtained from the amplitude

$$\mathfrak{M}=(\sqrt{2}G/\sqrt{2})(\sin\theta \cos\delta_V)p_\mu\bar{u}(r)\gamma_\mu v(s)/(4\omega_K\omega_\pi)^{1/2}\,, \quad (6)$$

where ω_K and ω_π are the energies of the kaon and pion, respectively. Then, from Eq. (6) and the experimental decay rate,[5] one has $\cos\delta_V \sim 0.87$ or $\delta_V=0.5$ which is inconsistent with a small value of δ_V.

3. AXIAL-VECTOR CURRENT

The effective coupling parameter for the $\Delta S=1$, $\Delta Q=0$ current $(j_\mu{}^A)_2{}^3$ is given by $G(\sin\delta_A \sin\theta)/\sqrt{2}$ and that of the $\Delta S=\Delta Q=1$ current $(j_\mu{}^A)_1{}^3$ is given by $G\sin\theta \cos\delta_A/\sqrt{2}$. The angle δ_A can be determined from

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[3] K. Tanaka, Phys. Rev. **140**, B463 (1965). The $\Sigma^+ \to p$ vertex is obtained from $\langle p|(j_\mu{}^V)_2{}^3|\Sigma^+\rangle$ with the additional assumption $k_\mu\langle p|j_\mu{}^V|\Sigma^+\rangle=0$. The matrix element is multiplied by a factor $M_1{}^2$ so that G has the right dimensions. We normalize $F_2=1$ which is equivalent to taking the transition magnetic moment equal to 2 n.m. Further references to $\Sigma \to p+\gamma$ and $K^+ \to \pi^+ + e^+ + e^-$ decays may be found in this reference.

[4] The Σ-N part of Eq. (5) corresponds to that of Eq. (1).

[5] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkus, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

the decay rates of $K^+ \to \pi^+ + \pi^0 + \gamma$ and $K^+ \to \pi^+ + \pi^- + e^+ + \nu$.

In contrast to the processes considered in Sec. 2, there are now two strongly interacting particles in the final state so that final-state interactions must be taken into account. The validity of the approximation of replacing form factors by constants is not considered here, but presumably the dependence of the effective coupling constants on the angles θ and δ_A is a consequence of the final state interactions.

The decay rate of $K^+(p) \to \pi^+(p_1) + \pi^0(p_2) + \gamma(k)$ depends on internal bremsstrahlung, $E1$ and $M1$ transitions to a final p-wave two-pion state, and the interference between internal bremsstrahlung and the $E1$ transition.[6,7] Which term is important is not known at present although the contribution from the internal bremsstrahlung can almost account for the decay rate.[8]

In order to obtain the angle δ_A, we neglect the internal bremsstrahlung [which cannot occur via $(j_\mu{}^A)_2{}^3$ as $K^+ \to \pi^+ + \pi^0$ is a $\Delta I = \frac{3}{2}$ transition] and its interference with $E1$, and consider the contributions to $K^+ \to \pi^+ + \pi^0 + \gamma$ from the $E1$ and $M1$ direct emission processes. The contributions are taken as equal. The $E1$ transition has an amplitude of the form[6]

$$\mathfrak{M}_e = e(g/\sqrt{2}M_2{}^3) \sin\theta \sin\delta_A) p_{1\mu} p_{2\nu} (k_\mu \epsilon_\nu - k_\nu \epsilon_\mu)\,, \quad (7)$$

where g is the effective coupling constant for $K_1{}^0 \to \pi^+ + \pi^-$ and M_2 is some mass characteristic of the volume of interaction.

The decay rate due to direct emission Γ_d can be expressed as

$$\begin{aligned}\Gamma_d(K^+ \to \pi^+\pi^0\gamma) &= \Gamma(K_1{}^0 \to \pi^+\pi^-)(M_\pi{}^8/2M_2{}^8) \sin^2\theta \sin^2\delta_A \\ &\qquad \times \int I_d(\omega_1)\phi(\omega_1) d\omega_1 \\ &= \Gamma(K^+ \to \text{all}) \times 0.6 \times 10^{-4} = 4.88 \times 10^3 \text{ sec}^{-1}\,, \end{aligned} \quad (8)$$

where $I_d = I_{E1} = I_{M1}$ and ϕ are given in Ref. 6 and ω_1 is the energy of π^+. The integral on the right-hand side of Eq. (8) over the range 0 to 108 MeV has the value 3.19×10^{-3}. The estimated branching ratio of $K^+ \to \pi^+ + \pi^0 + \gamma$ with respect to all K^+ decay[8] is $(2.2 \pm 0.7) \times 10^{-4}$ and that due to internal bremsstrahlung is 1.6×10^{-4}. The branching ratio due to direct emission is taken as 0.6×10^{-4}. We obtain from Eq. (8),

$$\sin\delta_A (M_\pi/M_2)^4 = 7.67 \times 10^{-2}\,. \quad (9)$$

The matrix element for K_{e4}, $K^+(p) \to \pi^+(p_1) + \pi^-(p_2) + e^+(s) + \nu(r)$, that occurs via $(j_\mu{}^A)_1{}^3$ and $(j_\mu{}^V)_1{}^3$ can be written as[9]

$$\begin{aligned}\mathfrak{M} = [\sqrt{2}G(\sin\theta \cos\delta_A)/M_2]\{&F_1(p_1+p_2)_\mu \\ &+ F_2(p_1-p_2)_\mu + F_3(\epsilon_{\mu\nu\gamma\delta} p_\nu p_{1\gamma} p_{2\delta}/M_2{}^2)\} \\ &\times \bar{u}(r)\gamma_\mu(1+\gamma_5)v(s)/(8\omega_K\omega_1\omega_2)^{1/2}\,, \end{aligned} \quad (10)$$

where M_2 is taken as the same mass that appears in Eq. (7). The two pions can be in states with $I=0$ and $I=1$. The form factors depend on the scalar products $p_1 \cdot p_2$, $p_1 \cdot p$, and $p_2 \cdot p$ of the momenta, but we will treat them as constants. The F_1 and F_2 arise from the axial-vector current, and F_3 from the vector current which is expected to contribute little so the decay rate and will be ignored.

The K_{e4} decay rate from Eq. (10) is[10]

$$\begin{aligned}\Gamma = [G^2 M_\pi{}^{10} (\sin^2\theta \cos^2\delta_A)(0.3)/3\pi^5 M_K{}^3 M_2{}^2] \\ \times (F_1{}^2 + 0.2F_2{}^2)\,, \end{aligned} \quad (11)$$

or[11]

$$\begin{aligned}\Gamma = [G^2 M_K{}^7 (\sin^2\theta \cos^2\delta_A)(0.0296)/2^8\pi^5 360] \\ \times (F_1{}^2 + 0.1F_2{}^2)\,. \end{aligned} \quad (12)$$

We put[12] $(F_1{}^2 + 0.2F_2{}^2) = 1$, and obtain from[13] $\Gamma = 3.49 \times 10^3$ sec^{-1} and Eq. (11)

$$(\cos\delta_A)(M_\pi/M_2) = 0.806\,. \quad (13)$$

Equations (9) and (13) have the solution

$$\delta_A = 0.17\,.$$

The correct decay rates are obtained by taking $M_1 = 170$ MeV. The same value of δ_A follows with the aid of Eqs. (9) and (12).

4. $\Sigma^- \to n + \pi^-$.

It is of interest of examine the weak vertex in terms of the nonleptonic decay amplitude and also estimate the branching ratio of $\Sigma^- \to n + \pi^- + \gamma$ with the aid of results obtained in Sec. 2.

One can estimate the effective coupling strength of the vector part of $\Sigma^- \to n$ by assuming that the decay Hamiltonian H_W of $\Sigma^- \to n + \pi^-$ transforms like the $\Delta Q = 0$, $\Delta S = 1$ component of unitary spin current, and using the algebra of current commutation relations,[14,15]

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[6] J. D. Good, Phys. Rev. **113**, 352 (1959).

[7] N. Cabibbo and R. Gatto, Phys. Rev. Letters **5**, 382 (1960); H. Chew, Phys. Rev. **123**, 377 (1961).

[8] D. Cline and W. F. Fry, Phys. Rev. Letters **13**, 101 (1964).

[9] Note that as in the matrix element (5), $\sin\theta$ has been factored out.

[10] E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **39**, 345 (1960) [English transl.: Soviet Phys.—JETP **12**, 245 (1961)].

[11] L. B. Okun and E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **37**, 1775 (1959) [English transl.: Soviet Phys.—JETP **10**, 1252 (1960)].

[12] The form factor F_1 is normalized to unity. Our form factors are related to those in Ref. 10 and 11 by $F_1 = f_1 + f_2$, $F_2 = f_2$ and $F_1 = f/2$, $F_2 = g/2$, respectively.

[13] The K_{e4} decay rate is obtained from R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, D. Cline, W. F. Fry, J. G. Gaidos, D. Murphree, C. T. Murphy, Phys. Rev. **139**, B1600 (1965).

[14] K. Tanaka, Phys. Rev. **151**, 1203 (1966).

[15] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966).

i.e.

$$A(\Sigma^- \to n\pi^-)_S = \lim_{k\to 0}(2k_0)^{1/2}\langle n\pi^-(k)|H_W|\Sigma^-\rangle_S = (g_r/\sqrt{2}M_p g_A)A(\Sigma^- \to n)_V = 4.5\times 10^{-7}, \quad (14)$$

where $g_A=1.18$ is the ratio of axial-vector to vector β-decay coupling constants, and $g_r=13.5$ is the strong pion-nucleon coupling constant.

We put

$$A(\Sigma^- \to n)_V = (G/\sqrt{2})(\sin\theta)m^3, \quad (15)$$

where m is some mass characteristic of the volume of interaction. Then, $m=293$ MeV$\sim M_\Sigma - M_n$ follows from Eqs. (14) and (15), which is a reasonable result.

When the magnetic moments of Σ^- and n and the transition magnetic moments are neglected, the decay rate $\Sigma^- \to n+\pi^-+\gamma$ depends on the internal bremsstrahlung process. It has been found that $\Sigma^- \to n+\pi^-$ occurs via S wave[16] which is predicted by the algebra of current commutation relations.

Let us take the internal bremsstrahlung contributions to the decay rates and consider the branching ratios $\Gamma_1(\Sigma^- \to n\pi^-\gamma)/\Gamma(\Sigma^- \to n\pi^-)$ and $\Gamma_1(K^+ \to \pi^+\pi^0\gamma)/\Gamma(K^+ \to \pi^+\pi^0)$.

These two ratios give no information of the effective coupling constants that has been considered so far. They have the common features that the hadrons in the final state are in S states and the available energies are the same, so that one would expect the following relation to hold[5]

$$\Gamma_1(\Sigma^- \to n\pi^-\gamma)/\Gamma(\Sigma^- \to n\pi^-) \approx \Gamma_1(K^+ \to \pi^+\pi^0\gamma)/\Gamma(K^+ \to \pi^+\pi^0) = 1.3\times 10^4\ \mathrm{sec}^{-1}/1.75\times 10^7\ \mathrm{sec}^{-1} = 0.82\times 10^{-3}. \quad (16)$$

The branching ratio for Σ^- decay is in agreement with a previous estimate[17] and also with experiments.[16] This result indicates that the direct-emission terms are not as important as the internal-bremsstrahlung terms.

[16] M. Bazin, H. Blumenfeld, U. Nauenberg, L. Seidlitz, R. T. Plano, S. Marateck, and P. Schmidt, Phys. Rev. **140**, B1358 (1965).

[17] S. Barshay, U. Nauenberg, and J. Schultz, Phys. Rev. Letters **12**, 76 (1964); **12**, 156(E) (1964).

5. REMARKS

The angle δ that gives the effective coupling constants $G\sin\theta\sin\delta/\sqrt{2}$ of the $\Delta S=\pm 1$, $\Delta Q=0$ processes and $G(\sin\theta\cos\delta)/\sqrt{2}$ of the $\Delta S=\Delta Q=\pm 1$ processes was obtained for the vector and axial-vector currents. Due to ambiguities in the calculation and lack of sufficient experimental data on these rare decay processes, a reliable estimate is not feasible at the present, but a rough estimation suggests

$$\delta_V \sim 0.05, \quad \delta_A \sim 0.17.$$

The dynamical reason why δ_V and δ_A are so small, i.e., $\Delta S=\pm 1$, $\Delta Q=0$ processes are so weak compared to $\Delta S=\Delta Q=\pm 1$ processes, is, of course, not known.

In regard to the weak vertex (Σ-N) the experimental data suggest that the vector current dominates the amplitude[1,2] $\Sigma^- \to n+e^-+\bar{\nu}$, which is consistent with $\Sigma^+ \to p+\gamma$ occurring also via the vector current. On the basis of the algebra of current commutation relations, the dominance of the vector part in $\Sigma \to N$ leads to dominance of the S-wave part of the nonleptonic decay $\Sigma^- \to n+\pi^-$ which agrees with recent experimental data.[16]

The relation (16) allows an estimate of the decay rate $\Sigma^- \to n+\pi^-+\gamma$ which agrees with a previous estimate[17] and experiments.[16]

Although we have been unable to provide any basic explanations, we can now understand phenomenologically the properties of a chain of decay processes which appeared hitherto unrelated.

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