Double-Octet Regge-Pole Model with Exchange Degeneracy for Charge- and Hypercharge-Exchange Reactions*

RICHARD C. ARNOLD
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois
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A model for pseudoscalar-meson–baryon and pseudoscalar-meson–baryon-resonance hpyercharge- and charge-exchange reactions at high energies is proposed, based on an octet of pairs of opposite-signature Regge poles. The model may be considered as an extension of the charge-exchange reaction model for K^-p used by Phillips and Rarita, incorporating $SU(3)$ symmetry and exchange degeneracy. In addition, the universality hypothesis is employed to extend the model to B^* production. The model thus extended contains only a few parameters (essentially only 2) and agrees with all available high-energy, small-momentum-transfer data on these reactions. The energy dependences (involving no free parameters) are very good; the universality hypothesis is satisfied within narrow experimental bounds, and $SU(3)$-symmetric normalizations are within a factor of 2 for most cross sections at high energy.

I. INTRODUCTION

At energies above 1.5 BeV/c lab momentum the angular distributions for reactions (K^- or $\pi^\pm$) $+p\rightarrow P+(B$ or $B^*)$ all show a peak in the forward direction of the meson,[1–4] indicative of a peripheral exchange process. (Here, as usual, P means pseudoscalar meson, B baryon, and B^* baryon resonance.) For K^-p reactions with $\Lambda\eta$, $\Lambda\pi^0$, $\Sigma^+\pi^-$, ΛX^0, and $\pi^-Y_1^{*+}$ final states the most obvious resonance in the t channel which could be responsible for the reaction is the well known $K_{1/2}^*(890)[J^P=1^-]$; exchange of a K meson is forbidden by parity for all the reactions considered. Similarly, the associated production reactions [e.g., $\pi^-p\rightarrow\Lambda K^0$] above 1.5 BeV/$c$ exhibit[5,6] a peripheral nature which has been interpreted since early times[7] as indicative of this exchange process.

In the case of the charge-exchange reaction $K^-+p\rightarrow \bar{K}^0+n$, a relatively simple model[8] involving ρ exchange can account for the angular distribution at momenta near 2 BeV/c.

A more complete Regge-pole fit to the K^-p and π^-p charge exchange (CE) and $\pi^-p\rightarrow\eta n$ reactions, and elastic and total cross sections, over a wide range of energies has been accomplished by Phillips and Rarita[9] only by including a second isovector pole, denoted as R by Pignotti[10] and Ahmadzadeh.[11] This pole also seems to be essential for a fit[11] to the energy dependences of the moderately large-angle np charge-exchange reaction [which also involves pion exchange as a small, sharply peaked contribution] and of $\sigma_{np}-\sigma_{pp}$. As first pointed out by Pignotti[10] on the basis of $SU(3)$-symmetric meson bootstrap equations, one expects a hypercharge-carrying trajectory Q associated with R (and the P' pole) such that R and Q belong to an octet representation of $SU(3)$. These trajectories have signature opposite to the octet of vector mesons [e.g. ρ, $K^*(890)$] and may recur as physical 2^+ resonances, some of which have been established. The status of the 2^+ mesons and their unitary symetry properties has been analyzed by Glashow and Scoolow.[12] The author[13] has also observed that such trajectories would be expected in a baryon-antibaryon bound-state model of mesons. In fact in such a model they would be approximately degenerate with the vector-meson trajectories; see for example Fig. 1 of Ref. 13. This picture suggests that a good high-energy model for hypercharge-exchange (HCE) reactions should involve both $K^*(890)$ and Q poles, since they would be expected to lie close together in the J plane for $t<0$.

An independent argument for such a double-pole model is provided by the appearance[1,3–5] of appreciable polarization of the final-state baryon in HCE reactions. A single pole cannot yield any polarization, since the helicity-flip and helicity-nonflip terms in the amplitude will have the same phase. We need then another independent term in the amplitude, of comparable magnitude, interfering with the K^* pole. If this second term were not *also* peripheral in character, there would be an appreciable reaction cross section away from the forward meson direction; but the complete amplitude

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[6] J. Kirz (private communication).

[7] J. Tiomno, in *Proceedings of the 1960 International Conference on High Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960).

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[9] R. J. N. Phillips and W. Rarita, Phys. Rev. **138**, B723 (1965); **138**, B1136 (1965).

[10] A. Pignotti, Phys. Rev. **134**, B630 (1964).

[11] A. Ahmadzadeh, Phys. Rev. **134**, B633 (1964).

[12] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[13] R. C. Arnold, Phys. Rev. Letters **14**, 657 (1965). For implementation of the exchange degeneracy idea, see A. Ahmadzadeh, Phys. Rev. Letters **16**, 952 (1966).

seems to be relatively small for large angles as the momentum is increased above 1.5 BeV/c .Thus a second exchange mechanism, with a large phase difference compared to the $K^*(890)$ pole [at least for small $(-t)$] is strongly suggested. Precisely this is supplied by a pole trajectory (e.g., Q) whose signature factor is opposite to that of K^*; the pole terms would be relatively imaginary if the trajectories were coincident. Note that the exchange of an *elementary* 2^+ meson would not resolve these difficulties, and would make the cross section grow rapidly with energy; this is contradictory to experimental evidence, as well as leading to the well-known conflict with unitarity at sufficiently high energy.

The polarization and angular distribution of the associated production reaction $\pi^- p \rightarrow K^0\Lambda$ has been analyzed in the low-energy region, below 1.5 BeV/c, by Hoff.[14] In that region, resonances in the s (direct) channel are very important; the Hoff model employs a superposition of elementary $K^*(890)$ exchange and $P_{1/2}$ and $F_{5/2}$ resonant s-channel amplitudes. Forward peaking is assured by the K^* contribution; the resonances provide shoulders or peaks in the total cross section as a function of energy, and polarization is generated by the interference of (real) K^* exchange with direct channel Breit-Wigner resonance terms. This approach is clearly complementary to the high-energy model we present, which is valid only above all important s-channel resonances. The transition between the two models is not completely clear, however, since Hoff has not included the Q pole, nor is any absorptive correction used, although competing channels (e.g., $\pi^- p \rightarrow \pi^+\pi^- n$) are certainly important.

There is evidence[15] in the HCE reactions near 2 BeV/c for resonances, which complicates the interpretation of those data. We will assume the model to be only qualitatively applicable in that energy region, in a spirit similar to that of Carroll, *et al.*,[16] in their analysis of the ρ-exchange contribution to $\pi^- p$ charge exchange in the vicinity of the $G_{7/2}(2190)$ πN $(T=\frac{1}{2})$ resonance. Note we do not appeal to any *a priori* theoretical justification for high-energy dominance of poles in the angular momentum plane, but simply adopt the viewpoint that peripheral processes are to be described by such poles since they are associated with particles and resonances in the t channel. Some remarks on the current ideas[17] of "absorptive unitarity corrections" will be made at the conclusion. We do not apply any explicit unitarity corrections in the text, but simply use the Regge pole form. An Appendix contains the necessary formalism for computing these corrections. They are found to have little effect on the consistency of the model.

Even with absorptive corrections, the pole approximation amplitude for the lowest partial waves in some cases violated limits on the magnitude of a unitary amplitude; see for example Jackson.[17] In general, we can only expect the pole approximations to give correctly the higher partial wave (or large impact parameter[8]) contribution to the reaction amplitude, but not necessarily the lower partial-wave (or small impact-parameter) components. From a practical point of view, this means that in comparing experiment with theory we should fit only the forward peak region and subtract out any contribution to the data which appear to come from low values of j (for example, resonance contributions) in the partial-wave decomposition. Only in favorable situations would the model completely describe the experimental data; for example, empirically there appears to be a great deal of low partial-wave contribution to the cross section for $K^- p \rightarrow \Lambda\pi^0$ at 2 BeV/c.

In the next section we develop briefly the general Regge-pole formalism for the reaction not involving B^*'s following Überall's analysis;[18] we then specialize to the double pole model. In Sec. III the threshold behavior and factorization properties of residues are elucidated. A simple linear approximation for the trajectories is adopted, as a minimally complicated expression adequate to fit the data, and a parameter-independent equation relating polarization in $\pi^- p \rightarrow \Lambda K^0$ and $K^- p \rightarrow \Lambda\pi^0$ is obtained. In Sec. IV we give the B^* reaction formalism; in V, we discuss symmetry considerations and present a comparison with the data. Finally in Sec. VI, some reactions other than those mentioned explicitly in Secs. II–IV are commented upon.

II. FORMALISM FOR P-B REACTIONS

The general Regge pole analysis of P-B HCE and CE amplitudes, including kinematic factors and isospin factors, has been given by Überall.[16] In addition, Iwao has discussed[19,20] the K^* Regge-pole terms[20] for $K^- p \rightarrow \Sigma^+\pi^-$ (as well as the baryon poles[19]). These authors have been concerned primarily with the case where one leading trajectory (K^* in particular) dominates the reaction. Such a case has been treated in detail by Wagner and Sharp.[21] It is necessary to supply a few more details here, while summarizing the formalism, to get a practically useful two-pole expression to compare with experiments.

For purposes of symmetrical notation we introduce the helicity-flip and helicity nonflip amplitudes for

[14] G. T. Hoff, Phys. Rev. **139**, B671 (1965).

[15] See e.g., R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters **16**, 1228 (1966).

[16] A. S. Carroll, I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, and W. S. C. Williams, Phys. Rev. Letters **16**, 288 (1966).

[17] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965); L. Durand, III, and Y. T. Chiu, Phys. Rev. **139**, B646 (1965).

[18] H. Überall, Nuovo Cimento **30**, 366 (1962).

[19] S. Iwao, Nuovo Cimento **25**, 973 (1962).

[20] S. Iwao, Nuovo Cimento **28**, 1246 (1963).

[21] W. G. Wagner and D. H. Sharp, Phys. Rev. **128**, 2899 (1962).

each P-B reaction in the s channel, G_- and G_+, respectively, such that

$$\frac{d\sigma}{dt}=|G_+|^2+|G_-|^2 \tag{1a}$$

$$\frac{d\sigma}{dt}P(\theta)=-2\,\mathrm{Im}(G_+^*G_-) \tag{1b}$$

where $P(\theta)$ is the final state baryon polarization. In terms of the amplitudes f_1, f_2 defined by Überall,[18] and the standard Mandelstam invariant amplitudes A and B (which are free of kinematic singularities and obey a Mandelstam representation for non-anomalous mass ratios) we have

$$\begin{aligned}G_+&=(f_1+f_2)\cos(\tfrac{1}{2}\theta)\,,\\ G_-&=(f_1-f_2)\sin(\tfrac{1}{2}\theta)\,,\end{aligned} \tag{2}$$

with

$$\begin{aligned}f_1&=(\kappa_1/8\pi)[A+(W-\bar{M})B]\,,\\ f_2&=(\kappa_2/8\pi)[-A+(W+\bar{M})B]\,,\end{aligned} \tag{3}$$

where

$$\kappa_1=\frac{\pi^{1/2}}{W}\left[\frac{E_2+M_2}{E_1-M_1}\right]^{1/2},\quad \kappa_2=\frac{\pi^{1/2}}{W}\left[\frac{E_2-M_2}{E_1+M_1}\right]^{1/2},$$

$$\bar{M}=\tfrac{1}{2}(M_1+M_2)\,.$$

Here E_1, M_1 are the total energy of the proton in the c.m. frame, and the proton mass; E_2, M_2 are the corresponding values for the final state baryon (Λ, n, or Σ). The total c.m. energy is W; s and t will denote usual Mandelstam variables, $s=W^2$ and $t=-\Delta^2$, where Δ is the 4-momentum transfer; θ is the c.m. angle of the final-state meson. These expressions are taken from Ref. 18, Sec. 4. We do not at this point assume $s\gg(M_1+M_2)^2$. Isospin is ignored here; a complete discussion may be found in Ref. 18.

To pick out the Regge poles in the crossed-channel reactions $P_1+P_2\to\bar{B}_1+B_2$, define helicity amplitudes $\mathfrak{F}_+$ and $\mathfrak{F}_-$ for the t channel by

$$\begin{aligned}\mathfrak{F}_+&=\sum_J (J+\tfrac{1}{2})\hat{f}_+^J(t)P_J(x)\,,\\ \mathfrak{F}_-&=\sum_J (J+\tfrac{1}{2})\hat{f}_-^J(t)P_J'(x)\,.\end{aligned} \tag{4}$$

Here x is the cosine of the reaction angle in the t-channel reaction center-of-mass system, which in terms of s is

$$x=[M_1^2+M_2^2-2(M_1^2+p^2)^{1/2}\times(M_2^2+q^2)^{1/2}-s]/2pq\,, \tag{5}$$

where (p,q) are t-channel c.m. momenta of (baryons, mesons), respectively. We have introduced partial-wave helicity amplitudes for the t channel, following Iwao;[19] in terms of the latter's amplitudes we have written

$$\begin{aligned}\hat{f}_+^J(t)&=(pq)^Jf_+^J(t)/p^2\,,\\ \hat{f}_-^J(t)&=(pq)^{J-1}f_-^J(t)/[J(J+1)]^{1/2}\end{aligned}$$

where the $f_\pm^J$ are defined in Sec. 2 of Ref. 17. The new amplitudes are chosen to yield plausibly to a Sommerfeld-Watson transform in the J plane and to simplify the kinematic factors somewhat.

The relation between A and B amplitudes and these helicity amplitudes in the crossed channel is then

$$\begin{aligned}A&=8\pi[-\mathfrak{F}_++(M_1M_2)^{1/2}qx\mathfrak{F}_-/p]\,,\\ B&=8\pi\mathfrak{F}_-\,.\end{aligned} \tag{6}$$

Applying the Sommerfeld-Watson transform to the partial-wave expansions (4) and retaining only the pole terms in the right-half J plane, we get the t-channel Regge-pole contributionsto $\mathfrak{F}_+$ and $\mathfrak{F}_-$:

$$\begin{aligned}\mathfrak{F}_+^{(P)}&=\sum_k \xi_k(t)b_k^{(+)}(t)P_{\alpha_k(t)}(-x)\,,\\ \mathfrak{F}_-^{(P)}&=\sum_k \xi_k(t)b_k^{(-)}(t)P_{\alpha_k(t)}'(-x)\,,\end{aligned} \tag{7}$$

where

$$\xi_k=[2\alpha(t)+1]\{1+\epsilon_k\exp[-i\pi\alpha_k(t)]\}/\sin[\pi\alpha_k(t)]$$

with $\epsilon_k=\pm1$ the signature of the kth pole, $\alpha_k(t)$ its trajectory function, and $b_k^{(\pm)}(t)$ residue functions.

Assuming now that the $K^*(890)$ (or ρ) and Q (or R) poles are dominant in the energy range under consideration, we have those two terms only in the sums (7). This is essentially the same as retaining all poles in the right-half J plane for small $(-t)$ if the picture of meson states proposed in Ref. (13) is correct. Such an assumption is the starting point for a two-pole model, but until further analysis of the terms is performed we cannot use this since it contains 4 independent functions of t for each reaction, plus 2 trajectory functions. Note the reactions considered in this paper, since they have a P-P state in the t channel, cannot have P trajectory exchange, or 1^+-meson trajectory exchange; and since quantum numbers (isospin, charge, or hypercharge) are transferred, isoscalar trajectories do not contribute. Of all known mesons, only the $[\rho,A_2]$ and $[K^*(890), K^*(1400)]$ trajectories can therefore contribute.

III. FACTORIZATION, CROSSING, THRESHOLD PROPERTIES, AND APPROXIMATIONS FOR RESIDUE AND TRAJECTORY FUNCTIONS

In order that we may compare the various HCE reactions it is desirable to factorize the residues $b^{(\pm)}$. To accomplish this it is only necessary to observe that the general factorization theorem[21,22] for Regge poles is applicable, and we can therefore write for a given reaction

$$b_k^{(\pm)}(t)=\sigma_k(t)\gamma_k^{(\pm)}(t)$$

where σ_k depends on the nature of the final-state meson, whereas the $\gamma_k^{(\pm)}$ depend on the nature of the final-state baryon.

[22] M. Gell-Mann, Phys. Letters 8, 262 (1962).

As observed by Wagner and Sharp,[21] in comparing Regge pole contributions to $K^-p \to \Lambda\pi^0$ and $\pi^-p \to \Lambda K^0$, we find (aside from isospin Clebsch-Gordan coefficients) the same residue functions, but the *sign* of each of the pole contributions for the latter reactions [compared to the former] is changed for poles with odd signature [e.g., K^*], while remaining unchanged for even signature [e.g., Q]. This leads to the possibility of considerable difference between the cross sections. There is, however, an equation relating the polarizations, Eq. (12) below. In addition, for very high energies such that $W \gg \bar{M}$, we can see from (3) and the preceding discussion that if $\alpha_Q(0) \cong \alpha_{K^*}(0)$ then

$$\frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{\pi^-p\to\Lambda K^0}\Bigg|_{\theta_k=0} \cong \left(\frac{d\sigma}{d\Omega}\right)_{K^-p\to\Lambda\pi^0}\Bigg|_{\theta_\pi=0}$$

but the shape of the angular distributions may be different. This relation does not seem to be satisfied as low as $W=2.2$ BeV, within errors, even if the large angle (nonperipheral) backgrounds are subtracted out from both forward peaks before comparison. The right-hand member of this relation after such a subtraction is 190 ± 50 μb/sr,[3,4] while on the left we have[5] 67 ± 8 μb/sr at this energy. (These errors are roughly estimated on the basis of statistical errors and the amount of background.) Presumably, resonant interference may play an important role at such an energy.[4]

It is convenient, if comparison with the shape of $d\sigma/dt$ for $t\neq0$ is contemplated, to adopt a simple functional form for the residues. To this end we consider the branch points of the pole residues at t-channel thresholds $q=0$ or $p=0$, following analyses of Desai[23] originally applied to the elastic-scattering pole models. The residues $b(t)$ have a threshold behavior [when $\alpha(\text{threshold})>\frac{1}{2}$] given by

$$b_k^{(\pm)}(t) \to (qp/m^2)^{\alpha_k(t)}\tilde{b}_k^{(\pm)}(t) \qquad (8)$$

as $p\to0$ or $q\to0$, where m^{-1} is the effective potential range in the t-channel reaction[23]; the reduced residues $\tilde{b}^{(\pm)}(t)$ are slowly varying in the threshold region. We now assume the $\tilde{b}^{(\pm)}(t)$ are slowly varying over the range of t encountered in our CE and HCE reactions, but possibly have a zero.

The masses m are estimated by examining the singularities nearest threshold for the t-channel reaction, e.g., $p\bar{Y}\to K^-\pi$; these are the baryon pole terms in our examples. We find that $m^2\approx2\mu M$, where μ is a mean meson mass and M is a mean baryon mass. We adopt below $m^2\cong0.30$ BeV^2 unless other comments are made.

For the energies and reactions discussed in the present work, $-x\gg1$ throughout the physical region and we can use the approximations $P_\alpha(-x)\cong(-x)^\alpha$ and

[23] B. R. Desai, Phys. Rev. **138**, B1174 (1965); T. Binford and B. R. Desai, *ibid.* **138**, B1167 (1965).

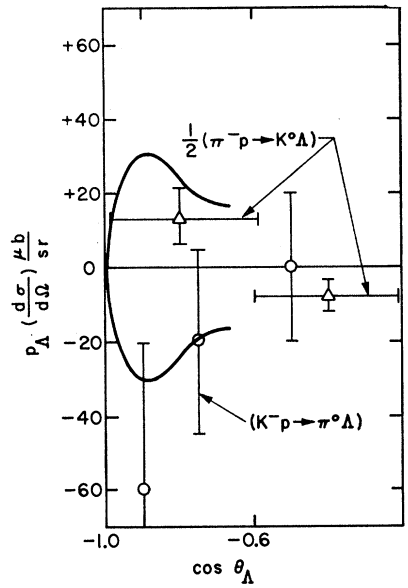

FIG. 1. Comparison of Λ polarization in $\pi^-p\to K^0\Lambda$ (Ref. 5) and $K^-p\to\pi^0\Lambda$ (Ref. 4), at $W=2.20$ BeV. Curves are typical fits to data with constant residues, and are meant only as a guide to the eye.

$-x\cong s/2pq$. Combining these approximations, we can write

$$\mathfrak{F}_\pm^{(P)} \cong \sum_k \xi_k(t)\tilde{\sigma}_k \begin{bmatrix} \tilde{\gamma}_k^{(+)}(s/2m^2)^{\alpha_k(t)} \\ \alpha_k\tilde{\gamma}_k^{(-)}(s/2m^2)^{\alpha_k(t)-1} \end{bmatrix}, \qquad (9)$$

where we have employed factorization to introduce slowly varying meson and baryon factors for the residues,

$$\tilde{b}_k^{(\pm)}(t) = \tilde{\sigma}_k(t)\tilde{\gamma}_k^{(\pm)}(t) \qquad (10)$$

under the additional assumption that m^2 is about the same for all the reactions considered.

Substituting these expressions into the relations (6), and defining $Z\equiv s/2m^2$, we obtain the high-energy double-Regge-pole forms:

$$A = 8\pi\sum_k \xi_k\tilde{\sigma}_k Z^{\alpha_k(t)} \times[-\tilde{\gamma}_k^{(+)} - (M_1M_2)^{1/2}(m^2/p^2)\alpha_k(t)\tilde{\gamma}_k^{(-)}] \qquad (11)$$
$$B = 8\pi\sum_k \xi_k\tilde{\sigma}_k Z^{\alpha_k(t)-1}[\alpha_k(t)\tilde{\gamma}_k^{(-)}].$$

There exists a relation between polarizations which follows from (11), when only two poles with opposite signatures are present. The polarization will be proportional to the product of V (i.e., ρ or K^*) and T (R or Q) residues. If we consider two reactions which differ by crossing the meson lines,[21] e.g., $K^-p\to\pi^0\Lambda$ and $\pi^-p\to K^0\Lambda$ (using isospin invariance also), the odd-signature trajectory contribution changes sign while the even-signature contribution does not. Ingoring the mass differences involved, we deduce then:

$$\left(P_\Lambda\frac{d\sigma}{dt}\right)_{K^-p\to\pi^0\Lambda} = -\frac{1}{2}\left(P_\Lambda\frac{d\sigma}{dt}\right)_{\pi^-p\to K^0\Lambda} \qquad (12)$$

at each corresponding energy and angle. Experimentally[3-5] the right- and left-hand terms in (12) are comparable in size and have the correct relative sign at $W=2.20$ BeV, although the errors are large; the comparison is shown in Fig. 1.

When $(M_2+M_1)^2 \ll (-t) \ll (M_2-M_1)^2$, which is true for most of the physical region in our reactions, we can approximate $p^2(t)$ by $-\bar{M}^2$.

Under the assumption of exchange degeneracy[13,24] for the baryon couplings of the even (T) and odd (V) signature trajectories, one has (for any pair of initial and final baryons)

either

$$\tilde{\gamma}_T^{(+)} = +\tilde{\gamma}_V^{(+)}$$

or

$$\tilde{\gamma}_T^{(+)} = -\tilde{\gamma}_V^{(+)}.$$

The relative sign, once determined, reverses if the baryon lines are crossed [since $\gamma_V^{\pm}$ change sign]. There is no clear *a priori* way to assign the relative sign; one must appeal to a specific dynamical model or to experiment. We choose the latter approach.

Similarly, for the $\tilde{\gamma}_V^{(-)}$ and $\tilde{\gamma}_T^{(-)}$ couplings, there is a choice of sign. These signs do not enter the calculation of the differential cross sections, as the V and T pole terms are relatively imagninary. However, when considering polarization we must assign a definite relative sign; one choice gives zero polarization, while the other gives maximal polarization. For the present, since it is irrelevant to most of our discussion, we will assign relative (+) signs for both $\tilde{\gamma}_V^{(+)}/\tilde{\gamma}_T^{(+)}$ and $\tilde{\gamma}_V^{(-)}/\tilde{\gamma}_T^{(-)}$, for baryons (not antibaryons). Also introducing $\alpha_V=\alpha_T$, and approximating $(M_1M_2)^{1/2}$ by $\bar{M}$, we obtain

$$\begin{aligned} f_1 &= \kappa_1 Z^{\alpha(t)} e^{-i\pi\alpha(t)/2} \{-\tilde{\gamma}^{(+)} + \alpha(t)\tilde{\gamma}^{(-)} \\ &\quad \times [(W-\bar{M})Z^{-1} + m^2\bar{M}^{-1}]\} F_M(t), \\ f_2 &= \kappa_2 Z^{\alpha(t)} e^{-i\pi\alpha(t)/2} \{\tilde{\gamma}^{(+)} + \alpha(t)\tilde{\gamma}^{(-)} \\ &\quad \times [(W+\bar{M})Z^{-1} - m^2\bar{M}^{-1}]\} F_M(t), \end{aligned} \tag{13}$$

where

$$F_M(t) = \{i\tilde{\sigma}_V(t)\sec[\pi\alpha(t)/2] + \tilde{\sigma}_T(t)\csc[\pi\alpha(t)/2]\} \times [2\alpha(t)+1],$$

the residues $\tilde{\gamma}$ depend on the final state baryon, and the $\tilde{\sigma}$'s depend on the initial and final meson; $(V,T) = (\rho,R)$ or (K^*,Q) according to the quantum numbers exchanged.

At high energies such that the center-of-mass momenta in initial and final states are both large compared to mass differences,

$$\kappa_1 - \kappa_2 \cong 2M\pi^{1/2}/kW,$$

$$\kappa_1 + \kappa_2 \cong 2E\pi^{1/2}/kW.$$

[24] A. Ahmadzadeh and C. H. Chan, Phys. Letters 22, 692 (1966).

Using these approximations, and $M \cong \bar{M}$, substituting (13) into (2), we obtain

$$\begin{aligned} G_+ &= (4\pi M^2/k^2 s)^{1/2} (\cos\tfrac{1}{2}\theta) \\ &\quad \times Z^{\alpha(t)} e^{-i\pi\alpha(t)/2} F_M(t) F_B^{(+)}(s,t), \\ G_- &= (E/M)(4\pi M^2/k^2 s)^{1/2} (\sin\tfrac{1}{2}\theta) \\ &\quad \times Z^{\alpha(t)} e^{-i\pi\alpha(t)/2} F_M(t) F_B^{(-)}(s,t), \end{aligned} \tag{14}$$

where

$$F_B^{+} = -\tilde{\gamma}^{(+)} + \alpha\tilde{\gamma}^{(-)}(m^2/M)[(2E/W)-(1-2M^2/s)],$$

$$F_B^{-} = -\tilde{\gamma}^{(+)} + \alpha\tilde{\gamma}^{(-)}(m^2/M) \times [(1-2M^2/s)+2M^2/EW].$$

Define $C^+(t)=\tilde{\gamma}^{(+)}(t)$, $C^-(t)=m^2\tilde{\gamma}^{(-)}(t)/M$. At sufficiently high energies we can use the following approximations:

$$\begin{aligned} F_B^{(+)} &\cong -C^+(t), \\ F_B^{(-)} &\cong -C^+(t) + \alpha(t) C^-(t). \end{aligned} \tag{15}$$

We will, for simplicity, use (15) in comparisons with experimental cross sections, but look for quantitative agreement only at high energies.

The differential cross section now becomes

$$\frac{d\sigma}{dt} = (8\pi M^2 m^2 k^{-2}) Z^{2\alpha(t)-1} |F_M(t)|^2 \{[C^+(t)]^2 \cos^2(\theta/2) + [C^+(t)+\alpha(t)C^-(t)]^2 (E^2/M^2) \sin^2(\theta/2)\}. \tag{16}$$

For sufficiently small $|t|$ we obtain

$$(d\sigma/dt)_0 \cong (8\pi M^2 m^2 k^{-2}) Z^{2\alpha(0)-1} |F_M(0)|^2 [C^+(0)]^2. \tag{17}$$

For conciseness, we denote $\tilde{\sigma}_{V,T}(0)$ by $\mu_{V,T}$ henceforth. Then

$$|F_M(0)|^2 = \{\mu_V^2 \sec^2[\pi\alpha(0)/2] + \mu_T^2 \csc^2[\pi\alpha(0)/2]\} \times [2\alpha(0)+1]^2. \tag{18}$$

IV. FORMALISM FOR P-B^* REACTIONS

The formalism for the B^* reactions is more complicated, as more helicity states are involved. We will consider only the $\frac{3}{2}^+$, SU_3-decuplet baryon resonances in this paper since they allow several predictions to be made on the basis of SU_3 symmetry in the residue functions. A concise discussion of these reactions has been given by Hara,[25] who showed that if a static model for the B^* states is used, one obtains a single amplitude for N^* production (instead of 4 independent helicity amplitudes in general), and that this is the same ($M1$) as obtained in a Regge pole model with a vector-octet exchange whose couplings are determined near $t=0$ by the universality assumption[26] for (ρNN^*) coupling.

We will immediately adopt this assumption, as it seems to be consistent with the data available. The assumption of exchange degeneracy of baryon residues

[25] Y. Hara, Phys. Rev. 140, B178 (1965).

[26] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters 11, 90 (1963); L. Stodolsky, Phys. Rev. 134, B1099 (1964).

then requires the $R(Q)$ pole residues to have the $M1$ coupling to baryons also.

Thus the reaction t-channel amplitudes for $P+B \to P+B^*$ will be given by a generalization of Eq. (3.3a) of Hara,[25] allowing two meson poles (ρ and R octets) of opposite signature but degenerate trajectory. After a rearrangement of terms we obtain the t-channel amplitude:

$$T_{\frac{3}{2},\frac{1}{2}}(s,t)=s^{1/2}Z^{\alpha(t)}\alpha(t)C'(t)e^{-i\pi\alpha(t)/2}F_M(t)\,, \quad (19)$$

where we have extracted an over-all phase factor as in (14), and introduced BB^* residue factors $C'(t)$. These will be related to the $C^{(-)}$ terms in ρNN couplings by universality.[26]

With this expression for t-channel amplitudes, the differential cross section in the s channel and the B^* density matrix can be obtained according to Gottfried and Jackson.[27] The density matrix (as in the pure ρ-exchange model) is the one corresponding to pure $M1$ coupling of a vector meson. The differential cross section for $\pi^+p\to\pi^0N^{*++}$ at small angles and high energy will be given in terms of the $\pi^-p\to\pi^0n$ helicity-flip amplitude by relation (1.4) of Hara,[25] assuming universal ρ couplings of the residues of the ρ pole;

$$(d\sigma/dt)_{(N^{*++})}=\tfrac{3}{2}(d\sigma/dt)_{\text{H.F.}(n)}\,. \quad (20)$$

The right-hand side will be given by the (ρ) analog of (16), using

$$\left(\frac{d\sigma}{dt}\right)_{\text{H.F.}(n)}=|G_-|_\rho{}^2 \quad (21)$$

and residues appropriate to the π^-p charge-exchange reaction. According to the analysis of Höhler *et al.*,[28] the C^+ terms are negligible compared to αC^- in $(G_-)_\rho$; so we can directly relate C' to $C_\rho{}^{(-)}(NN)$.

In analyzing B^* reactions we assume the $\tilde{\sigma}_{V,T}(t)$ are slowly varying for $|t|<0.50$, and replace them by $\mu_{V,T}$.

V. TRAJECTORY APPROXIMATIONS AND $SU(3)$

To fix the parameters of the model we assume the trajectories $\alpha(t)$ are linear functions of t in the regions of t relevant to peripheral reactions and physical resonances. With the assumption of exchange degeneracy the trajectories for (ρ,R) and (K^*,Q) can be determined by a linear extrapolation between (ρ,A_2) (masses)^2 and similarly for [$K^*(890)$, $K^*(1400)$]. The former extrapolation yields $\alpha_\rho=\alpha_R\cong0.5+0.90t$ for the isovector trajectories, and we will use

$$\alpha_{K^*}=\alpha_Q\cong0.25+0.90t \quad (22)$$

for the isodoublet trajectories, where t is in (BeV)^2.

The former is roughly consistent for small $|t|$ with the ρ trajectory directly determined by π^-p charge-exchange analysis. The latter shows a built-in feature of the model: hypercharge exchange will be suppressed relative to charge exchange by a factor which decreases with energy like $Z^{-0.50}$ in P-B final states. (The suppression factor for B^* production will be found to be a little more severe.)

The number of free parameters can be reduced much further under the assumption of exact $SU(3)$ symmetry for the residues.

For example, the ratios of $C^{(-)}$ for (Λ) and (Σ) can be fixed under the assumption that a 1^- meson octet containing ρ and K^* dominates the electromagnetic current operators, if we use the empirical ratio of neutron-proton magnetic moments. [No additional predictions concerning K^* couplings follow from assuming $SU(6)$ symmetry if we use this experimental magnetic moment ratio.] This yields a D/F ratio of $\frac{3}{2}$ for the total baryon magnetic moment form factors, corresponding to the $C^{(-)}$ coefficients (helicity flip).

The $C^{(+)}$ terms are proportional to the baryon charge form factors (at small q^2), and assuming a conserved F-spin current coupling for the vector octet[29] gives pure F coupling for these. An additional condition for the ratios $C^{(-)}/C^{(+)}$, might be obtained by assuming the residues vary little in the interval $0<t<M_{K^*}{}^2$ and employing the empirical value of the neutron (or proton) magnetic moment. However, such a condition is not found to be consistent with the ρ-exchange analysis of π^-p charge exchange.[28] This ratio must be determined from $t=0$ data.

The degree to which one can reasonably expect F coupling for the vector meson trajectories to hold can be estimated by considering the results of Barger and Olsson,[30] who analyzed total cross section differences at high energies. They conclude on the basis of neutral nonstrange vector meson contributions that $D/F=-0.5\pm0.1$ for the C^+ couplings of the vector octet, instead of $D/F=0$. Thus our SU_3 relations in what follows should be taken essentially only as a guide to interpretation by suggesting reasonable parameter values. With the limited accuracy presently available for experimental data, however, we must actually employ SU_3 relations to reduce the number of free parameters in the model. Analysis involving only K,

TABLE I. F and D coefficients required for utilizing SU_3 symmetry in residues $C^\pm$.

	F	D
$K^{*+}\Lambda$	$-(\frac{3}{2})^{1/2}$	$-6^{1/2}$
$K^{*0}\Sigma^+$	$+1$	-1
ρ^+n	$+1$	$+1$

[27] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

[28] G. Höhler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966).

[29] J. J. Sakurai, in *Theoretical Physics* (International Atomic Energy Agency, Trieste, 1963).

[30] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965).

TABLE II. Summary of cross-section data and model predictions for the polarization.

Reaction	Momenta (BeV/c)	Reference	Sign of predicted polarization ($\alpha_-=\frac{3}{2}$)
$\pi^-p\to\pi^0n$	2 –18	32	0
ηn	3 –18	34	0
π^0N^{*0}	1.7–2.5	16	
$K^0\Sigma^0$	1.5–4.2	5,6	$-$
$K^0\Lambda^0$	1.5–4.2	5,6	+
$K^0Y_1^{*0}$			
$\pi^+p\to\pi^0N^{*++}$	1.6–8.0	38	
ηN^{*++}	4,8	38	
$K^+p\to K^0N^{*++}$	2.3–5	40	
$K^+n\to K^0p$	2.3	35	
$K^-p\to\bar{K}^0n$	2 –10	2,33	$-$
$\bar{K}^0N^{*0}$	3,4.1,5.5	39,41	
K^-N^{*+}	4.1,5.5	39	
$\pi^0\Lambda$	2 –5.5	3,4,37,39,41	($-$) input
$\eta\Lambda$	2 –5.5	3,4,37,39,41	+
$\pi^-\Sigma^+$	2 –5.5	3,4,37,39,41	+
$\pi^-Y_1^{*+}$	2 –5.5	3,4,37,39,41	

π, and η ratios by Ahmadzadeh[24] shows that excellent agreement may be achieved.

Explicitly, in exact SU_3 symmetry, each $V_8B_8\bar{B}_8$ vertex part (C^+ or C^-) can be written

$$C_V^{\pm}(B)=\bar{C}^{\pm}[F_{Vp}{}^B+\alpha_{\pm}D_{Vp}{}^B], \qquad (23)$$

where $\alpha_-\cong\frac{3}{2}$ and $\alpha_+\cong 0$; $\bar{C}^{\pm}$ are common factors for the octet; F and D are symbols for $SU(3)$ Clebsch-Gordon coefficients and have values as shown in Table I. These were obtained from the tabulation of McNamee and Chilton.[31] The last line of Table I allows the connection with ρ exchange in $K^-p\to\bar{K}^0n$ and $\pi^-p\to\pi^0n$.

With the values for $\alpha_{\pm}$ given above, we find for example:

$$C^+(\Lambda)/C^+(\Sigma)=-(\tfrac{3}{2})^{1/2},\quad C^-(\Lambda)/C^-(\Sigma)=(\tfrac{3}{2})6^{1/2}. \qquad (24)$$

The SU_3-symmetric meson-meson vertex parts (μ's) are determined within the octets, since the even-parity trajectories (R,Q) have only D-type coupling to two pseudoscalar mesons, while the odd-parity trajectories (ρ,K^*) couple by F only to two pseudoscalars; these restrictions follow from symmetry of the P-P wave function. Thus we can uniquely predict within exact SU_3, again with the aid of Clebsch-Gordan coefficient tables,[31]

$$\mu_{K^*}(K\eta)/\mu_{K^*}(K\pi^0)=+3^{1/2},\quad \mu_Q(K\eta)/\mu_Q(K\pi^0)=-\tfrac{1}{3}{}^{1/2}. \qquad (25)$$

Note the qualitative prediction that the Λ polarization with η is comparable in magnitude but opposite in sign to that with π^0; this is supported by preliminary data[4] at small angles. Similarly, if $\alpha_-\approx\frac{3}{2}$, and $|C_-|\gg|C_+|$, the relations (26) imply that the polarization of Σ^+ (in $K^-p\to\pi^-\Sigma^+$) should be of opposite sign to that of the Λ_0 (in $K^-p\to\pi^0\Lambda^0$). This, too, is supported by experiment.[4]

[31] P. McNamee and F. Chilton, Rev. Mod. Phys. 36, 1006 (1964).

The reactions we will consider are listed in Table II, together with the energies at which some data is presently available.

We will emphasize the comparison of the model with cross sections as close to $t=0$ as possible, in order to isolate the helicity-nonflip amplitudes which in the high-energy, small-angle limit are proportional to the C^+ couplings. We ignore temporarily the question of the D/F ratio for the C^- terms; a brief discussion concerning polarizations will be given after the forward cross-section data have been examined.

As input to the model, we choose forward π^-p charge exchange[32] to determine $[C_p{}^+(n)]^2\times[\mu_\rho(\pi\pi)]^2$.

If the exchange-degeneracy arguments (based originally on baryon-antibaryon bound states) are extended to $K\bar{K}$ bound states (i.e., Regge poles), we expect (as discussed previously for baryon couplings) either

$$\mu_R(KK)=+\mu_\rho(KK)\quad\text{or}\quad \mu_R(KK)=-\mu_\rho(KK). \qquad (26)$$

If one of these is assumed to hold, the normalizations for all forward cross sections in Table II are determined, given a value of the D/F ratio α_+, in terms of one parameter, which we take to be the forward π^-p charge-exchange cross section. We will assume this is true in comparing with the data; it will be seen that the qualitative features of the forward cross sections are compatible with this assumption, although $SU(3)$ breaking may confuse the issue.

Note the $K\bar{K}$-exchange-degeneracy assumption as stated above involves a choice of sign. These cases may be distinguished by an examination of $[\sigma_T(K^+n)-\sigma_T(K^+p)]$ and $[\sigma_T(K^-n)-\sigma_T(K^-p)]$. As observed by Ahmadzadeh,[13,24] the former vanishes, while the second is roughly comparable to $[\sigma_T(\pi^-p)-\sigma_T(\pi^+p)]$. The ($KN,\bar{K}N$) cross-section differences are linearly related to ($\mu_\rho+\mu_R$) and ($\mu_\rho-\mu_R$), respectively; thus the vanishing of the KN difference implies $\mu_\rho=-\mu_R$ for K^+ reactions. (The author has been unable to discover any *a priori* reason for this sign assignment, as with the baryon couplings.)

This determination of relative sign is not important in computing the forward cross sections, since the V and T poles are relatively imaginary. However, in estimating polarization, the sign will depend on the relative sign of μ_V and μ_T.

Comparison[24] of the data on $\pi^-p\to\pi^0n$, K^-p charge exchange,[33] and $\pi^-p\to\eta n$ [34] (there is insufficient data

[32] I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters 14, 408 (1965); A. V. Stirling *et al.*, *ibid.* 14, 763 (1965); P. Sonderegger *et al.*, Phys. Letters 20, 75 (1966).
[33] P. Astbury *et al.*, Phys. Letters 16, 328 (1965).
[34] O. Guisan *et al.*, Phys. Letters 18 200 (1965).

on K^+n charge exchange[35]) using exchange degeneracy has shown striking agreement for all angles with the differential cross sections based on the first of these reactions. Thus, we concentrate on their energy dependence, the HCE reactions, and B^* cross sections.

The ratio (at $t\cong 0$)

$$\frac{d\sigma}{dt}(K^-p\to\Lambda\pi^0)\Big/\frac{d\sigma}{dt}(K^-p\to\Sigma^+\pi^-) \qquad (27)$$

is very sensitive to the D/F ratio (α_+) of the C^+ terms, and this ratio may be used as a free parameter. Then applying $SU(3)$ and universality, the near-forward cross sections (numerical values and energy dependence) of all reactions in Table II are determined by these two parameters.

The C's in B^* reactions are related by $SU(3)$ within the class of B^* reactions, and by universality of ρ couplings

$$[C_\rho'(pN^{*++})]^2=(\tfrac{3}{2})[C_\rho^-(pn)]^2. \qquad (28)$$

The separate factors μ and C can be normalized in any way such that their product yields the correct amplitudes. We choose to set $\mu_\rho(\pi\pi)=1$, and absorb the normalization factors in the C's.

The data will now be examined, and predictions of the model (based on the best known forward cross sections) will be compared with available cross sections for the reactions in Table II. A previous N^* comparison, based only on ρ exchange, by Roy[36] has been done, with more incomplete data; and Ahmadzadeh and Chan have shown[24] that the relations implied by (16) ,(22), and $SU(3)$ for nonstrange baryon reactions [i.e., not involving $\alpha\pm$] are well satisfied for $|t|<0.50$.

From $\mu_R(KK)$ [and $\mu_\rho(\pi\pi)=1$], all other μ's for the reactions of Table II can be obtained assuming $SU(3)$ symmetry;

$$\mu_{K^*}(K^-\pi^-)=2^{1/2}\mu_{K^*}(K^-\pi^0)=2^{-1/2} \qquad (29a)$$

$$\mu_{K^*}(K^-\eta)=-3^{1/2} \qquad (29b)$$

$$\mu_\rho(K^-\bar{K}^0)=\mu_R(K^-\bar{K}^0)=2^{-1/2} \qquad (29c)$$

$$\mu_R(\pi^-\eta)=(\tfrac{2}{3})^{1/2}\mu_R(K^-\bar{K}^0)=3^{-1/2} \qquad (29d)$$

$$\mu_Q(K^-\pi^-)=2^{1/2}\mu_Q(K^-\pi^0)=2^{-1/2} \qquad (29e)$$

$$\mu_Q(K^-\eta)=-1/(2\times 3^{1/2}). \qquad (29f)$$

The relation (29c) can be obtained from exchange degeneracy and the universality assumption, aside from exact $SU(3)$ symmetry. The relations (29b), (29d), and (29f) are based on a pure octet η (no singlet mixing).

Turning now to the C's, we obtain the following relations: for $\alpha_+=0$ (pure F coupling of C^+),

$$\begin{aligned}C_{K^*}{}^+(\Lambda)&=-(\tfrac{3}{2})^{1/2}C_\rho{}^+(n)\\ C_{K^*}{}^+(\Sigma^+)&=C_\rho{}^+(n)\end{aligned} \qquad (30)$$

and, as previously assumed (independent of α_+),

$$\begin{aligned}C_Q{}^+(\Sigma^+)&=C_{K^*}{}^+(\Sigma^+)\\ C_Q{}^+(\Lambda)&=C_{K^*}{}^+(\Sigma^+).\end{aligned} \qquad (31)$$

Then for every P-B HCE reactions we can write

$$\left(\frac{d\sigma}{dt}\right)_0\cong|F_M(0)|^2(4\pi M^2s/k^2s_0{}^2)Z^{2[\alpha(0)-1]}\times[C_{K^*}{}^+(B)]^2 \qquad (32)$$

where $\alpha=\alpha_{K^*}$. For CE reactions, $(K^*,Q)\to(\rho,R)$. If $\alpha_+=0$ were correct, we would obtain

$$\frac{(d\sigma/dt)_0(K^-p\to\pi^-\Sigma^+)}{(d\sigma/dt)_0(K^-p\to\pi^0\Lambda)}=\tfrac{4}{3}. \qquad (33)$$

According to available data, at 3.5 GeV/c [37] the left side of (33) is 1.2±0.1. This corresponds to $\alpha_+\cong 0$, the expected value. The (Σ/Λ) ratios are the only quantities sensitive to this ratio. This result may be compared with Barger and Olsson's[30] value of −0.50±0.10.

Now all the forward P-B cross sections (except for the input reaction, π^-p charge exchange) can be expressed in forms of the coupling parameter $[C_\rho{}^+(n)]^2$, and α_+. Since deviations of α_+ from 0 essentially affect only the (Σ/Λ) ratios, we use $\alpha_+=0$ to determine the Λ reactions for simplicity; the Σ^+ reactions at $t=0$ then are predicted always to be slightly larger than the Λ reactions.

Denoting by σ the value of $(d\sigma/dt)$ at $t=0$ divided by the value of $(d\sigma/dt)(\pi^-p\to\pi^0n)$ [at the same energy] at $t=0$, we obtain

$$\sigma(\pi^-p\to\eta n)=\mu_R{}^2(\pi\eta)\cot^2[\pi\alpha_\rho(0)/2]=\tfrac{1}{3}, \qquad (34a)$$

$$\sigma(K^+n\to K^0p)=\sigma(K^-p\to\bar{K}^0n), \qquad (34b)$$

$$\sigma(K^-p\to\bar{K}^0n)=\mu_p{}^2(KK)+\mu_R{}^2(KK)\times\cot^2[\pi\alpha_\rho(0)/2]=1, \qquad (34c)$$

$$\sigma(\pi^-p\to K^0\Lambda)=2\sigma(K^-p\to\pi^0\Lambda), \qquad (34d)$$

$$\begin{aligned}\sigma(K^-p\to\pi^0\Lambda)=(\tfrac{3}{2})Z^{-\Delta}&\left[\frac{\alpha_{K^*}(0)+\frac{1}{2}}{\alpha_\rho(0)+\frac{1}{2}}\right]^2\cos^2[\pi\alpha_\rho(0)/2]\\ &\times\{\mu_{K^*}{}^2(K\pi^0)\sec^2[\pi\alpha_{K^*}(0)/2]\\ &+\mu_Q{}^2(K\pi^0)\csc^2[\pi\alpha_{K^*}(0)/2]\}\cong\tfrac{3}{2}Z^{-\Delta}\\ &\times(\tfrac{3}{4})^2[\tfrac{1}{8}+\tfrac{1}{8}(8/\pi)^2]\cong 0.84Z^{-\Delta},\end{aligned} \qquad (34e)$$

$$\begin{aligned}\sigma(K^-p\to\eta\Lambda)=\tfrac{3}{2}Z^{-\Delta}&\left[\frac{\alpha_{K^*}(0)+\frac{1}{2}}{\alpha_\rho(0)+\frac{1}{2}}\right]^2\cos^2[\pi\alpha_\rho(0)/2]\\ &\times\{\mu_{K^*}{}^2(K\eta)\sec^2[\pi\alpha_{K^*}(0)/2]\\ &+\mu_Q{}^2(K\eta)\csc^2[\pi\alpha_{K^*}(0)/2]\}\cong 0.65Z^{-\Delta}.\end{aligned} \qquad (34f)$$

[35] I. Butterworth *et al.*, Phys. Rev. Letters **15**, 734 (1965). Interpretations of these results follow Ref. 9.

[36] D. P. Roy, Nuovo Cimento **40**, 513 (1965).

[37] N. Hague *et al.*, report to the Oxford International Conference on Elementary Particles, 1965 (unpublished) and unpublished report.

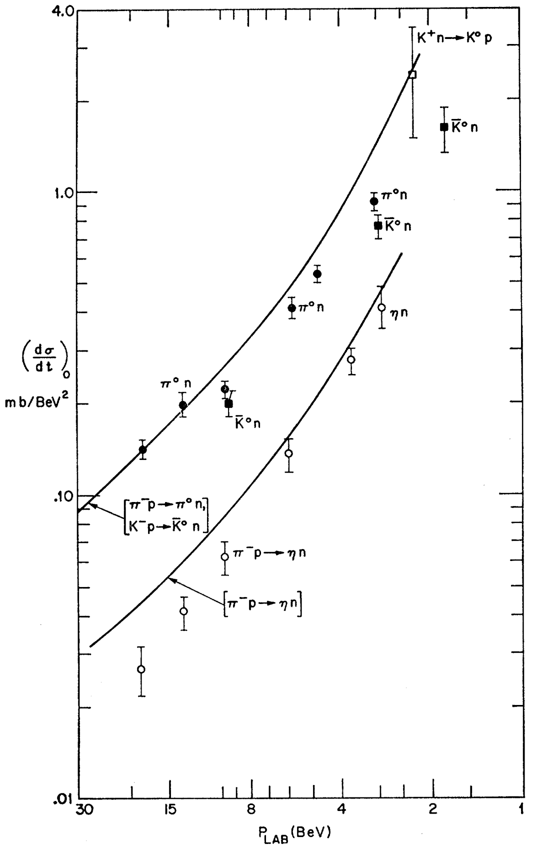

FIG. 2. Forward differential cross sections for *P-B* final states with charge exchange; reactions are identified in Table II. Solid curves are predictions of model, normalized to 18.5 BeV/c $\pi^-p\rightarrow\pi^0n$ datum.

Note: The prediction (34e) is particularly sensitive to the value of $\alpha_{K^*}(0)$. We have used $\alpha_{K^*}(0)=0.25$, as this corresponds to a slope of 1 BeV,$^{-2}$ parallel to the ρ trajectory. Ahmadzadeh obtained[13] $\alpha_{K^*}(0)=0.35$ by extrapolation. The difference will not be noticeable until an order of magnitude increase in accuracy of data can be achieved.

The predictions (34), with $\pi^-p\rightarrow\pi^0n$ as input, are compared in Figs. 2 and 3 with available data.[1–6,32–35,37–41]

In the HCE reactions at high energy the statistical accuracy is poor and in many cases only a total cross section is quoted. In these instances it was necessary to roughly estimate $(d\sigma/dt)_0$; a very crude estimate (the errors indicated on the graph are increased, over statistical errors, to $\pm50\%$ on these points) may be achieved, if one assumes that the cross sections of most 2-body states have an exponential behavior near $t=0$:

$$d\sigma/dt\approx(d\sigma/dt)_0e^{At},$$

where $A\approx10$ (BeV/c)$^{-2}$, as in elastic and charge-exchange scattering. Then the forward cross section can be roughly estimated from

$$(d\sigma/dt)_0\approx A\sigma_T.$$

Cross sections based on fewer than 10 events are not plotted in Figs. 2 and 3.

Roughly estimated data points (in Fig. 3) are marked with a dagger. At 4.1 BeV/c, $A\sim6$; at 3.5 BeV/c, $A\sim3.0-3.3$ as determined by fits to data.

For the B^* reactions, we employ (21.) and (30.) to obtain (with $|t|\ll M_\rho^2$) for $\pi^+p\rightarrow\pi^0N^{*++}$:

$$\frac{d\sigma}{dt}\cong\left(\frac{4\pi M^2s}{k^2s_0^2}\right)Z^{2[\alpha(t)-1]}\times\frac{3}{2}\left[\frac{EC_\rho^-(n)}{2M}\right]^2|F_{\pi\pi}(t)|^2\alpha^2(t)\sin^2\frac{\theta}{2}. \quad (35)$$

We have written $s_0=2m^2$. Note: the pure ρ-exchange reactions are predicted to have a zero in $(d\sigma/dt)$ at $\alpha_\rho(t)=0$, which occurs at $t\cong-0.50$. However, if R and Q exchanges are allowed, the B^* cross section will not have such a pronounced dip in the physical region, since the factor $\alpha(t)$ cancels with $(\sin\pi\alpha)^{-1}$, and is not present in those terms.

This includes $\pi^+p\rightarrow\eta N^{*++}$, which is predicted to have thereby a considerably larger width of the forward peak than $\pi^+p\rightarrow\pi^0N^{*++}$, as observed at 8 BeV/c.

The predicted zero in $d\sigma/dt(\pi^+p\rightarrow\pi^0N^{*++})$ will be charged by absorptive corrections into a pronounced dip.

To compare B^* cross sections at various energies it is convenient to express $\sin^2(\theta/2)$ in terms of t. For example, (35) can be written:

$$\frac{d\sigma}{dt}\cong(E^2/4k^2)(4\pi M^2s/k^2s_0^2)Z^{2[\alpha(t)-1]}(\tfrac{3}{2})\times[C_\rho^-(n)]^2|F_{\pi\pi}|^2\alpha^2(t)[-t/M^2] \quad (36)$$

(for $\pi^+p\rightarrow\pi^0N^{*++}$.)

Thus, *at fixed t*, these cross sections will have an energy dependence comparable to that of the helicity-flip contributions to the *P-B* CE [or HCE] reactions. To the extent that the (logarithmic) change in shape of $d\sigma/dt$ can be ignored, this will imply such an energy dependence in the total cross sections.

Since the accuracy of the data on angular distributions is poor at high energies, and since we do not wish

[38] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965); Aachen-Berlin-CERN collaboration, Phys. Letters **19**, 608 (1965); Saclay-Orsay-Bologna-Bari Collaboration, *ibid.* **13**, 341 (1964).

[39] For Y_1^* and N^* data: M. Derrick, T. Fields, F. Schweingruber, and other members of the Argonne National Laboratory-Northwestern University collaboration (private communication). See also M. Derrick *et al.*, in *Second Topical Conference on Resonant Particles* (Ohio University Press, Athens, Ohio, 1965). For $\Sigma^+\pi^-$ data: U. E. Kruse and J. Loos, preliminary values (private communication). For $\Delta\pi^0$ data: D. Reeder, preliminary values (private communication).

[40] G. R. Lynch *et al.*, Phys. Letters, **9**, 359 (1964); M. Ferro-Luzzi *et al.*, Nuovo Cimento **36**, 1101 (1965); E. Boldt *et al.*, Phys. Rev. **133**, B220 (1964).

[41] J. Badier *et al.*, paper presented at the International Conference on Elementary Particles, Oxford, 1965 (unpublished).

to compare the detailed behavior of the residue and trajectory functions for $t\neq 0$, it will suffice in the present work to qualitatively examine the ρ-exchange angular distributions to see if they are consistent with (36) for small $|t|$ (i.e., show a dip); we will quantitatively compare only the total cross sections for B^* reactions.

The consistency of the normalization condition (28) has been discussed by Carroll, *et al.*,[16] in the region around 2 BeV/c, where they subtract resonance contributions. At those energies consistency was verified. Here we concentrate on (1.) the energy dependence of N^* production total cross sections, and (2.) the ratio of predicted Y_1^* to predicted N^* cross sections at each energy.

According to (36), ignoring (logarithmic) changes in the shape of $d\sigma/dt$,

$$\sigma_T(N^*)\propto E^2k^{-4}s^{2\langle\alpha_\rho\rangle-1} \quad (37a)$$

$$\sigma_T(Y^*)\propto E^2k^{-4}s^{2\langle\alpha_{K^*}\rangle-1} \quad (37b)$$

where $\langle\alpha\rangle$ is an average of $\alpha(t)$ over the region of maximum contribution in $d\sigma/dt$ vs t. From an inspection of the data,[38] for ρ this is around $t\cong-0.10$, which yields $\langle\alpha_\rho\rangle\cong 0.40$; similarly for $\langle\alpha_{K^*}\rangle$, we estimate[37] (from $t\approx-0.2$)

$$\langle\alpha_{K^*}\rangle\cong 0.$$

Thus

$$\sigma_T(Y^*)/\sigma_T(N^*)\propto Z^{-0.80}. \quad (38)$$

The ratios of $SU(3)$ couplings for the reactions in Table II yield:

$$\frac{d\sigma}{dt}(\pi^0N^{*++})=|\alpha_\rho(t)F_{\pi\pi}(t)|^2H(s,t)$$
$$\cong\{\alpha_\rho(t)\sec[\pi\alpha_\rho(t)/2][2\alpha_\rho(t)+1]\}^2H \quad (39a)$$

$$\frac{d\sigma}{dt}(\eta N^{*++})=|\alpha_\rho(t)F_{\pi\eta}(t)|^2H(s,t)$$
$$\cong(\tfrac{1}{3})\{\alpha_\rho(t)\csc[\pi\alpha_\rho(t)/2][2\alpha_\rho(t)+1]\}^2H \quad (39b)$$

$$\frac{d\sigma}{dt}(K^0N^{*++})=|\alpha_\rho(t)F_{KK}(t)|^2H\cong\tfrac{1}{2}\alpha_\rho^2(t)[2\alpha_\rho(t)+1]^2$$
$$\times\{\sec^2[\pi\alpha_\rho(t)/2]+\csc^2[\pi\alpha_\rho(t)/2]\}H \quad (39c)$$

$$\frac{d\sigma}{dt}(\pi^-Y_1^{*+})=(\tfrac{1}{3})Z^{-\Delta}|\alpha_{K^*}(t)F_{K\pi}(t)|^2$$
$$\cong\tfrac{1}{3}Z^{-\Delta}\alpha_{K^*}{}^2(t)[2\alpha_{K^*}(t)+1]^2$$
$$\times\{\tfrac{1}{2}\sec^2[\pi\alpha_{K^*}(t)/2]+\tfrac{1}{2}\csc^2[\pi\alpha_{K^*}(t)/2]\}H \quad (39d)$$

where

$$H(s,t)=(\pi/2)(E^2m^{-2}k^{-2})Z^{2\alpha_\rho(t)-1}[\sin^2(\theta/2)]$$
$$\times\{3[C_\rho^-(n)]^2/2\}\mu_\rho^2(\pi\pi).$$

The reaction cross sections for $\pi^-p\to\pi^0N^{*0}$, $\pi^-p\to K^0Y_1^{*0}$, $K^-p\to\bar{K}N^{*0}$, and $K^-p\to K^-N^{*+}$ may be obtained from those given above by isospin invariance in the residues.

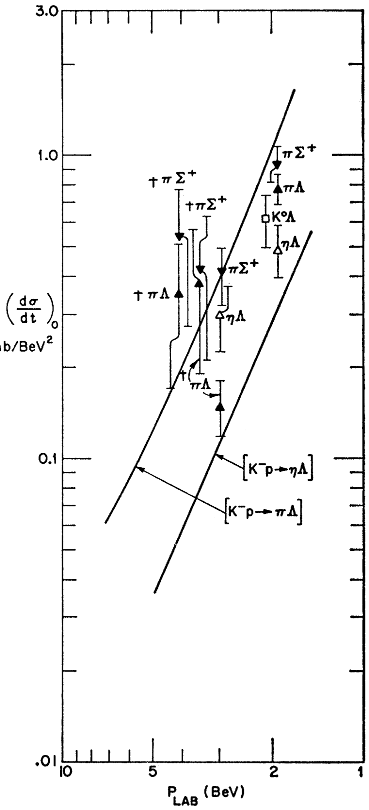

FIG. 3. Forward differential cross sections for *P-B* final states with hypercharge exchange; reactions are identified in Table II. Solid curves are predictions of model, with normalization taken (as in Fig. 2) from $\pi^-p\to\pi^0n$ at 18.5 BeV/c. K^+n charge exchange should coincide with K^-p CE results if model is correct.

Since there are too few data at high energies to compare the detailed shape ratios of (39a)–(39d), as was done[24] in the nonstrange *P-B* reactions, we will examine only the total cross sections.

The corresponding total cross sections are related by

$$\langle\sin^2[\pi\alpha_\rho/2]\rangle\sigma(\eta N^{*++})=\tfrac{1}{3}\langle\tan^2[\pi\alpha_\rho/2]\rangle$$
$$\times\sigma(\pi^0N^{*++}), \quad (40a)$$

$$\langle\sin^2[\pi\alpha_\rho/2]\rangle\sigma(K^0N^{*++})=\tfrac{1}{4}\langle\sec^2[\pi\alpha_\rho/2]\rangle$$
$$\times\sigma(\pi^0N^{*++}), \quad (40b)$$

$$\sigma(K^-p\to\pi^-Y_1^{*+})=\tfrac{1}{6}Z^{-\Delta}\sigma(\pi^0N^{*++})$$
$$\times\left\{\left\langle\left[\frac{\alpha_{K^*}(\alpha_{K^*}+\frac{1}{2})\cos[\pi\alpha_\rho/2]}{\alpha_\rho(\alpha_\rho+\frac{1}{2})\cos[\pi\alpha_{K^*}/2]}\right]^2\right\rangle+\left\langle\left[\frac{\alpha_{K^*}(\alpha_{K^*}+\frac{1}{2})\cos[\pi\alpha_\rho/2]}{\alpha_\rho(\alpha_\rho+\frac{1}{2})\sin[\pi\alpha_{K^*}/2]}\right]^2\right\rangle\right\}, \quad (40c)$$

where $\Delta\cong 0.80$ [cf. discussion preceding (38)].

Here $\langle\cdots\rangle$ means an average over t, weighted by H. To compare these relations with the data, we estimate

$$\langle\sin^2(\pi\alpha_\rho/2)\rangle=0.40, \quad (41a)$$

$$\langle\tan^2(\pi\alpha_\rho/2)\rangle=0.50, \quad (41b)$$

$$\langle\sec^2(\pi\alpha_\rho/2)\rangle=2.0, \quad (41c)$$

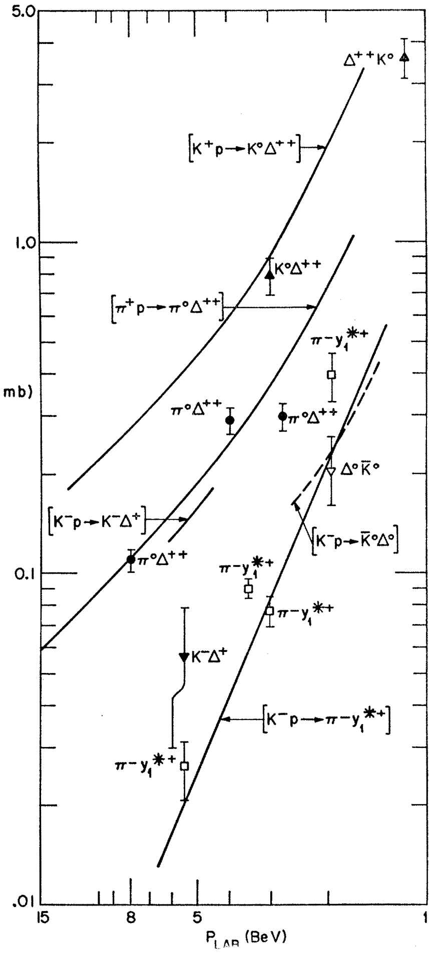

FIG. 4. Total cross sections for P-B^* final states; reactions may be identified in Table II. Curves are predictions of model, with normalization to 8 BeV/c $\pi^0\delta^{++}$ cross section (consistent with universality and π^-p CE, as discussed in text).

$$\left\langle\left[\frac{\alpha_{K^*}(\alpha_{K^*}+\frac{1}{2})\cos(\pi\alpha_p/2)}{\alpha_p(\alpha_p+\frac{1}{2})\cos(\pi\alpha_{K^*}/2)}\right]^2\right\rangle=0.0\,, \qquad (41\text{d})$$

$$\left\langle\left[\frac{\alpha_{K^*}(\alpha_{K^*}+\frac{1}{2})\cos(\pi\alpha_p/2)}{\alpha_p(\alpha_p+\frac{1}{2})\sin(\pi\alpha_{K^*}/2)}\right]^2\right\rangle=1.2\,. \qquad (41\text{e})$$

Note: The average (41e) strongly affects the ratio (40c); the predicted values seem to agree reasonably well with data, but this may be fortuitous. Comparison of $(d\sigma/dt)$ values at each t would be better.

From (37a) the cross section $\sigma(\pi^0N^{*++})$ should vary with energy like E^2k^{-4}. With 8-BeV/c lab momentum chosen as normalization point, Fig. 4 shows the comparison with this energy dependence and a comparison of the predictions (43) with available data.[3,37–41]

The relation of $\sigma(\pi^0N^{*++})$ to the π^-p charge-exchange cross section is not unambiguous, since we need to know C_p^- (or the helicity flip term) in the latter process. We previously have used only $C_p^{(+)}$. It appears, however,[25] that the helicity flip term actually dominates the *integrated* cross section (σ_T) for $\pi^-p\to\pi^0n$. Using this information, we obtain the approximate relation

$$\sigma_T(\pi^0N^{*++})\cong\tfrac{3}{2}\sigma_T(\pi^0n)\,. \qquad (42)$$

This comparison should be made at the highest available energy. Interpolating the right-hand side of (42) to 8 BeV/c [cf. Fig. 3 of Sonderegger *et al.*, Ref. 32] we obtain $\frac{3}{2}$ (66.5±2.0) μb=100±3 μb; on the left-hand side, at 8 BeV/c we find[38] 110±10 μb. This is in excellent agreement, probably better than should be expected since we have simply ignored the helicity nonflip cross section on the right of Eq. (42).

Finally, we consider the baryon polarizations, in the double pole model. From (14) and (15), if $\gamma_v^+/\gamma_t^+=\gamma_v^-/\gamma_t^-$ we find $\mathrm{Im}(G_+{}^*G_-)=0$. Thus the existence of appreciable polarization in the asymptotic energy region would preclude this choice of relative sign.

Assuming instead [since we desire polarization[4,6]] $[\gamma_v^{(-)}/\gamma_t^{(-)}]=-[\gamma_v^{(+)}/\gamma_t^{(+)}]$, we see it is sufficient for computing polarization to replace F_M by $F_M{}^*$ when it occurs multiplying C^- in (14), using (15). Then G_+ and G_- become

$$G_+\cong-(\cos\tfrac{1}{2}\theta)(8\pi M^2m^2/k^2)^{1/2}Z^{\alpha(t)-1/2}e^{-i\pi\alpha(t)/2}\times F_M(t)C^+(t)$$

$$G_-\cong(E/M)\sin(\theta/2)(8\pi M^2m^2/k^2)^{1/2}Z^{\alpha(t)-1/2}e^{-i\pi\alpha(t)/2}\times[-C^+(t)F_M(t)+\alpha(t)C^-(t)F_M{}^*(t)]\,.$$

For the polarization, we obtain

$$P(\theta)=\left(\frac{-E}{M}\sin\theta\right)\frac{\alpha(t)C^+(t)C^-(t)\{\mathrm{Im}[F_M{}^*(t)]^2/|F_M(t)|^2\}}{[C^+(t)]^2\cos^2(\frac{1}{2}\theta)+[C^+(t)-\alpha(t)C^-(t)]^2(E^2/M^2)\sin^2(\frac{1}{2}\theta)}\,.$$

Using the explicit form of F_M [at small t] in terms of μ_V and μ_T we obtain

$$P(\theta)=-(E/M)\sin\theta[\alpha(t)/\sin\pi\alpha(t)]\left\{\frac{2C^+C^-}{[C^+]^2\cos^2(\frac{1}{2}\theta)+[C^+-\alpha(t)C^-]^2(E^2/M^2)\sin^2(\frac{1}{2}\theta)}\right\}$$

$$\times\left\{\frac{2\mu_T\mu_V}{\mu_V{}^2\sec^2[\frac{1}{2}\pi\alpha(t)]+\mu_T{}^2\csc^2[\frac{1}{2}\pi\alpha(t)]}\right\}\,.$$

Note the factorization of $P(\theta)$ into meson and baryon terms.

We can compare various reactions differing by substitution of baryons assuming some value for $(D/F)_{\pm}=\alpha_{\pm}$, and the mesons can be changed without assumptions on (D/F). For example, the Λ polarization in $K^-p\rightarrow\pi^0\Lambda$ is opposite to the polarization in $K^-p\rightarrow\eta\Lambda$ because the product $\mu_V\mu_T$ changes sign; this does not depend on any D/F assumptions. Using $\alpha_+=0$ and $\alpha_-=\frac{3}{2}$, we can conclude that the polarization of Σ^+ in $K^-p\rightarrow\pi^-\Sigma^+$ is opposite in sign compared to the Λ in $K^-p\rightarrow\pi^0\Lambda$, since C^+ changes sign (but C^- does not) in comparing these two reactions. These facts seem to be consistent, within the rather large uncertainties, with data near 2 BeV/c.[3,4]

Note that the analysis of Phillips and Rarita of polarization in K^-p charge exchange[42] shows large polarizations at small angles; these authors did not consider the exchange degeneracy as a constraint, however, fitting to the differential cross section.

Also note that the detailed shape of the HCE forward peaks will depend strongly on α_-, since[25] $|C_\rho^-(n)|\gg|C_\rho^+(n)|$. Accurate measurements of angular distributions would then allow a determination of α_-.

The formalism for applying absorptive corrections to B final states in the model is given in the Appendix. There is some controversy over the domain of the applicability of the absorptive correction procedure[17] when vector-meson exchange is considered, and its relationship to the Regge-pole approach.[43] For the present application, it is plausible that such corrections will not seriously influence the ratios of the cross sections but may have considerable influence on the absolute magnitude of the amplitude.

IV. DISCUSSION AND REMARKS ON OTHER REACTIONS

In the CE P-B reactions (Fig. 2), it is apparent that the energy dependencies of the cross-sections are quite well described (at high enough momenta) by the value $\alpha(0)=0.50$ deduced from the physical A_2 and ρ masses. The absolute cross-section ratios are in error by two to four standard deviations. This can be interpreted in K reactions as $SU(3)$ breaking; in η production it is also possible that mixing with X^0 [$SU(3)$ singlet] changes the $SU(3)$ predictions sufficiently to account for the factor of two discrepancy observed. The exchange-degeneracy postulate may also not be accurate as concerns these couplings, and $\alpha_\rho(0)$ uncertainty may influence the comparison.[24]

In the HCE P-B reactions (Fig. 3), the available data on forward cross sections are too poor in accuracy to provide any definite tests. However, the decrease with energy and the order of magnitude of absolute cross section both support the model. More accurate data above 3 BeV/c are clearly needed.

In the P-B^* reactions, both CE and HCE, similar degrees of agreement and discrepancy may be noted in Fig. 4. Note in particular the good agreement with $\pi^+p\rightarrow\pi^0\Delta^{++}$ cross section at 4 BeV/c, and the more rapid decrease with energy of the $K^-p\rightarrow\pi^-Y_1^{*+}$ cross section (which roughly seems to be correctly normalized according to the model). As remarked previously, the universality [or $SU(6)_W$] prediction connecting Δ^+ with n is very well satisfied.

The polarization relation (Fig. 1) based on double poles without assuming exchange degeneracy clearly is consistent with the data, but does not provide much of a test of the model with presently available data. It would be very desirable to obtain even qualitative information on baryon polarizations at high energies, to see whether the polarization persists asymptotically, and if the signs indicated in Table II are satisfied.

The reactions $K^-p\rightarrow\Sigma^-\pi^+$, Ξ^-K^+, and Ξ^0K^0 also have been extensively studied[1,3] (at least up to 2 BeV/c), and are of the same kinematic type ($P_1+B_1\rightarrow B_2+P_2$ with exchange of hypercharge) as those discussed above. However, the double pole model proposed is inapplicable, since these reactions cannot proceed by the exchange of $T=\frac{1}{2}$, $Y=1$ states such as represented by $K^*(890)$ or Q trajectories.

Experimentally[1,3] there seems to be both forward and backward peaks in $\Sigma^-\pi^+$, with the forward peak [small $(-t)$] decreasing above 2 BeV/c. This is consistent with a picture containing t-channel exchange of a $T=\frac{3}{2}$ trajectory lying below $J=0$ at $t=0$, together with baryon-pole contributions which persist at high energies. The backward [large $(-t)$] peak is, however, much sharper than one would expect[44] from an elementary baryon-exchange formula; we will remark on this later. The fact that a $T=\frac{3}{2}$ meson trajectory (if present) must lie considerably lower than $J=0$ at $t=0$ is strongly suggested by the meson scheme of Ref. 13, as remarked therein.

In ΞK final-state reactions, little or no forward peak is observed above 2 BeV/c; a pronounced backward peak is present and apparently persists at higher energies. This picture is consistent with an absence of $|Y|=2$ mesons whose trajectories would be available for t-channel exchange; at least, such trajectories must lie very low in the J plane. The backward peak is most easily identified with baryon-pole contributions; in these reactions, the peaking is less sharp than in $\Sigma^-\pi^+$, but still sharper than expected from elementary baryon pole terms.

Aside from the appearance of this backward peak in the differential cross section, one other striking feature of the high-energy reaction $K^-p\rightarrow\Xi^-K^+$ is the large

[42] R. J. N. Phillips and W. Rarita, University of California Radiation Laboratory Report, 1965 (unpublished).

[43] J. S. Ball and W. R. Frazer, Phys. Rev. Letters **14**, 746 (1965). This paper contains references to earlier works. See also R. C. Arnold, Phys. Rev. **140**, B1022 (1965).

[44] C. H. Chan and Y. S. Liu, Nuovo Cimento **35**, 298 (1965); see also Iwao, Ref. 13.

average polarization of the Ξ^-, which apparently remains close to unity for K^- momenta above 1.8 BeV/c. If the reaction were dominated by a simple elementary baryon pole, no polarization would result.

This fact, together with the sharpness of the angular distribution, leads us to propose a reaction mechanism involving a pair of fermion Regge poles in the u channel with opposite signature but nearly coincident trajectories, analogous to our double pole model for the t channel. Such a pair of trajectories could be associated with the baryons and $\frac{3}{2}^+$ resonances; e.g., in $K^-p\to\Sigma^-\pi^+$ we expect n and $N_{\frac{3}{2}}{}^{*0}$ trajectories to contribute, while in $K^-p\to\Xi^-\pi^+$ we expect Λ, Σ^0 and $Y_1{}^{*0}(1385)$ trajectories as important contributions.

Other two-body reactions involving hypercharge exchange are the vector-meson production processes, e.g., $K^-p\to\phi\Lambda$, $\omega\Lambda$. In this class, some reactions (including the ones mentioned) apparently have important t-channel exchanges since there is a forward peaking of the vector meson. The parity selection rule forbidding P exchange is not present in these reactions, however, and we cannot rule out on *a priori* theoretical grounds considerable contribution from K exchange as well as from the K^* and Q. Since the K trajectory lies below $J=0$ at $t=0$, its contribution will decrease relatively to K^* and Q at sufficiently high energies. The relative contributions may be isolated by studying the density matrix of the vector meson.[45] In the $SU(3)$ analog reactions $K^\pm p\to K^{*\pm}p$, it is found that pseudoscalar exchanges are important [46] up to 1.5 BeV/c, but above 3 BeV/c the other terms are important.[47] In the experimental K^* production analyses to date it has been assumed that ω and ρ were the only vector exchanges that might dominate, but the $SU(3)$ octet companions of Q (which are P' and R) would on the basis of our double pole model be expected to be important.

If the double-pole picture survives experimental tests, it provides a common framework for analysis of many high-energy two-body nonelastic reactions, especially if augmented by pseudoscalar trajectory poles and absorptive corrections.[48,49] It appears that consideration of inelastic processes may be more fruitful than elastic scattering as concerns investigating the validity of the basic Regge-pole–dominance idea.

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APPENDIX A: ABSORPTIVE CORRECTIONS TO THE TWO-POLE MODEL

If we wish to apply the absorptive correction formulas of Jackson and Gottfried and of Durand and Chiu, each complex partial-wave helicity amplitude (defined below) as predicted by the model must be modified by a real multiplicative factor

$$S_J=1-C\exp(-J^2/J_0{}^2)\,, \tag{A1}$$

where C and J_0 are determined from the empirical K^-p elastic-scattering cross section at the same energy. Here we assume that final-state scattering is essentially the same as the initial-state (K^-p) scattering, that the elastic-scattering helicity-flip amplitudes are negligible, and that the bulk of the elastic scattering can be fitted by a simple exponential in t. The latter seems to be true down to 2 BeV/c. Explicitly, if the elastic scattering amplitude has an exponential behavior $\exp(At)$, and k is the center-of-mass K^- momentum, we have $J_0{}^2=4k^2A$ and $C=\sigma_{\text{tot}}/(8\pi A)$. For K^-p scattering at $p_{\text{lab}}=2$ BeV/c, $C\cong0.7$ and $J_0\cong5$.

The partial-wave helicity amplitudes $g_J{}^\pm(s)$ are defined by the decompositions (discussed by Jacob and Wick for pion-nucleon scattering):

$$\begin{aligned}G_+&=\cos\tfrac{1}{2}\theta\sum_{l=0}^{\infty}g_{l+1/2}{}^+[P_{l+1}'(\cos\theta)-P_l'(\cos\theta)]\\ G_-&=\sin\tfrac{1}{2}\theta\sum_{l=0}^{\infty}g_{l+1/2}{}^-[P_{l+1}'(\cos\theta)+P_l'(\cos\theta)]\end{aligned} \tag{A2}$$

which are more concisely written in terms of reduced rotation matrices:

$$G_\pm=\sum_J\,(J+\tfrac{1}{2})g_J{}^\pm d_{\pm\frac{1}{2},\frac{1}{2}}{}^J(\theta)\qquad J=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\cdots.$$

[45] J. D. Jackson and H. Pilkuhn, Nuovo Cimento 33, 906 (1964); see also Ref. 28.
[46] G. B. Chadwick *et al.*, Phys. Letters 6, 309 (1963).
[47] Birmingham–Glasgow–Imperial College–Oxford Collaboration, Phys. Letters 14, 338 (1965). Earlier works are referred to in this letter.
[48] J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. 139, B428 (1965).
[49] This point, as mentioned above, is controversial; see R. C. Arnold (this issue), Phys. Rev. 153, 1523 (1966).

Utilizing the orthogonality properties of the d^J functions, we obtain the uncorrected two-pole prediction for the g's through an inversion of these relations:

$$(J+\tfrac{1}{2})(g_J{}^{\pm})_{\rm pole}=\int_{-1}^{+1} d(\cos\theta)G_{\pm{\rm pole}}(s,\cos\theta)d_{\pm\frac{1}{2},\frac{1}{2}}{}^J(\theta)$$

where $G_{\pm{\rm pole}}$ is given by (14) in the text. In terms of Legendre polynomials, putting $z=\cos\theta$, $\bar{G}_+=G_+\cos(\theta/2)$ and $\bar{G}_-=G_-\sin(\theta/2)$, we have

$$(g_J{}^{\pm})_{\rm pole}=\int_{-1}^{+1} dz\, \bar{G}_{\pm{\rm pole}}(s,z)[P_{J+\frac{1}{2}}{}'(z)\mp P_{J-\frac{1}{2}}{}'(z)]. \quad (A3)$$

The corrected amplitudes G_+, G_- then are expressed as the sums (A2), where the g's are S_J from (A1) times $g_{\rm pole}$ from (A3). Our point of view is that these corrections are essential from an *a priori* standpoint.

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Strangeness-Changing Decay Processes†

K. Tanaka
Department of Physics, The Ohio State University, Colombus, Ohio
(Received 29 August 1966)

The effective coupling constant $G(\sin\theta\sin\delta)/\sqrt{2}$ of $\Delta S=1$, $\Delta Q=0$ processes and $G(\sin\theta\cos\delta)/\sqrt{2}$ of $\Delta S=\Delta Q=1$ processes is studied on the basis of the decay rates of the leptonic and photonic decay modes of hardons, where S is the strangeness quantum number, Q is the charge, and θ is the Cabibbo angle. The decay rates give information on $\sin\delta$ together with arbitrary mass factors. When the mass factors are eliminated, one finds both for the vector current and the axial-vector current a very small angle δ, which expresses a drastic reduction of the $\Delta S=1$, $\Delta Q=0$ processes. The decay modes connected with the weak vertex $(\Sigma^-\to n)$; $\Sigma^-\to n+\pi^-$, $\Sigma^-\to n+e^-+\bar{\nu}$, and $\Sigma^-\to n+\pi^-+\gamma$, are examined together and it is found that the vector-current dominance of $\Sigma^-\to n+e^-+\bar{\nu}$ decay is consistent with the S-wave decay of $\Sigma^-\to n+\pi^-$ and parity-conserving decay of $\Sigma^+\to p+\gamma$. The decay rate of $\Sigma^-\to n+\pi^-+\gamma$ is estimated and found to be in agreement with a previous estimate and experiments.

I. INTRODUCTION

The octet-current hypothesis[1] that the weak currents $(j_\mu)_i{}^j$ of strongly interacting particles transform according to an eight representation of $SU(3)$ and that the semi-leptonic effective Lagrangian transforms like members of these weak-current operators, has played a fundamental role in our understanding of elementary-particle weak interactions.

In particular, the vector and axial-vector currents written in the combination[1]

$$J_\mu=(\cos\theta)(j_\mu)_1{}^2+(\sin\theta)(j_\mu)_1{}^3,$$
$$(j_\mu)_i{}^j=(j_\mu{}^V)_i{}^j+(j_\mu{}^A)_i{}^j,$$

have been found to be very useful in relating leptonic decay processes of hadrons with $\Delta S=0$ to processes with $\Delta S=\pm1$, where S is the strangeness quantum number, and the $SU(3)$ content of the currents is given by

$$j_i{}^j\to\begin{bmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\eta}{\sqrt{6}} \end{bmatrix}.$$

The Cabibbo angle $\theta=0.26$ has the effect of decreasing the effective coupling constant $G(\sin\theta)/\sqrt{2}$ of the $\Delta S=1$ processes compared to the coupling constant $G(\cos\theta)/\sqrt{2}$ of the $\Delta S=0$ processes, so as to agree with the experimental results on hadron decays.[2]

It is also known that among the $\Delta S=1$ processes the decays with $\Delta Q=0$ are suppressed compared to decays with $\Delta Q=1$. We therefore make an attempt to de-

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[2] H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, G. A. Snow, and W. Willis, Phys. Rev. **136**, B1791 (1964).