$SU(6)_W$ and Baryon Resonances of Odd Parity*

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An $SU(6)_W$-symmetric dynamical model, used previously to predict quantum numbers and branching ratios of even-parity meson resonances, is extended to odd-parity baryon resonances. It is assumed that meson exchange forces of the tensor type dominate. The relative sizes of the tensor forces in all meson-baryon states are given by the assumed $SU(6)_W$ symmetry. The matrix elements of the tensor forces corresponding to the lowest orbital angular momentum in odd-parity states are S-D-wave transition elements. Those S-D states in which the largest forces exist are assumed to resonate. $SU(3)$ singlet states of $j=\frac{1}{2}$ and $\frac{3}{2}$ are predicted; these may be identified with the 1405- and 1520-MeV $Y_0{}^*$ particles. The predicted octet resonances correspond to the two distinct f/d ratios 1 and $-\frac{1}{3}$. The predicted results are compared with experimental data on the branching ratios of the 1660- and 1765-MeV $Y_1{}^*$ particles, and with the recent measurements of the relative phases of different $\bar{K}N\rightarrow\pi\Lambda$ resonant amplitudes by Smart, Kernan, Kalmus, and Ely. Some experiments are suggested that would test the model further.

If the MBB (meson-baryon-baryon) and MMM interaction vertices are evaluated in the limit of small three-momenta, the one-M exchange potential in any MM, MB, BB, or $B\bar{B}$ state may be written as a sum of a central and a tensor potential. In a previous paper, it was assumed that the MMM vertices satisfy $SU(6)_W$ symmetry, and that the (tensor) forces associated with S wave-D wave transitions in MM states are responsible for the lighter meson resonances of even parity.[1] The properties of the predicted resonances were examined. The purpose of the present paper is to make analogous predictions concerning odd-parity baryon resonances, and to compare them with experiment.

The baryons considered are the nucleon octet (N) and spin-parity $\frac{3}{2}^+$ resonance decuplet (D), associated with the $SU(6)_W$ multiplet **56**. The mesons are the pseudoscalar and vector-meson nonets (P and V), associated with the representation $\mathbf{35}\oplus\mathbf{1}$. Mass differences among the baryons, and among the mesons, are neglected. The $SU(6)_W$ singlet is the V singlet in the state of zero-spin component in the direction of an interaction vertex.

We consider the one-M-exchange contribution to any $MB\rightarrow MB$ amplitude. In the limit of small momenta of the real particles, vertices associated with virtual P and magnetically coupled V mesons may be written in the respective forms

$$i(G/m)\mathbf{S}\cdot\mathbf{q},\quad i(F/m)\mathbf{S}\times\mathbf{q}\cdot\mathbf{e}, \tag{1}$$

where G and F are interaction constants, $\mathbf{e}$ is the three-polarization vector of the virtual V meson, m and $\mathbf{q}$ are, respectively, the mass and momentum transfer of the real particles (mesons or baryons), and $\mathbf{S}$ is a conveniently chosen vector operator connecting the spin states of the real particles. It may be shown that the tensor potential resulting from one M exchange may be written in configuration space in the form

$$\sum_i\left(\frac{G_M{}^iG_B{}^i-F_M{}^iF_B{}^i}{4\pi m_Mm_B}\right)\left(\frac{3(\mathbf{S}_M\cdot\mathbf{r})(\mathbf{S}_B\cdot\mathbf{r})}{r^2}-\mathbf{S}_M\cdot\mathbf{S}_B\right)U(r),$$

where the sum is over the internal states of the exchanged P and V nonets, and $U(r)$ contains the radial dependence.[2] The dependence on the internal variables of the real particles has been suppressed; this potential is actually a matrix in the space of the coupled MB channels. Only one MMM interaction is allowed by our assumption of $SU(6)_W$ symmetry, the completely antisymmetric interaction involving only the representation **35**.[3] Therefore, the $SU(6)_W$ singlet is not involved in our model, and the ratio of any two elements of the tensor-potential matrix is specified by the symmetry.

The calculational technique, to be discussed in more detail in a later paper, involves limiting attention to S-wave and D-wave channels of fixed total angular momentum, and to S-D transition elements of the tensor potential. The S-D potential matrix is a constant matrix multiplied by a radial function. The eigenvalues of this constant matrix occur in pairs of equal magnitude and opposite sign. It is assumed that resonances correspond to the largest positive eigenvalues, and that the relative coupling of a resonance to two S-wave channels, or to two D-wave channels, is given by the relative amplitudes of these channels in the eigenvector that corresponds to the resonance.

The theorem of Ref. 1 shows that the only MB states with nonzero tensor forces correspond to the $SU(3)$ representations **1**, **8**, and **10** (but not **10***). We list the results for the singlet case first. There is only one positive eigenvalue λ_0 and two eigenvectors, associated with total angular momenta $\frac{1}{2}$ and $\frac{3}{2}$. (The magnitude of λ_0 is discussed at the end of the paper.) The S-

* Supported in part by the National Science Foundation.
[1] R. H. Capps, Phys. Rev. Letters **16**, 1066 (1966).
[2] The formulation of the $SU(6)_W$-symmetric, one-M-exchange forces in terms of tensor and central potentials is introduced and discussed in detail by R. H. Capps [Phys. Rev. **150**, 1263 (1966)].
[3] The $SU(6)_W$-symmetric MMM interaction is discussed by R. H. Capps [Phys. Rev. **148**, 1332 (1966)].

and D-wave parts of the eigenvectors are,

$$j=\tfrac{1}{2},\quad \psi_S=(\tfrac{3}{4})^{1/2}(PN)-(\tfrac{1}{4})^{1/2}(VN),\quad \psi_D=(VN)_3, \quad (2)$$

$$j=\tfrac{3}{2},\quad \psi_S=(VN),$$
$$\psi_D=(\tfrac{3}{8})^{1/2}(PN)-(\tfrac{1}{2})^{1/2}(VN)_3-(\tfrac{1}{8})^{1/2}(VN)_1, \quad (3)$$

where the symbols PN and VN denote normalized states with the appropriate $SU(3)$ transformation properties.[4] The subscripts of the (VN) symbols in the D-wave terms denote twice the total intrinsic angular momenta.

In the decuplet system, there are $j=\frac{5}{2}$ and $\frac{3}{2}$ states of eigenvalue $(35/32)^{1/2}\lambda_0$, $j=\frac{3}{2}$ and $\frac{1}{2}$ states of eigenvalue $(27/32)^{1/2}\lambda_0$, and a $j=\frac{1}{2}$ state of eigenvalue $(\frac{5}{8})^{1/2}\lambda_0$. We list these eigenvectors below, together with the eigenvalues λ.

$$j=\tfrac{5}{2},\quad \lambda=(35/32)^{1/2}\lambda_0;\quad \psi_S=(VD),$$
$$\psi_D=(4/35)^{1/2}(PN)+(\tfrac{1}{4})^{1/2}(PD)-(2/15)^{1/2}(VN)_3 +(4/105)^{1/2}(VN)_1-(\tfrac{2}{5})^{1/2}(VD)_5 +(1/60)^{1/2}(VD)_3+(1/21)^{1/2}(VD)_1. \quad (4)$$

$$j=\tfrac{3}{2},\quad \lambda=(35/32)^{1/2}\lambda_0,\quad \psi_S=-(\tfrac{5}{8})^{1/2}(PD)+(\tfrac{3}{8})^{1/2}(VD),$$
$$\psi_D=(1/140)^{1/2}(PN)+(1/14)^{1/2}(PD)+(3/35)^{1/2}(VN)_3 -(27/140)^{1/2}(VN)_1-(3/20)^{1/2}(VD)_5 -(27/70)^{1/2}(VD)_3+(3/28)^{1/2}(VD)_1. \quad (5)$$

$$j=\tfrac{3}{2},\quad \lambda=(27/32)^{1/2}\lambda_0;$$
$$\psi_S=(5/24)^{1/2}(PD)-(4/9)^{1/2}(VN)+(25/72)^{1/2}(VD),$$
$$\psi_D=(1/36)^{1/2}(PN)+(5/18)^{1/2}(PD)-(1/27)^{1/2}(VN)_3 -(1/108)^{1/2}(VN)_1+(7/12)^{1/2}(VD)_5 -(1/54)^{1/2}(VD)_3+(5/108)^{1/2}(VD)_1.$$

$$j=\tfrac{1}{2},\quad \lambda=(27/32)^{1/2}\lambda_0;$$
$$\psi_S=(\tfrac{1}{3})^{1/2}(PN)-(\tfrac{1}{9})^{1/2}(VN)+(5/9)^{1/2}(VD),$$
$$\psi_D=(5/36)^{1/2}(PD)+(2/27)^{1/2}(VN)_3 +(\tfrac{1}{3})^{1/2}(VD)_5-(49/108)^{1/2}(VD)_3.$$

$$j=\tfrac{1}{2},\quad \lambda=(\tfrac{5}{8})^{1/2}\lambda_0;\quad \psi_S=(\tfrac{1}{4})^{1/2}(PN)+(\tfrac{3}{4})^{1/2}(VN),$$
$$\psi_D=(\tfrac{1}{4})^{1/2}(PD)-(\tfrac{3}{5})^{1/2}(VD)_5-(3/20)^{1/2}(VD)_3.$$

All the VD and PD states in all the decuplet eigenvectors are the following linear combination of states involving the meson octets and singlets.

$$(MD)=(\tfrac{4}{5})^{1/2}(M_8D)+(\tfrac{1}{5})^{1/2}(M_1D), \quad (6)$$

where M denotes either a V or P.

We now turn to the octet states. Two different types of VN and PN states are involved. These are denoted by α and β, and are listed below.

$$(MN)^\alpha=(\tfrac{5}{16})^{1/2}(M_8N)^s+(\tfrac{9}{16})^{1/2}(M_8N)^a +(\tfrac{1}{8})^{1/2}(M_1N), \quad (7)$$

$$(MN)^\beta=(\tfrac{5}{8})^{1/2}(M_8N)^s-(\tfrac{1}{8})^{1/2}(M_8N)^a -(\tfrac{1}{4})^{1/2}(M_1N), \quad (8)$$

where s and a denote the symmetric (d type) and antisymmetric (f type) states, respectively.[4] The f/d ratios for the α and β states are 1 and $(-\frac{1}{3})$, respectively.[5] There are two sets of octet states with nonzero tensor forces, "singlet-type" and "decuplet-type" states. The eigenvalues of the singlet-type states are identical to those that exist in the $SU(3)$ singlet case, and the eigenvectors may be obtained by replacing the symbols (VN) and (PN) in Eqs. (2) and (3) by $(VN)^\alpha$ and $(PN)^\alpha$. The eigenvalues of the other octet states are identical to those of the decuplet states, and the eigenvectors may be obtained by replacing (VN) and (PN) by $(VN)^\beta$ and $(PN)^\beta$ in the equations for the eigenvectors.

Although the eigenvalue $(35/32)^{1/2}\lambda_0$ is slightly larger than that associated with the singlet-type states, it is not clear which set of states should be lighter, since the singlet-type states do not involve the heavier baryon multiplet D. For purposes of comparing with experiment we will assume that the states corresponding to the two larger eigenvalues λ_0 and $(35/32)^{1/2}\lambda_0$ are the lighest. The two predicted $SU(3)$ singlet resonances may be associated with the 1405- and 1520-MeV Y_0^* particles.[6] We will discuss in detail only the experimental data associated with the 1660-MeV and 1765-MeV Y_1^* particles, (assumed to correspond to $j^P=\frac{3}{2}^-$ and $\frac{5}{2}^-$). These are ideal for checking branching ratio predictions, since the phase space factors of the $\pi\Lambda$, $\pi\Sigma$, and $\bar{K}N$ decay modes are not very different. Since the data are not accurate, we will maintain simplicity by neglecting phase-space differences.

The Y_1^* (1660) must be associated with the predicted singlet-type $j=\frac{3}{2}$ octet, since the observed PN modes of this particle are large, while no PD mode is clearly established. As seen from Eq. (5), spin-$\frac{3}{2}$ particles of the decuplet type are coupled very weakly to PN states, and relatively strongly, both in S-wave and D-wave modes, to PD states. We associate the Y_1^* (1765) with the $j=\frac{5}{2}$ octet, rather than the decuplet, because the decuplet assignment would lead to predicted $\pi\Lambda/\bar{K}N$ and $\pi\Sigma/\bar{K}N$ branching ratios as large or larger than one, in strong contradiction to experiment.[6]

[4] The necessary $SU(3)$ Clebsch-Gordan coefficients are given by P. McNamee, S. J. Chilton, and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[5] The f/d ratio used here is the conventional one, and is related to the parameter α of J. J. de Swart [Rev. Mod. Phys. **35**, 916 (1963)], by the equation $f/d=\alpha/(1-\alpha)$.

[6] Except where otherwise noted, the experimental data used here are those of the compilation of A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos [Rev. Mod. Phys. **37**, 633 (1965)]. The decay momenta are also listed in this reference.

The predictions of the model are compared with experiment in Table I. The $\pi\Sigma/\pi\Lambda$ decay ratio is proportional to $(f/d)^2$. The $\bar{K}N$ ratios in parentheses in Table I are the values that would result if the sign of f/d were opposite to that predicted by the model. The predicted signs provide the best fit, although an appreciable admixture of some other state is necessary in the model in order to produce the small $\bar{K}N$ mode of the 1660-MeV Y_1^*. Some PD modes of the $\frac{5}{2}^-$ octet particles should be observed, despite the small phase space of these modes. The predicted ratio of squares of Clebsch-Gordan coefficients corresponding to $\pi Y_1^*/\pi\Lambda$ [of Y_1^* (1765)] is 7/3.

In a recent experiment, Smart, Kernan, Kalmus, and Ely examined the $\bar{K}N \to \pi\Lambda$ amplitudes in the energy region of the four Y_1^* resonances at 1660, 1765, 1915, and 2030 MeV.[7] It was found that the relative signs of the resonance contributions to the amplitude are $-$, $+$, $+$, and $-$, respectively. By convention, we will define the sign associated with the 2030-MeV particle to be negative. In an analysis of this effect, Kernan and Smart assumed that the Y_1^* (2030) is a decuplet state [as is required by the assumption that it is the Regge recurrence of the Y_1^* (1385)], and showed that the sign of any corresponding octet amplitude should be negative if and only if f/d is positive and greater than one.[8] The positive sign of the 1915-MeV amplitude is then explained by the assumption that the f/d ratio for this particle is similar to that of its Regge parent, the Σ.[8] Our model predicts correctly the positive sign of the 1765-MeV amplitude. Since f/d for the Y^* (1660) is predicted here to be exactly one, the observed negative sign of this amplitude is a condition on the sign of the admixture responsible for the $\bar{K}N$ decay. It is worth noting, however, that f/d values as large as one are fairly unusual in theoretical models of baryon resonances.

One may extend this argument to the $\bar{K}N$–$\pi\Sigma$ amplitudes associated with these resonances. If the signs of the $\bar{K}N$–$\pi\Sigma$ and $\bar{K}N$–$\pi\Lambda$ amplitudes associated with the 2030-MeV decuplet state are defined to be the same, it may be shown that the signs of these two amplitudes in an octet are the same if and only if f/d is negative. The prediction of the present model is that the signs of the $\bar{K}N$–$\pi\Sigma$ amplitudes of the lighter two resonances are opposite to those of the heavier two resonances.[9] This prediction should be tested.

[7] W. M. Smart, A. Kernan, G. E. Kalmus, and R. P. Ely, Jr., Phys. Rev. Letters **17**, 556 (1966).

[8] A. Kernan and W. M. Smart, Phys. Rev. Letters **17**, 832 (1966). The parameter α of this work is that of Ref. 5.

TABLE I. Experimental and predicted branching ratios of 1660- and 1765-MeV Y_1^* particles.[a]

Ratio	$Y^*(1660)$ Expt	$Y^*(1660)$ Predicted $(f/d=1)$	$Y^*(1765)$ Expt	$Y^*(1765)$ Predicted $(f/d=-\frac{1}{3})$
$\pi\Sigma/\pi\Lambda$	≈ 6	6	$\lesssim 0.5$	$\frac{2}{3}$
$\bar{K}N/(\pi\Lambda+\pi\Sigma)$	≈ 0.4	0 (6/7)	≈ 3	8/5 ($\frac{2}{5}$)

[a] The $\bar{K}N$ ratios in parentheses are the values that would result if the sign of f/d were opposite to that predicted by the model.

Of the other resonance multiplets predicted here, the spin-$\frac{1}{2}$ octet should be striking; the average mass of this multiplet should be comparable to or less than that of the multiplet of the Y_1^* (1660). The predicted f/d ratio is one. If there is no appreciable η-X mixing, this implies that the $\eta\Xi$ probability in the Ξ^* wave function is four times the corresponding η probability for the other members of this multiplet.[10] The $\eta\Xi$ system should be investigated experimentally.

Finally, we discuss the magnitude of the eigenvalue λ_0. If the MMM and MBB interactions are related by the universality hypothesis, and the normalization convention of Ref. 1 is used, then $\lambda_0=(5/9)^{1/2}$.[11] This means that the tensor force corresponding to λ_0 is $\frac{2}{3}$ as strong as that of the meson octet states discussed in Ref. 1. The presence of t- and u-channel forces in the MM system is taken into account in this comparison. Since the reduced mass is somewhat larger in the MB system it is reasonable that the potential that produces the resonances is smaller in this system.

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[9] If the $Y_1^*(1660)$ is not close to a pure state in our model, the prediction could be wrong for this particle. This possibility is suggested because the $\Xi^*(1816)$ is usually regarded as a member of the same multiplet as the $Y_1^*(1660)$, yet the present experimental evidence concerning the $\pi\Xi:\bar{K}\Lambda:\bar{K}\Sigma$ branching ratios of this particle cannot be fitted with a pure octet assignment. In our model, a complicated admixture of the predicted octet and decuplet states would be needed for this Ξ^*.

[10] There is some experimental evidence for peaks in ηN and $\eta\Lambda$ states near the thresholds; these may be associated with members of a $j^P=\frac{1}{2}^-$ octet. For discussions of the ηN and $\eta\Lambda$ peaks, respectively, see Peter N. Dobson, Jr., Phys. Rev. **146**, 1022 (1966); D. Berley *et al.*, Phys. Rev. Letters **15**, 641 (1965).

[11] The universality hypothesis is discussed by R. H. Capps [Phys. Rev. **144**, 1182 (1966)]. It is pointed out in this reference that universality relates the quantities G/m and F/m of Eq. (1), rather than the constants G and F.