Meson-Meson Coupling Constants in Broken $SU(3)$ and the Algebra of Currents*

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Sum rules are obtained for meson-meson coupling constants in broken $SU(3)$ with symmetry-breaking effects taken into account to first order, using the algebra of currents and the hypothesis of partial conservation of axial-vector currents. In particular, enlarged sets of coupling-constant sum rules are obtained for the vector-octet–pseudoscalar-octet–vector-octet (V_8-P_8-V_8) coupling constants and for the V_8-P_8-P_8 coupling constants. The corresponding two-parameter families of symmetry-breaking interaction Hamiltonians are also given.

RECENTLY,[1] it was shown that from the algebra of currents[2] and the hypothesis of partial conservation of axial-vector currents (PCAC), one could derive sum rules for meson-baryon coupling constants in broken $SU(3)$ with symmetry-breaking effects taken into account to first order. Moreover, the sum rules derived in this manner are much stronger than those obtained previously from purely group-theoretic considerations,[3] since one finds that the perturbed vertex is now characterized by fewer independent parameters than before with a corresponding increase in the number of coupling-constant identities. Since we find no bar[4] to the extension of the methods of Ref. 1 to the case of the meson-meson couplings as well (except for a slight modification in the case of the VPP coupling as discussed below), we wish to report on such an extension in this paper.[5]

Let us consider the meson-meson vertex $V'\to M+P$, where P is a pseudoscalar meson, V a vector meson, and M either a vector meson [cases (i) and (ii)] or a pseudoscalar meson [case (iii)]. Writing the broken-symmetry correction to $f_0(V'MP)$ [the vertex in the limit of exact $SU(3)$] as

$$f_1(V_i'M_jP_k)=\lambda\langle M_jP_k|S_8|V_i'\rangle, \quad (1)$$

and proceeding as in Ref. 1, we find

$$f(V_i'M_jP_k)=f_0(V_i'M_jP_k)+id_{k8k}\lambda C\langle M_j|S_k{}^5|V_i'\rangle, \quad (2)$$

where the matrix element $\langle M_j|S_k{}^5|V_i'\rangle$ is to be evaluated in the $SU(3)$ limit. Now in case (i) ($M_j=V_j$, $j=1,\cdots,8$),[6] from the group-theoretic considerations of MG[7] we are led to the characterization

$$f(V_i'V_jP_k)=d_{ijk}(G_0+d_{k8k}\lambda G_1), \quad (3)$$

where the independent parameters, G_0, G_1, are proportional to the reduced matrix elements coming from $f_0(V_i'V_jP_k)$ and $\langle V_j|S_k{}^5|V_i'\rangle$, respectively. In case (iii) ($M_j=P_j$), we wish to consider $\lim_{q^j\to 0}(2q_0{}^j)^{1/2}\times\langle P_j|S_k{}^5|V_i'\rangle$, but cannot carry the reduction process further since the jth meson must be P wave, as we infer from the R antisymmetry of the interaction Hamiltonian of MG; so we antisymmetrize with respect to the pseudoscalar-meson indices j, k. Hence, factoring away the expected momentum dependence (in the rest system of the vector meson) of the reduced matrix elements, we find

$$f(V_i'P_jP_k)=f_{ijk}[H_0+\tfrac{1}{2}(d_{j8j}+d_{k8k})\lambda H_1]. \quad (4)$$

We now present our results for the various cases of interest.

(i) Vector-octet–pseudoscalar-octet–vector-octet. The two-parameter formula (3) yields[8]

$$g_{\rho^+\omega^{(8)}\pi^-}=(\sqrt{\tfrac{2}{3}})g_{\bar{K}^{0*}K^{+*}\pi^-}=-\frac{1}{\sqrt{5}}\left(G_0+\frac{1}{\sqrt{3}}\lambda G_1\right), \quad (5a)$$

$$g_{K^{-*}\omega^{(8)+}K}=-\tfrac{1}{2}(\sqrt{\tfrac{2}{3}})g_{\rho^-\bar{K}^{0*}K^+}=\frac{1}{2\sqrt{5}}\left(G_0-\frac{1}{2\sqrt{3}}\lambda G_1\right), \quad (5b)$$

$$g_{K^{-*}K^{+*}\eta}=g_{\omega^{(8)}\omega^{(8)}\eta}=-g_{\rho^0\rho^0\eta}=\frac{1}{2\sqrt{5}}\left(G_0-\frac{1}{\sqrt{3}}\lambda G_1\right), \quad (5c)$$

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[1] S. K. Bose and Y. Hara, Phys. Rev. Letters **17**, 409 (1966).

[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[3] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963), hereafter referred to as MG; the matrix representation for the octets, B, P, and V are not correctly given in this paper. [See J. J. Sakurai, Phys. Rev. **132**, 434 (1963).]

[4] In particular, we are neglecting off-mass-shell effects (Ref. 1).

[5] Note that if we take seriously the hypothesis of partial conservation of tensor currents (PCTC) [R. F. Dashen and M. Gell-Mann, Caltech report (unpublished); S. Fubini, G. Segrè, and J. D. Walecka, Stanford report (unpublished)], then for the broken-symmetry correction to the vector-meson–baryon-baryon vertex, one finds

$$\lim_{q^k\to 0}(2q_0{}^k)^{1/2}\langle B_jV_k|S_8|B_i'\rangle=\text{const}\,\langle B_j|[\epsilon^\mu(q^k)T_\mu{}^k,S_8]|B_i'\rangle$$
$$=\text{const}\,d_{k8k}\epsilon_\mu\langle B_j|S_k{}^\mu|B_i'\rangle,$$

which leads to sum rules obtainable from those of Ref. 1 under the replacement: $\pi\to\rho$, $K\to K^*$, $\eta\to\omega^{(8)}$.

[6] The ϕ-ω mixing may readily be taken into account by means of the relation between the pure $SU(3)$ states ($\omega^{(0)},\omega^{(8)}$) and the physical states (ω,ϕ):

$$|\omega^{(0)}\rangle=\cos\lambda|\omega\rangle+\sin\lambda|\phi\rangle,$$
$$|\omega^{(8)}\rangle=-\sin\lambda|\omega\rangle+\cos\lambda|\phi\rangle,$$

together with the empirical result, $\tan\lambda\simeq 1/\sqrt{2}$. [See G. Segrè and J. D. Walecka, Stanford report (unpublished).]

[7] From MG we see that the most general symmetry-breaking interaction in this case has even R symmetry (R: $V\to V^T$, $P\to P^T$). (See Ref. 2.)

[8] We have absorbed common constant factors into the previously defined parameter pairs.

and the five sum rules, Eq. (5c), and

$$-g_{\rho^+\omega^{(8)}\pi^-}+g_{K^{-*}\omega^{(8)}K^+}+\frac{1}{\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}+\frac{2}{\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}=0, \quad (6a)$$

$$-2g_{\rho^0\rho^0\eta}+g_{K^{-*}K^{+*}\eta}+\frac{4}{\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}-\frac{1}{\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}=0, \quad (6b)$$

$$-2g_{\omega^{(8)}\omega^{(8)}\eta}+\frac{2}{3\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}+\frac{1}{3\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}+g_{K^{-*}K^{+*}\eta}+2g_{K^{-*}\omega^{(8)}K^+}=0. \quad (6c)$$

The sum rules, Eqs. (6a)–(6c), are just those of MG. From these, we may derive the auxiliary sum rule

$$g_{\rho^0\rho^0\eta}-\frac{4}{3\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}+\frac{1}{3\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}=0, \quad (7)$$

which is free of $\phi\,\omega$ mixing.[9] Making use of the relation

$$\tfrac{1}{8}\,\mathrm{Tr}\lambda_k\{\lambda_i,\{\lambda_j,\lambda_8\}\}-\tfrac{1}{6}\,\mathrm{Tr}\lambda_i\lambda_k\,\mathrm{Tr}\lambda_j\lambda_8=d_{j8j}d_{ijk}, \quad (8)$$

together with the identity given by Eq. (9) of MG, we find the two-parameter family of symmetry-breaking interaction Hamiltonians[2] which gives rise to this enlarged set of coupling-constant identities, namely, $g_2=-g_3=-6g_1$.

(ii) Vector-singlet–pseudoscalar-octet–vector-octet. We find a two-parameter formula for the vertex which yields the sum rule

$$4g_{\omega^{(0)}K^{-*}K^+}-3g_{\omega^{(0)}\omega^{(8)}\eta}-g_{\omega^{(0)}\rho^+\pi^-}=0. \quad (9)$$

(iii) Vector-octet–pseudoscalar-octet–pseudoscalar-octet. We find a two-parameter formula for each vertex:

$$g_{K^-K^+\omega^{(8)}}=(\sqrt{\tfrac{3}{2}})g_{\bar{K}^0K^+\rho^-}=-\frac{1}{4\sqrt{2}}\left(H_0-\frac{1}{2\sqrt{3}}\lambda H_1\right), \quad (10a)$$

$$g_{\pi^-\bar{K}^0K^{+*}}=-\frac{1}{4\sqrt{3}}\left(H_0+\frac{1}{4\sqrt{3}}\lambda H_1\right), \quad (10b)$$

$$g_{\eta K^-K^{+*}}=-\frac{1}{4\sqrt{2}}\left(H_0-\frac{3}{4\sqrt{3}}\lambda H_1\right), \quad (10c)$$

$$g_{\pi^+\pi^0\rho^-}=-\frac{1}{2\sqrt{6}}\left(H_0+\frac{1}{\sqrt{3}}\lambda H_1\right), \quad (10d)$$

with the three sum rules

$$g_{\pi^+\pi^0\rho^-}-(4/\sqrt{2})g_{\pi^-\bar{K}^0K^{+*}}-(1/\sqrt{2})g_{\bar{K}^0K^+\rho^-}+(3/\sqrt{3})g_{K^-K^+\omega^{(8)}}=0, \quad (11a)$$

$$g_{K^-K^+\omega^{(8)}}-g_{\eta K^-K^{+*}}+(1/\sqrt{6})g_{\bar{K}^0K^+\rho^-}-(1/\sqrt{6})g_{\pi\ \bar{K}^0K^{+*}}=0, \quad (11b)$$

$$-(\sqrt{\tfrac{2}{3}})g_{K^-K^+\omega^{(8)}}+g_{\bar{K}^0K^+\rho^-}=0. \quad (11c)$$

The coupling-constant identities, Eqs. (11a) and (11b), are just those of MG; the identity (11c) is new. Since it follows that

$$g_{\pi^+\pi^0\rho}\ -2\sqrt{2}g_{\pi^-K^0K^{+*}}+\sqrt{2}g_{K^0K^+\rho^-}=0, \quad (12)$$

we have the possibility of determining the coupling constant $g_{\bar{K}^0K^+\rho^-}$ from the observed widths, $\Gamma(\rho^+\to\pi^+\pi^0)$, $\Gamma(K^{-*}\to\pi^-\bar{K}^0)$. Finally, from the relation

$$\tfrac{1}{8}\,\mathrm{Tr}\lambda_k[\lambda_i+\lambda_j,\ \{\lambda_i-\lambda_j,\ \lambda_8\}]=-if_{ijk}(d_{i8i}+d_{j8j}), \quad (13)$$

we find the appropriate two-parameter family of MG symmetry-breaking interaction Hamiltonians[2]; it is that given by $g_1=-g_2$.

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[9] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); S. L. Glashow, *ibid.* **11**, 48 (1963).