Inelastic Model of the n-p Mass Difference*

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(Received 13 July 1966)

A simple model of the n-p mass difference is exhibited, in field-theoretic terms, in order to illustrate one way in which the inelastic feedback mechanism may be expected to operate.

RECENTLY, two new derivations of the n-p mass difference have been given[1,2] which attempt to include inelastic Compton effects by exhibiting a so-called feedback term with the effect of reversing the over-all sign of earlier estimates. The purpose of this note is to give a simple, field-theoretic example of how the feedback mechanism may be expected to operate. We consider the difference, $\Delta\Sigma(\omega)$, of the nucleon self-energies obtained by iterating the difference, $\Delta\Sigma^{(0)}(\omega)$, of the Feynman-Speisman (FS)-type[3] electromagnetic self-energies between an increasing number of virtual pions. This difference of sums over the subset of proper self-energy rainbow (or ladder) graphs, illustrated in Fig. 1, is supposed to approximate the n-p mass difference, and provide the mechanism whereby inelastic states of one photon plus many pions can react on the strong-interaction determination of the nucleon masses. These graphs are chosen only because they can be generated by the iteration of a linear integral equation, which we do not attempt to solve.[4] Rather, we observe that, if a solution exists for which $\mathrm{Im}\Delta\Sigma$ is not pathological near threshold and vanishes rapidly for large ω, then the feedback mechanism can be sufficient to reverse the sign of the initial $\Delta\Sigma^{(0)}$ estimate. When evaluated with the aid of reasonable charge and moment form factors, the latter leads to the incorrect result for $\Delta m = m_p - m_n$, of amount $\Delta m^{(0)} = -Z\Delta\Sigma^{(0)}(m) \sim +\frac{1}{2}$ MeV, where Z denotes the nucleon wave-function renormalization constant.

We represent by $\delta\Sigma = x_p\delta\Sigma_p + x_n\delta\Sigma_n$ the sum of such self-energy graphs for the nucleon, obtained by the iteration of the simpler, electromagnetic $\delta\Sigma^{(0)} = x_p\delta\Sigma_p^{(0)} + x_n\delta\Sigma_n^{(0)}$. Here, $x_{p,n}$ denote isotopic projection operators for proton and neutron, respectively, while $\delta\Sigma_{p,n}$ and $\delta\Sigma_{p,n}^{(0)}$ are the corresponding self-energy functions. $\delta\Sigma^{(0)}$ is illustrated by the first term on the right side of Fig. 1; it is of order e^2 and is understood to be given in terms of realistic charge and moment distributions of the electromagnetic vertex. The model may thus be defined by the equation[5]

$$\delta\Sigma(-i\gamma\cdot p) = \delta\Sigma^{(0)}(-i\gamma\cdot p) + \frac{ig^2}{(2\pi)^4}\sum_{i=1}^{3}\int \frac{d^4k}{k^2+\mu^2}\gamma_5\tau_i \frac{1}{m+i\gamma\cdot(p+k)} \times \delta\Sigma[-i\gamma\cdot(p+k)]\frac{1}{m+i\gamma\cdot(p+k)}\gamma_5\tau_i, \quad (1)$$

where we neglect the variation of pion and nucleon masses in Eq. (1) because $\delta\Sigma$ is already of order e^2. Both e^2 and g^2 are taken as renormalized, with $g^2/4\pi = 15$. Inserting isotopic representations, one easily finds, for $\Delta\Sigma = \delta\Sigma_p - \delta\Sigma_n$,

$$\Delta\Sigma(-i\gamma\cdot p) = \Delta\Sigma^{(0)}(-i\gamma\cdot p) - \frac{ig^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mu^2}\gamma_5 \times \frac{1}{m+i\gamma\cdot(p+k)}\Delta\Sigma[-i\gamma\cdot(p+k)]\frac{1}{m+i\gamma\cdot(p+k)}\gamma_5, \quad (2)$$

where $\Delta\Sigma^{(0)} = \delta\Sigma_p^{(0)} - \delta\Sigma_n^{(0)}$.

It is useful to employ for $\Delta\Sigma(\omega)$ the form of the very general representations[6] valid for $\Sigma(\omega)$,

$$\Delta\Sigma(\omega) = \int_{m+\mu}^{\infty} dn \left[\frac{\Delta\rho_+(n)}{n-\omega} + \frac{\Delta\rho_-(n)}{n+\omega}\right], \quad (3)$$

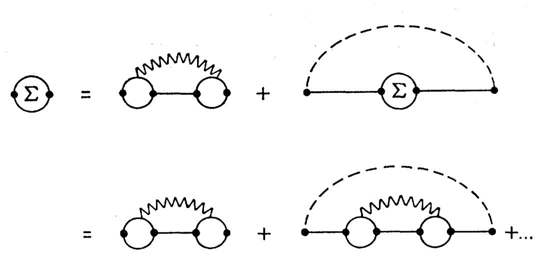

FIG. 1. Pictorial representation of an approximate integral equation for $\delta\Sigma$, and its iterative solution.

* Supported in part by the U. S. Atomic Energy Commission.

[1] H. Pagels, Phys. Rev. **144**, 1261 (1966).

[2] H. M. Fried and T. N. Truong, Phys. Rev. Letters **16**, 559 (1966); **16**, 884(E) (1966); Phys. Rev. (to be published). The relation of this method to that of the conventional calculation is described in the latter paper.

[3] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954). References 1 and 2 contain or lead to an exhaustive list of the relevant literature.

[4] For simple $\Delta\Sigma^{(0)}$, this equation has been studied by many authors; e.g., D. Falk, Phys. Rev. **115**, 1069 (1959).

[5] More precisely, one should begin with the unrenormalized m_0, g_0, and include self-energy corrections to the pion and nucleon propagators and to the ps vertex; extract the appropriate renormalization constants which convert g_0 to g; and then approximate the renormalized propagators by their respective pole terms, and each renormalized ps vertex by $\gamma_5\tau_i$.

[6] See, for example, M. Ida, Phys. Rev. **136**, B1767 (1964).

where $\Delta\rho_\pm(n)=\pm(n\mp m)^2\Delta\tau_\pm(n)$. Insertion of Eq. (3) into Eq. (2), followed by the computation of an elementary Feynman integral, produces

$$\Delta\Sigma(\omega)=\Delta\Sigma^{(0)}(\omega)+\frac{g^2}{8\pi^2}\int_{m+\mu}^{\infty}dn \times\left[\frac{\Delta\rho_+(n)}{n-m}f_+(n,\omega,\mu)+\frac{\Delta\rho_-(n)}{n+m}f_-(n,\omega,\mu)\right], \quad (4)$$

where

$$f_\pm(n,\omega,\mu)=\int_0^1 dx \times\left\{\frac{xm[m-(1-x)\omega]}{\mu^2(1-x)+xm^2-x(1-x)\omega^2}-\frac{1}{2}\frac{[n\mp(1-x)\omega]}{(n\mp m)} \times\ln\left[\frac{\mu^2(1-x)+xn^2-x(1-x)\omega^2}{\mu^2(1-x)+xm^2-x(1-x)\omega^2}\right]\right\}. \quad (5)$$

These relations provide a linear equation for $\Delta\rho_\pm$, in terms of the absorptive part of $\Delta\Sigma^{(0)}(\omega)$ and the cuts of $f_\pm$. If a solution exists, we are interested in $\Delta\Sigma(m)$,

$$\int_m^\infty dn\,[(n-m)\Delta\tau_+(n)-(n+m)\Delta\tau_-(n)] =\Delta\Sigma^{(0)}(m)+\frac{g^2}{8\pi^2}\int_m^\infty dn\,[(n-m)\Delta\tau_+(n)f_+(n,m,0) -(n+m)\Delta\tau_-(n)f_-(n,m,0)], \quad (6)$$

where the $\Delta\rho_\pm$ have been replaced by the $\Delta\tau_\pm$ and, for simplicity, all dependence on μ has been dropped: this latter approximation does not lead to an infrared logarithm in $\Delta\Sigma(m)$, but introduces errors proportional only to $(\mu/m)^2$. Writing

$$f_\pm(n,m,0)=1-l_\pm(n), \quad (7)$$

where

$$l_\pm(n)=\frac{1}{2}\int_0^1 dx\left[\frac{n\mp(1-x)m}{n\mp m}\right]\ln\left(1+\frac{n^2-m^2}{xm^2}\right), \quad (8)$$

and defining the average value l of Eq. (8),

$$l\equiv\int_m^\infty dn\,[(n-m)\Delta\tau_+(n)l_+(n) -(n+m)\Delta\tau_-(n)l_-(n)]\Big/ \int_m^\infty dn\,[(n-m)\Delta\tau_+(n)-(n+m)\Delta\tau_-(n)], \quad (9)$$

we may use the definition $\Delta m=-Z\Delta\Sigma(m)$ to obtain

$$\Delta\Sigma(m)=\Delta\Sigma^{(0)}(m)+\frac{g^2}{8\pi^2}(1-l)\Delta\Sigma(m),$$

or

$$\Delta m=\Delta m^{(0)}\left(1-\frac{g^2}{8\pi^2}(1-l)\right)^{-1}. \quad (10)$$

For negative l, or for sufficiently small positive l ($l<0.57$), Eq. (10) exhibits the desired change in sign. The value of l depends on the detailed solution to the model, but it is apparent that the effect depends upon reasonable threshold behavior together with strong damping, at higher energies, of $\Delta\tau_\pm$.[7] For example, if a solution exists with $\Delta\tau_+\approx\Delta\tau_-=\Delta\tau$, where $\Delta\tau$ increases from threshold in a smooth way and is essentially cut off for $n>2m$, then we may approximate $\Delta\tau$ by

$$\Delta\tau(n)\approx2\Delta\tau_0\left\{\left(\frac{n}{m}-1\right)\theta\left(\frac{3}{2}-\frac{n}{m}\right)\theta\left(\frac{n}{m}-1\right) +\left(2-\frac{n}{m}\right)\theta\left(2-\frac{n}{m}\right)\theta\left(\frac{n}{m}-\frac{3}{2}\right)\right\},$$

where $\Delta\tau_0$ is some constant. This, together with the relation

$$\frac{1}{2}\int_0^1 dx\,(1-x)\ln\left(1+\frac{n^2-m^2}{xm^2}\right)\approx\frac{n}{m}-1,$$

approximately valid in the region $2m>n>m$, leads to the value $l=\frac{1}{2}$, satisfying our criterion. The magnitude of Eq. (10) should not be taken seriously; the model itself has only the virtue of providing a simple example of the feedback mechanism.

It is a pleasure to acknowledge many relevant conversations with Professor T. N. Truong; it is also a pleasure to thank Professor S. Drell for the hospitality extended by SLAC, where this article was written.

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[7] Similar damping is essential to the success of the feedback mechanisms of Refs. 1 and 2. Even if a solution with these properties does not exist here, it might still be possible to find a sufficiently small l; otherwise, a damping factor, corresponding, e.g., to the use of more realistic *ps* vertex functions, must be included.