approximate ImF by the contribution from the $l=0$, $I=0$ channel and use $\delta_0{}^0 \simeq [(s-4)/s]^{1/2} a_0 \mu$ and get

$$6h\mu^4 \cong -2a_0{}^2\mu^2 + \mu^2/\pi \int_{6\mu}^{\infty} \frac{\mathrm{Im}F(\nu')}{\nu'(\nu'^2+\frac{1}{2}\mu^2)} d\nu' . \quad (62)$$

If we ignore the second term in (62) and assume it to be a fraction of $\mu^2/c_\pi{}^2 \cong 8\pi/9$, we obtain on substituting (62) into (53)

$$a_0 = -(1/32\pi\mu)[-7(\mu^2/c_\pi{}^2) - 10a_0{}^2\mu^2] . \quad (63)$$

This last equation has two roots for a_0. One will, to within 2%, give us back the same answer as before, $a_0 \cong 0.2\mu^{-1}$. The other root is ridiculously large, $a_0 \cong 10\mu^{-1}$, and clearly unphysical. The latter root will also give a very large value for a_2.

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πN Polarization and Regge Poles*

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We show that the recent high-energy π^-p polarization data from CERN are explained in a natural way by the three–Regge-pole model. The prediction of this model for π^+p polarization differs greatly from that for π^-p polarization in the region where $|t| < 0.6$ (GeV/c)^2. In particular, in this region, the π^+p polarization has an opposite sign and comparable magnitude to that for π^-p.

THIS paper shows that recent high-energy π^-p polarization data from CERN[1] are explained in a natural way by the three–Regge-pole model.[2] The prediction of the model for π^+p polarization has an opposite sign and comparable magnitude to that for π^-p.

Elastic πN scattering at small momentum transfer is dominated, in this model, by three Regge poles in the crossed channel. Thus it is a more complicated problem than the charge-exchange reactions, with only one or two poles, for which the Regge hypothesis has had great success.[2–7] However, this complication is largely compensated by the greater variety of data available.

The data we use are total cross sections,[8] differential cross sections for elastic[9,10] and charge-exchange[11,12] scattering, Coulomb interference measurements of the phase of the forward elastic amplitude,[13] and π^-p elastic polarization.[1] These data are from 5.9 GeV/c upward, and with squared momentum transfer $|t| < 1$ (GeV/c)^2. For $d\sigma/dt$ data, we worked with a representative subset of 141 elastic points in the interval $-1 < t < -0.1$ and

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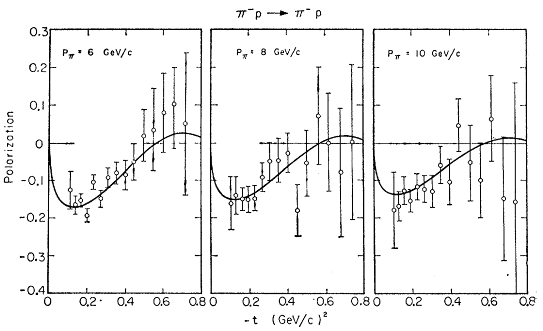

FIG. 1. $\pi^- p$ polarization data of Ref. 1 at 6, 8, and 10 GeV/c compared with the Regge fit of solution (a).

charge-exchange points in $-0.9<t<0$. There is also an indirect datum, i.e., from dispersion relations one can place a constraint on the zero intercept of P' trajectory.[14] The constraints on the amplitudes at the ρ by relating ρ-meson coupling constants to nucleon electromagnetic structure, as described in Refs. 2 and 6, are also included.

We set aside the recent charge-exchange polarization data.[15] In the present model, such polarization is necessarily zero, but it seems the observed nonzero values can be explained by small background effects—either the tails of s-channel resonances[16] or another low-lying trajectory[17]—which have negligible effect on other experimental quantities.

We assume scattering is dominated by the first and second vacuum Regge trajectories P and P' (presumably associated with the f and f' mesons) and the isovector ρ trajectory. Following Refs. 2 and 5, we parametrize these contributions of the trajectories to the invariant amplitudes A' and B of Singh[18]:

$$A'=C_0\exp(C_1t)\alpha(\alpha+1)\xi(E_L/E_0)^{\alpha}\quad\text{for } P \text{ and } P'$$
$$=C_0[(1+C_2)\exp(C_1t)-C_2](\alpha+1)\xi(E_L/E_0)^{\alpha}\quad\text{for } \rho;\qquad(1)$$

TABLE I. Regge-pole parameters.

Parameters	and units	Solution (a) P	P'	ρ	Solution (b) P	P'	ρ
C_0	(mb GeV)	7.43	16.6	1.49	8.88	16.4	1.49
C_1	(GeV)$^{-2}$	1.68	6.17	2.01	2.49	2.42	1.98
C_2		⋯	⋯	1.79	⋯	⋯	1.80
D_0	(mb)	−27.4	−83.0	29.2	−3.55	−8.99	29.1
D_1	(GeV)$^{-2}$	4.94	7.96	0.12	0.41	−2.08	0.13
$\alpha(0)$		1.00	0.72	0.576	1.00	0.65	0.576
α'	(GeV)$^{-2}$	0.34	0.34	1.02	0.23	0.93	1.02

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TABLE II. Data fitted.

Type	Number of data points	χ^2 Solution (a)	Solution (b)
$P(\pi^- p)$	45	28	33
$\sigma_T(\pi^\pm p)$	16	10	7
$d\sigma/dt(\pi^\pm p)$	141	133	161
$d\sigma/dt(\pi^- p\rightarrow\pi^0 n)$	56	87	87
$\mathrm{Re}A'(0)/\mathrm{Im}A'(0)$	9	16	15
$\alpha_{P'}(0)$	1	0.1	0.3
ρ-coupling constraints	2	1	2
Total	270	275	305

$$B=D_0\exp(D_1t)\alpha^2(\alpha+1)\xi(E_L/E_0)^{\alpha-1}\quad\text{for } P \text{ and } P'$$
$$=D_0\exp(D_1t)\alpha(\alpha+1)\xi(E_L/E_0)^{\alpha-1}\quad\text{for } \rho;\qquad(2)$$

$$\xi(t)=-[\exp(-i\pi\alpha)\pm1]/\sin\pi\alpha;\qquad(3)$$

$$\alpha(t)=\alpha(0)+\alpha' t.\qquad(4)$$

Here $\alpha(t)$ is the trajectory, $\xi(t)$ is the signature factor (with signature + for P and P', − for ρ), E_L is the total pion lab energy, and E_0 is a scale factor chosen to be 1 GeV. C_0, C_1, C_2, D_0, D_1, $\alpha(0)$, and α' are adjustable parameters. The factor α^2 introduced here in contrast to that α used in Ref. 2 for the B_P and $B_{P'}$ amplitudes is at this stage purely *ad hoc*. It was made in formal analogy to the behavior of the ρ amplitudes at $\alpha_\rho=0$ as discussed in Ref. 5.

For definiteness, let us fix C_0 and D_0 to mean the values for $\pi^- p$ elastic scattering. Then for $\pi^+ p$ scattering, P and P' stay the same while ρ changes sign; for charge exchange, P and P' terms vanish while ρ is multiplied by $-\sqrt{2}$.

Experimental quantities are given by

$$\sigma_T(s)=\mathrm{Im}A'(s,t=0)/p,\qquad(5)$$

$$\frac{d\sigma}{dt}(s,t)=\frac{1}{\pi s}\left(\frac{M_N}{4k}\right)^2\left\{\left(1-\frac{t}{4M_N^2}\right)|A'|^2-\frac{t}{4M_N^2}\left(\frac{4M_N^2p^2+st}{4M_N^2-t}\right)|B|^2\right\},\qquad(6)$$

$$P(s,t)=-\frac{\sin\theta}{16\pi\sqrt{s}}\frac{\mathrm{Im}A'B^*}{d\sigma/dt},\qquad(7)$$

where s is the square of total c.m. energy, p is the pion lab momentum, k is the c.m. momentum, θ is the c.m. angle, and $P(s,t)$ is the polarization, defined relative to the direction $\mathbf{p}_{\mathrm{in}}\times\mathbf{p}_{\mathrm{out}}$ with $\mathbf{p}_{\mathrm{in}}$ and $\mathbf{p}_{\mathrm{out}}$ being the momenta of the pion.

We adjusted the Regge-pole parameters for an optimum over-all fit to the data. The results are shown in Tables I and II and Fig. 1. Table I gives the best-fit Regge parameters. Some parameters are not well determined, in particular D_0, D_1, and α' for both P and P', so ranges of solutions are possible. Table I shows two examples. Case (a) has moderate slopes for the P and P' trajectories, as in Ref. 2. Case (b) has a slightly smaller slope for P but a much bigger one for P', and has the interesting property[19] of allowing a secondary bump in $d\sigma/dt$ for elastic scattering, similar to that observed.[20] Table II lists the number of data of each kind that we used, with the corresponding contributions to χ^2 for both solutions. Notice the individual χ^2 values tabulated are comparable for both solutions. Figure 1 shows the fit to the new $\pi^- p$ polarization data for the solution (a). The corresponding fit for solution (b) is not illustrated here. It differs little from the one shown for $|t|<0.6$, but beyond this region the fit of solution (b) rises more quickly. For instance, at $t=-0.8$, solution (b) gives 16% for 10 GeV/c. The rest of the data fitting is also not illustrated, but the quality of the fitting is essentially as good as in Refs. 2 and 5.

The $\pi^\pm p$ polarizations are proportional to the cross products of the A' amplitudes $(A_P'+A_{P'}'\mp A_\rho')$ with the B amplitudes $(B_P+B_{P'}\mp B_\rho)$. The A_ρ' and B_ρ amplitudes are mainly determined by charge-exchange data, which require a small A_ρ' amplitude relative to A_P' and $A_{P'}'$ and a strong B_ρ amplitude. Since the A' and B amplitudes have the same phase for a given trajectory, ignoring the A_ρ' amplitude one is left with terms $(A_P'+A_{P'}')\times B_\rho$, $(A_P'\times B_{P'})$, and $(A_{P'}'\times B_P)$ for the $\pi^- p$ polarization. The sign of B_ρ is plausibly determined by ρ-meson coupling constants,[2] and the sign of $(A_P'+A_{P'}')$ at $t=0$ is fixed by σ_T. So the sign of $(A_P'+A_{P'}')\times B_\rho$ is fixed. From $d\sigma(\pi^\pm p)/dt$ data, the magnitude of $(A_P'+A_{P'}')\times B_\rho$ in the small $|t|$ region (say for $|t|<0.6$) also is essentially determined. It then turns out that this term alone accounts for the most of the observed $\pi^- p$ polarization. The dominance of this term is also suggested by the vanishing of the observed polarization near $t=-0.6$, where the B_ρ amplitude

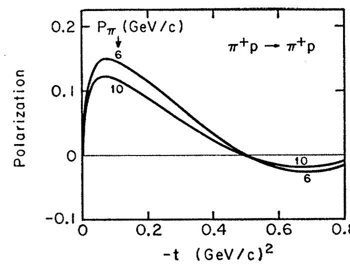

FIG. 2. Predictions of $\pi^+ p$ polarization at 6 and 10 GeV/c from Regge model of solution (a).

vanishes. Although at this stage the individual terms $A_P'\times B_{P'}$ and $A_{P'}'\times B_P$ are not well determined (see Table I), in order to account for the observed polarization for $|t|<0.6$, the *sum* of the contributions from these two terms has to be small. Now the $(A_P'+A_{P'}')\times B_\rho$ term has opposite signs for $\pi^- p$ and $\pi^+ p$ polarizations. Since this term dominates, one is led to predict that $\pi^+ p$ polarizations for $|t|<0.6$ should be comparable in magnitude and opposite in sign to that for $\pi^- p$. The prediction for solution (a) is illustrated in Fig. 2 at 6 and 10 GeV/c, where for $|t|<0.6$ the $\pi^+ p$ polarization appears with opposite sign. Beyond this region, solution (a), as illustrated, predicts a small negative polarization. The corresponding prediction for solution (b) is not illustrated here. It is similar to the one shown for $|t|<0.6$, but it remains positive near $t=-0.6$, and rises quickly beyond this region. For instance, this prediction gives 17% $\pi^+ p$ polarization at $t=-0.8$ for 10 GeV/c; this value is roughly the same as mentioned above for $\pi^- p$ polarization with the (b) fit. The relatively large polarization with the same sign for both $\pi^+ p$ and $\pi^- p$ at $t=-0.8$ for solution (b) is due to the fact that here the contribution due to the sum of $(A_P'\times B_{P'})$ and $(A_{P'}'\times B_P)$ is large and dominates.

To conclude, our results show that the recent CERN $\pi^- p$-polarization data are readily explained by the $P+P'+\rho$ Regge-pole model. This model predicts that $\pi^+ p$ polarization will be comparable in magnitude and opposite in sign to $\pi^- p$ results for $|t|<0.6$.

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