Universality Principles with 1^- and 2^+ Dominance

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It is assumed that the photon and graviton form factors are dominated by the vector and tensor mesons, which leads to the result that the vector- and tensor-meson couplings are renormalized in the lowest order of the $SU(3)$-symmetry-breaking interaction, but in a definite way. This result is different from the one obtained by Gatto and Ademollo, where vector currents are unrenormalized. The consequences of this assumption for the weak interaction, universality of meson couplings, and meson-baryon cross sections are discussed. The results are compared with experimental widths, and the agreement is found to be good.

I. INTRODUCTION

THE $SU(3)$ symmetry for strongly interacting particles has been successful in describing the classification[1,2] of hadrons, to some extent their electromagnetic and weak interactions,[3] and in the broken version their masses.[2,4] However, while the unbroken $SU(3)$ scheme with singlet octet mixing does explain the gross features of the decay rates of hadrons,[5] at least for the tensor and vector mesons the theoreticla predictions for the decay rates differ significantly from the experimental values, sometimes by as much as 50%. Note that the mixing phenomenon itself is due to $SU(3)$-breaking interaction and hence it is not consistent to allow for mixing while using $SU(3)$ symmetric currents. One may then hope that broken $SU(3)$ calculations can explain the above differences. However, it has been proved[6] that if one equates the vector current due to 0^- mesons, which couples to the vector mesons, with the electromagnetic current, then the vector currents are unrenormalized to first order in the symmetry breaking which is assumed to transform as the eighth component of an octet. In view of this, it remains a problem to reconcile the details of the experimental and the predicted rates of the vector-meson decays.

An analogous situation exists[7] for 2^+ mesons which couple to the other systems via their generalized energy momentum tensor densities. The role of the photon is now played by the graviton which couples universally with all the systems.

In this note we point out that some of the above difficulties can be resolved if one performs a broken $SU(3)$ calculation where the number of unknown amplitudes is sufficiently reduced by using universality of photon and graviton couplings so as to make the agreement with experimental number significantly nontrivial. The assumptions of the calculation are as follows:

(1) universality of electromagnetic and gravitational couplings;

(2) meson pole dominance: It will be assumed that the form factors for electromagnetic vertices are almost completely dominated by the poles due to ω, ρ, and ϕ mesons while for gravitational couplings the dominance is by the poles due to f^0 and f'. In field-theoretical language this means that the electromagnetic and gravitational properties are dominated by the direct couplings

$$A^\mu(\rho_\mu{}^0+(1/\sqrt{3})\omega_\mu{}^8) \tag{1}$$

and

$$Gh_{\mu\nu}f_1{}^{\mu\nu}, \tag{2}$$

respectively, where A^μ and $h_{\mu\nu}$ are the photon and the graviton fields; ρ^0 and ω^8 are the neutral components of the vector octet and f_1 is the unitary singlet 2^+ meson. This assumption differs from the one on which the work of Gatto and Ademollo[6] is based: Usually the electromagnetic current is taken to be the same as the vector current to which the neutral component of the vector octet couples. This would be the same as our assumption if the vector-meson masses were all taken as equal. We feel that equating the two currents, one coupled to strongly interacting massive vectons and the other to a photon, is not as reliable as our dynamical assumption of the form factors being dominated by neutral vector-meson poles. Similar comments apply to the graviton and 2^+ mesons.

(3) We assume that the "effective" medium strong interaction which breaks $SU(3)$ symmetry transforms as a sum of the eighth component of an octet and a singlet.

(4) The physical ω and ϕ and f^0 and f' are mixtures of singlets and the $I=0$ component of the octets, and the mixing parameters are determined by the observed tensor and vector meson masses.

With the above assumptions we will discuss the broken vector and tensor meson couplings.

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II. VECTOR-MESON COUPLINGS

A. *PPV* Couplings

The coupling of a vector meson α to two pseudoscalar mesons i and j is given by[8]

$$g_\alpha^{ij}=aC\begin{pmatrix}8&8&8_f\\ i&j&\alpha\end{pmatrix}+\sum_x b_xC\begin{pmatrix}8&8_f&x\\ i&j&\alpha\end{pmatrix}C\begin{pmatrix}8&x&8_d\\ \alpha&\alpha&0\end{pmatrix}\tag{3}$$

if the vector meson belongs to the octet, where 0 stands for $Y=0$, $I=0$ component, x runs over 8, 10, and $\overline{10}$; C denotes generalized Clebsch-Gordan coefficients, and subscripts f and d signify the usual f and d couplings when three octets occur. If the vector meson is a singlet, then the coupling is given by a single amplitude

$$f_0^{ij}=BC\begin{pmatrix}8&8&8_f\\ i&j&0\end{pmatrix}C\begin{pmatrix}8&1&8\\ 0&0&0\end{pmatrix}.\tag{4}$$

Then our assumption (2) that vector mesons dominate the electromagnetic form factors implies that the charges are proportional to

$$r\left\{C\begin{pmatrix}8&8&8\\ i&j&3\end{pmatrix}+\frac{1}{\sqrt3}C\begin{pmatrix}8&8&8_f\\ i&j&0\end{pmatrix}\right\}=\frac{g_3^{ij}}{m_\rho^2}+\frac{1}{\sqrt3 m_\omega^2}\sin\theta(\cos\theta\, f_0^{ij}+\sin\theta\, g_0^{ij})+\frac{1}{\sqrt3 m_\phi^2}\cos\theta(\cos\theta\, g_0^{ij}-\sin\theta\, f_0^{ij}),\tag{5}$$

where θ is the mixing angle between the singlet and the octet[9] and is taken to be 40°. Equation (5) is satisfied if and only if

$$b_{10}=b_{\overline{10}}=0,\tag{6}$$

and

$$a\left[\frac{1}{m_\rho^2}-\frac{\sin^2\theta}{m_\omega^2}-\frac{\cos^2\theta}{m_\phi^2}\right]+b_8\left[\frac{C_1}{m_\rho^2}-C_2\left(\frac{\sin^2\theta}{m_\omega^2}+\frac{\cos^2\theta}{m_\phi^2}\right)\right]=B\left(\frac{1}{m_\omega^2}-\frac{1}{m_\phi^2}\right)\sin\theta\cos\theta\, C_3,\tag{7}$$

where

$$C_1=C\begin{pmatrix}8&8&8_d\\ 3&0&3\end{pmatrix},\quad C_2=C\begin{pmatrix}8&8&8_d\\ 0&0&0\end{pmatrix},$$

and

$$C_3=C\begin{pmatrix}8&1&8\\ 0&0&0\end{pmatrix}.$$

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Note that $\sin\theta\cos\theta(1/m_\omega^2-1/m_\phi^2)B$ is of the second order in smallness. Using the experimental values for masses, Eq. (7) reduces to

$$a+2.6b_8=-0.22B.\tag{8}$$

We are now in a position to compare our results with the experimental numbers. Using (6) we have

$$\begin{aligned}\Gamma(\rho\to\pi\pi)&=t(a+0.45b_8)^2,\\ \Gamma(K^*\to K\pi)&=0.29t(a-0.22b_8)^2,\\ \Gamma(\phi\to K\bar K)&=0.062t[0.54(a-0.45b_8)-0.16B]^2.\end{aligned}\tag{9}$$

Taking $\Gamma(K^*\to K\pi)=50$ MeV and $\Gamma(\phi\to KK)=3.0$ MeV and using result (8), we get

$$\Gamma(\rho\to\pi\pi)=107\text{ MeV}\tag{10}$$

which is a significant improvement over the result of 170 MeV obtained from unbroken $SU(3)$ or if one equates the electromagnetic current to the current which couples to the vector mesons as in Ref. 6. Thus the universality principle as stated in assumption (2) is satisfied by the vector-meson decays.[10]

Let us consider the consequences of this principle for weak vector currents. If we make the assumption that the weak-interaction vector currents are dominated by vector-meson poles, then in consequence of result (6) the weak vector coupling is given by

$$h_\alpha^{ij}=C\begin{pmatrix}8&8&8_f\\ i&j&\alpha\end{pmatrix}\left[\frac{a}{m_\alpha^2}+\frac{b_8}{m_\alpha^2}C\begin{pmatrix}8&8&8_d\\ \alpha&\alpha&0\end{pmatrix}\right].\tag{11}$$

Equation (7) to first order in smallness is

$$a\left(\frac{1}{m_\rho^2}-\frac{1}{m_8^2}\right)+b_8\frac{(C_1-C_2)}{m^2}=0,\tag{12}$$

where m_8 is the mass of the eighth component of the octet, obtained from Gell-Mann–Okubo mass formula, and m is the $SU(3)$ symmetric mass of the octet. Using (12), it is easy to see that h_α^{ij} is unrenormalized to first order in smallness. This result is the same as the one obtained by Gatto and Ademollo.

B. *BBV* Couplings

The coupling of a vector meson α to two baryons i and j is of the same form as (3) except that there are two unperturbed coupling constants and x now runs over 1, 8, 10, $\overline{10}$, 27. Using C invariance, b_{10} and $b_{\overline{10}}$ are related and there are two 8's, one symmetric and the other antisymmetric in baryon indices. Similarly, (4)

[10] If we assume that the quarks are integrally charged, then the photon–vector-meson coupling is

$$A_\mu\left(\frac{1}{\sqrt2}\rho_0^\mu+\frac{1}{\sqrt3}\omega_0^\mu+\frac{1}{\sqrt6}\omega_8^\mu\right).$$

This gives, for $\Gamma_{K^*\to K\pi}=49$ MeV and $\Gamma_{\phi\to K\bar K}=3.3$ MeV, the $\rho\to2\pi$ width as $\Gamma_{\rho\to2\pi}=177$ MeV, which is not in good agreement with the experimental result, as we have assumed in the main text that the quarks are fractionally charged.

also has two other additional terms, one perturbed and the other unperturbed. Use of universality in the form (5) then implies that to first order in smallness only a, b_1, and b_8 of (3) are nonzero and a and b_8 have the same value as for pseudoscalars. This means that ρ and K^* mesons couple universally, i.e., their couplings to pseudoscalars and baryons are the same, and also the weak vector currents due to baryons are unrenormalized to first order in smallness and their coupling strength is the same as for pseudoscalars.

III. 2^+ MESON COUPLINGS

We can now perform the calculation with the same technique as before. However, for pedagogical reasons, we shall use a slightly different approach.[6] The effective interacting Legrangian can be written as

$$\mathcal{L}=f_1T^1+f_8T^8+\tfrac{1}{2}m_1^2f_1f_1+\tfrac{1}{2}m_8^2f_8f_8+m_{18}f_1f_8+Gh_{\mu\nu}f_1^{\mu\nu},$$

where T^1 and T^8 are the quantities which couple to the $I=Y=0$ singlet f_1 and octet f_8, respectively, in the broken $SU(3)$ interaction:

$$(\sqrt{6})a_1(\mathrm{Tr}F\{M,M\})+a_2(\mathrm{Tr}FM\lambda_8M)+a_3(\mathrm{Tr}FM)(\mathrm{Tr}M\lambda_8)+a_4(\mathrm{Tr}F\lambda_8)(\mathrm{Tr}MM)+b_1f_1(\mathrm{Tr}MM)+b_2f_1(\mathrm{Tr}M\lambda_8M).$$

In the above expression, F and M are, respectively, the 2^+ and 0^- octet matrices. The space-time indices are such that

$$(MM)_{ab}{}^{\mu\nu}=\tfrac{1}{2}(M^\mu M^\nu+M^\nu M^\mu)_{ab}-g^{\mu\nu}\tfrac{1}{2}[M^\lambda M_\lambda+\tfrac{1}{2}(m_a^2+m_b^2)MM]_{ab}.$$

In terms of the physical particles

$$f=f_1\cos\theta+f_8\sin\theta,$$
$$f'=-f_1\sin\theta+f_8\cos\theta,\quad \theta\sim29°$$

we rewrite $\mathcal{L}$ as

$$\mathcal{L}=f(T^1\cos\theta+T^8\sin\theta)+f'(-T^1\sin\theta+T^8\cos\theta)+\tfrac{1}{2}m_f^2ff+\tfrac{1}{2}m_f'^2f'f'+Gh(f\cos\theta-f'\sin\theta).$$

Because of universality, the effective graviton-matter coupling must be of the form

$$gh_{\mu\nu}(T_\pi{}^{\mu\nu}+T_K{}^{\mu\nu}+T_\eta{}^{\mu\nu}),$$

where the $T^{\mu\nu}$'s are the energy-momentum tensor densities for the corresponding particles. This principle, when applied to π, K, and η, successively, gives the following relations:

$$-g/G=A\left(b_1+\frac{b_2}{\sqrt{3}}\right)+B\left(2a_1+\frac{1}{3\sqrt{2}}a_2+\sqrt{2}a_4\right),$$

$$-g/G=A\left(b_1-\frac{b_2}{2\sqrt{3}}\right)+B\left(-a_1-\frac{\sqrt{2}}{3}a_2+\sqrt{2}a_4\right),$$

and

$$-g/G=A\left(b_1-\frac{b_2}{\sqrt{3}}\right)+B\left(-2a_1+\frac{1}{\sqrt{2}}a_2+\frac{\sqrt{2}}{3}a_3+\sqrt{2}a_4\right)$$

where

$$A=\cos^2\theta/m_f^2+\sin^2\theta/m_f'^2,$$

and

$$B=\sin\theta\cos\theta(1/m_f^2-1/m_f'^2).$$

As the coupling constant G is not fixed in this calculation,[11] we have effectively only two relations, which are taken as

$$a_3=-3a_2,$$

and

$$\tfrac{1}{2}\sqrt{3}Ab_2=-B(3a_1+(1/\sqrt{2})a_2).$$

There are six independent amplitudes, so hence, we need four additional pieces of information as the input data. The results are tabulated in Table I. It is seen that our results are in fair agreement with the experimental data, even though we have less success here than with the 1^- mesons in reducing some of the widths.

TABLE I. Results of the calculation.

Decay modes	Widths[a] (MeV)	Expt. widths[b] (MeV)
$f^0\to\pi\pi$	110 (input)	118±20
$f^0\to K\bar{K}$	7.8	2±2
$f^0\to\eta\eta$	0.4	?
$A_2\to K\bar{K}$	4.5 (input)	4.5±1.5
$A_2\to\pi\eta$	8.5	3±2
$K^{**}\to K\pi$	35 (input)	39±20
$K^{**}\to K\eta$	1.5	2±2
$f'\to\pi\pi$	0 (input)	0±6
$f'\to K\bar{K}$	41	51±25
$f'\to\eta\eta$	13	?

[a] We obtain $a_1=0.206$, $a_2=0.017$, $a_3=-0.051$, $a_4=-0.013$, $b_1=0.79$, and $b_2=-0.110$.
[b] From Ref. 5.

If we assume that f is the Pomeranchuk trajectory in the Regge theory, then the ratio of the π-p total cross section to that of the K-p is determined by the quantity

$$R=\mathrm{Amplitude}(f\to\pi\pi)/\mathrm{Amplitude}(f\to KK)=1.36.$$

This is an improvement over the value given by Desai and Freund[12] but still not in agreement with the experimental situation.

This gravitational universality principle can be applied to the baryons in the similar manner. There, we obtain three relations among the independent amplitudes. However, in view of the lack of application for the time being, we shall not discuss it further.

[11] Had we assumed that the effective symmetry breaking transforms as the eighth component of an octet only, we would have had an extra condition $-g/G=b_1/m_1^2$, where m_1 is the unbroken mass of the singlet. However, we would not then have good fit with the experimental data.

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IV. *PP* AND *PB* CROSS SECTIONS

Let us consider the consequences of assumption (2) for meson-meson and meson-baryon total cross sections. Let us ignore symmetry breaking and also identify the unitary singlet 2^+ meson with the Pomeranchon. It is easily seen that the assumption (2) then implies that the Pomeranchon couples to P(pseudoscalar) mesons and B(baryons) with equal strength. Therefore, in the $SU(3)$ limit, the PP, PB, BB total cross sections are all equal which is in disagreement with experiments. This then suggests one of the following possibilities:

(a) $SU(3)$ symmetry breakings are large.

(b) Our assumption (2) is wrong.

(c) Perhaps the most likely possibility is that the Pomeranchon cannot be identified with any of the 9 known 2^+ mesons.[12]

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Meson-Baryon Coupling Constants in a Current-Current Model

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Methods recently developed by Chiu and Schechter for evaluation of matrix elements of bilinear products of hadron currents are combined with the PCAC (partially conserved axial-vector current) reduction technique to obtain approximate values of the meson-baryon coupling constants. Several current-current-type symmetry-breaking Hamiltonians are examined, and the results are compared with recent calculations using a Hamiltonian proportional to the space integral of the eighth component of the quark scalar density.

RECENTLY Chiu and Schechter[1] have investigated the assumption that the medium-strong (ms) baryon mass-splitting interaction is described by one of the following bilinear products of octet currents:

$$H_{sb}^{(a)}=g_{ms}^{(a)}V_3^3V_3^3\,, \tag{1a}$$

$$H_{sb}^{(b)}=g_{ms}^{(b)}\times\tfrac{1}{2}[V_c^3,V_3^c]_+\,, \tag{1b}$$

$$H_{sb}^{(c)}=g_{ms}^{(c)}(V_3^3V_3^3+P_3^3P_3^3)\,, \tag{1c}$$

$$H_{sb}^{(d)}=g_{ms}^{(d)}\times\tfrac{1}{2}([V_c^3,V_3^c]_+ +[P_c^3,P_3^c]_+)\,. \tag{1d}$$

They have obtained numerical values for the mass splittings which are in reasonable agreement with experiment for all the Hamiltonians in Eq. (1). The symmetry-breaking coupling constant $g_{ms}^{(k)}$ was obtained from baryon mass data.

The purpose of this article is to point out that their calculation can be extended to give estimates of the meson-baryon coupling constants in broken $SU(3)$.

We examine each of the Hamiltonians of Eq. (1) as a possible symmetry-breaking interaction, taking the values of $g_{ms}^{(k)}$ from baryon mass data. We assume that the coupling constants can be written

$$g^{(k)}(BB'P)\bar{U}_{B'}\gamma_5U_B\phi_\pi= \langle B|H_{sb}^{(k)}|B'P\rangle+g_0(BB'P)\bar{U}_{B'}\gamma_5U_B\phi_\pi\,, \tag{2}$$

where $g_0(BB'P)$ is the unrenormalized coupling constant resulting from the $SU(3)$ symmetric interaction and $g^{(k)}(BB'P)$ is the renormalized coupling constant.

Following Ref. 1 we substitute each of the Hamiltonians of Eq. (1) into the right-hand side of Eq. (2). Using the generalized hypothesis of partially conserved axial-vector current (PCAC)

$$\partial_\mu J_{5\mu}{}^k=c_kP_k\,,$$

where

$$c_k=\frac{(M_B+M_{B'})\mu^2}{g(BB'P)}(-G_A/G_V)\,, \tag{3}$$

we reduce the matrix elements of H_{sb} in the usual way[2,3] getting

$$(2q_0{}^k)^{1/2}\langle B'P|H_{sb}|B\rangle =\frac{-i}{c_k}\int d^4x\exp(-iq^{(k)}x)(\Box^2-\mu^2) \times\langle B'|[\partial_\mu J_{5\mu}{}^{(k)},H_{sb}]|B\rangle\theta(-x_0)\,, \tag{4}$$

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