Constraints on πN Phase-Shift Analysis from Angular Distributions of Production Reactions*

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A model-independent angular-momentum decomposition of the $\pi N \to \pi\pi N$ reaction amplitudes is exploited to relate angular distributions of each final-state particle with the absorption parameters $\eta_{l\pm}{}^I$ used in πN-scattering phase-shift analyses. This relation provides constraints in the form of lower bounds on $(1-\eta_{l\pm}{}^2)$ for specific sets of angular momentum and parity scattering states. Available data on production reactions from threshold to 600 MeV are examined and the consistency of various phase-shift solutions with these constraints is discussed. The Cence solution does not appear to agree with the energy dependence of the inelastic cross section, as deduced from available data on total, elastic, and charge-exchange πN cross sections. None of the existing phase-shift solutions at 290 MeV provides for absorption in partial waves with $J>\frac{3}{2}$, while our analysis shows that production data require such absorption.

I. INTRODUCTION

THE inelasticity of πN scattering in the low-energy region is dominated by single-pion production even above the threshold for two-pion production, up to $T_{\text{lab}}(\pi)\sim 650$ MeV. As a consequence, the inelasticity parameters $\eta_{l\pm}{}^I$ appearing in phase-shift analyses of πN elastic scattering are completely determined in the region up to 650 MeV by the amplitudes for the processes $\pi N \to \pi\pi N$. These amplitudes cannot be determined uniquely by a knowledge of production cross sections and final-state angular distributions. Nevertheless, such information, obtained from data on the production reactions, can serve to place consistency restrictions on allowed sets of inelasticity parameters. Such restrictions are much stronger than simple total-cross-section consistency requirements. In this paper we formulate this idea in precise terms and apply it to the comparison of published phase-shift sets with available data on the production reactions below 650 MeV.

The practical analyses of production reactions (with 3-body final states) previously published have been concerned, in the main, with specific models such as final-state resonance production and one-meson exchange mechanisms. Although considerable success for such models (e.g., the isobar model of Olsson and Yodh[1] and one-pion exchange[2] for $\pi^+p\to\pi^+\pi^+n$) has been noted in restricted energy regions and for certain charge states, we do not consider such models in our analysis. Our objective is the determination of experimentally correct phase shifts and inelasticities with no built-in prejudices concerning angular-momentum states or energy dependences.

The analysis is based on an exact formalism for production reactions proposed by Branson, Landshoff, and Taylor (BLT)[3] that does not involve a decomposition in terms of relative angular momenta of two-body subsystems in the final state. Similar 3-body formalisms have been used by Omnes for 3-body scattering[4] and bound-state problems, and by Berman and Jacob[5] in their discussion of 3-body decays of resonant states with definite angular momentum and parity.

II. FORMALISM

The reaction amplitude for $\pi_1 N\to\pi_2\pi_3 N'$ in a definite charge state with conservation of momentum is determined by 5 independent continuous momentum variables and two independent discrete variables describing the spins of the initial and final nucleons. Following BLT, we first analyze the matrix element connecting a state with a definite initial nucleon helicity μ to a state of definite nucleon spin projection λ along the normal to the "decay" plane; from these, amplitudes will be constructed that represent transitions between states of definite parity and total angular momentum.

The angle and energy variables are chosen in a rigid-body final state coordinate system[3]; we define the *normal* system as follows: Let the outgoing nucleon (or any one of the desired final-state particles) define the x axis, and the normal to the decay plane the z axis, of a coordinate system whose origin moves with the center of mass. The configuration of the other two final-state particles in the production plane is determined by specifying any two final-state particle energies, say ω_1 and ω_2, plus the over-all center-of-mass energy $w=s^{1/2}$. The orientation of such a rigid-body coordinate system with respect to the initial-state momenta can be specified by three Euler angles (Φ,θ,Ψ). The polar and azimuthal coordinates of the incident pion (beam) in the rigid-body coordinate system are θ, and Φ, respectively; the angle Ψ, representing a rotation around the beam axis, must disappear from final expressions for squares of matrix elements (observables) since rotational invariance is assumed for the matrix element. In

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[1] M. Olsson and G. B. Yodh, Phys. Rev. **145**, 1309 (1966); **145**, 1327 (1966).

[2] J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **126**, 763 (1962).

[3] D. Branson, P. V. Landshoff, and J. C. Taylor, Phys. Rev. **132**, 902 (1963). We shall refer to this reference as BLT.

[4] R. Omnes, Phys. Rev. **134**, B1358 (1964).

[5] S. M. Berman and M. Jacob, Phys. Rev. **139**, B1023 (1965).

terms of these variables such a production matrix element may be written[3] as $A_\lambda{}^\mu(s,\omega_1\omega_2;\Phi\theta\Psi)$.

Initial states of definite parity may be obtained by forming symmetric and antisymmetric linear combinations of helicity states. The phase conventions are important here; following BLT, we use the phase convention of Wick[6] instead of that of Jacob and Wick.[7] Utilizing parity conservation for the production reaction and the angular-momentum decomposition of BLT, we obtain for the matrix element between an initial state of definite parity (and a final state with the same parity) the following representation:

$$A_\lambda{}^{m\pm}(s,\omega_1\omega_2;\Phi\theta\Psi)=\sum_{J=0}^{\infty}\sum_{\Lambda=-J}^{+J}\left(\frac{2J+1}{8\pi}\right)^{1/2}D_{\Lambda m}{}^J(\Phi\theta\Psi)B_{J\Lambda}{}^m(s,\omega_1\omega_2;\lambda)\times[1\pm\eta_2(-1)^{\Lambda+1/2}]. \quad (1)$$

In the representation, J is the total angular momentum, Λ is its projection along the normal to the production plane, and m is the spin projection of the initial nucleon (target) along the beam direction; η_2 is the intrinsic parity of the final nucleon, here positive; the ($\pm$) refers to the parity of the initial state. As before, λ is the final spin projection on the "body-fixed" z axis.

The $B_{J\Lambda}{}^m$ are complex partial-wave production amplitudes providing a complete description of the production process in a manner analogous to the description of two-body scattering by partial-wave scattering amplitudes. The contribution to the inelasticity parameters $(1-\eta^2)$ for each J and parity state of πN scattering can be obtained by integration of a sum of squares of the partial-wave production amplitudes for the same J^P over allowed values of ω_1 and ω_2 (i.e., over the Dalitz plot). If only a few scattering states have $\eta^2<1$, only the corresponding J^P states will have $|B_{J\Lambda}|^2\neq 0$, thus limiting the complexity of the angular distributions of the production process. Note that the ($\pm$) parity of the initial state has the effect of setting $B_{J\Lambda}=0$ if $(\Lambda\pm\frac{1}{2})$ is an odd integer.

With the normalization of BLT, the differential cross section for production (with a definite initial spin projection m, definite parity, and definite final spin projection λ) is

$$\frac{\partial^4\sigma(m,\lambda)}{\partial\omega_1\partial\omega_2\partial(\cos\theta)\partial\Phi}=\frac{\pi^3}{16W^3k}|A_\lambda{}^m|^2, \quad (2)$$

where the Ψ dependence cancels out.

Instead of a polar angle defined with respect to the normal production plane, as in the normal system, it is often more convenient (at least when examining previously published data) to consider the distribution of the angle between the incident beam direction and one of the outgoing particles. The form of such distributions can be obtained by a rotation of coordinate systems, and we will give this form after considering special J^P cases relevant to the specific energy region and production process under consideration. First, however, it is necessary to expand $|A|^2$ in terms of spherical harmonics, and integrate the coefficients (formally) over the Dalitz plot.

The index λ will be suppressed in what follows, since the dependence of $|A^2|$ on this index is purely dynamical and has no relation in our formalism to the explicit kinematic factors. Furthermore, since unpolarized targets have been exclusively used in production experiments, we will average $|A|^2$ over $m=\pm\frac{1}{2}$. The result after employing parity conservation in the production matrix elements can be written

$$\langle A^2\rangle_{\rm av}\equiv\tfrac{1}{2}\sum_m|A_\lambda{}^m|^2=\frac{1}{8\pi}\sum_{JJ',\Lambda\Lambda'}\{[(2J+1)(2J'+1)]^{1/2}B_{J\Lambda}(\omega_1\omega_2)B_{J'\Lambda'}{}^*(\omega_1\omega_2)\times e^{i(\Lambda'-\Lambda)\Phi}[d_{\Lambda\frac{1}{2}}{}^J(\theta)d_{\Lambda'\frac{1}{2}}{}^{J'}(\theta)+(-1)^{J+J'+\Lambda+\Lambda'}d_{\Lambda-\frac{1}{2}}{}^J(\theta)d_{\Lambda'-\frac{1}{2}}{}^{J'}(\theta)]\}, \quad (3)$$

where we have used[8]

$$D_{\Lambda\mu}{}^J(\Phi\theta\Psi)=e^{-i\Lambda\Phi}d_{\Lambda\mu}{}^J(\theta)e^{-i\mu\Psi},$$

and suppressed the index $m=\pm\frac{1}{2}$ in the $B_{J\Lambda}$'s, and the variable s. The parity of each state in the sum is characterized implicitly by the occurrence of either even or odd values of $(\Lambda+\frac{1}{2})$, $(\Lambda'+\frac{1}{2})$ [see Eq. (1)].

The products of d functions in (3) can be expanded[9] in spherical harmonics, and the result is

$$\langle A^2\rangle_{\rm av}=\frac{1}{4\pi^{1/2}}\sum_{JJ',\Lambda\Lambda'}\Bigg\{[(2J+1)(2J'+1)]^{1/2}B_{J\Lambda}(\omega_1\omega_2)B_{J'\Lambda'}{}^*(\omega_1\omega_2)(-1)^{\Lambda'-\frac{1}{2}}\times\sum_{|J-J'|\leqslant j\leqslant J+J'}(2j+1)^{1/2}\begin{pmatrix}J & J' & j\\ \Lambda & -\Lambda' & \Lambda'-\Lambda\end{pmatrix}Y_j{}^{\Lambda-\Lambda'}(\theta,\Phi)^*\left[\begin{pmatrix}J & J' & j\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}-(-1)^{J+J'+\Lambda+\Lambda'}\begin{pmatrix}J & J' & j\\ -\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}\right]\Bigg\}. \quad (4)$$

The 3-j symbols[10] have been used for conciseness of notation.

[6] G. C. Wick, Ann. Phys. (N. Y.) 18, 65 (1962).
[7] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).
[8] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957), Chap. 4.
[9] Reference 8, Sec. 4.3.
[10] Reference 8, Sec. 3.7.

Integrating ω_1, ω_2 over the Dalitz plot yields the angular-distribution function for the normal system;

$$W(\theta,\Phi)\equiv\int\int d\omega_1 d\omega_2 \langle A^2(\theta,\Phi;\omega_1,\omega_2)\rangle_{\text{av}},$$

which can be written in the form

$$W(\theta,\Phi)=\sum_{jM}\left(\frac{2j+1}{4\pi}\right)^{1/2} Y_j^{M*}(\theta,\Phi)W_M{}^j. \quad (5)$$

In the normal system [appropriate to decomposition (1)], the coefficients $W_M{}^j$ can be expressed as

$$W_M{}^j=\frac{1}{2\pi}\sum_{JJ',\Lambda\Lambda'}\langle J\Lambda,J'\Lambda'\rangle G_M{}^j(J\Lambda,J'\Lambda'), \quad (6)$$

where

$$\langle J\Lambda,J'\Lambda'\rangle\equiv\int\int d\omega_1 d\omega_2 B_{J\Lambda}(\omega_1\omega_2)B_{J'\Lambda'}{}^*(\omega_1\omega_2).$$

The set of parameters $\langle J\Lambda,J'\Lambda'\rangle=\langle J'\Lambda',J\Lambda\rangle^*$ contains the dynamics of the production process; they are essentially density matrix elements. Each total energy w and each alternative choice of rigid-body coordinate system (here the normal system has been employed) leads to a different set. The G factors are independent of dynamics.

If the polar angle distribution for a given single particle in the final state is desired, a different coordinate system can be utilized. The simplest way to obtain the required distributions, however, is to define the matrix elements $B_{J\Lambda}$ with respect to a normal system with the desired particle momentum along the x axis of the rigid-body system and, after obtaining the corresponding distribution function (5), rotate the coordinate system by $\pi/2$ around the y axis. Each term in the sum over j in (5) transforms as an irreducible representation of the rotation group, so the transformed angular distribution can be written

$$\bar{W}(\theta',\Phi')=\sum_{jM}\left(\frac{2j+1}{4\pi}\right)^{1/2} Y_j^{M*}(\theta',\Phi')\bar{W}_M{}^j, \quad (7)$$

where θ' is now the polar angle between outgoing particle and incident beam, and

$$\bar{W}_M{}^j=\sum_{M=-j}^{+j} W_{M'}{}^j D_{M'M}{}^j(0,\pi/2,0). \quad (8)$$

The values of the required D functions are tabulated by Edmonds for $j<3$. Integrating (7) over Φ' we obtain the distribution function for the angle θ (dropping primes here) between the desired outgoing particle and the incident beam;

$$\frac{\partial\sigma}{\partial(\cos\theta)}=\bar{W}(\theta)=\sum_j(2j+1)P_j(\cos\theta)(2\pi\bar{W}_0{}^j). \quad (9)$$

The expressions (6) and (9) form the basis of our discussion of the published data. The parameters $\langle J\Lambda,J'\Lambda'\rangle$ are determined by production data.

It can be seen from (6) that only even j values are present when states of the same parity contribute to the production process; odd j values are associated with interference between odd and even parity amplitudes. Also, if J_M is the maximum value of angular momentum for which $B_{J\Lambda}\neq 0$ [and hence $(1-\eta^2)\neq 0$], the maximum complexity of (9) will be determined by $j_{\max}\leq 2J_M$. Thus, if a $\frac{3}{2}^-$ *or* $\frac{3}{2}^+$ state dominates (for example), the distribution (9) can only have the form $[1+aP_2(\cos\theta)]$.

III. INELASTICITY PARAMETERS AND PRODUCTION PROPERTIES IN THE $I=\frac{1}{2}$ STATE

A. Total Inelasticity

The total reaction cross section for πN processes in each isotopic spin state is related to the set of inelasticity parameters in that isospin state by

$$\sigma_r=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{\infty}[(l+1)(1-\eta_{l+}{}^2)+l(1-\eta_{l-}{}^2)]. \quad (10)$$

The simplest constraint relating production data and η values is the equality of $(\sigma_r/\pi\lambda\!\!\!^{-2})$ from (10) at each energy with the empirically determined value. To obtain σ_r, a subtraction of elastic and charge-exchange cross sections from the total must be carried out at each energy.

We have compiled $I=\frac{1}{2}$ reaction cross sections[11] and compared the results with the predictions of five published phase-shift analyses.[12–16] The results are shown in Fig. 1.

It is clear from the figure that the inelasticities of Auvil *et al.*[13] are in best agreement with the data. Since small changes in the inelasticity parameters will give large percentage changes in the (small) reaction cross section, it is probably not correct to discard all of the other solutions on the basis of the poor agreement in the 450–600 MeV solution. The Cence[13] solution, however, shows an energy dependence (with a sharp "break" near 450 MeV) that is clearly inconsistent with experiment. Consequently, on the basis of the $I=\frac{1}{2}$ total reaction cross section alone, (and subject to the qualifications discussed in the Appendix) we find the Cence solution unacceptable.

B. Angular Distributions

It will be convenient, in discussing angular distributions, to modify slightly the notation of Eq. (6) by

[11] For discussion and references, see the Appendix.
[12] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).
[13] J. Cence, Phys. Letters **20**, 306 (1966).
[14] P. Auvil, C. Lovelace, A. Donnachie, and A. T. Lea, Phys. Letters **12**, 76 (1964).
[15] P. Bareyre, Proc. Roy. Soc. (London) **A289**, 463 (1966).
[16] B. H. Bransden, R. G. Moorhouse, and P. J. O'Donnell, Proc. Roy. Soc. (London) **A289**, 538 (1966).

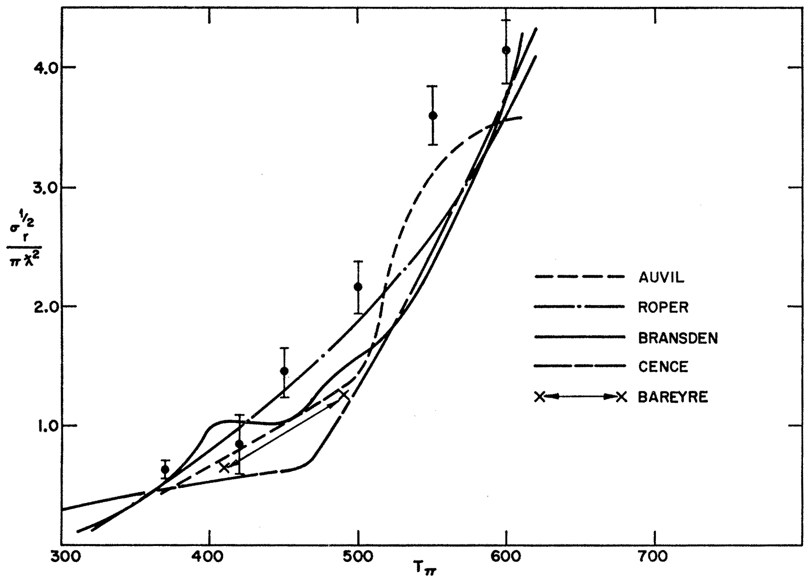

FIG. 1. Experimental values of $T=\frac{1}{2}$ reaction cross section (see Appendix) compared with predications of 5 phase-shift solutions (Refs. 12, 13, 14, 15, and 16). Energy scale is laboratory pion kinetic energy in MeV; ordinate is $T=\frac{1}{2}$ reaction cross section divided by the value corresponding to complete absorption in the $s_{1/2}$ state [see Eq. (10)].

writing

$$\langle a,b\rangle \equiv B_b^* \cdot B_a, \tag{11}$$

where a, b stand for the quantum numbers $J\Lambda$, $J'\Lambda'$. Here $B_b^*B_a$ stands for the integrated (over ω_1, ω_2) and summed (over final spins) product. It is easy to show that such products satisfy the Schwartz inequality

$$|B_b^* \cdot B_a|^2 \leqslant |B_b|^2 |B_a|^2.$$

Let us now define the quantities

$$A_j = \pi(2j+1)\bar{W}_0{}^j, \tag{12a}$$

$$B_\pm = B_{\frac{1}{2},\pm\frac{1}{2}}, \tag{12b}$$

$$C_{2\Lambda} = B_{\frac{3}{2},\Lambda}. \tag{12c}$$

In the case where only the incident $j=\frac{1}{2}$ and $\frac{3}{2}$ states contribute to the absorption, the A_j's take the form

$$\begin{aligned}
A_0 &= |B_+|^2 + |B_-|^2 + \sum_{\Lambda=-\frac{3}{2}}^{3/2} |C_{2\Lambda}|^2, \\
A_1 &= -\tfrac{2}{3}\,\mathrm{Re}\{B_+^*B_- + 2^{-1/2}(B_+^*C_{-1} - B_-^*C_1) + (\tfrac{3}{2})^{1/2}(B_-^*C_{-3} - B_+^*C_3) + \tfrac{1}{5}[2C_1^*C_{-1} + 3^{1/2}(C_3^*C_1 + C_{-3}^*C_{-1})]\}, \\
A_2 &= \tfrac{1}{10}\,\mathrm{Re}\{|C_3|^2 + |C_{-3}|^2 - |C_1|^2 - |C_{-1}|^2 + 4\sqrt{2}(B_-^*C_{-1} - B_+^*C_1) \\
&\qquad + 2\times 6^{1/2}(B_+^*C_{-3} - B_-^*C_3) - 2\times 3^{1/2}(C_3^*C_{-1} + C_1^*C_{-3})\}, \\
A_3 &= (3/7)\,\mathrm{Re}\{C_3^*C_{-3} + \tfrac{3}{5}[C_1^*C_{-1} - 3^{-1/2}(C_3^*C_1 + C_{-1}^*C_{-3})]\}.
\end{aligned} \tag{13}$$

The quantities B_+, C_1, C_{-3} are related to the incident odd-parity (s and d) states, whereas the remaining coefficients belong to the states of even parity. It is easy to see, therefore, that the coefficients satisfy the same "maximum complexity" conditions that one has in elastic scattering.

The corresponding coefficients in the normal system [before rotation (8) is performed] are listed in Table I; we did not make use of that system in the following discussion.

We now make use of the expressions in Eq. (13) to examine some phase-shift solutions near 300-MeV incident-pion kinetic energy (laboratory). For this purpose we shall use the experiment by Batusov *et al.*[17] on

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n \tag{14}$$

with 290-MeV incident pions. At this energy, the process (14) provides about 80% of the inelasticity in the $T=\frac{1}{2}$ state.[1] It will be convenient, therefore, when discussing phase-shift analyses at this energy to assume that the inelasticity parameters relate only to the process (14).

In the Batusov experiment it was found that the neutron angular distribution was strongly peaked in the backward direction. We have made a least-mean-square Legendre polynomial fit to the angular distribution with

[17] Yu. A. Batusov, N. Bagachev, S. Bunyatov, V. Sidorov, and Y. Yarba, Dokl. Akad. Nauk SSSR **133**, 52 (1960) [English transl.: Soviet Phys.—Doklady **5**, 731 (1961)]; Zh. Eksperim. i Teor. Fiz. **40**, 460 (1961) [English transl.: Soviet Phys.—JETP **13**, 320 (1961)].

the following results (arbitrary units):

$$a_0=10.1\pm1.4\,,\quad a_3=5.5\pm3.2\,,$$
$$a_1=14.0\pm3.0\,,\quad a_4=-0.4\pm0.2\,. \qquad (15)$$
$$a_2=10.8\pm3.5\,,$$

Here a_l is the coefficient of $P_l(\cos\theta)$ in the expansion. The χ^2 for the best fit is 11.5 for 5 deg. of freedom. Taken at face value, the results suggest that absorption is taking place in states with j at least as high as $\frac{5}{2}$, which is itself a surprising result at this energy.[18] However, we shall proceed by tentatively ignoring the a_3 and a_4 coefficients on the grounds that they are statistically poorly determined.

Turning now to the phase shift analyses, we have the values in Table II predicted for the η parameters at 310 MeV. Values of $(j+\frac{1}{2})(1-\eta^2)$ are in parentheses.

On the other hand, from the expressions in Eq. (13) we can derive the following inequalities for A_1 and A_2.

$$|A_2|\leqslant\tfrac{1}{5}[\rho_{3+}{}^2+\rho_{3-}{}^2+(14)^{1/2}(\rho_{1+}\rho_{3-}+\rho_{1-}\rho_{3+})]\,, \qquad (16a)$$

$$|A_1|\leqslant\tfrac{2}{3}[\rho_{1+}\rho_{1-}+\tfrac{4}{5}\rho_{3+}\rho_{3-}+\sqrt{2}(\rho_{1+}\rho_{3+}+\rho_{1-}\rho_{3-})]\,. \qquad (16b)$$

Here we have defined

$$(\rho_{2j+1,\pm})^2=(j+\tfrac{1}{2})(1-\eta_{j\pm}{}^2)\,, \qquad (17)$$

where the $\pm$ refers to $j=l\pm\frac{1}{2}$.

Using the values in Table II we find that the maximum possible values of A_1/A_0 and A_2/A_0 are as shown in Table III. As a matter of fact, the maximum possible value of $|A_1|/|A_0|$, if only $j=\frac{1}{2}$ and $\frac{3}{2}$ participate is about 0.5. Thus, it is apparent that higher partial waves (at least $j=\frac{5}{2}$) are playing an appreciable role in the absorption process at an energy as low as 290 MeV.

It is of some interest to note that at increased energies there is also evidence for the importance of high partial waves to two-pion production. For example, in the 604-MeV experiment of Vittitoe *et al.*[19] we find that

TABLE I. Coefficients of $[4\pi/(2j+1)]^{1/2}Y_j{}^{M*}$ in $W(\theta,\phi)$ with normal coordinate system. The B and C notation is described in the text; M values not explicit here may be obtained by complex conjugation.

j	M	
0	0	$\lvert B_+\rvert^2+\lvert B_-\rvert^2+\lvert C_3\rvert^2+\lvert C_1\rvert^2+\lvert C_{-1}\rvert^2+\lvert C_{-3}\rvert^2$
1	1	$\frac{1}{5}(\frac{2}{3})^{1/2}[C_3C_1{}^*+2C_1C_{-1}{}^*+C_{-1}C_{-3}{}^*]$ $-\frac{1}{3}[3^{1/2}C_3B_+{}^*+C_1B_-{}^*-\sqrt{2}B_+B_-{}^*-3^{1/2}B_1C_{-3}{}^*-B_+C_{-1}{}^*]$
2	0	$\frac{1}{5}[\lvert C_1\rvert^2+\lvert C_{-1}\rvert^2-\lvert C_3\rvert^2-\lvert C_{-3}\rvert^2$ $+\sqrt{2}(B_+C_1{}^*+C_1B_+{}^*)-\sqrt{2}(B_-C_{-1}{}^*+C_{-1}B_-{}^*)]$
2	2	$-\frac{1}{5}\sqrt{2}[C_3C_{-1}{}^*+C_1C_{-3}{}^*-\sqrt{2}(C_3B_-{}^*-B_+C_{-3}{}^*)]$
3	1	$-(6/35)[C_3C_1{}^*+C_{-1}C_{-3}{}^*-3^{1/2}C_1C_{-1}{}^*]$
3	3	$-(6/35)[5^{1/2}C_3C_{-3}{}^*]$

[18] However, we find that this sort of behavior was predicted by G. F. Chew in his remarks at the Berkeley Conference on Strong Interactions reported in Rev. Mod. Phys. **33**, 361 (1961).

[19] C. N. Vittitoe, B. R. Riley, W. J. Fickinger, V. P. Kenney, J. G. Mowat, and W. D. Shephard, Phys. Rev. **135**, B232 (1964).

TABLE II. Inelasticities for $T=\frac{1}{2}$ as given by published phase-shift sets. Numbers in parentheses are values of $(j+\frac{1}{2})(1-\eta^2)$. Lower number of each pair is value of η at 310 MeV.

	s_1	p_1	p_3	d_3	d_5	f_5
Roper[a]	(0.006)	(0.120)	(0.044)	(0.020)		
	0.997	0.938	0.989	0.995	1	1
Cence[b]	(0)	(0)	(0.190)	(0.079)		
	1	1	~0.95	0.98	1	1
Auvil[c]	(0)	(0.154)	(0)	(0.195)		
	1	0.92	1	0.95	1	1
Bransden[d]	(0.006)	(0.084)	(0.067)	(0.004)		
	0.997	0.952	0.983	0.999	1	1

[a] See Ref. 12.
[b] See Ref. 13.
[c] See Ref. 14.
[d] See Ref. 15.

Legendre polynomials through P_9 are required to fit the neutron angular distribution in the process (14). Thus, it appears that at this energy both $j=\frac{9}{2}$ states are contributing to the production amplitude. Although we have not attempted a quantitive study comparable to that performed at 290 MeV, it would appear that only the Roper phase shifts have inelasticity in high enough angular-momentum states to be in reasonable accord with experiment.

IV. CONCLUSIONS

It is probably not correct to draw excessively firm conclusions concerning the nonvalidity of existing phase-shift solutions on the basis of the work presented here. Rather, it has been our intention to show how one can use production angular distributions to test phase-shift analyses of elastic-scattering experiments. For this purpose we have focused our attention upon one experiment performed at an energy where the situation is extremely simple in that one channel dominates the inelasticity. Clearly, more sophisticated application of the techniques presented here would require considerably more work than we have done. In addition, there is a clear need for more measurements of inelastic angular distributions in the 300–550 MeV (incident-pion laboratory kinetic energy) range in order to provide a firm basis for the conclusions tentatively offered here.

These conclusions are twofold. First, we believe that the Cence[13] phase shifts may be rejected solely on the basis of their inconsistency with the measured energy

TABLE III. Production-reaction complexity at 290 MeV from published phase-shift analyses.

	$\lvert A_1\rvert/\lvert A_0\rvert$	$\lvert A_2\rvert/\lvert A_0\rvert$
Roper	0.50	0.40
Cence	0.24	0.20
Auvil	0.44	0.10
Bransden	0.36	0.45
Experimental	1.4±0.4	1.1±0.4

dependence of the isospin-$\frac{1}{2}$ reaction cross section in the 500-MeV region. We suspect that the "no resonance" feature of the Cence solutions may disappear if the reaction cross sections of Fig. 1 are used as a constraint.

Our second conclusion is that none of the existing phase shift solutions provides for absorption in sufficiently high partial waves. In the vicinity of 300 MeV, for example, we believe that there is an appreciable contribution from one or both of the $j=\frac{5}{2}$ states to the reaction cross section that is not accounted for in the existing solutions. It is our conjecture that a correct description of the high partial-wave absorption will require a reduction (in existing phase-shift solutions) of the $1-\eta^2$ parameter belonging to the p_{11} state. Changing the η parameters will in turn require modification of the real part of the phase shift. We believe, therefore, that our present considerations reopen the question of the existence and position of the Roper p_{11} resonance.

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APPENDIX: EXPERIMENTAL DETERMINATION OF $\sigma_r^{1/2}$

The experimental points plotted in Fig. 1 were obtained from the data obtained from Refs. 20–29 and listed in Table IV. We have invoked the theoretician's license to select the data according to our own prejudices which, generally speaking, were in favor of the measurements with the smallest quoted errors. This selection may have led us astray in the case of the 550-MeV charge-exchange cross section where we used the measurement of Bulos *et al.*[28] Their cross-section was about 2 mb lower than values quoted by several other authors, so that the reaction cross section plotted in Fig. 1 may be about 10% too high at this energy. Clearly, however, a more sophisticated analysis of the experimental data is indicated and we hope that some experimentalist will soon undertake this task.

Table IV gives, for the total, elastic, and charge-exchange cross sections the quoted value in mb and, in parentheses, the references from which they are taken. Where more than one reference is given we have used the unweighted average. The quoted reaction cross sections are derived from the other cross sections in the usual manner. The notation is

$\sigma_T{}^{\pm}=\pi^{\pm}+p$ total cross section;
$\sigma_e{}^{\pm}=\pi^{\pm}+p$ elastic scattering cross section;
$\sigma_{\rm ex}=$ cross section for $\pi^-+p\rightarrow\pi^0+n$;
$\sigma_r{}^{1,3}=$ total reaction cross section in the $I=\frac{1}{2},\frac{3}{2}$ state.

Note that, as a matter of taste, we have chosen the subtraction method for obtaining reaction cross sections and our values for σ_r are not in good agreement with determinations based on direct measurements of all single pion production cross sections. Our preference was based upon the (perhaps naive) belief that contributions to σ_r can be missed in production experiments. Discrepancies between the two methods, therefore, indicate that our conclusions regarding the total inelasticity constraint must not be taken as final; we only wish to point out the relevance of such constraints.

Some of the phase-shift fits[13] have, in fact, included inelastic cross sections as obtained by the second method above as constraints. In those cases the disagreements indicated in Fig. 1 may then reflect experimental inconsistencies rather than theoretical difficulties.

TABLE IV. Cross-section data.

T_π (MeV)	$\sigma_T{}^+$	$\sigma_e{}^+$	$\sigma_r{}^3$	$\sigma_T{}^-$	$\sigma_e{}^-$	σ_{ce}	$\sigma_r{}^1$
370	40.39±1.62(20)	38.74±0.73(24)	1.65±1.78	27.9 ±1.2(20,21)	10.6 ±0.1(24,25)	12.03±0.51(22,23)	7.0 ±0.9
420	32.7 ±1.1(20)[a]	29.02±0.66(24)[a]	3.7 ±1.3	29.4 ±1.4(20)	11.62±0.16(24)[a]	11.30±0.71(23)	7.9 ±2.7
450	29.44±1.63(26)	24.39±0.49(24)	5.1 ±1.7	33.03±0.69(26)	12.19±0.26(24)	10.8 ±0.8(23)	12.4 ±1.8
500	21 ±1(27)	18.69±0.46(24)	2 ±1	36.01±0.61(26)	14.08±0.35(24)	10.3 ±0.9(23)[a]	16.4 ±1.7
550	17.88±0.82(20)[a]	14.38±0.19(24)	3.50±0.85	41.86±0.81(26)	16.98±0.37(24)	7.61±0.62(28)[a]	24.15±1.68
600	16.06±0.85(20)[a]	11.06±0.18(24)[a]	5.00±0.87	46.20±0.84(26)	19.87±0.34(24)	8.88±0.34(23,29)	24.96±1.56

[a] Interpolated value.

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