Gravitational Collapse and Causality

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The relativistic dynamics of a thin spherical shell of dust collapsing under its own weight is developed on the hypothesis that the shell rebounds elastically after imploding to a point. The resulting description involves certain causal anomalies. The question of whether these anomalies can lead to actual logical contradictions is investigated. Possible similarities between the results obtained for the special case considered here and the characteristics of general asymmetric collapse are discussed. Reasons are given for believing (i) that singularities do not arise in the course of an asymmetric collapse except in special cases; (ii) that collapse of a mass M does not proceed irreversibly to zero volume, but is an oscillatory phenomenon in which the object never becomes smaller than a critical linear size of order GM/c^2; (iii) that when the object is near this critical size, a region develops in its interior where past and future cannot be globally distinguished; and (iv) that since the law of baryon conservation cannot be globally valid in such a pathological region, large-scale annihilation of baryons is possible.

1. INTRODUCTION

THE fate of a spherical body collapsing under its own weight has recently been a subject of renewed astrophysical interest.[1]

According to the conventional relativistic interpretation,[2] the body shrinks asymptotically to its Schwarzschild radius, reaching it at infinite coordinate time.

A different interpretation of the relativistic formulas has recently been suggested by the author.[3] In its superficial aspects, it closely resembles the Newtonian picture of an elastic pulsation with finite period, but actually it involves causal anomalies of an unexpected nature.

All such anomalies occur within a radius comparable to the Schwarzschild radius, and are thus beyond the reach of practical observation.

Nonetheless, the new picture obviously cannot be correct if these causal anomalies lead to demonstrable logical contradictions. The object of the present paper is to test for logical contradictions in the special case where the collapsing body is a thin spherical shell of dust.

Sections 2–6 develop the dynamical equation of the shell and deal with the general characteristics of the motion as seen by interior and exterior observers. In Secs. 7 and 8 the question of possible causal violations is considered. The paper concludes with some comments on the possible relevance of the results obtained here to the characteristics of general asymmetric collapse.

2. INTERIOR AND EXTERIOR LINE ELEMENTS

Let the time-like 3-space Σ be the history of a thin spherical shell of dust.

The spherical symmetry of Σ means that its intrinsic metric is expressible in the form

$$(ds^2)_\Sigma=[R(\tau)]^2d\Omega-d\tau^2. \qquad (1)$$

Here, τ is proper time along the world-line (θ, ϕ= const) of a dust particle, $d\Omega\equiv d\theta^2+\sin^2\theta d\phi^2$ and R is the shell's radius. It should be noted that (1) defines the radius purely intrinsically by the statement that the area of the shell at instant τ is $4\pi[R(\tau)]^2$.

Birkhoff's theorem[4] asserts that every spherically symmetric vacuum field can be represented by a metric of Schwarzschild's form with a suitable constant m. The line element in the region exterior to the shell may accordingly be written

$$(ds^2)_{\text{ext}}=(1-2m/r)^{-1}dr^2+r^2d\Omega-(1-2m/r)dt^2, \qquad (2)$$

where m is the gravitational mass (total energy) of the shell. For the interior region, Birkhoff's theorem together with the requirement of regularity at $r=0$ leads to the flat line element

$$(ds^2)_{\text{int}}=dr^2+r^2d\Omega-dT^2. \qquad (3)$$

Both (2) and (3) must induce the same intrinsic metric, namely (1), on the imbedded hypersurface Σ. Because of the tangential character of the parametric lines of θ, ϕ at each point of Σ, we may compare coefficients of $d\Omega$ in (1), (2), (3). This shows that the equation of Σ is

$$r=R(\tau) \qquad (4)$$

with respect to both interior and exterior radial coordinates r. Comparison of the induced metrics along the streamlines θ, ϕ= const yields

$$d\tau^2=dT^2-dR^2=(1-2m/R)dt^2-(1-2m/R)^{-1}dR^2. \qquad (5)$$

If $m>0$, the applicability of the exterior r, θ, ϕ, t chart is restricted to the domain $r>2m$: the coordinate t becomes singular on the Schwarzschild sphere $r=2m$. To pursue the history of a collapsing shell through this

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[1] *Quasistellar Sources and Gravitational Collapse*, edited by I. Robinson, A. Schild, and E. L. Schucking (University of Chicago Press, Chicago, Illinois, 1964); Ya. B. Zel'dovich and I. D. Novikov, Usp. Fiz. Nauk **84**, 377 (1964) [English transl.: Soviet Phys.—Usp. **7**, 763 (1965)]; Usp. Fiz. Nauk **86**, 447 (1965) [English transl.: Soviet Phys.—Usp. **8**, 522 (1966)].

[2] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455 (1939).

[3] W. Israel, Nature **209**, 66 (1966); Phys. Rev. **143**, 1016 (1966); Phys. Letters **21**, 47 (1966); Nature **211**, 466 (1966).

[4] See, e.g., J. L. Synge, *Relativity: The General Theory* (North-Holland Publishing Company, Amsterdam, 1960), p. 276.

TABLE I. Collapse of thin spherical shell of dust.

Positive binding energy: $a=\cos\alpha<1$. [Shell falls from rest at finite radius $R_{\max}=m/2a(1-a)$.]	Negative binding energy: $a=\cosh\alpha>1$. [Shell impelled from infinity.]
$R(\Theta)=\frac{1}{2}m\sec\alpha\csc^2\alpha(\cos\alpha-\cos\Theta)$	$R(\Theta)=\frac{1}{2}m\,\mathrm{sech}\alpha\,\mathrm{csch}^2\alpha(\mathrm{csch}\Theta-\cosh\alpha)$
$\tau(\Theta)=\frac{1}{2}m\sec\alpha\csc^3\alpha(\Theta\cos\alpha-\sin\Theta)+\mathrm{const}$	$\tau(\Theta)=\frac{1}{2}m\,\mathrm{sech}\alpha\,\mathrm{csch}^3\alpha(\Theta\cosh\alpha-\sinh\Theta)+\mathrm{const}$
$T(\Theta)=\frac{1}{2}m\sec\alpha\csc^3\alpha(\Theta-\cos\alpha\sin\Theta)+\mathrm{const}$	$T(\Theta)=\frac{1}{2}m\,\mathrm{sech}\alpha\,\mathrm{csch}^3\alpha(\cosh\alpha\sinh\Theta-\Theta)+\mathrm{const}$
$t(\Theta)=-\frac{1}{2}m\csc^3\alpha(\sin\Theta+B\Theta\sec\alpha)$ $+2m\ln[\sin\frac{1}{2}(3\alpha-\Theta)/\sin\frac{1}{2}(3\alpha+\Theta)]+\mathrm{const}$	$t(\Theta)=\frac{1}{2}m\,\mathrm{csch}^3\alpha(\sinh\Theta+B\Theta\,\mathrm{sech}\alpha)$ $+2m\ln[(e^{\Theta}-e^{3\alpha})/(e^{\Theta}-e^{-3\alpha})]+\mathrm{const}$
$u(\Theta)=2m\csc2\alpha\sin\frac{1}{2}(\Theta+3\alpha)\exp G(\Theta)$	$u(\Theta)=2m\,\mathrm{csch}2\alpha\sinh\frac{1}{2}(\Theta+3\alpha)\exp[-G(\Theta)]$
$z(\Theta)=4m\csc\alpha\sin\frac{1}{2}(\Theta-3\alpha)\exp[-G(\Theta)]$	$z(\Theta)=4m\,\mathrm{csch}\alpha\sinh\frac{1}{2}(\Theta-3\alpha)\exp G(\Theta)$
$G(\Theta)=\frac{1}{8}\sec\alpha\csc^3\alpha[B\Theta+\sin(\Theta-\alpha)]+\mathrm{const}$	$G(\Theta)=\frac{1}{8}\mathrm{sech}\alpha\,\mathrm{csch}^3\alpha[B\Theta+\sinh(\Theta-\alpha)]+\mathrm{const}$
$B=\cos^2 2\alpha-2\cos^2\alpha$	$B=\cosh^2 2\alpha-2\cosh^2\alpha$
Zero binding energy: $a=1$ [Shell falls from rest at infinity.]	Shell of photons: $a\to\infty$, $b\to 0$, $ab=m$
$R(\lambda)=m(\lambda^2-\frac{1}{4})$	$R=\lvert T\rvert+\mathrm{const}$
$\tau(\lambda)=m(\frac{2}{3}\lambda^3-\frac{1}{2}\lambda)+\mathrm{const}$	$R+2m\ln(R-2m)=\lvert t\rvert+\mathrm{const}$
$T(\lambda)=m(\frac{2}{3}\lambda^3+\frac{1}{2}\lambda)+\mathrm{const}$	$\tau=0$
$t(\lambda)=m(\frac{2}{3}\lambda^3+\frac{5}{2}\lambda)+2m\ln[(\lambda-\frac{3}{2})/(\lambda+\frac{3}{2})]+\mathrm{const}$	$uz=-8m^2\ln z+\mathrm{const}$ (for collapse)
$u(\lambda)=2\sqrt{2}m(\lambda+\frac{3}{2})\exp G(\lambda)$	$u=\mathrm{const}$ (for expansion)
$z(\lambda)=\sqrt{2}m(\lambda-\frac{3}{2})\exp[-G(\lambda)]$	
$G(\lambda)=-\frac{1}{6}\lambda^3+\frac{1}{4}\lambda^2-\frac{5}{8}\lambda+\mathrm{const}$	
Self-collision for $\lambda(X)=1.6050$, $R(X)=2.3260m$	Self-collision for $R(X)=2.5568m$

sphere, one requires a coordinate system which covers the complete exterior manifold down to the geometrical singularity at $r=0$. In terms of one such maximal coordinate system,[5] the Schwarzschild metric (2) takes the form

$$(ds^2)_{\mathrm{ext}}=2dudz+(z^2/2mr)du^2+r^2d\Omega\,, \qquad (6)$$

with

$$r=2m+uz/4m\,. \qquad (7)$$

The relation between the coordinates (u,z) and (r,t) is

$$t=r-2m\ln(u/2z)\,, \qquad (8)$$

$$u=[8m(r-2m)]^{1/2}\exp[(r-t)/4m]\,,$$
$$z=[2m(r-2m)]^{1/2}\exp[(t-r)/4m]\,, \qquad (9)$$

in the domain $(r>2m)$ where t is defined.

3. GENERAL CHARACTERISTICS OF THE MOTION

In a previous paper[6] it was shown that the equation of motion of the shell has the first integral

$$1+(dR/d\tau)^2=(a+b/2R)^2\,. \qquad (10)$$

The constant b was there interpreted as the *nucleonic mass* of the shell, i.e., the total mass of the constituents when infinitely dispersed and at rest. The constant

$$a=m/b$$

is the ratio of total mass (including contributions from the kinetic and potential energies) to nucleonic mass. Thus, $(1-a)b$ measures the *binding energy* of the shell.

It will be assumed throughout that b and m are positive.[7] Then the inequality

$$a+m/(2aR)\geq 1\,,$$

implied by (10), places an upper bound

$$R_{\max}=m/2a(1-a) \qquad (11)$$

on the radius of the shell if $a<1$; for $a\geq 1$ it is trivially satisfied. Thus, as expected, expanding shells with positive binding energy fall back after reaching a finite maximal radius; shells with negative binding energy can expand to infinity.

From (11),

$$R_{\max}\geq 2m\,,$$

a result to be expected, since a particle on the outer boundary with a stationary $r<2m$ would have a spacelike world line. We conclude that *no shell of dust can remain permanently submerged within its Schwarzschild sphere.* In Sec. 6 this statement will be considerably strengthened.

4. INTEGRATION OF THE EQUATIONS

The results of integrating (10) are conveniently expressed in parametric form. We distinguish two cases.

For shells with positive binding energy $(0<a<1)$, we

[5] W. Israel, Phys. Rev. **143**, 1016 (1966). Equivalent completions have been given by M. D. Kruskal, Phys. Rev. **119**, 1743 (1960); G. Szekeres, Publ. Math. Debrecen **7**, 285 (1960); C. Fronsdal, Phys. Rev. **116**, 778 (1959).

[6] W. Israel, Nuovo Cimento **44B**, 1 (1966).

[7] Since the causal peculiarities associated with the Schwarzschild "singularity" $r=2m$, $t=\pm\infty$ arise only for $m>0$, the case $m<0$ (even if physically meaningful) is not relevant to the main purpose of this paper.

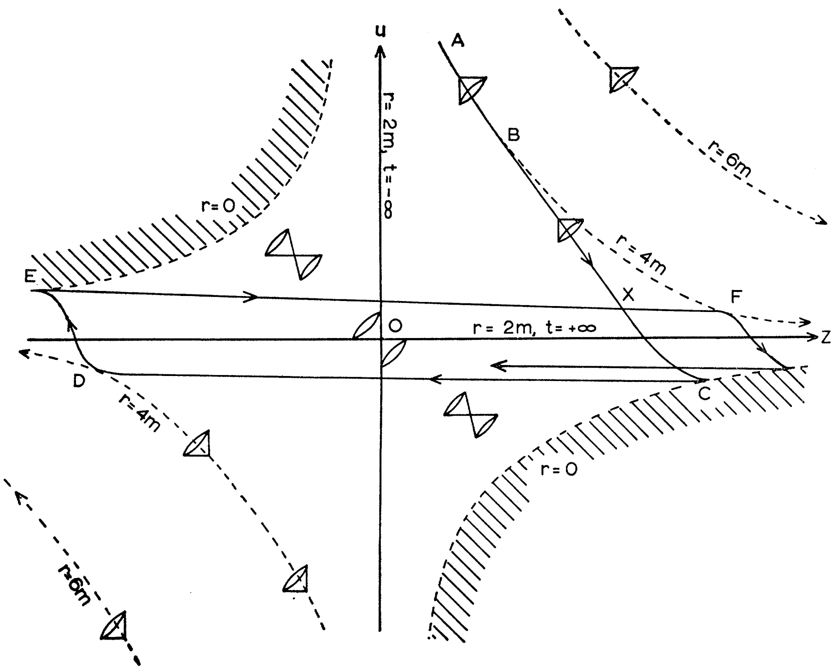


FIG. 1. The uz plane, showing (not to scale) the history $ABCDEF\cdots$ of a particle in the wall of a spherical shell which has been held at rest (section AB), then released and allowed to pulsate elastically under its own weight. According to the elliptic interpretation, the events (u,z,θ,ϕ) and $(-u, -z, \theta, \phi)$ are physically identical, so that one half of the uz plane is superfluous. We take advantage of this circumstance to represent the two diametrically opposite 2-spaces $\theta=c_1$, $\phi=c_2$ and $\theta=\pi-c_1$, $\phi=\pi+c_2$ on the same figure. Thus, points in quadrants I and III represent events with $r>2m$ in the spaces $\theta=c_1,\phi=c_2$ and $\theta=\pi-c_1,\phi=\pi+c_2$, respectively. Points in quadrants II and IV represent events with $r<2m$ in either of these spaces. The two spaces are hinged together at the singular curve $r=0$, so that the particle crosses over to the diametrically opposite space at each passage through $r=0$.

define a variable Θ by

$$R(\Theta)=\frac{m}{2a(1-a)}\frac{a-\cos\Theta}{a+1}, \tag{12}$$

and an acute angle α by

$$a=\cos\alpha. \tag{13}$$

Integration of (10) yields $\tau(\Theta)$. Then $T(\Theta)$, $t(\Theta)$ are found from (5), and $u(\Theta)$, $z(\Theta)$ from (9).

For shells with negative binding energy ($a>1$) a new Θ and a new α are introduced by replacing $\cos\Theta$, $\cos\alpha$ in (12) and (13) by their hyperbolic equivalents. The integration proceeds as before.

Table I summarizes the results.

5. REBOUND FROM THE SINGULARITY

The formulas of Table I enable us to follow the career of a collapsing shell, from both internal and external points of view, until its implosion to zero radius. At this point a singularity develops in the geometry, and the subsequent history is a matter of conjecture.

The remainder of this paper will be devoted to exploring the consequences of what seems (in view of the time-symmetry of the field equations) the simplest hypothesis: namely, that the shell rebounds elastically and reversibly from $R=0$ with conservation of the number of particles. Then (10)—with unchanged values of a and b—continues to hold for the subsequent re-explosion.[8]

To see in detail how the continuation through $R=0$ is made, let us consider for definiteness the exterior description of the case $a<1$ (first column of Table I). We observe that there is an arbitrary additive constant in the function $G(\Theta)$. For $R>2m$, changes in this constant correspond to time-translations $t(\Theta)\rightarrow t(\Theta)+\text{const}$. Let us set the constant equal to some definite value (say zero) for the phase of collapse, $-\pi<\Theta<-\alpha$. At total collapse, the value of u is $u(-\alpha)=m\sec\alpha\exp G(-\alpha)$. Re-expansion begins at $\Theta=+\alpha$. Its analytical description is the same as that of the preceding collapse, but may involve different additive constants in $t(\Theta)$ and $G(\Theta)$, say $G_{\text{new}}(\Theta)=G(\Theta)+C$. The value of u at the initial moment of re-expansion is

$$u(\alpha)=2m\exp[G(\alpha)+C].$$

Assuming that maximum implosion $R=0$ is represented by a single point in the exterior coordinate map, we must have

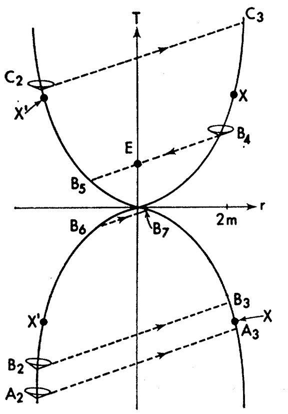
$$u(-\alpha)=u(\alpha),$$

and this fixes

$$C=-\tfrac{1}{8}\sec\alpha\csc^3\alpha(2B\alpha+\sin2\alpha)+\ln(\tfrac{1}{2}\sec\alpha),$$

where B is defined in Table I.

[8] Basically, our assumption is that at the instant of infinite compression $R=0$, and at all "self-collisions," the particles of the shell interpenetrate freely without nuclear interactions or random scattering. While physically absurd, this attempt to isolate the purely kinematical aspects is perhaps excusable in a first survey. Inspection of Fig. 1 shows that for self-collisions inside $r=2m$, each particle of the shell meets itself (or its diametrically opposite partner) traveling backwards in time. If this has any counterpart in reality, it implies that the conservation of baryon number (considered as a *global* law) is subject to breakdown inside the Schwarzschild sphere, and that matter-antimatter annihilation plays a significant role in realistic examples of gravitational collapse. Compare Secs. 8 and 9.



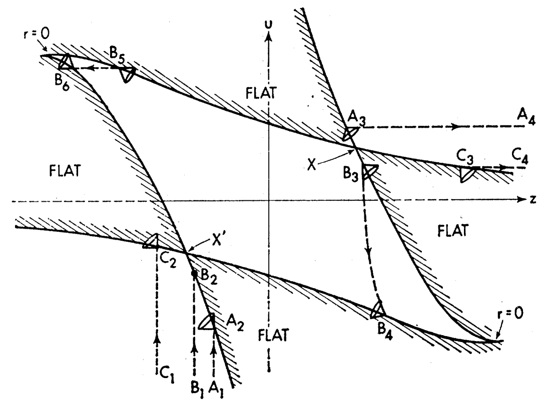


FIG. 2. The history of a spherical shell of dust, as imbedded (a) in the interior flat space and (b) in the exterior Schwarzschild space. Quadrants I and III of (b) represent two radially opposite 2-spaces with $r>2m$ (see caption of Fig. 1) corresponding to the right and left halves of Fig. 2(a). The light-like probe B, shot radially inwards from B_1, penetrates the contracting left wall at B_2 and traverses the interior of the shell (segment B_2B_3). Its world line emerges into the exterior domain through the shell's future cone at B_3 and enters the expanding right wall through the shell's *future* cone at B_4. It now traverses the interior of the shell moving backwards in T time. It continues to zigzag indefinitely and ever more closely toward the singularity at $R=0$. The exterior manifold of Fig. 2(b) is non-orientable, and it is the mapping of this region onto a plane that causes the apparent discontinuous flip of the exterior normal vector on passage through X or X'.

6. NATURE OF THE PULSATION

Our hypothesis of elastic rebound leads to a pulsating model for a shell with positive binding energy. The exterior coordinate time for a complete pulsation (proportional to the period actually measured by an external stationary clock) is, by (8),

$$t(\pi)-t(-\pi)=2m\ln[u(-\pi)z(\pi)/u(\pi)z(-\pi)]$$
$$=-\pi mB\sec\alpha\csc^3\alpha-4mC.$$

If $R_{\max}\gg 2m$, this reduces to $\pi m/\alpha^3\sim\pi(R_{\max}{}^3/m)^{1/2}$, in agreement with the Newtonian prediction.

Figure 1 illustrates a segment of the history Σ of a pulsating shell with $R_{\max}=4m$.

Figure 2 illustrates, from both internal and external points of view, the history Σ of a shell which executes a single pulsation with $R_{\max}>2m$. The figure shows vividly how the difference in the *local* geometry of the imbeddings of Σ in the interior and exterior domains entails remarkable differences of a topological nature. On Σ as imbedded in the interior flat space, there are two 2-spheres $r=R(X)$ with trace X, X' in the r, T plane. They are associated with opposite values of T and they are distinct. In the space with metric (2), on the other hand, an external self-adhesion of Σ makes these 2-spheres coincide. In plain physical terms, *an external observer sees the shell collide with itself at radius* $R(X)$*; an internal observer does not notice this.*

If the shell is permitted to execute more than one pulsation, the number of self-collisions increases rapidly. All occur within a radius comparable to the Schwarzschild radius. In general, the radius of the outermost self-collision is subject to

$$2m\leq R(X)\leq 2.5568m,$$

the upper bound being attained for a shell of photons.

To an external observer with $r>R(X)$, only the outermost V-shaped segments $A_2X'C_2\cdots$ and $A_3XC_3\cdots$ of Σ are optically observable. Light from other sections (e.g., from B_3) cannot reach him directly. He therefore describes the sequence of events as follows: the shell collapse from $R_{\max}$ to a minimal radius $R(X)>2m$, then rebounds elastically. It is never observed to enter the Schwarzschild sphere.

7. TRANSMISSION OF PROBES THROUGH THE SHELL

At the root of our hypothesis of elastic rebound at $R=0$ is the so-called "elliptic interpretation"[5,9,10] of the extended Schwarzschild manifold. In this interpretation, the points (u,z,θ,ϕ) and $(-u,-z,\theta,\phi)$ are considered to represent the same physical event.

For the behavior of probes and light-signals in the Schwarzschild *vacuum* field, the elliptic interpretation leads to quite bizarre predictions.[5,10] A radially falling probe reaches $r=2m$ at coordinate time $t=+\infty$, although the proper time of descent to $r=2m$ and, indeed, to $r=0$ is finite. It is in conformity with the elliptic interpretation to assume that the probe's momentum carries it through the singularity $r=0$ onto the opposite radius. The probe then re-emerges from $r=2m$ at $t=-\infty$. Now, the coordinate t is proportional to the time actually registered by stationary clocks with $r>2m$. It follows that observers hovering just outside the Schwarzschild sphere may see the probe re-emerge from this sphere before it entered!

[9] F. J. Belinfante, Phys. Letters **20**, 25 (1966); J. M. Souriau, Bull. Soc. Math. France **93**, 193 (1965); W. Rindler, Phys. Rev. Letters **15**, 1001 (1965); W. Israel, Nature **211**, 466 (1966).

[10] J. L. Anderson and R. Gautreau, Phys. Letters **20**, 24 (1966).

It should be kept in mind that we are concerned here with nothing more than a mathematical oddity—a curious property of time-like curves in a manifold which represents pure empty space apart from a singular curve $r=0$. *The singularity $r=0$ is nonphysical:* It is space-like, so it cannot be the history of a particle. There is no such thing as a point-mass in general relativity. In fact, it is not hard to see that the property we obtained in Sec. 6 for the shell holds for any spherical body: to an external observer the body never appears smaller than its Schwarzschild sphere.[11]

Thus, physically significant paradoxes arise only if it can be shown that the anomalous behavior described above is reproducible in the presence of the mass distribution which creates the exterior vacuum field. This question will now be investigated for the case where the central mass is a free-falling spherical shell.

We consider the radial motion of a probe which encounters no resistance in passing through the shell. Is it possible for the probe to re-emerge at an earlier Schwarzschild time t than its moment of entry? The prospect of such behavior seems particularly auspicious for a thin shell, since here the Schwarzschild "singularity" $r=2m$, $t=\pm\infty$ is real: sections of the u and z axes are actually exposed as part of the domain with metric (2). Conceivably a probe crossing either of these sections might re-emerge "before" it enters.

To follow the motion in detail, one requires junction conditions to connect the paths in the exterior and interior regions when the probe's world line cuts through Σ, the history of the shell wall. In order to formulate these conditions, we observe that there is a well-defined past-future arrow on the world line of the probe and also at each event on Σ. (But these cannot be extended to a globally consistent past-future distinction in the domain $r<2m$ of the exterior space.[5]) We postulate that, on passage through Σ, the scalar product[12] of the (normalized) future-pointing tangent vectors is preserved. Physically this implies continuity of the 3-velocity of the probe relative to the shell wall: only the acceleration is discontinuous.

Figure 3 shows the null paths of three massless probes A, B, C which are fired radially into the left wall of the shell. (For probes of finite mass, which have time-like paths, the conclusions are qualitatively the same.) A probe reaching the left wall at any moment later than the self-collision at X' (such as C_2) or sufficiently earlier than X' (such as A_2) passes uneventfully through the expanding or collapsing shell, and emerges from the right wall at a later t time.

On the other hand, a probe (such as B) which enters the left wall just before X' will emerge from the opposite wall with a radial coordinate less than $2m$, i.e., within the Schwarzschild "singularity" of the exterior space. It continues on an endless waning zig-zag course in time and space, passing repeatedly through the shell and rapidly sinking towards the singularity $R(0)=0$. It never re-enters the region $r>2m$.

Thus, there are no causal anomalies in the region $r>R(X)$ outside the shell.

8. OTHER POSSIBILITIES FOR CAUSAL VIOLATIONS

Causal difficulties might be anticipated from the clash of the arrow on the segment B_4B_5 of probe B's world line with the arrows of bodies that have evolved in the expanding interior of the shell.

Actually it is easy to see that the segment B_4B_5 will be nonexistent for a macroscopic probe in a realistic physical situation. For simplicity,[13] let us idealize the probe as a coherent stream of identical baryons with 4-velocity u^μ. The assembly is characterized by an energy tensor $T^{\mu\nu}=\rho u^\mu u^\nu$ and a numerical flux vector $M^\mu=\rho u^\mu$, both of which are divergenceless, expressing the conservation of 4-momentum and of baryon number, respectively. An observer having 4-velocity v^μ measures $T^{\mu\nu}v_\mu v_\nu$ as the energy density and $-M^\mu v_\mu$ as proportional to the baryon density. If we consider the case where v^μ and u^μ point into opposing halves of the null cone, we see that a probe "traveling backwards in time" is characterized by positive energy density and negative baryon number, and thus has all the properties conventionally attributed to antimatter. It follows that the probe and the shell wall will interact as matter and anti-matter at B_4, with annihilation of the probe.

The single remaining possibility is the formation of a closed causal chain by two organisms in the rhomboidal sector $r<R(X)$ of the exterior space [Fig. 2(b)]. Two bodies, one of which has evolved in the exterior domain $r>R(X)$, the other in the expanding interior during the period $0<T<T(X)$, and which have been launched into the rhomboidal sector, will meet there with their arrows in opposition. If the bodies are intelligent, it is difficult to see how they could be prevented from conspiring together so that each can change his own past.

While this paradox cannot be categorically precluded, its realizability is nevertheless rendered unlikely by the following considerations:

(i) It requires the evolution of intelligence in the interior from a state of chaos at $T=0$ within a time of the order of the characteristic Schwarzschild time $2GM/c^3$ (where M is the mass of the shell in grams). The single instance of terrestrial life is of course not a valid basis for generalization, but it suggests that the time actually required may be of the same order as the present age of the universe.

(ii) To form a closed causal chain, organisms sufficiently sensitive and complex to make observations and simple decisions would have to survive in the

[11] We may expect this result to remain qualitatively valid for asymmetric situations which do not deviate too far from spherical symmetry. Thus, any freely collapsing object of mass M will appear to an external observer to pulsate with a minimal linear size of order GM/c^2.

[12] Or simply its sign, if one of the tangent vectors is null.

[13] See, e.g., W. Israel [J. Math. Phys. 4, 1163 (1963)] for a more general discussion.

rhomboidal sector for a time interval of the order $c^{-1}m$, where the length $m \equiv GM/c^2$. Causality is basically a statistical phenomenon. In any situation which has a significant probability of actually occurring, the bounce and self-collision of the shell will cause an appreciable fraction of its mass to be scattered into the rhomboidal sector (3 volume $\sim m^3$) in the form of high-speed particles and antiparticles with density not much less than Mm^{-3} and intensity $\sim cMm^{-3}$. A minimal condition for survival is clearly that the total mass of antiparticles encountered should not exceed the organism's mass μ. This leads to the requirement

$$a^2(cMm^{-3})(c^{-1}m) \lesssim \mu ,$$

where a is a characteristic linear dimension of the organism. In terms of the organism's density ρ, this can be written

$$M \gtrsim (c^4/G^2)\mu^{-1/3}\rho^{-2/3}$$
$$\sim 1.8\times 10^{56}\mu^{-1/3}\rho^{-2/3} \text{ (g)},$$

with μ, ρ in conventional cgs units. For the survival of organisms not inconceivably more dense and massive than those familiar from terrestrial experience this demands that the shell's mass be tremendously large. (The mass of the observable universe is of the order of 10^{56} g.) Massive shielding of the organism which increases μ by a factor of 10^3 will reduce the required M by a factor of 10, but also reduces the sensitivity by a much larger factor.

Both (i) and (ii) point to the conclusion that causal violations in the rhomboidal region are physically realizable only if the mass of the shell is very large, perhaps comparable with that of the observable universe. It might be argued that this constraint is sufficiently overwhelming to be considered a limitation of principle.

9. GENERAL (NONSYMMETRIC) COLLAPSE AND THE OCCURRENCE OF SINGULARITIES

As already emphasized,[8] the artificiality of the model discussed in this paper precludes its direct applicability to realistic examples of gravitational collapse. The results are of physical interest only insofar as they can provide clues to the characteristics of more general situations.

General nonsymmetric collapse has recently been the subject of two independent lines of attack.[14,15] In particular, the question of whether physical singularities inevitably develop in the course of the collapse has been studied by both groups, with results which appear at first sight to conflict.

On the one hand, it has been strongly contended[14] that the occurrence of singularities in the known mathematical solutions representing collapse is entirely due to the symmetry or homogeneity of these solutions. For instance, in spherically symmetric collapse, the singularity develops because all particles are focused toward a single point, and would be smoothed out by a small perturbation. In support of their contention, it has been shown by these authors that, whereas a general solution of Einstein's field equations for an ultrarelativistic fluid involves 8 physically arbitrary functions of three variables (spatial coordinates), a solution with singularity contains at most 7 arbitrary functions. The initial conditions leading to a singular collapse thus form a subset of measure zero in the manifold of all possible initial conditions.

On the other hand, Penrose[15] has established that a singularity is necessarily present (or space-time is incomplete) if the following conditions are met: (a) The energy density is non-negative definite; (b) space-time contains a "trapped surface" (defined as a closed, space-like 2-space S with the property that, of the four systems of null geodesic rays emanating normally from S, two—drawn into the same half of the null cone—converge locally); (c) there exists a noncompact space-like hypersurface which intersects every time-like and null line exactly once; (d) the null cones form two separate systems, past and future.

There is no known internal reason to doubt the mathematical correctness of either of these analyses, but the apparent incompatibility of the results has led to the suspicion that one or other must be in error. I wish to suggest here that actually both are correct. Indeed, suppose it is accepted that in general no singularity develops. According to Penrose's theorem, in a nonsingular collapse which satisfies (a) and (b), there must be a violation of (c) or (d). Thus, *in a general nonsymmetric collapse satisfying conditions (a) and (b), a portion of the collapsing matter will pass into an abnormal region of space-time in which past and future cannot be globally distinguished.* This shows that the two analyses are not necessarily incompatible and is also perfectly consonant with what we have found in this paper for a special highly symmetric model.

If such abnormal regions really exist, baryon conservation would be reduced in them to a law of merely local validity. It would, for instance, be possible for two time lines whose arrows point into the future null cone in a normal region of space-time, to intersect in an abnormal region with their arrows in opposition. Thus, two equal masses, both originating as ordinary matter in a normal region could nevertheless interact as matter and antimatter if they arrive, by paths which differ suitably, at the same event of an abnormal region. The fraction of the total number of baryons annihilated in this way presumably increases with the degree of symmetry of the collapse. Conceivably, it is this mechanism which is basically responsible for the prodigious outputs of high-energy quanta and relativistic particles observed in quasistellar sources and supernovae.

[14] See the review by E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk **80**, 391 (1963) [English transl.: Soviet Phys.—Usp. **6**, 495 (1964)].

[15] R. Penrose, Phys. Rev. Letters **14**, 57 (1965). See also S. W. Hawking, *ibid*. **15**, 689 (1965); S. W. Hawking and G. F. R. Ellis, Phys. Letters **17**, 247 (1965).