Quasiparticle Model for La^{140}†

GORDON L. STRUBLE
Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California

(Received 13 June 1966; revised manuscript received 6 September 1966)

A model is proposed for odd-odd nuclei in which the independent excitations are quasiparticles defined by a special Bogoliubov transformation and in which a finite-range two-body central force acts as the residual interaction. The model is applied to La^{140}, whose energy levels, wave functions, (d,p) spectroscopic factors, and $M1$ transition rates are computed and compared with experiment.

I. INTRODUCTION

In recent years many models of the nuclear many-body problem have been described in terms of independent excitations called quasiparticles. Kisslinger and Sorenson[1] studied systems of single-closed-shell nuclei in 1960. Since that time, several attempts have been made to extend these techniques in heavier nuclei to systems that contain both neutrons and protons outside magic configurations.[2–5] The neutron-proton interaction was either assumed to affect the single-particle energies in some systematic way or was taken explcitly into account in first-order perturbation theory. Although Kisslinger and Sorenson[5] concluded that their calculations were consistent with the limited amount of information on odd-odd nuclei available at that time, no detailed calculations using an odd-odd quasiparticle model were attempted.

In the preceding paper[6] we analyzed data from measurements on the reaction $La^{139}(d,p)La^{140}$. These data and data from other experiments[7,8] give information about (d,p) cross sections and gamma-ray branching ratios, as well as the spins and parities of twelve states. Therefore we have decided to make a quantitative comparison between experiment and theory for an odd-odd quasiparticle model.

In this model we assume that the 50-proton 82-neutron system forms an inert core. The self-consistent single-particle energies for the 83rd neutron are determined from experimental data. For the seven valence protons we make a special Bogoliubov transformation to define quasiparticles. The energies and wave functions for these excitations are determined by solving the self-consistent equations which give the minimum energy for each quasiparticle state. Finally the neutron-proton interaction is considered by diagonalizing it in a truncated quasiparticle basis. Since none of the experimental data[6–8] indicate any phonon admixture in states which have been characterized, we neglect those terms in the Hamiltonian introducing quasiparticle correlations which generate collective vibrations. In Secs. II and III we give the mathematical details of the model, discuss how we fit the parameters of the model to the experimental data, and give the results of the calculation. In Sec. IV we compute (d,p) spectroscopic factors, gamma transition probabilities, and compare the results with experiment.

II. THEORY

We consider that the system $^{82}Sn_{50}^{132}$ forms an inert core with respect to single-particle and collective excitations. It contributes only to the self-consistent potential for nucleons outside this core. With this approximation, the m-scheme Hamiltonian for the system may be written in the occupation number representation as

$$H=H_p+H_n+H_{np}, \tag{1}$$

where

$$H_p=\sum_\alpha \epsilon_\alpha C_\alpha^\dagger C_\alpha+\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|V_{pp}|\gamma\delta\rangle C_\alpha^\dagger C_\beta^\dagger C_\delta C_\gamma,$$

$$H_n=\sum_a \epsilon_a C_a^\dagger C_a,$$

$$H_{np}=\sum_{\alpha a\beta b}\langle\alpha a|V_{np}|\beta b\rangle C_\alpha^\dagger C_a^\dagger C_\beta C_b.$$

In this equation the ϵ's are the single-particle energies, the C's are annihilation operators for shell-model particles, Greek letters designate the quantum numbers for a complete set of compatible observables in the assumed self-consistent potential for protons, and Latin letters designate these quantum numbers for the neutron.

If we assume that the single-particle states are generated from the usual shell-model harmonic-oscillator potential with a single-particle spin-orbit term, then $|\alpha\rangle=|\psi(p)_{n_\alpha s l_\alpha j_\alpha m_\alpha}\rangle$,[9] and if we assume a truncated space of only one oscillator shell, the proton summation is performed over the $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$

† This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, 9 (1960).
[2] M. K. Pal and D. Mitra, Nucl. Phys. **42**, 221 (1963).
[3] L. Silverberg, Arkiv Fysik **20**, 341 (1962).
[4] L. Silverberg, Nucl. Phys. **60**, 483 (1964).
[5] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963).
[6] J. Kern, G. Struble, and R. K. Sheline, preceding paper, Phys. Rev. **153**, 1331 (1966).
[7] J. S. Geiger, R. L. Graham, and G. T. Ewan, Bull. Am. Phys. Soc. **6**, 71 (1961).
[8] J. Burde, M. Rakavy, and G. Adam, Nucl. Phys. **68**, 561 (1965).
[9] We adopt the following conventions for notational convenience: $\alpha=n_\alpha s l_\alpha j_\alpha m_\alpha$ and $j_\alpha=n_\alpha s l_\alpha j_\alpha$.

orbitals while the sum for the neutron is over the $2f_{7/2}$, $3p_{3/2}$, $1h_{9/2}$, $3p_{1/2}$, $2f_{5/2}$, and $1i_{13/2}$ orbitals. Even this highly truncated shell-model calculation would be very laborious and probably not physically illuminating. However, since in this region of the nuclear periodic table the description of even- and odd-A systems in terms of independent quasiparticles which are defined by a special Bogoliubov transformation is very good,[5] we make the transformation

$$\xi_\alpha^\dagger = u_\alpha C_\alpha^\dagger - s_\alpha v_\alpha C_{-\alpha}. \tag{2}$$

Here $\xi_\alpha^\dagger$ is a creation operator for quasiparticles. The transformation coefficients are chosen so that the quasiparticle operators obey the anticommutation relations for fermions and that they transform under rotations in the same manner that shell-model particles do. A set of sufficient conditions for the transformation to be canonical and for the system to be spherically symmetric are that u_α and v_α are real and that

$$\begin{aligned} &v_\alpha > 0, \\ &u_\alpha = u_{j_\alpha}, \\ &v_\alpha = v_{j_\alpha}, \\ &u_\alpha^2 + v_\alpha^2 = 1, \\ &s_\alpha = (-1)^{j_\alpha - m_\alpha}. \end{aligned} \tag{3}$$

By defining s_α this way, we have adopted the Condon-Shortley phase conventions.[10]

The state vector for a one-quasiparticle system can be written

$$|\gamma^0\rangle \equiv \xi_\gamma^\dagger |\bar{0}\rangle = C_\gamma^\dagger \prod_{\alpha>0,\ \alpha\neq\gamma} (u_{\gamma\alpha} + s_\alpha v_{\gamma\alpha} C_\alpha^\dagger C_{-\alpha}^\dagger)|0_p\rangle, \tag{4}$$

where $|\bar{0}\rangle$ is the quasiparticle vacuum and $|0_p\rangle$ is the proton shell-model vacuum. In the case of $^{83}La_{57}^{140}$ the vacuum represents the fifty protons in the inert $^{82}Sn_{50}^{132}$ core. The transformation coefficients are labeled with respect to the single-quasiparticle state in order to indicate that blocking will now be considered explicitly. The coefficients $v_{\gamma\alpha}$ are determined so that the expectation value, $\langle\gamma^0|H_p'|\gamma^0\rangle$, is a minimum, where

$$H_p' = H_p - \lambda N_p. \tag{5}$$

It is necessary to use the primed auxiliary Hamiltonian because the trial vectors given by Eq. (4) are not eigenvectors of N_p, the counting operator for shell-model protons. Using the Lagrange multiplier λ guarantees that the number of protons will be conserved on the average. The equations which must be solved self-consistently for the values of $v_{\gamma\alpha}$ are

$$v_{j_\gamma j_\alpha}^2 = \frac{1}{2}\left\{1 - \frac{\epsilon_{j_\alpha} - \lambda - \mu'_{j_\gamma j_\alpha}}{(\epsilon_{j_\alpha} - \lambda - \mu'_{j_\gamma j_\alpha} + \Delta_{j_\gamma j_\alpha}^2)^{1/2}}\right\}, \tag{6}$$

$$n = 1 + \sum_{j_\alpha \neq j_\gamma} \hat{j}_\alpha v_{j_\gamma j_\alpha}^2 + (\hat{j}_\gamma - 2) v_{j_\gamma j_\alpha}^2,$$

where

$$\begin{aligned} \mu'_{j_\gamma j_\alpha} = &-\sum_J \sum_{j_\beta \neq j_\gamma} \frac{\hat{J}}{\hat{j}_\alpha} G(j_\alpha j_\beta j_\alpha j_\beta J) v_{j_\gamma j_\beta}^2 \\ &- \frac{\hat{j}_\gamma - 2}{\hat{j}_\gamma} \sum_J \frac{\hat{J}}{\hat{j}_\alpha} G(j_\alpha j_\gamma j_\alpha j_\gamma J) v_{j_\gamma j_\gamma}^2 \\ &- \frac{1}{\hat{j}_\gamma} \sum_J \frac{\hat{J}}{\hat{j}_\alpha} G(j_\alpha j_\gamma j_\alpha j_\gamma J), \end{aligned}$$

$$\begin{aligned} \Delta_{j_\gamma j_\alpha} = &-\sum_{j_\beta \neq j_\gamma} \left(\frac{\hat{j}_\beta}{\hat{j}_\alpha}\right)^{1/2} \langle j_\alpha j_\alpha 0 | V | j_\beta j_\beta 0\rangle u_{j_\gamma j_\beta} v_{j_\gamma j_\beta} \\ &- \left(\frac{\hat{j}_\gamma}{\hat{j}_\alpha}\right)^{1/2} \left(\frac{\hat{j}_\gamma - 2}{\hat{j}_\gamma}\right) \langle j_\alpha j_\alpha 0 | V | j_\gamma j_\gamma 0\rangle \\ &\times u_{j_\gamma j_\gamma} v_{j_\gamma j_\gamma}, \end{aligned}$$

$$\begin{aligned} G(j_\alpha j_\beta j_\alpha j_\beta J) = &\langle j_\alpha j_\beta J | V_{pp} | j_\alpha j_\beta J\rangle - (-1)^{j_\alpha + j_\beta + J} \\ &\times \langle j_\alpha j_\beta J | V_{pp} | j_\beta j_\alpha J\rangle, \end{aligned}$$

$$\hat{j}_\gamma \equiv 2j_\gamma + 1.$$

The energy of an N-particle system in the state $|\gamma^0\rangle$ is

$$\begin{aligned} \langle\gamma^0|H_p|\gamma^0\rangle = &\lambda N + (\epsilon_{j_\gamma} - \lambda) - \mu_{j_\gamma j_\gamma} \\ &+ \sum_{j_\alpha \neq j_\gamma} \hat{j}_\alpha [(\epsilon_{j_\alpha} - \lambda - \tfrac{1}{2} u_{j_\gamma j_\alpha}) v_{j_\gamma j_\alpha}^2 \\ &- \tfrac{1}{2} u_{j_\gamma j_\alpha} v_{j_\gamma j_\alpha} \Delta_{j_\gamma j_\alpha}] \\ &+ (j_\gamma - 2)[(\epsilon_{j_\gamma} - \lambda - \tfrac{1}{2}\mu_{j_\gamma j_\gamma}) v_{j_\gamma j_\gamma}^2 \\ &- \tfrac{1}{2} u_{j_\gamma j_\gamma} v_{j_\gamma j_\gamma} \Delta_{j_\gamma j_\gamma}], \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mu_{j_\gamma j_\alpha} = &-\sum_J \sum_{j_\beta \neq j_\gamma} \frac{\hat{J}}{\hat{j}_\alpha} G(j_\alpha j_\beta j_\alpha j_\beta J) v_{j_\gamma j_\beta}^2 \\ &- \frac{\hat{j}_\gamma - 2}{\hat{j}_\gamma} \sum_J \frac{\hat{J}}{\hat{j}_\alpha} G(j_\alpha j_\gamma j_\alpha j_\gamma J) v_{j_\gamma j_\gamma}^2. \end{aligned}$$

More extensive treatments of quasiparticle descriptions for systems of fermions can be found in several standard references.[11,12]

For numerical simplicity and in order to reduce the number of parameters in the theory, we use a pairing force, i.e.,

$$\langle\alpha\beta|V_{pp}|\gamma\delta\rangle = G(-1)^{j_\alpha + j_\gamma + l_\alpha + l_\gamma + m_\alpha + m_\gamma} \delta_{\beta-\alpha}\delta_{\delta-\gamma}. \tag{8}$$

In order to minimize the energy of the ground state, the use of the pairing force requires that

$$u_\alpha = (-1)^{l_\alpha}(1 - v_\alpha^2)^{1/2}. \tag{9}$$

Equations (3) and (9) specify all sign conventions. It has been shown that this is often an acceptable approxi-

[10] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963).

[11] A. M. Lane, *Nuclear Theory* (W. A. Benjamin, Inc., New York, 1964).

[12] D. J. Thouless, *The Quantum Mechanics of Many-Body Systems* (Academic Press Inc., New York, 1961).

mation for the actual two-body force in pairing-type matrix elements.[13] We further assume that the quasiparticle representation is good in the sense that we can neglect the correlations between the quasiparticles. It is known that the lowest energy states that arise from such correlations are collective and vibrational in character.[5] It is also known that for single-closed-shell even-even nuclei in the mass-140 region the energy of this state is about 1.5 MeV but as neutrons are added, the neutron-proton long-range quadrupole force causes the energy of the lowest vibrational state to drop sharply.[14] However, since La^{140} is a single-closed-shell-plus-one nucleus and because there are experimental assignments for only the first 0.5 MeV of excitation, we assume that we can neglect vibrational particle mixing.

The basic states for the odd-odd system are chosen to be

$$|j_\alpha^0 j_a JM\rangle=[\xi_\alpha^\dagger C_a^\dagger]^J{}_M|0'\rangle, \tag{10}$$

where the bracket notation denotes vector coupling and $|0'\rangle$ is the direct product $|\bar{0}\rangle|0_n\rangle$.

Assuming a truncation so that a state with a particular angular momentum, parity, and charge occurs only once, matrix elements of H_{np} are given by

$$\langle j_\alpha^0 j_a J|H_{np}|j_\beta^0 j_b J\rangle=F(j_\alpha j_\beta)\Big\{u_{j_\beta j_a}u_{j_\alpha j_\beta}\langle j_\alpha j_a J|V_{np}|j_\beta j_b J\rangle$$

$$-v_{j_\beta j_\alpha}v_{j_\alpha j_\beta}\sum_{J'}\hat{J}'\begin{Bmatrix} j_\beta & j_a & J' \\ j_\alpha & j_b & J\end{Bmatrix}\langle j_\alpha j_b J'|V_{np}|j_\beta j_a J'\rangle+\delta_{j_\alpha j_\beta}\delta_{j_a j_b}\sum_{j_\gamma J'}\frac{\hat{J}'}{\hat{j}_a}\langle j_\gamma j_a J'|V_{np}|j_\gamma j_b J'\rangle v_{j_\alpha j_\gamma}{}^2\Big\}, \tag{11}$$

where

$$F(j_\alpha j_\beta)=\prod_{j_\tau\neq j_\alpha,j_\beta}(u_{j_\alpha j_\tau}u_{j_\beta j_\tau}+v_{j_\alpha j_\tau}v_{j_\beta j_\tau})^{j_\tau+1/2}(u_{j_\alpha j_\alpha}u_{j_\beta j_\alpha}+v_{j_\alpha j_\alpha}v_{j_\beta j_\alpha})^{j_\alpha-1/2}(u_{j_\alpha j_\beta}u_{j_\beta j_\beta}+v_{j_\alpha j_\beta}v_{j_\beta j_\beta})^{j_\beta-1/2}. \tag{12}$$

Equation (11) is a special case of the matrix element for the neutron-proton interaction in the case where both proton and neutron are quasiparticles.

$$\langle j_\alpha^0 j_a^0 J|H_{np}|j_\beta^0 j_b^0 J\rangle=F(j_\alpha j_\beta)F(j_a j_b)\Big\{(u_{j_\beta j_\alpha}u_{j_a j_\beta}u_{j_b j_a}u_{j_a j_b}+v_{j_\beta j_\alpha}v_{j_a j_\beta}v_{j_b j_a}v_{j_a j_b})\langle j_\alpha j_a J|V_{np}|j_\beta j_b J\rangle$$

$$-(u_{j_\beta j_a}u_{j_a j_\beta}v_{j_b j_a}v_{j_a j_b}+u_{j_b j_a}u_{j_a j_b}v_{j_\beta j_a}v_{j_a j_\beta})\sum_{J'}\hat{J}'\begin{Bmatrix} j_\beta & j_a & J' \\ j_\alpha & j_b & J\end{Bmatrix}\langle j_\alpha j_b J'|V_{np}|j_\beta j_a J'\rangle$$

$$+\delta_{j_a j_\beta}\delta_{j_a j_b}\sum_{j_c J'}\frac{\hat{J}'}{\hat{j}_\alpha}\langle j_\alpha j_c J'|V_{np}|j_\beta j_c J'\rangle(u_{j_\beta j_\alpha}u_{j_a j_\beta}-v_{j_\beta j_\alpha}v_{j_a j_\beta})v_{j_a j_c}{}^2$$

$$+\delta_{j_\alpha j_\beta}\delta_{j_a j_b}\sum_{j_\gamma J'}\frac{\hat{J}'}{\hat{j}_a}\langle j_\gamma j_a J'|V_{np}|j_\gamma j_b J'\rangle(u_{j_b j_a}u_{j_a j_b}-v_{j_b j_a}v_{j_a j_b})v_{j_a j_\gamma}{}^2$$

$$+\delta_{j_\alpha j_\beta}\delta_{j_a j_b}\sum_{j_\gamma j_c J}\hat{J}\langle j_\gamma j_c J|V_{np}|j_\gamma j_c J\rangle v_{j_a j_\gamma}{}^2 v_{j_a j_c}{}^2\Big\}. \tag{13}$$

The formulas that we have presented assume only that H_{np} is a two-body interaction. At present there are three different methods for treating this interaction. One may take a highly realistic interaction, i.e., one which explains two-nucleon scattering—the deuteron problem—and gives the correct results in nuclear-matter calculations, and then attempt to explain differences between experiment and theory by systematic corrections to the model. The second approach is to invent an interaction which has desirable analytic properties and can explain certain systematic features observed in many nuclear spectra. The well-known pairing-plus-quadrupole model is an example of this. Finally, one may choose an interaction which is sufficiently simple to allow extensive calculations and even parameter-fitting but which is complicated enough to simulate the properties of the realistic force. Such an interaction will be model-dependent, for it will depend on the assumptions of the model (e.g., the stability of the inert core), the extent to which the basic states approximate the Hartree-Fock self-consistent orbitals and the truncation of the Hartree-Fock single-particle orbitals. The latter will depend on the region of the nuclear periodic table, the number of valence particles, and the size of the computer. If the model is untested and there is only a limited amount

[13] Mannque Rho, Ph.D. thesis, University of California Lawrence Radiation Laboratory Report No. UCRL-11080, 1963 (unpublished).

[14] *Nuclear Data Sheets*, compiled by K. Way, *et al.* (Printing and Publishing, Office, National Academy of Sciences—National Research Council, Washington, D. C.).

of experimental data, the third approach is perhaps the most useful since it not only allows one to draw conclusions about the physical assumptions built into the model but also makes predictions which can be tested experimentally.

For the neutron-proton interaction, the Pauli principle does not exclude any states of relative motion. Therefore the number of parameters for a reasonably sophisticated interaction soon exceeds the number of available data. We therefore choose the simplest possible force capable of explaining the data. The simplest reasonable neutron-proton interaction is the δ function. However, it fails to reproduce the experimental levels in La^{140}. The next simplest force is the finite-range central interaction with Gaussian shape. We chose this interaction but restricted it so that all exchange components of the force have the same range parameters. With this approximation

$$\langle \vec{j}_\alpha \vec{j}_a J | V_{np} | \vec{j}_\beta \vec{j}_b J\rangle = \tfrac{1}{2}\{(V_{TE}+V_{TO})+(V_{TE}-V_{TO})(-1)^{j_\beta+j_b+J}P_{\vec{j}_\beta\vec{j}_b}\}\langle \vec{j}_\alpha \vec{j}_a J | e^{-r_{pn}^2/r_0^2} | \vec{j}_\beta \vec{j}_b J\rangle$$
$$-\tfrac{1}{2}\{(V_{TE}-V_{SE}+V_{TO}-V_{SO})+(V_{TE}+V_{SE}-V_{TO}-V_{SO})(-1)^{j_\beta+j_b+J}P_{\vec{j}_\beta\vec{j}_b}\}\langle \vec{j}_\alpha \vec{j}_a J | e^{-r_{pn}^2/r_0^2}\pi_S | \vec{j}_\beta \vec{j}_b J\rangle, \quad (14)$$

where for notational convenience

$$|\vec{j}_\alpha \vec{j}_a J\rangle \equiv |n_\alpha n_a (\tfrac{1}{2}l_\alpha) j_\alpha (\tfrac{1}{2}l_a) j_a J M\rangle. \quad (15)$$

In this equation, π_S is the singlet projection operator, $P_{\beta b}$ is an operator which exchanges the proton and neutron quantum numbers in the ket vector, the V's are the strengths of the two-body potential in the triplet-even, triplet-odd, singlet-even, and singlet-odd states, and r_0 is the range of the force.

$$\langle \vec{j}_\alpha \vec{j}_a J | e^{-r_{pn}^2/r_0^2} | \vec{j}_\beta \vec{j}_b J\rangle = \sum_k f_k F_k, \quad (16)$$

where

$$F_k = \frac{1}{\hat{k}} \int\int R_{n_\alpha l_\alpha}(r_p) R_{n_a l_a}(r_n) v_k(r_p, r_n) R_{n_\beta l_\beta}(r_p) R_{n_b l_b}(r_n) r_p^2 r_n^2 dr_p dr_n, \quad (17)$$

and

$$f_k = (-1)^{j_\alpha+j_\beta+J}(\hat{j}_\alpha \hat{j}_a \hat{j}_\beta \hat{j}_b)^{1/2}\hat{k}\begin{Bmatrix} j_\alpha & j_a & J \\ j_b & j_\beta & k \end{Bmatrix}\begin{pmatrix} j_\alpha & k & j_\beta \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} j_a & k & j_b \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad (18)$$

with the restriction that both $(l_\alpha+k+l_\beta)$ and (l_a+k+l_b) must be even, otherwise $f_k=0$. The $R(r)$'s are the radial wave functions[15] and $v_k(r_p,r_n)$ is the Legendre transform of the Gaussian.

$$\langle \vec{j}_\alpha \vec{j}_a J | e^{-r_{pn}^2/r_0^2}\pi_S | \vec{j}_\beta \vec{j}_b J\rangle = \tfrac{1}{4}\sum_k q_k F_k, \quad (19)$$

where

$$q_k = 2(-1)^{j_\alpha+j_\beta+J+1}\hat{k}(\hat{j}_\alpha \hat{j}_\beta \hat{j}_a \hat{j}_b \hat{l}_\alpha \hat{l}_\beta \hat{l}_a \hat{l}_b)^{1/2}\begin{Bmatrix} J & j_\beta & j_b \\ \frac{1}{2} & l_b & l_\beta \end{Bmatrix}\begin{Bmatrix} l_\alpha & j_\alpha & \frac{1}{2} \\ j_a & l_a & J \end{Bmatrix}\begin{Bmatrix} l_\beta & J & l_b \\ l_a & k & l_\alpha \end{Bmatrix}\begin{pmatrix} l_\alpha & k & l_\beta \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_a & k & l_b \\ 0 & 0 & 0 \end{pmatrix}. \quad (20)$$

Finally let us consider the problem of conservation of particle number in the proton quasiparticle wave functions. Equation (4) corresponds to an ensemble of nuclei each with a different number of protons. However, the average number of protons taken over the ensemble is equal to the number of valence protons. In order for this to be true for each proton quasiparticle state, the BCS equations must be solved for each state with proper regard for conserving the average number of particles. In doing this one blocks,[11] i.e., neglects, conjugate states one of which is occupied by the odd particle. In general, the v's for the various single-particle states depend on the quasiparticle state, but since we consider states with only one quasiparticle, their different spins and parities automatically make them orthogonal. For many nuclear properties simpler treatments than blocking are justified, but the errors in the eigenvalues are not small if the neutron-proton interaction is treated with basis states generated by operating with the various quasiparticle creation operators on the same BCS solution. If the v's are chosen so that the expectation value of the number operator is n in the ground state, then a one-quasiparticle state for an orbital well below the Fermi surface gives an expectation value $\sim(n-1)$. Those states for orbitals well above the Fermi surface have an expectation value of $\sim(n+1)$. Therefore, the neutron in La^{140} interacts with a different average number of protons depending on the state of the quasiproton. Thus when blocking is not considered, states which are predominantly hole have a higher energy relative to particle states. In model calculations, Nilsson[16] has shown that the blocking method may unduly reduce the pairing correlation and that corrections due to blocking are often spurious when states with different seniority are being compared. However, since we are considering only seniority-one states in the proton system, these considerations are not important.

[15] We adopt the convention that n equals the number of nodes (excluding $r=0$ but including $r=\infty$) and that the nuclear-size parameter ν equals the $m\omega/\hbar$ which is twice the value used by de-Shalit and Talmi (Ref. 10).

[16] S. G. Nilsson, Nucl. Phys. **55**, 97 (1964).

III. NUMERICAL SOLUTIONS FOR La^{140}

Before evaluating the n-p force matrix elements, we seek a self-consistent iterative solution for the coupled BCS equations to determine the quasiproton energies and wave functions. To solve these equations it is necessary to specify a p-p interaction (we have chosen a pairing force) and a set of single-particle levels. We would like to take single-particle proton energies (and the single-particle neutron energies) from experimental data for one nucleon outside the Sn^{132} core. Unfortunately this is not now possible and so we must resort to empirical or calculated values. In all quasiparticle calculations the choice of single-particle levels is perhaps the most uncertain feature and for this reason we have performed our calculation with three rather different sets of levels. Two of these sets are those used by Kisslinger and Sorensen[1,5] and the third is that of Tamura and Udagawa.[17] In all cases the value of the pairing force matrix element was that used by Kisslinger and Sorensen[5] and the $1g_{7/2}$-$2d_{5/2}$ level spacing was altered so that the calculation reproduces the experimental separation in La^{139}. The final results of the calculation are very insensitive to the choice of single-particle levels and so we present here only the results using the second scheme. The parameters and the quasiparticle energies and occupation probabilities are given in Table I. By using such an empirical set of levels, we have already included the first-order effects of self-energy contributions and so have neglected them in this calculation.

For the neutrons we have taken the position of the single-particle levels from the experimental work of Fulmer *et al.*,[18] on Ce^{141}. Since this nucleus has one neutron and eight protons outside the Sn^{132} core, it is necessary to correct for the effect of n-p residual interaction on the spacing of these levels. Considering only first-order effects, we find that

$$\epsilon_{j_a}=\epsilon_{j_a}'-\sum_{j_\gamma J'}\frac{\hat{J}'}{\hat{j}_a}\langle j_\gamma j_a J'|V_{np}|j_\gamma j_a J'\rangle v_\gamma^2+c, \quad (21)$$

where ϵ_{j_a} is the value for Sn^{133}, ϵ_{j_a}' is the experimental value for Ce^{141}, and c is a constant chosen so that $\epsilon'_{2f_{7/2}}=0$. The v_γ's represent the unblocked solution of Ce^{140} (see Table I). Finally for the theory as formulated, we need to specify five parameters for the n-p interaction. Four of these are the strengths of the interaction in the triplet-even, triplet-odd, singlet-even, and singlet-odd states of relative motion. The fifth parameter is denoted by X and is given by

$$X=\nu r_0^2. \quad (22)$$

Although ν and r_0 have different physical meanings, only X has mathematical relevance. As we explained in the last section, the values of these parameters are sensitive to the truncation of the shell model and the quasiparticle states. Since we expect that the model will have validity only for the fourteen negative-parity states that are predominantly from the configurations generated by the $1g_{7/2}$ and $2d_{5/2}$ proton quasiparticle states and the $2f_{7/2}$ neutron state, we have truncated the calculation to include only the $1g_{7/2}$, $2d_{5/2}$, and the $2d_{3/2}$ quasiproton states and the $2f_{7/2}$, $3p_{3/2}$, $2f_{5/2}$ neutron-shell-model states. Each of these three quasiproton states is specified by the five shell-model orbitals that are listed in Table I. To specify these five n-p force parameters we have used a least-squares search technique to find the minimum of χ^2 in a six-dimensional parameter space, where the last parameter specifies the optimum normalization and where χ^2 is the sum of the squares of deviations in energy. We chose not to normalize to the ground state because experimentally[6] it is found that the 3− ground state is strongly admixed between the $1g_{7/2}$ and $2d_{5/2}$ proton states and so its energy position will be sensitive to the magnitude of an off-diagonal matrix element and also to the relative energy spacing of the $1g_{7/2}$ and $2d_{5/2}$ quasiproton states. This latter quantity might be expected to change slightly from La^{139} to La^{140} because the neutron-proton quadrupole interaction will lower the energy of the vibrational phonon. In Fig. 1 we compare the experimental level scheme with the theoretical predictions for these states. In Table II we list the energies and wave functions for all levels below 600 keV in excitation energy. The mean deviation between the experimental and theo-

TABLE I. Quasiparticle energies, parameters, and occupation probabilities.

Quasiparticle state	Single-particle energy (MeV)	Quasiparticle energy (MeV)	λ_p (MeV)	$\lvert\Delta_p\rvert$ (MeV)	V coefficients $1g_{7/2}$	$2d_{5/2}$	$1h_{11/2}$	$2d_{3/2}$	$3s_{1/2}$
Vacuum	...	...	0.381	0.796	0.8460	0.5134	0.2173	0.1481	0.1363
$1g_{7/2}$	0.00	0.0	0.324	0.648	0.8507	0.4482	0.1758	0.1194	0.1098
$2d_{5/2}$	0.81	0.166	0.169	0.651	0.7912	0.3870	0.1634	0.1135	0.1049
$1h_{11/2}$	2.08	1.268	0.128	0.698	0.7681	0.3886	0.1708	0.1197	0.11075
$2d_{3/2}$	2.98	2.131	0.121	0.710	0.7643	0.3902	0.1729	0.1214	0.1123
$3s_{1/2}$	3.22	2.364	0.120	0.712	0.7636	0.3906	0.1733	0.1217	0.

[17] T. Tamura and T. Udagawa, Progr. Theoret. Phys. (Kyoto) **26**, 947 (1961).
[18] R. H. Fulmer A. L. McCarthy, and B. L. Cohen, Phys. Rev. **128**, 1302 (1962).

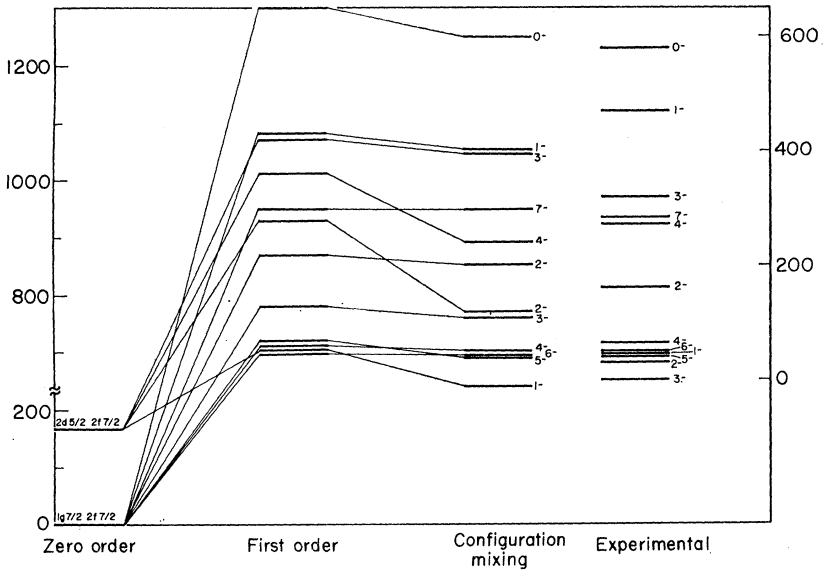

FIG. 1. Comparison of energies between calculated and experimentally determined states in ^{140}La.

retical spectra is 54 keV. This spectrum was generated with a neutron-proton force having $V_{TE}=-37.00$ MeV, $V_{TO}=-14.80$ MeV, $V_{SE}=-29.60$ MeV, $V_{SO}=-11.10$ MeV, and $X=1.0$.

IV. WAVE FUNCTIONS

A. (d,p) Spectroscopic Factors

Under certain general assumptions,[19] the differential cross section for the (d,p) reaction may be written as

$$\frac{d\sigma^\gamma}{d\Omega}=\frac{\hat{J}_f}{\hat{J}_i}\sum_l S_l{}^\gamma\phi(l,Q,\theta), \tag{23}$$

where J_i is the spin of target nucleus, J_f is the spin of the residual nucleus, and $S_l{}^\gamma$ is the spectroscopic factor. The last factor, called the single-particle cross section, is assumed to depend on the Q value, the orbital angular momentum of the stripped neutron, and the angle of the outgoing proton. The spectroscopic factor is a sum of overlap integrals

$$S_l{}^\gamma=\sum_{j=l\pm\frac{1}{2}} I_\gamma{}^2(l,j). \tag{24}$$

If we write a state vector in La^{140} as

$$\Psi_{JM}{}^\gamma\equiv|\gamma JM\rangle=\sum_{j_\alpha j_a} C_{j_\alpha j_a J}{}^\gamma|\,j_\alpha{}^0 j_a JM\rangle \tag{25}$$

then the overlap integral for the state labeled γ is

$$I_\gamma(l,j)=\langle\Psi_{JM}{}^\gamma|[\Psi_\beta\psi_b]^J{}_M\rangle=C_{j_\beta j_b J}{}^\gamma. \tag{26}$$

Here Ψ_β denotes the target ground state which we assume to be a pure $1g_{7/2}$ quasiproton state, and ψ_b repre-

TABLE II. Energies and wave functions for all levels below 600 keV in excitation energy.

		State vectors								
Spin-parity	Energy (MeV)	$1g_{7/2}$ $2f_{7/2}$	$1g_{7/2}$ $3p_{3/2}$	$1g_{7/2}$ $2f_{5/2}$	$2d_{5/2}$ $2f_{7/2}$	$2d_{5/2}$ $3p_{3/2}$	$2d_{5/2}$ $2f_{5/2}$	$2d_{3/2}$ $2f_{7/2}$	$2d_{3/2}$ $3p_{3/2}$	$2d_{3/2}$ $2f_{5/2}$
0−	0.597	0.9750					−0.1595		0.1548	
1−	−0.011	0.1613		0.0036	0.9735	0.0041	0.1239		0.1044	−0.0076
1−	0.404	0.9738		−0.1034	−0.1678	−0.0952	0.0138		0.0482	−0.0373
2−	0.120	0.2615	−0.0454	−0.0018	−0.9038	−0.2210	−0.0698	0.2348	0.0241	0.0577
2−	0.202	0.9553	−0.1046	−0.0488	0.2662	0.0399	0.0191	−0.0133	0.0226	−0.0234
3−	0.108	0.9892	−0.1336	−0.0368	−0.0176	−0.0124	0.0031	0.0327	0.0258	−0.0042
3−	0.395	0.0209	0.0346	−0.0099	0.9883	0.1275	0.0228	0.0312	0.0595	0.0170
4−	0.053	0.9931	−0.1032	−0.0155	−0.0311	−0.0306	−0.0036	0.0280		−0.0103
4−	0.241	0.0405	−0.0209	−0.0073	0.9216	0.3640	0.0601	−0.1047		−0.0370
5−	0.042	0.9897	−0.1358	−0.0181	−0.0026		0.0139	0.0388		
5−	0.513	0.0064	0.0646	−0.0024	0.9865		−0.0468	0.1427		
6−	−0.147	0.0831		0.0033	−0.9965					
6−	0.043	0.9964		0.0167	0.0831					
7−	0.298	1.0								

[19] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).

sents the spin-angle part of the stripped neutron's wave function.

No experimental evidence exists from the (d,p) reaction[6] for l mixing in the states observed to 580-keV excitation. Therefore experimental spectroscopic factors are extracted by assuming that the $2f_{7/2}$ state is the only single-particle neutron component in the wave functions and by assuming that the 7− state at 284 keV is entirely from the $1g_{7/2}$ quasiproton configuration.

In Table III the experimental S_3 values are compared with the calculated values from the wave functions that are listed in Table II. Also in Table III we compare the theoretical and experimental differential cross sections normalized to the value for the 7− state at 284 keV. In computing the theoretical cross sections, we have used the distorted-wave Born approximation (DWBA) single-particle cross sections obtained from fitting angular distributions for 65° in the laboratory frame.[6] The experimental intensities for the 31-, 43-, and 38-keV states were deduced by assuming that the $1g_{7/2}$ and $2d_{5/2}$ proton states and the $2f_{7/2}$ neutron states were the only components in the state vectors. This was consistent with the total unresolved intensity of these three states. This calculation underestimates configuration mixing between the $1g_{7/2}$ and $2d_{5/2}$ proton quasiparticle states for spins 1, 2, and 3 and this is consistent with the incorrect energy ordering in the lowest states. If one allows the spacing between the $1g_{7/2}$ and $2d_{5/2}$ quasiproton states to be an additional parameter, then a slightly improved energy fit and much better (d,p) predictions can be obtained. Insofar as this parameter has any physical significance, it probably simulates the first-order effects of particle-phonon coupling. The predictions for the other states seem to be in good agreement with the (d,p) data.

TABLE III. Calculated and experimental values of S_3 and relative intensity.

Energy (keV)	$J\pi$	$[S_3]_{\text{theor}}$	$[S_3]_{\text{expt}}$	Relative intensity (theoretical)	Relative intensity (experimental)
579	0−	0.95	1.0	0.06	0.05
43	1−	0.03	0.66	0.01	0.13
467	1−	0.95	0.35	0.19	0.07
31	2−	0.07	0.48	0.02	0.16
162	2−	0.91	0.52	0.31	0.17
0	3−	0.98	0.81	0.47	0.38
319	3−	0.000	0.18	0.00	0.09
63	4−	0.99	1.0	0.60	0.58
38	5−	0.98	1.0	0.74	0.73
49	6−	0.99	1.0	0.86	0.85
284	7−	1.0	1.0	1.0	1.0

B. $M1$ Transition Probabilities

This discussion of gamma transition probabilities is based on Chap. 17 of de-Shalit and Talmi.[10]

In the long-wave approximation, the transition probability is given by

$$T_t(L)=\frac{8\pi(L+1)}{L[(2L+1)!!]}\frac{1}{\hbar}\left(\frac{\Delta E}{\hbar c}\right)^{2\lambda+1}B_t(L), \quad (27)$$

where L is the multipolarity of the transition, ΔE is the energy of the transition, t is an index so that $t=e$ denotes an electric multipole and $t=m$ denotes a magnetic multipole. The reduced transition rate is given by

$$B_t(L)=\sum_{M,M_f}|\langle\alpha_f J_f M_f|O_M{}^{tL}(p)+O_M{}^{tL}(n)|\alpha_i J_i M_i\rangle|^2. \quad (28)$$

Here $O_M{}^{tL}(\tau)$ is the one-body multipole moment operator where $\tau=1$ denotes a proton and $\tau=2$ denotes a neutron. In general, for any one-body operator, we have

$$\langle\vec{j}_\alpha{}^0\vec{j}_a J_f M_f|O_M{}^{tL}(\tau)|\vec{j}_\beta{}^0\vec{j}_b J_i M_i\rangle = R(\vec{j}_\alpha\vec{j}_\beta{}^\tau)\langle\vec{j}_\alpha\vec{j}_a J_f M_f|O_M{}^{tL}(\tau)|\vec{j}_\beta\vec{j}_b J_i M_i\rangle. \quad (29)$$

The factor $R(\vec{j}_\alpha\vec{j}_\beta{}^\tau)$ is given by

$$R(\vec{j}_\alpha\vec{j}_\beta{}^\tau)=F(\vec{j}_\alpha\vec{j}_\beta)[u_{j_\beta j_\alpha}u_{j_\alpha j_\beta}-(-1)^T v_{j_\beta j_\alpha}v_{j_\alpha j_\beta}]\delta_{\tau,1} +(1-\delta_{\tau,1}), \quad (30)$$

where $(-1)^T=-1$ for operators which are odd under time reversal and $(-1)^T=1$ for operators which are even. Eq. (28) can then be written as

$$B_t(L)=\frac{1}{\hat{J}_i}\left|\sum_{\substack{\vec{j}_\alpha\vec{j}_a\\ \vec{j}_\beta\vec{j}_b\tau}} C_{\vec{j}_\alpha\vec{j}_a J_f}{}^{\alpha_f}C_{\vec{j}_\beta\vec{j}_b J_i}{}^{\alpha_i}R(\vec{j}_\alpha\vec{j}_\beta{}^\tau)\langle\vec{j}_\alpha\vec{j}_a J_f\|O^{tL}(\tau)\|\vec{j}_\beta\vec{j}_b J_i\rangle\right|^2. \quad (31)$$

After decoupling the reduced matrix element, Eq. (31) may be written as

$$B_t(L)=\hat{J}_f\left|\sum_{\substack{\vec{j}_\alpha\vec{j}_a\\ \vec{j}_\beta\vec{j}_b}} C_{\vec{j}_\alpha\vec{j}_a J_f}{}^{\alpha_f}C_{\vec{j}_\beta\vec{j}_b J_i}{}^{\alpha_i}\left[(-1)^{j_\alpha+j_a+J_i+L}R(\vec{j}_\alpha\vec{j}_\beta 1)\begin{Bmatrix} j_\alpha & J_f & j_a\\ J_i & j_\beta & L\end{Bmatrix}\langle\vec{j}_\alpha\|O^{tL}(p)\|\vec{j}_\beta\rangle\delta_{\vec{j}_a,\vec{j}_b}\right.\right.$$
$$\left.\left.+(-1)^{j_\alpha+\vec{j}_b+J_f+L}\begin{Bmatrix} j_a & J_f & j_\alpha\\ J_i & j_b & L\end{Bmatrix}\langle\vec{j}_a\|O^{tL}(n)\|\vec{j}_b\rangle\delta_{\vec{j}_\alpha,\vec{j}_\beta}\right]\right|^2. \quad (32)$$

TABLE IV. Reduced transition probabilities and transition probabilities for all possible $M1$ transitions between states below 579 keV in excitation.

Spin-parity (initial)	Energy (MeV)	Spin-parity (final)	Energy (MeV)	Transition energy (MeV)	Type	Rate Th.-I (sec^{-1})	Relative rate Th.-I	Rate Th.-II (sec^{-1})	Relative rate Th.-II	Rate Th.-III (sec^{-1})	Relative rate Th.-III	Experimental branching ratios
0−	0.580	1−	0.466	0.114	$M1$	9.63×10^{10}	0.40	3.70×10^{10}	0.0051	3.98×10^{10}	0.0051	0.00069
		1−	0.043	0.537	$M1$	2.41×10^{11}	1.0	7.28×10^{12}	1.0	7.84×10^{12}	1.0	1.0
1−	0.466	2−	0.162	0.304	$M1$	8.65×10^{11}	1.0	2.59×10^{12}	1.0	1.86×10^{12}	1.0	1.0
		1−	0.043	0.423	$M1$	2.59×10^{11}	0.30	1.90×10^{12}	0.73	1.24×10^{12}	0.67	0.56
		2−	0.030	0.436	$M1$	1.08×10^{12}	1.2	1.52×10^{12}	0.59	5.34×10^{11}	0.29	0.53
2−	0.162	1−	0.043	0.119	$M1$	1.75×10^{10}	0.12	9.54×10^{8}	0.004	1.04×10^{8}	0.0005	0.011
		2−	0.030	0.132	$M1$	1.11×10^{9}	0.0077	7.56×10^{9}	0.032	6.98×10^{9}	0.036	0.047
		3−	0.	0.162	$M1$	1.46×10^{11}	1.0	2.36×10^{11}	1.0	1.96×10^{11}	1.0	1.0
1−	0.043	2−	0.030	0.013	$M1$	2.26×10^{8}	1.0	1.74×10^{8}	1.0	1.35×10^{8}	1.0	1.0
2−	0.030	3−	0.	0.030	$M1$	8.58×10^{7}	1.0	8.03×10^{5}	1.0	3.65×10^{7}	1.0	1.0

Experimental information exists only for $M1$ transition probabilities.[6] The reduced single-particle diagonal matrix element for the $M1$ multipole operator is

$$\langle n\tfrac{1}{2}lj\|O^{m1}\|n\tfrac{1}{2}lj\rangle=[(3/4\pi)j(j+1)(2j+1)]^{1/2}\times g_{lj}(e\hbar/2Mc), \quad (33)$$

where g_{lj} is the gyromagnetic ratio for the state that has orbital angular momentum l and spin j. The nondiagonal elements with our phase conventions are given by

$$\langle n\tfrac{1}{2}lj\|O^{m1}\|n'\tfrac{1}{2}l'j'\rangle=(-1)^{j-l+1/2}(g_l-g_s)\times\left[\frac{3}{4\pi}\frac{2l(l+1)}{(2l+1)}\right]^{1/2}\frac{e\hbar}{2Mc}\delta_{l,l'}\delta_{n,n'}, \quad (34)$$

where g_s and g_l are the spin and oribtal angular momentum gyromagnetic ratios. For convenience we define dimensionless reduced transition probabilities B' by

$$B_m'(1)=(4\pi/3)(e\hbar/2Mc)^{-1}B_m(1). \quad (35)$$

If the transition energy is expressed in MeV, then

$$T_m(1)=4.25\times10^{12}(\Delta E)^3B_m'(1)\ \text{sec}^{-1}. \quad (36)$$

Since details of the interaction (e.g., neglect of tensor force) might considerably influence the amount of configuration mixing, we computed transition probabilities not only for the theoretical wave functions but also for experimental wave functions extracted from the (d,p) experiment.[6] In Table IV we tabulated the reduced transition probabilities and the transition probabilities for all possible $M1$ transitions between states below 579 keV in excitation. In theory I we use the wave functions from Table II and Schmidt-limit gyromagnetic ratios. In theories II and III we used wave functions extracted from the (d,p) experiment.[6] Of course only absolute values of the state vector's coefficients can be obtained. However, since there are only two components in the expansion (see Ref. 6, Table V), we can specify the vectors if we assume a sign for the off-diagonal matrix element of the Hamiltonian matrices of angular momentum one, two, and three. We choose this sign to agree with that of the theoretical matrix element. Theory II uses Schmidt-limit gyromagnetic ratios and theory III uses effective gyromagnetic ratios. These were deduced from magnetic moment data[20] on the nuclei $La^{139}(1g_{7/2})$, $Pr^{141}(2d_{5/2})$, and $Ce^{141}(2f_{7/2})$. The relative rates are normalized to the most intense gamma ray that depopulates each parent state. The experimental data are taken from the work of Geiger *et al.*[7] The very limited experimental data do not offer a very good test of the theoretical wave functions or of the consistency of our model with (d,p) and gamma-ray data. However, both theories II and III give acceptable results. Underestimating the configuration mixing probably also explains the discrepancies that occur in theory I.

V. DISCUSSION

In this paper we have investigated the possibility of describing La^{140} in terms of the interaction between a quasiproton and a shell-model neutron. Because of the seniority-zero correlations between pairs of protons inherent in the quasiparticle description, a one-quasiparticle state is partly particle in nature and partly hole. The diagonal neutron-proton matrix elements in the La^{140} problem reflect this by not only taking into account the energy of the interaction between the neutron and conjugate pairs of protons but also by forming a linear combination of two-body particle-particle and particle-hole matrix elements. Thus the calculation predicts that the lowest energy of the $|(1g_{7/2})^0 2f_{7/2};JM\rangle$ configuration is 6−. This is consistent with the Brennan and Bernstein rule[21] for the ground state of particle-hole odd-odd nuclei. The lowest member of the $|(2d_{5/2})^0\times 2f_{7/2};JM\rangle$ configuration is also 6− which is consistent with the rules for particle-particle configurations. These results occur because the BCS calculations show that for 57 protons the $1g_{7/2}$ orbital is predominantly full and the $2d_{5/2}$ orbital is predominantly empty (see Table I).

[20] I. Lindgren, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965).

[21] M. H. Brennan and S. A. Bernstein, Phys. Rev. **120**, 927 (1960).

The most plausible explanation for a 3− state being the ground state is configuration interaction due to the neutron-proton force. Experimentally[6] the ground state appears to have large configuration admixtures but theoretically we get a rather pure wave function for the lowest 3− state particularly with respect to admixtures of the $2d_{5/2}$ proton orbital. In the case of the 1− and 2− states the magnitude of the matrix element connecting the $1g_{7/2}$ and $2d_{5/2}$ configurations is also much too small. This may be due to the fact that all these off-diagonal matrix elements will be large for forces capable of producing spin flips but the least-squares force has strong attractive singlet components. Moreover, off-diagonal matrix elements connecting proton configurations near the Fermi level tend to be reduced due to the u and v coefficients.

Another problem is posed by the 1− states. The experimental data seem to demand that the state at 43 keV has $1g_{7/2}$ as its largest proton component. However, the calculation (in accordance with the empirical rules of Brennan and Bernstein) computes the largest component to be $2d_{5/2}$. This discrepancy might be explained by the inclusion of a tensor force. The theoretical model also predicts that there are appreciable admixtures of higher configurations in some of the low-energy states. The energy resolution of the (d,p) experiment[6] was sufficiently poor that angular distributions had to be made on groups of peaks. Therefore even appreciable admixtures of higher neutron configurations in the lower spin states would not be detected in the angular distributions. However the energies of some of these configurations are comparable with three-quasiproton states and thus their *a priori* neglect is probably not justified.

In the present calculation, there are ambiguities in the choice of force, single-particle energies, and truncation. Nevertheless the results are encouraging since with a central two-body force we have been able to explain the qualitative features of twelve low-lying levels in a complicated eight-body system. Moreover use of model wave functions obtained from the (d,p) experiment gives remarkably good results when used to compute $M1$ branching ratios. Thus a more complete treatment of the mass-140 region using realistic interactions and Hartree-Fock–Bogoliubov theory would be worthwhile.

ACKNOWLEDGMENTS

The author wishes to thank Professor John O. Rasmussen for many helpful suggestions and critically reading the manuscript. The advice and assistance of Dr. Jean Kern throughout the course of this work is also gratefully acknowledged.