Mixing of the Ground-State and Gamma-Vibrational Bands in Er^{166} and Er^{168}*

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Gamma-ray intensities were measured in Er^{166} and Er^{168} with an accuracy of a few percent, using a Ge(Li) detector. Several new γ transitions, not previously reported, were observed. The half-life of the 1095-keV state in Er^{168} was measured to be 115.7 ± 3.3 nsec. The $E2$ branching ratios from the gamma band to the ground-state band are in excellent agreement with the band-mixing theory. Our z_2 values are 0.0460 ± 0.0010 for Er^{166} and 0.0388 ± 0.0015 for Er^{168}. These z_2 parameters as well as some recent values for other even-even nuclei in the $150<A<190$ regions are compared with calculated values according to different theories. The theories and the experiments give the same mass dependence of z_2. However, the experimental values are up to a factor of 2 higher than the theoretical ones. The ratios of the intrinsic $E2$ matrix elements, $\langle 2|M'(E2,0)|2\rangle/\langle 0|M'(E2,-2)|2\rangle$ are derived as 9.8 ± 0.3 for Er^{166} and 9.2 ± 0.7 for Er^{168}. Using Coulomb-excitation data, we obtain for the ratio of the quadrupole moments, $Q_0(K=2)/Q_0(K=0)$, the values 1.38 ± 0.15 for Er^{166} and 1.15 ± 0.14 for Er^{168}. Finally, the γ transition rates from the negative-parity states are discussed in terms of the collective nuclear model.

I. INTRODUCTION

THE reduced γ-transition probabilities $B(E2)$ between γ-vibrational and ground-state bands in strongly deformed nuclei are known to deviate systematically from the theoretical values for totally decoupled bands (adiabatic approximation).[1] The interband $B(E2)$ values are quantities which are very sensitive to band mixing. Therefore, further refinements to the theory of the unified model of the nucleus, taking into account the interaction between different bands, are necessary.

More specifically, it was suggested that these deviations are mainly caused by the mixing of the ground-state and γ-vibrational bands.[2] The reason is that these admixtures give rise to collective transitions which are known to be strongly enhanced for the deformed nuclei. If this holds, the modifications of the transition rates between all the rotational states of the two bands should be described by only one additional parameter (usually denoted by z_2).

The most systematic experimental investigation to verify this theory has been made by Gallagher *et al.*[3] These authors derive z_2 from various branching ratios in Er^{166}, and find agreement for one single-valued parameter with a precision of approximately 15%. Other z_2 values for several cases have been obtained by different investigators.[4–6] However, more precise measurements of z_2 parameters, obtained from several different branching ratios in the same nucleus, would be of great interest. Such high-precision measurements, made possible with the newly developed Ge(Li) detectors, are reported here.

For this investigation, we selected the isotopes Er^{166} and Er^{168}, which are almost ideal cases for several reasons: Both nuclei are in the middle of the deformed-nuclei region; in both cases the radioactive elements which populate the γ-vibrational bands up to high spin levels are readily available; aside from these bands, only a few intrinsic excited states are populated, so that the γ spectra are relatively simple.

The excited states of Er^{166}, populated by the decay of Ho^{166m}, have been studied by several authors, the most recent investigations being the ones by Gallagher *et al.*,[3] and Reich and Cline.[7] The latest study of the decay of Tm^{168} to Er^{168} was made by Reidy *et al.*[8] Reference to earlier works can be found in these papers.

In the following, the results of the γ-spectrum investigations are presented. The experimental transition rates from the γ-vibrational to the ground-state bands are discussed in terms of the interaction between the two bands. The predictions of different theories for such an admixture are discussed.

The intrinsic quadrupole moments of the γ-vibrational bands are then calculated within the framework of the strong-coupling model from the experimental intensity ratios of the interband to intraband transitions. Finally, the negative-parity states are discussed.

II. EXPERIMENTAL PROCEDURES

A. Source Preparation

The Ho^{166m} ($T_{1/2}=1200$ y) was prepared by neutron irradiation of Ho^{165}. At the time of investigation, the source material was about 4 years old. The only remain-

* This work was performed under the auspices of the U. S. Atomic Energy Commission. Prepared under contract No. (04-3)-63 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

[1] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **34**, No. 5 (1954).
[2] P. Hansen, O. Nielsen, and R. Sheline, Nucl. Phys. **12**, 389 (1959).
[3] C. J. Gallagher, Jr., O. B. Nielsen, and A. W. Sunyar, Phys. Letters **16**, 298 (1965).
[4] O. B. Nielsen, in *Proceedings of the Rutherford Jubilee Conference*, edited by J. B. Berks (Academic Press Inc., New York, 1961), p. 317.
[5] Y. Yoshizawa, B. Elbek, B. Herskind, and M. C. Olesen, Nucl. Phys. **73**, 273 (1965).
[6] R. Graetzer, G. B. Hagemann, K. A. Hagemann, and B. Elbek, Nucl. Phys. **76**, 1 (1966).
[7] C. W. Reich and J. E. Cline, Phys. Rev. **137**, B142 (1965).
[8] J. J. Reidy, E. G. Funk, and J. W. Mihelich, Phys. Rev. **133**, B556 (1964).

ing contaminations were Eu^{162} and Eu^{164} (approximately 5%). These ratioactive elements were removed by an ion-exchange technique, as described by Helmer and Burson.[9]

The Tm^{168} ($T_{1/2}=85$ days) was prepared by the reaction[10] $Er^{170}(p,3n)Tm^{168}$. One hundred mg of Er_2O_3 were bombarded with 28-MeV protons for a total of 15.0 μA h. The produced activity was approximately 200 μCi. At the beginning of this tudy, 2 weeks after the irradiation, the only detectable contamination was Tm^{167} (initial contamination approximately 6%). Therefore no radiochemical separation was necessary.

B. Detector–Electronic Equipment

The γ spectra were investigated using a 1 $cm^2 \times 4$ mm Ge(Li) detector. The experimental arrangement was similar to that used by Ewan and Tavendale.[11] The Ge(Li) was connected to an ORTEC 105 XL, low-noise preamplifier, a 220 ORTEC bias amplifier, and a 4096 Nuclear Data multichannel analyzer.

Since the resolution of a Ge(Li) detector is determined almost solely from the noise in the associated electronics, it is practically independent of the energies of the γ rays. In this experiment, the resolution varied

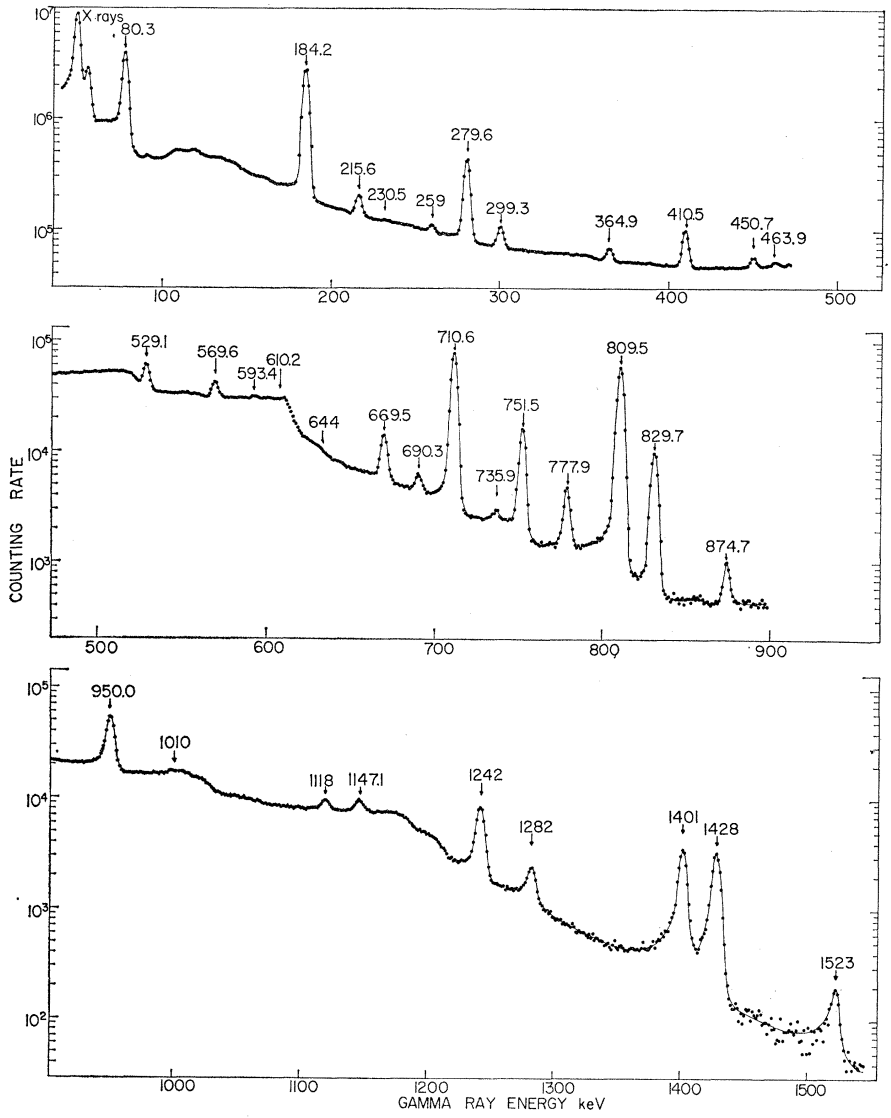

FIG. 1. The gamma-ray spectum of Er^{166}.

[9] R. G. Helmer and S. B. Burson, Phys. Rev. **123**, 978 (1961).
[10] G. V. S. Rayudu and L. Yaffe, Can. J. Chem. **41**, 2544 (1963).
[11] G. T. Ewan and A. J. Tavendale, Can. J. Phys. **42**, 2286 (1964).

between 3.7 and 4.8 keV, for energies varying from 80 to 1500 keV.

The calibration of the detector was made using radionuclei which have precisely known relative gamma-ray intensities. A complete description of the calibration method is given in another publication.[12] The systematic error in the intensity calibration was estimated to be less than 0.5% per 100 keV, for energies above 100 keV. For energies lower than 100 keV a greater uncertainty exists.

III. EXPERIMENTAL RESULTS

A. The Gamma-Ray Spectrum of Er^{166}

A typical γ-ray spectrum of Er^{166} obtained in one of the several measurements is shown in Fig. 1. The γ spectra were analyzed in the following way: The background under each line was subtracted graphically and each γ line was then fitted to a Gaussian, using points above $\frac{1}{3}$ of the total height. For each γ line, several independent measurements were analyzed. The results for the intensities were consistent within the statistical errors.

The energies of the γ transitions were determined using as calibration lines the values of Geiger *et al.*[13] indicated in Table I, as well as the standard lines of Mn^{54} and Co^{60}. Table I shows the γ-transition energies together with the relative intensities. The errors quoted for the intensities are derived from the statistical errors and the uncertainties in the background. Possible systematic errors are not included. The relative intensities are compared with the results reported by Reich and Cline.[7] For this comparison, we recalculated the γ-ray intensities for transitions below 300 keV, using the theoretical conversion coefficients of Sliv and Band.[14]

In addition to the transitions reported by Reich and Cline,[7] the following γ rays were observed: 644.4, 1010.5, 230.5, 735.9, and 1522.8 keV. The first two lines were observed previously by Gallagher *et al.*,[3] who interpreted them as decays from the 8+ γ-vibrational state. This interpretation is confirmed by our observation of the 230.5-keV transition from the 6− (1784 keV) level to this 8+ level. The two remaining γ lines are included in the decay scheme, Fig. 2.

Our γ-ray intensities show that the 6+, $K=2$, and the 8+ and 6+, $K=0$, levels are populated by β^- decay. The intensities and logft values, derived for a half-life of 1200 yr[15] for Ho^{166m}, are shown in Fig. 2.

Finally, a coincidence measurement was made to confirm the γ transitions from the two highest energy levels (1825 and 1784 keV) to the next two levels (1692 and 1664 keV), which were reported by Reich and Cline. In this measurement, we detected the γ rays between 1100 and 1500 keV with a 3 in.×3 in. NaI(Tl) detector. The low-energy spectrum in coincidence with these lines was detected with the Ge(Li) detector. The result of this measurement is shown in Fig. 3.

TABLE I. Energies and relative intensities of the gamma rays in Er^{166}.

Energies[a] (keV)	Relative intensities This work	Reich & Cline[b]
80.3±0.1	14.55±0.45	15.6
94.2±1.0		
120.1±1.0		
134.0±1.0		
160.7±1.0		
184.2±0.1[c]	100 ±1	88.9 ±7.3
215.6	4.15±0.06	4.6 ±2.4
230.5±1.0	0.32±0.05	
258.9±0.8	1.42±0.10	1.1 $\pm_{0.6}^{2.4}$
279.6±0.2[c]	43.6 ±0.4	35.5 ±3.6
299.3	5.45±0.05	4.2 ±1.2
364.9	3.72±0.07	2.1 ±1.2
410.5±0.4[c]	16.8 ±0.2	15.2 ±2.4
450.7	4.30±0.08	3.3 ±1.2
463.9	1.66±0.08	2.1 ±1.2
529.1±0.3[c]	13.0 ±0.4	14.4 ±1.2
569.6	7.08±0.14	8.7 ±2.4
593.4	0.74±0.10	1.0 ±0.5
610.2	1.59±0.32	3.0 ±1.2
644.4±1.0	0.31±0.03	
669.5	7.35±0.29	7.0 ±1.2
690.3	1.62±0.08	1.7 ±1.0
710.6±0.3[c]	71.5 ±0.7	70.5 ±7.3
735.9	0.50±0.05	
751.5	15.2 ±0.3	17.6 ±2.4
777.9	3.88±0.06	4.5 ±1.8
809.5±0.4[c]	76.4 ±0.8	72.6 ±7.3
829.7	12.9 ±0.3	12.9 ±2.4
874.7	0.91±0.04	1.0 ±0.2
950.0	3.16±0.12	4.0 ±0.6
1010.5±1.0	0.11±0.03	
1118.3±1.5	0.26±0.02	0.37±0.12
1147.1	0.26±0.02	0.25±0.12
1241.8±1.5	1.06±0.04	1.22±0.36
1282.2	0.22±0.02	0.37±0.12
1401.2±2.0	0.72±0.02	0.85±0.36
1427.7±2.0	0.69±0.02	0.73±0.36
1522.8±2.0	0.03±0.01	

[a] The energies are given with a precision of ±0.5 keV, unless otherwise noted.
[b] See Ref. 7.
[c] These energies are taken from Geiger *et al.*, Ref. 13.

B. The Gamma-Ray Spectrum of Er^{168}

The γ-ray spectrum following the decay of Tm^{168} is shown in Fig. 4. For the energy calibration we used from 0 to 450 keV, the energies given by Jacob *et al.*[16] Above 450 keV, γ lines in the decay of Na^{22}, Cs^{137}, Mn^{54}, and Co^{60} were used.

Table II shows the energies and relative intensities of the observed lines. Our intensities are compared with the result of Reidy *et al.*[8] Except for some of the very weak lines, the agreement is satisfactory.

Figure 5 shows the decay scheme of Er^{168}. The γ lines with energies of 173.7, 348.8, and 421.9 keV had pre-

[12] C. Gunther and H. Ryde (to be published).
[13] J. S. Geiger, R. L. Graham, and G. T. Ewan, Nucl. Phys. **30**, 409 (1962).
[14] *Alpha, Beta, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), Appendix 5.
[15] K. T. Faler, J. Inorg. Nucl. Chem. **27**, 25 (1965).
[16] K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960).

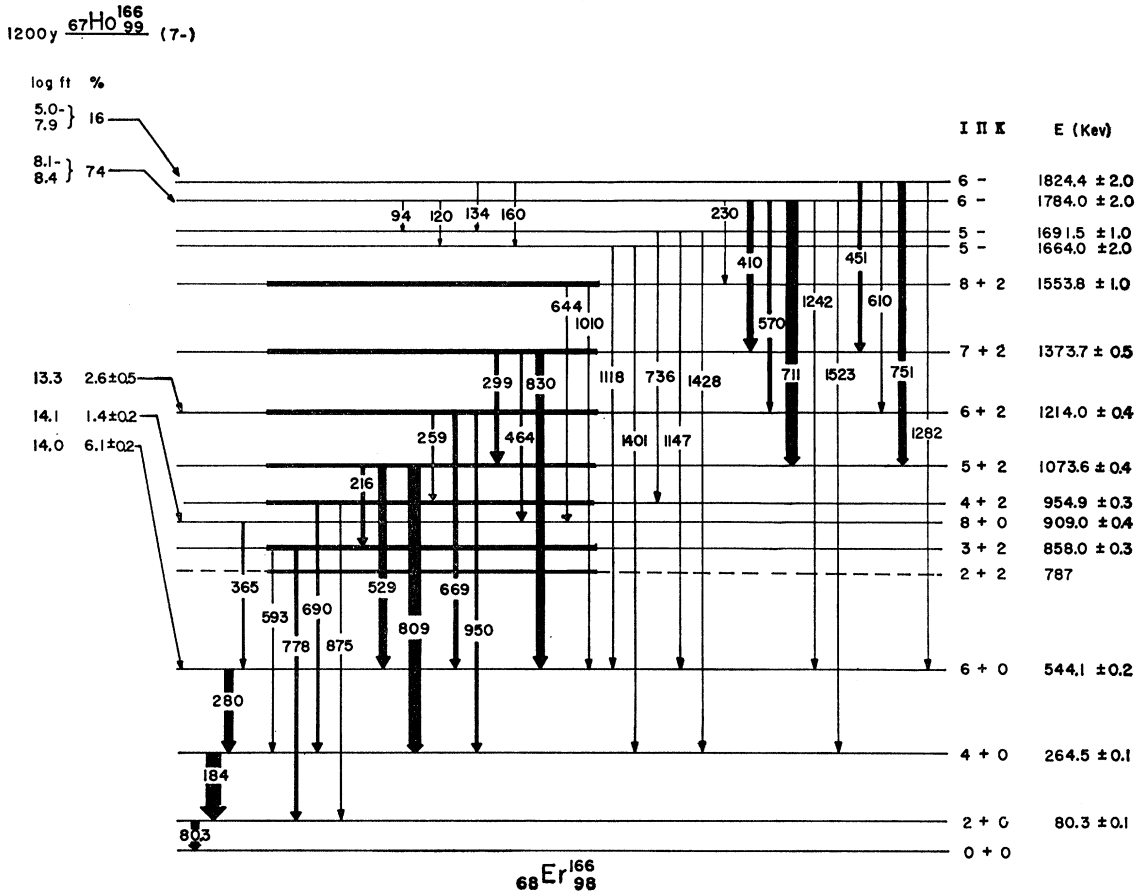

FIG. 2. The decay scheme of Ho^{166m}.

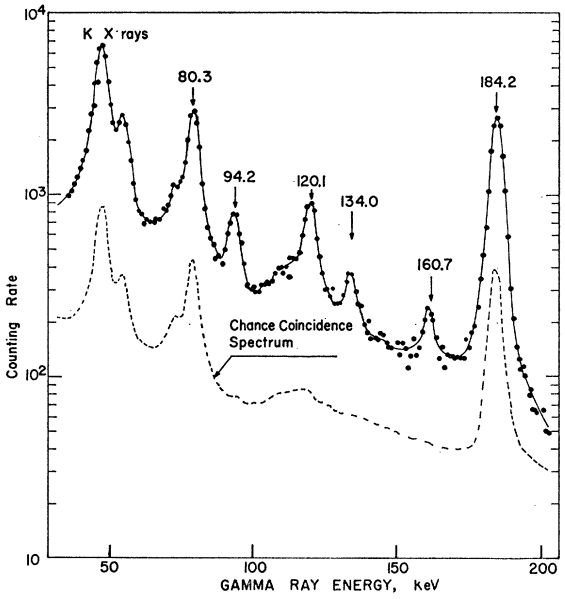

FIG. 3. The coincidence spectrum of the low-energy region in coincidence with gamma rays between 1100 and 1500 keV in Er^{166}.

viously only been seen in the conversion electron spectrum.[16] Reidy *et al.* report two γ lines with energies of about 1180 and 1365 keV which probably correspond to our lines at 1167±2 and 1353±3 keV. Two other lines, reported by the same authors at about 1085 and 1525 keV, were not observed here. Our intensity limits for these lines are a factor of ~5 and 30, respectively, below the values reported by Reidy *et al.* However, since our observed intensities for the weak 1015 and 1167 keV lines are also nearly a factor of 5 below those of Reidy *et al.* we cannot exclude the existence of a 1085-keV line.

Finally, we observed new lines with energies of 316.6, 674.8, 749.6, 930.0, 1324.2, and 1593.4 keV. From successive measurements we found that all the weak lines, which cannot readily be introduced into the decay scheme, decay with half-lives between 50 and 100 days. No radioisotope with such half-lives was found which has the observed γ lines.

Some of the above-mentioned weak lines can be introduced in the decay scheme by postulating new energy levels at 1194.3, 1616.0, and 1571.6 keV (see

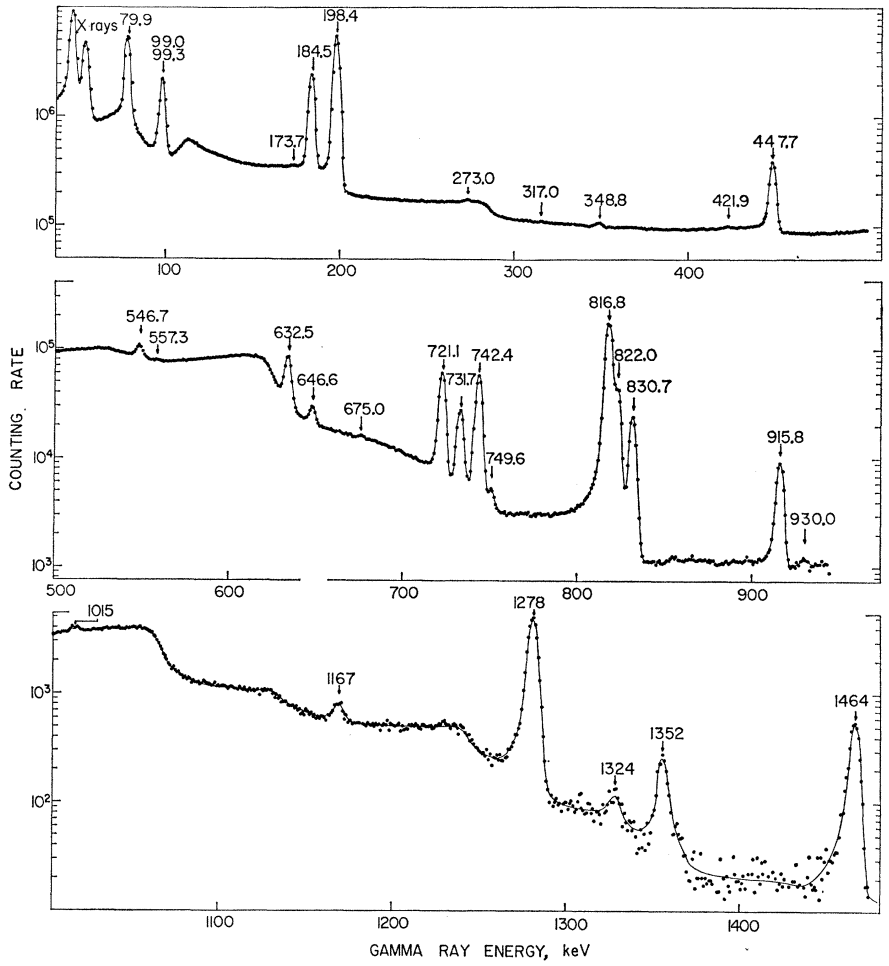

FIG. 4. The gamma-ray spectrum of Er^{168}.

Fig. 5). The first two levels were observed by Koch,[17] who investigated the γ spectrum following the reaction $Er^{167}(n\gamma)$ Er^{168}. The last level is based on the energy difference and comparable intensity of the two depopulating lines.

C. The Half-Life of the 1095-keV Level

The half-life of the 1095-keV level has been reported by Jacob *et al.*[16] as 120±20 nsec. We have remeasured this half-life (Fig. 6). For this measurement, we used the fast-slow coincidence technique, with a time-to-amplitude converter, EG and G TH 200. Coincidences were measured between the 448- and the 198-keV transitions.

Our value for the half-life of the 1095 keV level is

$$T_{1/2}=(115.7\pm3.3)\times10^{-9}\text{ sec}.$$

This value is in agreement with the measurement reported by Jacob *et al.*

[17] H. R. Koch, Z. Physik **192**, 142 (1966).

IV. DISCUSSION

A. The *E2* Branching Ratios between the γ-Vibrational and the Ground-State Bands

As was recognized earlier,[18] the *E2* branching ratios for γ transitions between the γ-vibrational and the ground-state bands deviate systematically from the simple ratios of Clebsch-Gordon coefficients predicted by the strong-coupling model. A number of theoretical attempts have been made to describe these deviations, either by taking into account the Coriolis interaction,[19] or the nonaxiality of the nuclear shape,[18] or the interaction between the rotation and vibration of the nucleus.[20,21] Basically, all these theories account for the anomalies in the *E2* branching ratios by an admixture of the ground-state and vibrational bands, caused by the coupling of the intrinsic and rotational motions.

[18] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).
[19] D. R. Bès, P. Federman, E. Marqueda, and A. Zuker, Nucl. Phys. **65**, 1 (1965).
[20] A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. **70**, 33 (1965).
[21] I. M. Pavlichenkov, Nucl. Phys. **55**, 225 (1964).

TABLE II. Energies and relative intensities of the gamma rays in Er^{168}.

Energies (keV)	Relative intensities This work	Reidy *et al.*[a]
79.9±0.1[b]	49.8 ±2.0	51 ±8.7
99.0	25.3 ±0.8	15.6 ±7.8
99.3		
173.7±0.3	0.25 ±0.04	
184.5	100 ±1	344 ±35
198.4	308 ±3	
273.0±0.4	0.73 ±0.11	<8.7
284.6	<0.1	
316.6±1.0	0.45 ±0.09	
348.8	2.21 ±0.33	
421.9±0.6	1.92 ±0.19	
447.7±0.7[b]	138 ±3	121
547.7±0.5[c]	16.6 ±0.5	21.6 ±6.5
557.3	1.27 ±0.45	
632.5	46.6 ±0.9	65 ±13
646.6	7.71 ±0.31	
674.8±1.0	1.1 ±0.2	
721.1	62.9 ±1.3	
731.7	27.4 ±0.8	182 ±26
742.4	65.4 ±1.3	
749.6	2.12 ±0.15	
816.8	261 ±5	
822.0	61.0 ±1.8	400 ±35
830.7	35.7 ±0.7	
915.8	15.6 ±0.3	18.2 ±2.6
930.0	0.24 ±0.04	
1015.0±1.0	0.45 ±0.07	1.47±0.43
~1085	<0.15	0.86±0.35
1167.2±2.0	0.41 ±0.04	2.16
1278.0±1.0	9.07 ±0.18	11.7 ±7
1324.2±3.0	0.102±0.030	
1352.5±3.0	0.527±0.026	1.0
1463.7±2.0	1.35 ±0.03	1.6±0.6
~1525	<0.04	1.3
1593.4±3.0	0.078±0.022	

[a] See Ref. 8.
[b] These energies up to 447.7 keV included, are taken from Jacob *et al.* (Ref. 16).
[c] The energies above 547.7 keV are determined with an accuracy of ±0.5, unless otherwise noted.

Recently a general theory of this coupling has been formulated by expanding the nuclear Hamiltonian in a power series of the total angular momentum I[22,23];

$$H=H_{\text{intr}}+H_{\text{rot}}+\sum_{i=1}^{\infty}H^{(i+1)}, \tag{1}$$

with

$$H^{(i+1)}=h_{+i}I_{-}{}^{i}+h_{-i}I_{+}{}^{i}.$$

I_+ (I_-) is the angular-momentum operator in the intrinsic coordinate system which lowers (raises) K by one, and $h_{\pm i}$ operate on the intrinsic wave functions changing K by i units. The term $H^{(3)}$ in first order gives rise to an admixture of state with equal spin and parity, whose K quantum numbers differ by 2. For the $E2$ transitions from the γ-vibrational band to the ground-state band, only mutual admixtures of these two bands are considered since they give rise to collective transitions.

Using the normalized wave functions

$$|I,K\rangle=\left(\frac{2I+1}{16\pi^2(1+\delta_{KO})}\right)^{1/2}\times\{\chi_K D_{MK}{}^{I}+(-)^{(I+K)}\chi_{-K}D_{M}{}^{I}{}_{-K}), \tag{2}$$

and the relations[23]

$$I_{\pm}D_{MK}{}^{I}=[(I\pm K)(I\mp K+1)]^{1/2}D_{MK\mp 1}{}^{I},$$
$$\langle\chi_{-K}|h_{-K}|\chi_0\rangle=(-)^{K}\langle\chi_{+K}|h_{+K}|\chi_0\rangle, \tag{3}$$

one gets, in first order, for the perturbed wave functions

$$\psi_{\text{gr}}=|I,K=0\rangle-[2(I-1)I(I+1)(I+2)]^{1/2}\epsilon_2|I,K=2\rangle,$$
$$\psi_{\gamma}=|I,K=2\rangle+\frac{1+(-)^{I}}{2}\times[2(I-1)I(I+1)(I+2)]^{1/2}\epsilon_2|I,K=0\rangle, \tag{4}$$

with the reduced coupling amplitude

$$\epsilon_2=\frac{\langle\chi_2|h_{+2}|\chi_0\rangle}{E_{K=2}-E_{K=0}}.$$

The E_K are the ground-state energies of the two bands, whose moments of inertia are assumed to be equal. With these functions, the $E2$ transition matrix elements are readily calculated using formula (4.3.2) of Ref. 22. If one neglects terms quadratic in ϵ_2, and assumes equal intrinsic quadrupole moments for the ground-state and γ-vibrational bands, one gets for the reduced $E2$ transition probabilities

$$B(E2,I_i\to I_f)=B(E2,I_iK=2\to I_fK=0)\times[1+z_2f_2(I_iI_f)]^2, \tag{5}$$

with

$$z_2=-(24)^{1/2}\frac{\langle 0|\,\mathfrak{M}'(E2,0)\,|0\rangle}{\langle 0|\,\mathfrak{M}'(E2,2)\,|2\rangle}\epsilon_2=-\left(\frac{15}{2\pi}\right)^{1/2}\frac{Q_0}{\langle 0|\,\mathfrak{M}'(E2,2)\,|2\rangle}\epsilon_2,$$

$$f_2(I_iI_f)=(24)^{-1/2}\left\{[(I_f-1)I_f(I_f+1)(I_f+2)]^{1/2}\begin{pmatrix}I_i & 2 & I_f\\ 2 & 0 & -2\end{pmatrix}\Big/\begin{pmatrix}I_i & 2 & I_f\\ 2 & -2 & 0\end{pmatrix}\right.$$
$$\left.-[(I_i-1)I_i(I_i+1)(I_i+2)]^{1/2}\begin{pmatrix}I_i & 2 & I_f\\ 0 & 0 & 0\end{pmatrix}\Big/\begin{pmatrix}I_i & 2 & I_f\\ 2 & -2 & 0\end{pmatrix}\right\}.$$

Tabulations of the function f_2 are given in the literature.[2]

In Table III our experimental results are compared with the above theory. All transitions between the two bands are assumed to have $E2$ multipolarity. This is justified by the angular-correlation measurements of

[22] O. Nathan and S. G. Nilsson, Ref. 14, Chap. X.
[23] A. Bohr and B. R. Mottelson, in *Lectures on Nuclear Structure and Energy Spectra* (Institute for Theoretical Physics and NORDITA, Copenhagen, 1962).

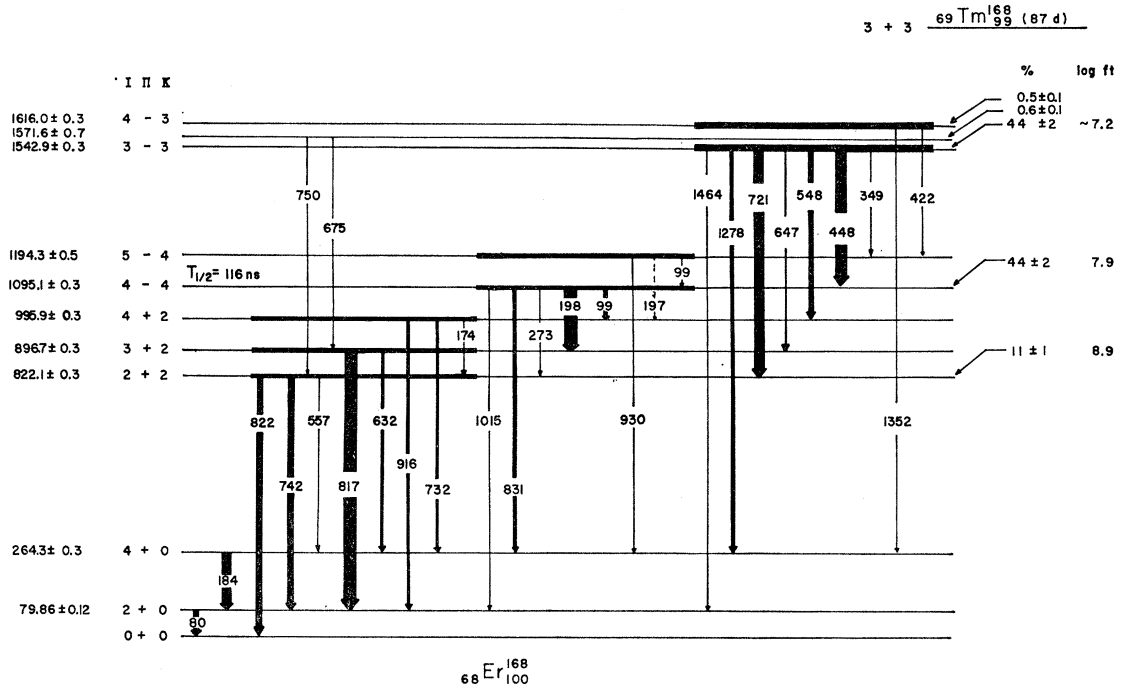

FIG. 5. The decay scheme of Tm^{168}.

TABLE III. Experimental $B(E2)$ ratio and z_2 values for Er^{166} and Er^{168}.

Transition	Experimental transition ratios Er^{166}	Experimental transition ratios Er^{168}	Theoretical ratios (strong-coupling limit)	Correction factor band mixing	$10^3 z_2$ Er^{166}	$10^3 z_2$ Er^{168}
$\frac{2\to 0}{2\to 2}$		0.563±0.020	0.700	$\left(\frac{1-z_2}{1+2z_2}\right)^2$		36.8±6.3
$\frac{2\to 2}{2\to 4}$		12 ±4	20	$\left(\frac{1+2z_2}{1+9z_2}\right)^2$		44_{-28}^{+52}
$\frac{3\to 2}{3\to 4}$	1.31 ±0.10	1.56 ±0.04	2.500	$\left(\frac{1-z_2}{1+6z_2}\right)^2$	52 ±7	36.7±2.1
$\frac{4\to 2}{4\to 4}$	0.171 ±0.011	0.185±0.005	0.3395	$\left(\frac{1-5z_2}{1+2z_2}\right)^2$	45.1±4.0	40.4±1.8
$\frac{5\to 4}{5\to 6}$	0.701 ±0.022		1.750	$\left(\frac{1-3z_2}{1+8z_2}\right)^2$	45.4±1.7	
$\frac{6\to 4}{6\to 6}$	0.0748±0.0042		0.2692	$\left(\frac{1-9z_2}{1+2z_2}\right)^2$	47.0±1.5	
$\frac{7\to 6}{7\to 8}$	0.425 ±0.024		1.500	$\left(\frac{1-5z_2}{1+10z_2}\right)^2$	45.4±2.0	
$\frac{8\to 6}{8\to 8}$	0.037 ±0.012		0.2395	$\left(\frac{1-13z_2}{1+2z_2}\right)^2$	43.5±5.0	

Refs. (7) and (8). As can be seen, the experimental results are in excellent agreement with the theory. To illustrate this even more clearly, the correction factors vs z_2 are shown in Fig. 7, together with the experimental values for the various branching ratios. From our data we get the average values

$$z_2(Er^{166}) = 0.0460 \pm 0.0010,$$

$$z_2(Er^{168}) = 0.0388 \pm 0.0015.$$

Our z_2 value for Er^{166} is in agreement with the values

found by Gallagher *et al.*[3] (0.048±0.008), and Yoshizawa *et al.*[5] (0.05±0.04). In contrast to our result, the previously reported value of z_2 for Er^{168} showed some discrepancies outside the limits of error for the different transition ratios.[4] Our z_2 value for that case agrees with the value reported by Yoshizawa *et al.*[5] (0.07±0.04). Branching ratios in Er^{168} have also recently been reported by Koch,[17] with an accuracy of 20%. They agree with our values within the experimental errors.

In view of the surprisingly good agreement of our experimental data with the band-mixing theory, it seems worthwhile to check the assumptions made in the theory.

In the above derivation, only the correction term $H^{(3)}$ of the Hamiltonian (1) was considered in first-order perturbation theory. Assuming the term $H^{(i+1)}$ to be of order i, one gets corrections to the wave funcfunction of order n from all terms with $i \leq n$, in $(n-i+1)$th order of perturbation theory. In general one can expect that the corrections to the transition rates are of the same order as those to the wave functions except for the admixtures, which lead to collective transitions. Thus, in our case, one would have to regard the corrections from $H^{(2)}$ in first- and second-order perturbation theory, and those from $H^{(3)}$ in first-order perturbation theory as first-order corrections. These terms have been considered by Mikhailov.[24] This author gets, in addition to the term $z_2 f_2(I_i I_f)$, a term $z_2' f_2'(I_i I_f)$ in $B(E2, I_i \rightarrow I_f)$. This latter term had also previously been derived by Lipas[25] for an admixture of the β-vibrational band to the γ-vibrational and ground-state bands. Of course, there are second-order correction terms which might lead to corrections in the transition rate of different I dependence. However, these corrections are expected to be of the order[24] $10^{-2}-10^{-3}$ in comparison with the first-order terms, and therefore negligible. From a fit to our data for Er^{166}, we get $z_2'/z_2 = -0.01 \pm 0.03$.

B. Comparison of Experimental z_2 Values with Different Nuclear Models

As mentioned above, several mechanisms for the mixing of bands with $\Delta K = 2$ have been considered, the most direct one being the inclusion of a rotation-vibration interaction term into the strong-coupling Hamiltonian. Two different approaches have been used for the inclusion of this interaction. In the hydrodynamical model, the Hamiltonian of a liquid drop, including harmonic oscillations around the axially symmetric equilibrium shape, is investigated in detail.[20] This model is thus basically a macroscopic model. In the second approach, the microscopic model, the interaction between the vibrations and rotations is derived from a Hamiltonian including pairing and quadrupole interaction.[21]

An alternative is the consideration of an axially asymmetric rotator. As it can easily be seen, this leads to a term H' which couples, in first order, states with $\Delta K = 2$ [26]:

$$H' = \tfrac{1}{4}\hbar^2\left(\frac{1}{J_1} - \frac{1}{J_2}\right)(I_+^2 + I_-^2), \tag{6}$$

where $J_{1,2}$ are the components of the moment of inertia along the 1 and 2 axis of the intrinsic coordinate system.

Finally, the mixing between the two bands can be caused by the Coriolis interaction in second order via intermediate states with $K = 1$.[19] In the following, we discuss first the hydrodynamical model together with the axially asymmetric rotor model, since both lead, for small nonaxiality, to the same result. The two microscopic models are discussed subsequently.

The Hydrodynamical Model of Rotation-Vibration Interaction and the Asymmetrical-Rotor Model

An extensive comparison of the two models has been given by Faessler and Greiner.[27] In Table IV we collect the relevant quantities used in our formulas (4) and (5). The quantities η and ξ are[27]

$$\eta = \sqrt{\frac{3}{2}\frac{(\hbar^2/2J_0)}{E_0}}, \quad \xi = \frac{1}{\sqrt{2}}\sin\gamma. \tag{7}$$

As is evident from Table IV, both models lead for small values of η and ξ to the same results. In order to compare the hydrodynamical model with the experi-

TABLE IV. Theoretical predictions of the asymmetrical rotor model and of the hydrodynamical rotation-vibration interaction model for the z_2 parameter.

Model	ϵ_2	$\langle 0\|\mathfrak{M}'\|0\rangle/\langle 0\|\mathfrak{M}'\|2\rangle$	z_2	E_{22}/E_{20}
Hydrodyn. rot.-vibr. model	$\frac{1}{\sqrt{3}}\frac{4\eta^3}{3-8\eta^2}$	$\frac{1}{\sqrt{2}\eta}$	$\frac{8\eta^2}{3-8\eta^2}$	$\frac{1}{4\eta^2}+\frac{3+8\eta^2+64\eta^4}{9-24\eta^2-64\eta^4}$
Asymmetric rotor model	$\frac{1}{\sqrt{6}}\frac{4\xi^3}{3-8\xi^2}$	$\frac{1}{\xi}$	$\frac{8\xi^2}{3-8\xi^2}$	$\frac{1}{4\xi^2}+\frac{3+32\xi^4}{9-24\xi^2-32\xi^4}$

[24] V. M. Mikhailov, Bull. Acad Sci. USSR, Phys. Ser. **28**, 225 (1964).
[25] P. O. Lipas, Nucl. Phys. **39**, 468 (1962).
[26] J. P. Davidson, Rev. Mod. Phys. **37**, 105 (1965).
[27] A. Faessler and W. Greiner, Z. Physik **177**, 190 (1964).

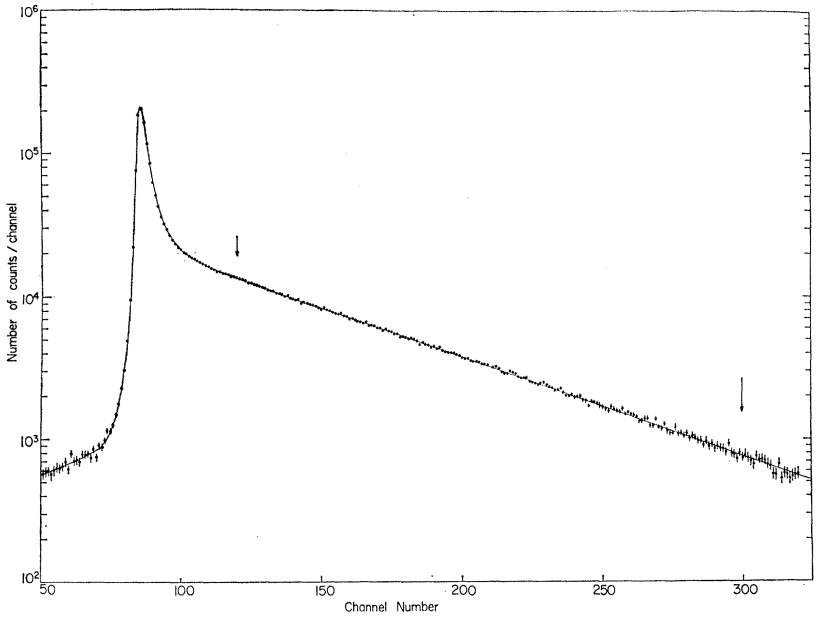

FIG. 6. Measurement of the half-life of the 1095-keV state in Er^{168}. The arrows represent the range of the least-squares fit.

mental values, we have calculated the parameter η from the energies as well as from the $B(E2)$ values. The results are collected in Table V. For the ratio of the reduced matrix elements we used the values of Ref. 5. The reduced coupling amplitudes ϵ_2, are derived from z_2 according to formula (5). As can be seen from Table V, the values for η derived from the coupling amplitudes and reduced matrix elements disagree with those from the energies, and also with each other. This might be surprising in view of the good agreement which Faessler and Greiner find for the $B(E2; 22 \rightarrow 20)/B(E2, 22 \rightarrow 00)$ ratio.[27] However, as is apparent from Fig. 7, the transition rates from the low-spin states are much less sensitive to the band mixing as those from the high-spin states. Also it is apparent from Table IV that z_2 is not so strongly dependent on η as ϵ_2. With $\eta=0.163$, in the case of Er^{166}, one gets $z_2=0.076$. The $B(E2)$ ratio from the 3+ state, calculated with this value deviates from the experimental value by 30%, whereas the ratios from the 7+ state differ by more than a factor of 2.

The formulas of Table IV are derived from perturbation theory. Recently the wave functions of the vibrational states have been calculated exactly within the hydrodynamical model.[20] In Table VI, we compare the

TABLE V. Comparison of the experimental z_2 values for Er^{166} and Er^{168} with the rotation-vibration model.

	Experimental value of η from		
Isotope	$\frac{\langle 0\|\mathfrak{M}'\|0\rangle}{\langle 0\|\mathfrak{M}'\|2\rangle}$	ϵ_2	$\frac{E_{22}}{E_{20}}$
Er^{166}	0.100±0.010	0.056±0.002	0.163
Er^{168}	0.088±0.008	0.051±0.002	0.159

TABLE VI. Comparison of theoretical and experimental admixtures of the ground-state band to the γ vibrational band.

	Amplitude of $K=0$ admixture to $K=2$ band	
I	Theory	Experiment
2	−0.020	−0.009
4	−0.071	−0.036
6	−0.135	−0.077
8	−0.204	−0.133

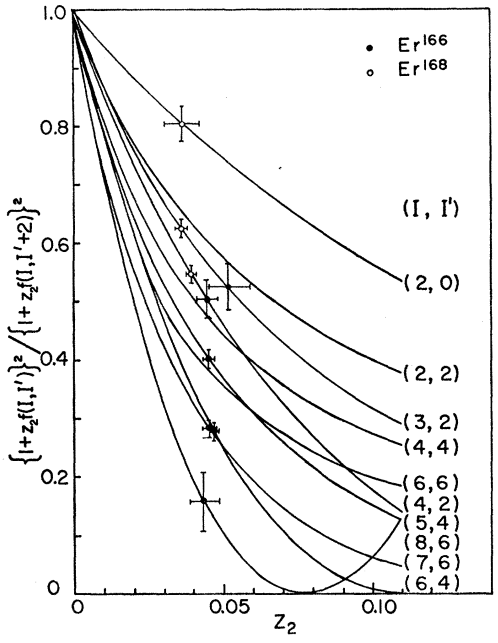

FIG. 7. Comparison of the experimental and theoretical correction factors for the $B(E2)$ ratios for the transitions from the γ-vibrational to the ground-state band.

wave functions of the γ-vibrational states in Er^{166} with the experimental ones, derived from formulas (4) and (5). There is no significant improvement in the agreement between the experimental and theoretical values in comparison with the perturbation theory.

As is evident from Table IV, the asymmetric rotor model gives, in first-order perturbation theory for small γ, the same results for ξ calculated from z_2 and E_{22}/E_{20} as does the hydrodynamical model for η. The ξ value calculated from the ratio of reduced transition rates is a factor of 1.4 larger than the corresponding η value; it is therefore in good agreement with the ξ value from the energy ratio. However, the value for ξ derived from ϵ_2 is only slightly different from its corresponding η value, and thus in disagreement with the other ξ values.

It must be mentioned here, however, that in the above-discussed treatment of the asymmetric rotor model, the β vibrations are not included properly. As it has been pointed out by Davidson and Davidson,[28] an exact treatment of the β vibrations is necessary to account for the observed level structure. This might lead to appreciable changes in the $E2$ branching ratios in comparison with the first-order perturbation treatment.[28] No theoretical calculations exist for the branching ratios from higher spin states, so that a meaningful comparison of our data with this theory was not possible.

Microscopic Model of Rotation-Vibration Interaction

Calculation of the z_2 parameter within the framework of the microscopic model of rotation-vibration interaction have been performed by Pavlichenkov.[21] This author derives for this interaction the expression

$$h_{\pm 2}=-\frac{\Gamma_2}{4J_0}A_{\mp 2}^{\dagger}, \tag{8}$$

where J_0 is the moment of inertia and $A_{\mp 2}^{\dagger}$ is the phonon production operator. The quantity Γ_2 is a function of the pairing energies of neutrons and protons, of the energy of the γ-vibrational band, and of the nuclear deformation. With experimental values for these parameters, as well as for the reduced $E2$ transition probability for transitions from the two 2+ states to the ground state, Pavlichenkov calculated the z_2's.

In Fig. 8 we compare the z_2 values of Pavlichenkov with some experimental data.[12,29] Some of the theoretical values were recalculated with the data of Ref. 5 for the reduced transition rates, experimental values for z_2, and Pavlichenkov's values for B_γ. Although the theoretical values have the right order of magnitude and mass dependence, there is a systematic discrepancy with the experimental values. This is

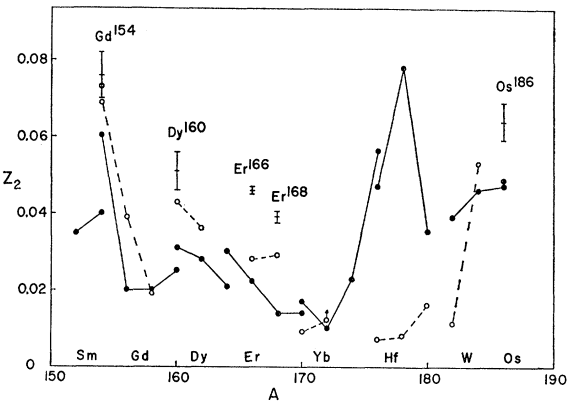

FIG. 8. Comparison of experimental and theoretical z_2 values in the region of deformed nuclei with $150<A<190$. ● Theoretical values of Bès *et al.*, Ref. 19. ○ Theoretical values of Pavlichenkov, Ref. 21. + Experimental values.

especially true in the middle of the deformed-nuclei region, where the band mixing is the smallest.

Strong-Coupling Model with Coriolis Interaction

In the rotor model of Bohr and Mottelson the quantities $h_{\pm 2}(q)$ vanish. As was pointed out by Nathan and Nilsson,[22] $H^{(3)}$ can be considered as representing some second-order effects of the Coriolis force. In fact, it has been shown by Bès *et al.*[19] that the second-order terms of this Coriolis force, which are connected with collective transitions, lead to the form (5) of the reduced transition rate. Bès *et al.* have also calculated theoretical values of z_2, using the Nilsson wave functions, and taking into account pairing and quadrupole interactions.

The theoretical values for z_2 of Bès *et al.* are included in Fig. 8. As can be seen from this figure, there is a good qualitative agreement between the two theoretical calculations for $A<172$. For $174<A<184$, the values of Bès *et al.* are up to a factor of 10 bigger than those of Pavlichenkov. This must be due to some close-lying $K=1$ states which strongly couple by Coriolis interaction to the γ-vibrational and ground-state bands. Unfortunately, no accurate experimental values are available in this region. The known experimental values seem to be in somewhat better agreement with the calculations of Pavlichenkov. The above discussion seems to indicate that neither the vibration-rotation interaction nor the Coriolis interaction can account completely for the quantitative admixtures of the $K=0$ and $K=2$ bands. There are two possibilities for the discrepancies; either the mixing of the two bands is in fact larger than predicted by the models, or there are admix-

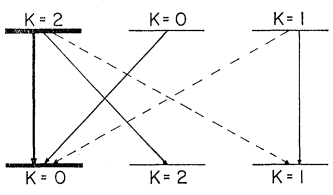

FIG. 9. Schematic diagram of the admixtures contributing to the $E2$ transition rates from the γ-vibrational to the ground-state band.

[28] J. P. Davidson and M. G. Davidson, Phys. Rev. **138**, B316 (1965).

[29] G. T. Emery, W. R. Kane, M. McKeown, M. L. Perlman, and G. Scharff-Goldhaber, Phys. Rev. **129**, 2597 (1963).

tures from other states which contribute, giving the same I dependence to the correction terms in the reduced $E2$ transition rates.

As mentioned above, the most important admixtures of other states are those with $\Delta K=1$, which are admixed in first order by the Coriolis force. Only the collective part of these admixtures has been taken into account by Bès *et al.* as is schematically shown in Fig. 9. It can be shown that the $\Delta K=1$ transitions (dashed lines in Fig. 9) lead to a correction term in $B(E2)$ with the same I dependence as the collective terms, and thus would give the desired renormalization of z_2. An estimate shows that these $\Delta K=1$ admixtures lead to z_2 terms the size of which are comparable to the size of the collective terms, if the reduced $E2$-matrix elements are approximately $\frac{1}{10}$ of the collective matrix elements. However, the $E2$ matrix elements for $\Delta K=1$ transitions are expected to be small and these terms thus in general negligible.[30] Thus, we have to conclude that the large values of z_2 seem to indicate a stronger mixing of the ground-state and γ-vibrational bands than calculated from the models, although the influence of $\Delta K=1$ admixtures seems at present not quite clear.

C. The Intraband to Interband $E2$-Transitions, and the Quadrupole Moments of the γ-Vibrational State

The ratio of the $E2$ transition rates within the γ-vibrational band to those between the γ-vibrational and the ground-state bands are of particular interest. They allow, together with the Coulomb excitation data, the derivation of the intrinsic quadrupole moments of the γ-vibrational band.[31] From the intensity ratios, one gets the quantity

$$R_1=\left|\frac{\langle 2|\,\mathfrak{M}'(E2,0)\,|2\rangle}{\langle 0|\,\mathfrak{M}'(E2,-2)\,|2\rangle}\right| =2\left[\frac{2I'+1}{2I-3}\right]^{1/2}\begin{pmatrix} I & 2 & I' \\ 2 & -2 & 0\end{pmatrix}\Big/\begin{pmatrix} I & 2 & I-2 \\ 2 & 0 & -2\end{pmatrix} \times[1+z_2 f(I,I')]\left[\frac{B(E2,I2\rightarrow I-2,2}{B(E2,I2\rightarrow I'0)}\right]^{1/2}. \quad (9)$$

TABLE VII. Experimental ratios of the reduced matrix elements for $E2$-transitions within the γ-vibrational band and between the γ-vibrational and the ground-state band.

Nucleus	I	I'	R_1
Er^{166}	5	4	10.1±0.5
	5	6	10.0±0.5
	6	4	9.3±0.8
	6	6	9.0±0.8
	7	6	9.7±0.5
	7	8	9.8±0.5
	Average		9.7±0.3
Er^{168}	4	2	9.2±0.7
	4	4	9.3±0.7
	Average		9.2±0.7

Our data for Er^{166} and Er^{168} are collected in Table VII. For the z_2 parameter, we used our experimental values. Because of the great energy difference between the interband and the intraband transitions we included an error of 0.5% per 100 keV for the intensity calibration. Using the values of Yoshizawa *et al.*[5] for the $E2$ matrix elements to the 2+ states we get for the ratios of the intrinsic quadrupole moments

$$\frac{Q_0(K=2)}{Q_0(K=0)}=1.38\pm0.15, \quad \text{for} \quad Er^{166};$$
$$=1.15\pm0.14, \quad \text{for} \quad Er^{168}.$$

These results seem to indicate a somewhat larger deformation of the γ-vibrational states than the ground states, as was already noted by Gallagher *et al.*[3] These authors pointed out that this result is in agreement with the larger moment of inertia of the γ band.

D. The Negative-Parity States in Er^{166}

The spins and parities of the upper states in Er^{166} as shown in the decay scheme (Fig. 2), are taken from Ref. 7. Table VIII summarizes the $B(E1)$ ratios for transitions from these states to the γ-vibrational and ground-state bands. The ratios for the K-forbidden transitions are derived from formula (4.4.12) of Ref. 22,

TABLE VIII. Comparison of experimental and theoretical $B(E1)$ ratios in Er^{166}.

Transition ratio $IK\rightarrow I'K'$	Experimental values		Theoretical values $K=1$	$K=2$	$K=3$
	Initial state				
	1662 keV	1692 keV			
$(5K\rightarrow 60)/(5K\rightarrow 40)$	0.709±0.005	0.714±0.005	0.83	0.83	0.83
$(5K\rightarrow 42)/(5K\rightarrow 40)$		4.8 ±0.5			
	1784 keV	1824 keV			
$(6K\rightarrow 72)/(6K\rightarrow 52)$	1.21 ±0.04	1.30 ±0.04	3.09	1.20	0.24
$(6K\rightarrow 62)/(6K\rightarrow 52)$	0.192 ±0.006	0.195±0.039	3.72	0.23	0.93
$(6K\rightarrow 62)/(6K\rightarrow 60)$	69.0 ±4.1	64 ±15			

[30] A. Bohr (private communication.).
[31] B. S. Dzhelepov, Bull. Acad. Sci. USSR, Phys. Ser. **28**, 3 (1964).

TABLE IX. Partial half-lives of the transitions depopulating the 1095-keV state in Er^{168}.

Transition (keV)	$\alpha_K^{expt}/\alpha_K^{th}$ [a] E1	M2	Adopted multipolarity	Partial half-life (sec)	Hindrance factor[b]	Forbiddenness $\Delta K-\lambda$
99			E1	$(2.9 \pm 0.6)\times10^{-6}$	1.3×10^{7}	1
198	1.04	0.025	E1	$(1.41\pm0.07)\times10^{-7}$	5.0×10^{6}	1
273	30	0.95	M2	$(5.9 \pm 0.9)\times10^{-5}$	9	0
831	1.3	0.1	E1	$(1.21\pm0.06)\times10^{-6}$	3.1×10^{9}	3
1015	~10	~1	M2	$(9.6 \pm 1.5)\times10^{-5}$	1.0×10^{4}	2

[a] Experimental conversion coefficients α_K^{expt} calculated from our γ intensities and from the electron intensities of Ref. 8. Theoretical conversion coefficients α_K^{th} from Rose.
[b] Relative to Weisskopf estimate.

which is based on admixtures of states with $\Delta K=\pm1$ due to the Coriolis interaction.

Our values confirm the conclusions of Reich and Cline, especially the surprising agreement of the $B(E1)$ ratios from the 6− states with the values predicted for $K=2$. Reich and Cline also give an extensive discussion of the possible configuration assignments for the negative-parity states.

E. The Negative-Parity States in Er^{168}

While the parity of the 1095-keV state has been shown to be negative, the spin of this state has not yet been determined unambiguously. Angular-correlation data are in agreement with spin 2, 3, or 4. Jacob *et al.*[16] postulate spin 3 for this state, based on the assignment of $E1$ multipolarity to the 273-keV transition from the measured K/L conversion electron ratio. However, there are strong arguments for a spin 4 of the 1095-keV state:

(1) The 1095-keV state decays much stronger to the 3+ than to the 2+ γ-vibrational state. Assuming $E1$ multipolarities for the 198-keV and the 273-keV radiation, we get from our intensities

$$\frac{B(E1, I\rightarrow 3)}{B(E1, I\rightarrow 2)}=(1.1\pm0.2)\times10^{3}.$$

This value is at least a factor of 100 bigger than the expected ratio for $I=3$ (see Table X).

(2) The conversion coefficients of the 273-keV and 1016-keV transitions are only in agreement with $M2$ multipolarity for these transitions (see Table IX).

(3) For $I=3$ the $E1$ transitions from the 1095-keV state either to the ground state or to the γ-vibrational state would be allowed. As can be seen from Table IX all $E1$ transitions from the 1095-keV state are highly hindered.

Therefore, we conclude that the 1095-keV state has $I=4$ and mainly $K=4$. The same conclusion was obtained by Koch[17] from the energy spacing of the rotational band built on the 1095-keV state.

Previous investigations have shown that the 1543-keV state has spin and parity 3−.[8] Reidy *et al.*[8] conclude from their intensity data, that the most probable K value for the 1563-keV state is $K=3$, although they find only a fair agreement of their intensity data with the theoretical predictions for $K=3$.

In Table X our reduced transition probability ratios for transitions from the 1543-keV state are compared with theoretical predictions for different K values of the 1543-keV state. All transitions are assumed to be pure $E1$. This is justified by the experimental observation that the $3-\rightarrow(2+, K=2)$ and the $3-\rightarrow(4+, K=0)$ transitions have $E1$ multipolarity, with $<5\%$ and $<0.1\%$ $M2$, respectively.[8] Although there is no quantitative agreement, the values for the transition ratios to the γ-vibrational band seem to make $K=3$ the most probable assignment. On the other hand, an assignment of $K=2$ would account better for the low hindrance of the transitions to the ground-state band relative to those to the γ-vibrational band.

From Table X it is apparent that, for both mainly $K=2$ and 3 of the 1543-keV state, small K admixtures might change the transition ratios to the γ band appreciably. In the following we will show that the experimental transition ratios can be understood equally well assuming that the 1543-keV state has mainly $K=2$ or $K=3$.

The effect of K admixtures on the transition ratios of K-allowed transitions has been investigated by Mikhailov.[24] This author considers for electric dipole transitions admixtures with $\Delta K=\pm1$. The branching ratios derived from his formulas, assuming the 3− state to have mainly $K=2$ or $K=3$, are given in columns 2 and 3 of Table XI. From this table it is clear that the admixtures of $\Delta K=\pm1$ cannot account for the observed transition ratios, if the 3− state has mainly $K=3$, since both ratios are changed in the same direction. The four solutions for mainly $K=2$ of the 1543-keV state lead

TABLE X. Comparison of the experimental and theoretical $B(E1)$ ratios of transitions depopulating the 1543-keV state in Er^{168}.

Transition ratio $IK\rightarrow I'K'$	Experimental $B(E1)$ ratio	Theoretical ratios $K=0$	$K=1$	$K=2$	$K=3$
$(3K\rightarrow32)/(3K\rightarrow22)$	0.170±0.008		8.75	1.40	0.35
$(3K\rightarrow42)/(3K\rightarrow22)$	0.135±0.008		11.25	1.80	0.05
$(3K\rightarrow40)/(3K\rightarrow20)$	10.1 ±0.3	1.33	0.75	0.75	0.75
$(3K\rightarrow22)/(3K\rightarrow20)$	392±12				
$(3K\rightarrow42)/(3K\rightarrow40)$	22.6 ±1.1				

TABLE XI. Theoretical $B(E1)$ ratios for different K admixtures of the 3-state in Er^{168}.

Transition ratio	Admixtures with $\Delta K=\pm1$ 3− state mainly $K=2$	Admixtures with $\Delta K=\pm1$ 3− state mainly $K=3$	Admixtures with $\Delta K=\pm1$, ±2 to $K=3$
$(3\rightarrow3)/(3\rightarrow2)$	$\frac{7}{5}\left(\frac{1-4a_1+4a_2}{1-a_1-5a_2}\right)^2$	$\frac{7}{20}\left(\frac{1+2a_1}{1-a_1}\right)^2$	$\frac{7}{20}\left(\frac{1-10y_1+2\sqrt{2}y_2}{1+2y_1-\sqrt{2}y_2}\right)^2$
$(3\rightarrow4)/(3\rightarrow2)$	$\frac{9}{5}\left(\frac{1+6a_1+2a_2}{1-a_1-5a_2}\right)^2$	$\frac{1}{20}\left(\frac{1+6a_1}{1-a_1}\right)^2$	$\frac{1}{20}\left(\frac{1+30y_1+6\sqrt{2}y_2}{1+2y_1-\sqrt{2}y_2}\right)^2$

to appreciable admixtures of $K=1$ and $K=3$ if all intrinsic matrix elements are taken to be equal. If one assumes that the intrinsic matrix elements from the $K=2$ component to the γ-vibrational band is 10 times smaller than the two others, all four solutions lead to $\gtrsim95\%$ of the $K=2$ component.

In order to investigate whether admixtures with $|\Delta K|>1$ can account for the observed $B(E1)$ ratios if the 3− state has predominantly $K=3$, one has to consider, in addition to the $\Delta K=\pm1$ components, admixtures of $K=1$ to the 3− state and $K=4$ to the γ-vibrational state. The latter contributes only to the 4+ vibrational state, and since no positive parity $K=4$ states are expected at reasonably low energies, we have neglected this admixture. Using the perturbation Hamiltonian (1) one gets for the amplitudes of the admixtures

$$\epsilon_{I,K\pm\Delta K}=\left[(1+d_{K,0}+d_{K,1})\frac{(I\mp K)!\,(I\pm K+\Delta K)!}{(I\pm K)!\,(I\mp K+\Delta K)!}\right]^{1/2}\epsilon_{K\pm\Delta K},$$

with

$$\epsilon_{K\pm\Delta K}=\frac{\langle\chi_{K\pm\Delta K}|h_{\pm\Delta K}|\chi_K\rangle}{E_{K\pm\Delta K}-E_K}. \tag{10}$$

The $B(E1)$ ratios, calculated with the above-mentioned admixtures, are given in column 4 of Table XI. The parameters have the form

$$y_1=\frac{x_1}{1+\sqrt{2}x_3},\quad y_2=\frac{x_2+x_3}{1+\sqrt{2}x_3},$$

$$x_{1,2}=\epsilon_{1,2}\frac{\langle K=2|\mathfrak{M}'|K'=1,2\rangle}{\langle K=2|\mathfrak{M}'|K=3\rangle},$$

$$x_3=\epsilon_3\frac{\langle K'=3|\mathfrak{M}'|K=3\rangle}{\langle K=2|\mathfrak{M}'|K=3\rangle}.$$

From the four solutions for y_1 and y_2, the most likely predicts an almost pure admixture of $K=1$: $y_1=0.025$, $y_2=-0.004$. Assuming equal intrinsic matrix elements, this would correspond to $\sim4\%$ admixture.

The transition ratio from the 3− state to the ground-state band levels is even more difficult to understand. Neither the admixture of $\Delta K=\pm1$ components to the levels involved, nor a pure $K=1$ admixture to the 3− state can account for the measured ratio (see Table X). We have calculated the effect of an additional admixture of the $K=2$ band to the ground-state band. However, this leads exactly to the same ratio as the pure $K=1$ admixture to the 3− state, as might be expected. Thus, more complicated admixtures seem to be necessary. A more detailed investigation is not worthwhile because of the limited amount of experimental data.

The two negative-parity states are interpreted by Pyatov and Soloviev,[32] as the components of a doublet with the two-quasiparticle configurations pp $523\uparrow\pm411\downarrow$. This interpretation is in agreement with the assignments of the 1095-keV state and suggests an assignment of $K=3$ to the 1543-keV state. No obvious possibility exists for a $K^\pi=1-$ admixture to the pp $523\uparrow-411\downarrow$ state, since the low-lying two-quasiparticle 1− states are neutron states.[33] The octupole $K^\pi=1-$ state is also expected to be mostly composed of these neutron states.[34] However, the lowest 1− neutron state is expected to be very close to the 1543-keV state and might then be admixed.

As an alternative the two states could be the nn $633\uparrow\pm521\downarrow$ doublet. Then an admixture of the close lying $K=1$ nn $633\uparrow-512\downarrow$ state could be easily understandable, for example, by Coriolis interaction via the nn $642\uparrow-521\downarrow$ state. An argument against this interpretation is the high experimental logft value of 7.2, for the electron-capture transition to the 1543-keV state, in comparison with the calculated value of 6.0 for the nn $633\uparrow-521\downarrow$ state.[31]

There is no apparent possibility for a $K=2$ interpretation since the β decay to all low-lying $K^\pi=2-$, two-quasiparticle states, as well as to the $K=2$, octupole state, is forbidden. This forbiddenness of course, could be weakened by admixtures of $K=1$ and $K=3$ states.

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Decay of 10.4-min Gd^{162}†

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A short-lived Gd^{162} isotope has been produced by double neutron capture during the Savannah River high-flux reactor experiment. The new isotope decays with a half-life of 10.4±0.2 min to the 7.4-min isomer of Tb^{162}. Gamma rays of 410 and 428 keV were observed in the decay of 10.4-min Gd^{162}.

INTRODUCTION

ISOTOPICALLY enriched samples of several nuclides were irradiated for varying periods during the high flux experiment[1] at Savannah River Laboratory, to produce new double neutron capture products or to better characterize previously known nuclides. A long-lived Gd^{162} activity was previously reported,[2] but subsequent attempts to duplicate the results were unsuccessful.[3] In the course of the present experiments, a short-lived Gd^{162} activity was observed.

EXPERIMENTAL

A series of special targets was prepared by direct deposition on high purity graphite cloth of about 50 μg of Gd^{160} in the Argonne isotope separator. The starting material for the sample preparation was gadolinium enriched to 94% Gd^{160} at Oak Ridge National Laboratory. For irradiation, the sample was enclosed in a loop of DuPont radiation resistant polymer PS-1, sealed in a screw-cap graphite inner capsule, and aluminum outer capsule.[4] The gadolinium samples were irradiated for 20 or 30 min in the mechanical rabbit facility at thermal neutron fluxes of $\sim 2\times 10^{15}$ n cm^{-2} sec^{-1}. The aluminum capsule was opened under 4 ft of water, and the graphite inner container removed to a lead-shielded cave for remote opening.

To obtain a gadolinium fraction free of trace impurities (Cu^{64}, Na^{24}, Mn^{56}, Cl^{38}) and the Tb^{161} daughter activity, a reversed phase chromatographic column separation was made using 1N HNO_3 as eluant, based on the procedure of Sochacka and Siekierski.[5] The unusually high separation factor of 5.5 for Gd and Tb provides an excellent separation on a 100 mm×2 mm column. The progress of the separation was followed by use of an eluant flow detector with rate meter and chart recorder, to permit selection of fractions for counting purposes. With this procedure, the purified gadolinium fraction is eluted from the column in less than 20 min.

RESULTS

Gamma spectroscopy was carried out with a 75 mm×75 mm NaI(Tl) detector and a 400-channel RIDL analyzer with punched tape output. The beta decay was followed with a Pilot B scintillator assembly and conventional electronics.

Representative spectra from the decay of the separated Gd fraction are shown in Fig. 1. The gamma spectra show an unresolved complex peak at ~400 keV with a half-life of 10 min, and an initial growth followed by equilibrium decay of 7-min Tb^{162}. Figure 2 shows a comparison of the separated gadolinium and terbium fractions. Concurrent beta-decay curves permit an estimate of the upper limit of branching to 2-h Tb^{162} of <2%. Least-squares analysis of the beta-decay curves gave a half-life for Gd^{162} of 10.4±0.2 min.

The composite Gd^{162} gamma peak was matched by synthesis to two lines of equal intensity at 410±5 keV and 428±5 keV, respectively. Study with high-resolution detectors will be required to analyze the complex peak. The calculated gamma-ray abundance

† Based on work performed under the auspices of the U. S. Atomic Energy Commission.
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