from F. Ajzenberg-Selove *et al.*,[2] J. Allen *et al.*,[11] and M. Thomas *et al.*[12] We note a close agreement between the energy and spacing of the levels of F^{19} and Ne^{19}.

[11] J. Allen, A. Howard, D. Bromley, and J. Olness, Phys. Rev. **140**, B1245 (1965).

[12] M. Thomas, J. Lopes, R. Ollerhead, A. Poletti, and E. Warburton, Nucl. Phys. **78**, 298 (1965).

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Coriolis-Coupling Model Prediction of Moments and Transition Rates for Deformed Odd Nuclei in the $1f_{7/2}$ Shell*

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The magnetic dipole and electric quadrupole moments and reduced transition probabilities have been computed for the Coriolis-coupling model using the wave functions obtained in an earlier paper. These wave functions were obtained from a linear combination of the ten rotational bands based on the ten available single-particle and core excited states in the $1f$-$2p$ shell, and the corresponding eigenvalues were consistent with the observed level spectra of the odd nuclei in the $1f_{7/2}$ shell. The moments and transition rates predicted here are also in good agreement with experiment. Thus, the observed magnetic moments can no longer be considered as evidence supporting the validity of solely the spherical shell model for these nuclei. Contributions to the magnetic moment from single-particle terms and from cross terms between the bands are found to be important and have been included. The observed inhibition of the $M1$ transitions (which are allowed in the collective model) is a consequence of the strong band mixing which results in strong cancellation between the contributions from the direct and the cross terms. The good agreement between the predicted and observed values of the quadrupole moments and $B(E2)$ values has been achieved using the free proton charge of unity and neutron charge of zero. The contribution to the quadrupole moment from the single-particle term is found to be important, and on occasion constitutes as much as 50% of the total quadrupole moment. Furthermore, the collective spectroscopic quadrupole moment, with proper inclusion of the contribution from different bands, in some cases has a different sign from the intrinsic quadrupole moment. Proper inclusion of the contributions to the quadrupole moment from different bands and of the single-particle term leads to the important result that odd nuclei may yet have a very small quadrupole moment despite having large deformations.

I. INTRODUCTION

RECENT investigations[1,2] of the negative-parity level spectra of the odd-even nuclei in the mass region $43 \leq A \leq 53$ have shown that the strong-coupling symmetric-rotator model, with Coriolis coupling between the ten possible single-particle or single-hole excited bands in the $1f$-$2p$ shell, is applicable to nuclei in the $1f_{7/2}$ shell. In particular, this model can reproduce the observed ground-state spins for all nuclei including the anomalous cases of Ti^{47}, Cr^{49}, and Mn^{51} which have a $\frac{5}{2}^-$ ground state. Observed level densities and measured spin values for negative-parity states are in good agreement with model predictions[2] in contrast to the results of shell-model calculations based on the $(1f_{7/2})^n$ configuration with residual interaction.[3] This implies that the observed ground-state spins of these nuclei can no longer be adduced as evidence supporting the validity of solely the spherical shell model in the $1f_{7/2}$ region.

In a previous application of the Coriolis-coupling model to V^{51} (Ref. 1), the wave functions obtained from the energy-level fit have been used to compute static moments $B(M1)$ and $B(E2)$ reduced transition rates. The computed values are in good agreement with experiment. This result is achieved without the use of an effective charge as is required in shell-model calculations based on the $(1f_{7/2})^n$ configuration.[4,5] In this paper the magnetic dipole and electric quadrupole moments and the transition rates in $1f_{7/2}$ shell nuclei have been computed using the wave functions obtained in Ref. 2. Because of the large admixtures of a number of rotational bands based on a number of intrinsic states into the final ground- or excited-state wave function, it is necessary to use, as in the case of V^{51}, the complete expressions for the moments and transition

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[1] W. Scholz and F. B. Malik, Phys. Rev. **147**, 836 (1966).

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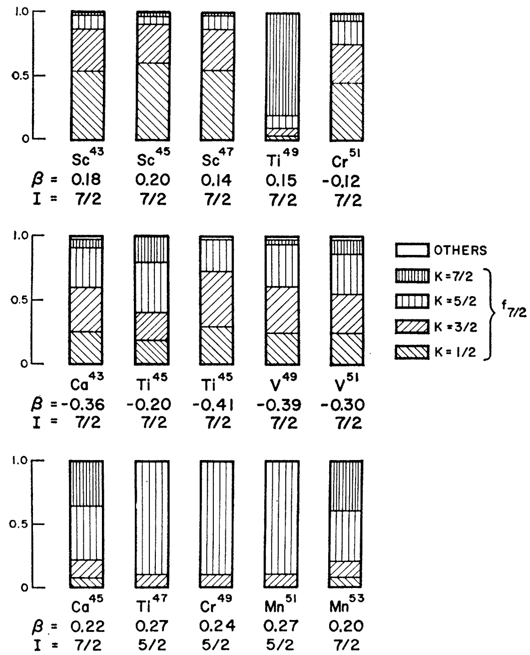

FIG. 1. The squared amplitudes $|C_{K,\nu}|^2$ of different bands for the ground states of odd nuclei in the $1f_{7/2}$ shell. The ground-state wave function is normalized to unity. The ground-state spin and the values of the deformation β used for each nucleus are shown below each column. The bands marked "others" include the sum of the squared amplitudes of six bands based on the Nilsson levels originating from the $2p_{3/2}$, the $1f_{5/2}$, and the $2p_{1/2}$ subshells.

rates with the full inclusion of the cross terms between amplitudes of different collective and single-particle contributions. These expressions are given in the next section together with a short outline of the method used to calculate the wave functions in Ref. 2. The two final sections contain a discussion of the results and the conclusion.

II. THEORY

The model wave function obtained in Ref. 2 for an eigenstate of a nucleus is given by

$$\Psi(I,M)=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\sum_{K=\Omega}\sum_{\nu}C_{K,\nu}[D_{MK}{}^{I}(\theta)\chi_{\Omega,\nu}+(-1)^{I-1/2}D_{M,-K}{}^{I}(\theta)\chi_{-\Omega,\nu}]\phi_c\,,\quad(1)$$

where I, M, and K refer, respectively, to the total spin and its projection on the space- and on the body-fixed z axis of a particular state. Ω is the projection of the intrinsic angular momentum of the last particle on the body-fixed z axis and is equal to K for a symmetric rotator. $D_{MK}{}^{I}(\theta)$, $\chi_{\Omega,\nu}$, and ϕ_c are, respectively, the rotational wave function, the intrinsic wave function of the last odd nucleon in the body-fixed coordinates and the core wave function, which depends implicitly on the total spin and on the particle state $\chi_{\Omega,\nu}$ reflecting the effect of the residual interaction. The $C_{K,\nu}$ are the amplitudes of the ten possible single-particle and single-hole excited states in the $N=3$ major oscillator shell.[2] The extra index ν distinguishes among different Nilsson levels with the same projection quantum number Ω.

These wave functions and the corresponding eigenvalues were computed as described in detail in Ref. 2. Nilsson[6] energy levels and wave functions for a symmetric deformed oscillator potential in the $1f-2p$ shell were computed for the usual value of the oscillator energy $\hbar\omega_0=41/A^{1/3}$ MeV, and a spin-orbit strength $C=-0.26\hbar\omega_0$; this latter gives the observed spin-orbit splitting in Ca^{41}. Two values of the well-flattening parameter $D=-0.06\hbar\omega_0$, and $-0.035\hbar\omega_0$, have been used in the calculation for different nuclei, the latter being the smallest absolute value which preserves the shell-model level ordering at zero deformation. Band-head energies were calculated using the complete expression[2,6] to sum over the occupied Nilsson levels. The rotational constant $A=\hbar^2/2\mathcal{I}$ was taken from the mean of the observed energies of the first 2^+ state in the adjacent even-even nuclei, assuming a rotational character for these states. For each nucleus only one rotational constant was used for all bands as well as for the strength of the Coriolis coupling in that nucleus. The final spectrum and the wave functions were obtained by diagonalizing the Coriolis coupling term with the rotational wave functions based on the ten available single-particle and single-hole levels in the $1f-2p$ shell.

The moments and the transition rates are computed here by transforming the dipole magnetic and the quadrupole electric tensors from the space-fixed to the body-fixed coordinates and using the wave functions (1) relevant to a particular state.

The magnetic moment is given by

$$\mu(I)=g_RI+\frac{1}{I+1}\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{K,\nu_f}KG_0+\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{-K+1,\nu_f}(-1)^{I-1/2}(I1K-1|I\,K-1)\times G_-/\sqrt{2}+\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{-K-1,\nu_f}(-1)^{I+1/2}\times(I1K1|I\,K+1)G_+/\sqrt{2}\quad(2)$$

where

$$G_0=(g_s-g_l)\langle s_0\rangle+(g_l-g_R)\langle j_0\rangle\,,\qquad G_\pm=(g_s-g_l)\langle s_\pm\rangle+(g_l-g_R)\langle j_\pm\rangle\quad(3)$$

where g_l, g_s, and g_R are the nucleonic orbital and intrinsic and the core gyromagnetic ratios, respectively. $\langle j_0\rangle$, $\langle j_+\rangle$, and $\langle j_-\rangle$ and $\langle s_0\rangle$, $\langle s_+\rangle$, and $\langle s_-\rangle$ are the expectation values of the components of the total intrinsic momentum j and the spin s, respectively. The explicit expressions for these quantities are given in Ref. 1.

[6] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

The quadrupole moment Ω of a state of spin I is given by

$$Q=Q_{\rm coll}+(I2I0|II)(q(0)+q(2)+q(-2)),\quad (4)$$

where the collective contribution to the quadrupole moment, $Q_{\rm coll}$ is given by

$$Q_{\rm coll}=Q_0\sum_{K,\nu}|C_{K,\nu}|^2\frac{3K^2-I(I+1)}{(I+1)(2I+3)}.\quad (5)$$

Q_0 is the intrinsic moment of the core and to the second order in the deformation β is given by

$$Q_0=(3/(5\pi)^{1/2})e(Z-1)R^2\beta(1+0.16\beta),\quad (6)$$

where R is the nuclear radius for which we have chosen the usual value of $1.2A^{1/3}$ F.

The terms $q(0)$, $q(2)$, and $q(-2)$ in (4) represent the contributions of the single particle to the quadrupole moment and are explicitly noted in Ref. 1.

The reduced magnetic dipole transition rate is given by

$$B(M1;i\rightarrow f)=\frac{3}{4\pi}\{g_R(I_i(I_i+1))^{1/2}\delta_{I_iI_f}\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{K,\nu_f}+\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{K,\nu_f}(I_i1K0|I_fK)G_0$$
$$+\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{-K+1,\nu_f}(-1)^{I_f-1/2}(I_i1K-1|I_f\,K-1)G_-/\sqrt{2}$$
$$+\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{-K-1,\nu_f}(-1)^{I_f+1/2}(I_i1K1|I_f\,K+1)G_+/\sqrt{2}\}^2.\quad (7)$$

The reduced electric quadrupole transition rate is given by

$$B(E2;i\rightarrow f)=(5/16\pi)[Q_c+q(0)+q(1)+q(-1)+q(2)+q(-2)]^2,\quad (8)$$

where Q_c represents the collective contribution and is given by

$$Q_c=Q_0\sum_{K,\nu_i,\nu_f}C_{K,\nu_i}C_{K,\nu_f}(I_i2K0|I_fK).\quad (9)$$

$q(0)$, $q(1)$, $q(-1)$, $q(2)$, and $q(-2)$ represent the contribution of the single-particle transition amplitude to the quadrupole matrix element and are given in Ref. 1.

The above formulas are derived assuming that the core overlap integral (ϕ_c,ϕ_c') is normalized to unity. In cases where this integral differs from one, i.e., for nuclei with odd nucleon count 5 above the $2s$-$1d$ shell, the cross terms in the above expressions are to be multiplied by its actual value as given in Ref. 2.

III. RESULTS AND DISCUSSION

In Fig. 1 the squared amplitudes of the wave functions calculated in Ref. 2 are reproduced. As mentioned in Ref. 2 the natural classification scheme in the Coriolis-coupling model involves grouping of the odd nuclei in the $1f_{7/2}$ shell in terms of the number of the odd nucleon count above the $2s$-$1d$ shell. The observed similarity in the level spectra of the members of each such group lends further support to this classification. The same similarity is exhibited in the composition of the wave functions for members of a particular group. Thus we find approximately the same ground-state wave function for the scandium isotopes (Group I) and again for Ca^{43}, Ti^{45}, V^{47}, V^{49}, V^{51} (Group II). In Group III, all nuclei with a ground-state spin of $\frac{7}{2}$, i.e., Ca^{45} and Mn^{53}, on the one hand and all nuclei with a ground-state spin of $\frac{5}{2}$ on the other hand, i.e., Ti^{47}, Cr^{49}, and Mn^{51}, have essentially the same admixture of different rotational bands in their ground states. In Group IV (Ti^{49} and Cr^{51}), although we have given the wave function of Cr^{51} for a negative deformation, the present experimental data are also consistent with a positive deformation; in that case the Cr^{51} wave function is similar to the one of Ti^{49}. All amplitudes have the same phase in the ground-state wave function. The two wave functions for Ti^{45} correspond to core overlap integrals of one ($\beta=-0.41$) and of 0.75 ($\beta=-0.2$). Although the core overlap of 0.75 produces no major change in the ground-state wave function, it is sufficient to place the first $\frac{3}{2}^-$ state in the spectrum about 300 keV higher than that obtained with a core overlap of unity[2] and it indicates an additional $\frac{7}{2}^-$ state around 1 MeV of excitation energy.

Using these wave functions the electromagnetic moments and the transition rates are computed with the help of the expressions given above. The collective gyromagnetic ratio g_R is taken to be Z/A, as given by the hydrodynamic model.[7] The intrinsic gyromagnetic ratios for bound nucleons in a nucleus may differ somewhat from the free ones. Migdal[8] has estimated this effective quenching of the gyromagnetic ratio reflecting the spin-dependent part of the nucleon-nucleon interaction to be 10 to 40%. Consequently we have chosen g_s to be 75% of the free gyromagnetic ratio, and have used the free-nucleon orbital gyromagnetic ratio for all odd nuclei. It is interesting to note that this choice of the nucleonic gyromagnetic ratios yields effective proton and neutron gyromagnetic ratios of 1.46 and -0.41, respectively, if used in the shell-model treatment with $(1f_{7/2})^n$ configuration only. This is in very close agree-

[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[8] A. B. Migdal, Nucl. Phys. **75**, 441 (1966).

TABLE I. The computed and observed magnetic moments of odd nuclei in the $1f_{7/2}$ shell. The contribution of the individual terms as given by Eq. (2) are listed separately. The experimental data are taken from Ref. 10 unless mentioned otherwise.

	Nucleus	Ground-state spin	Deformation β	Contribution of individual terms in nm μ_R	μ_0	μ_-	μ_+	Magnetic moment in nm Theory	Measured
Odd-proton	Sc^{43}	$\frac{7}{2}$	+0.18	1.709	0.422	1.408	1.419	4.958	4.61±0.04[a]
	Sc^{45}	$\frac{7}{2}$	+0.20	1.633	0.332	1.441	1.472	4.879	4.756
	Sc^{47}	$\frac{7}{2}$	+0.14	1.564	0.475	1.464	1.471	4.974	5.33±0.02[a]
	V^{47}	$\frac{7}{2}$	−0.39	1.713	0.643	1.195	1.157	4.707	
	V^{49}	$\frac{7}{2}$	−0.39	1.713	0.643	1.195	1.157	4.707	4.46
	V^{51}	$\frac{7}{2}$	−0.31	1.578	0.835	1.271	1.255	4.940	5.148
	Mn^{51}	$\frac{5}{2}$	+0.27	1.226	1.723	0.046	0.456	3.451	
	Mn^{53}	$\frac{7}{2}$	+0.20	1.651	1.712	0.684	0.581	4.623	5.050
Odd-neutron	Ca^{43}	$\frac{7}{2}$	−0.36	1.628	−0.633	−1.115	−1.086	−1.206	−1.317
	Ti^{45}	$\frac{7}{2}$	−0.20	1.711	−1.036	−0.768	−0.814	−0.907	±(0.095±0.002)[b]
	Ca^{45}	$\frac{7}{2}$	+0.22	1.556	−1.440	−0.575	−0.520	−0.980	
	Ti^{47}	$\frac{5}{2}$	+0.27	1.170	−1.565	−0.042	−0.417	−0.854	−0.788
	Cr^{49}	$\frac{5}{2}$	+0.24	1.225	−1.594	−0.042	−0.424	−0.836	
	Ti^{49}	$\frac{7}{2}$	+0.15	1.571	−1.964	−0.591	−0.271	−1.254	−1.104
	Cr^{51}	$\frac{7}{2}$	−0.12	1.647	−0.483	−1.257	−1.246	−1.338	

[a] R. G. Cornwell, W. Happer, Jr., and J. D. McCullen, Phys. Rev. **141**, 1106 (1966).
[b] R. G. Cornwell and J. D. McCullen, Phys. Rev. **148**, 1157 (1966).

ment with the corresponding values of 1.499 and −0.39 obtained from a least-squares fit of the experimental moments.[9]

Table I lists the computed magnetic moments of the ground states together with measured ones[10] for odd nuclei in this mass region. The contribution of the four individual terms in expression (2) are listed separately as μ_R, μ_0, μ_-, and μ_+, respectively. Considering that the uncertainty in the magnetic-moment operator due to the exchange current is of the order of 0.3 nm (Refs. 11, 12); theoretical predictions are in good accord with the experimental values. The only notable disagreement between theory and experiment occurs for the case of Ti^{45} which has a very small magnetic moment. The moments in general, however, are very sensitive to the composition of the wave function for nuclei in Group III because of the large cancellation among the contributions from different terms. It is worth noting that it is also difficult to reproduce the observed magnetic moment in the shell model with the residual interaction and in the method of generator coordinates in the limit of zero deformation.[13] Because of the small magnitude of the observed magnetic moment in Ti^{45}, various refinements, e.g., the contribution of the exchange currents, which have not been considered so far in any theoretical treatment, may become important. It is worth noting that inclusion of the diagonal terms alone in the computation of the magnetic moment yields poor results; this emphasizes once more the importance of incorporating the Coriolis coupling properly in any treatment of nucleonic motion in a deformed potential for light nuclei; without the proper band mixture one cannot get the observed magnetic moment for any of the odd nuclei in the $1f_{7/2}$ shell. The collective contribution μ_R to the magnetic moment constitutes only a fraction of the total magnetic moment; as a result the computed values are relatively insensitive to the choice of g_R. Interesting to note is the fact that the contributions of the single-particle terms μ_0, μ_+, and μ_- add coherently and incoherently to the collective part for odd-proton and for odd-neutron nuclei, respectively.

The calculated magnetic moment for the first excited state of V^{51} is 3.45 nm as compared with the observed value of (4.2±0.7) nm.[14]

The most important success of the present model is the essentially correct prediction of the ground-state quadrupole moments for all nuclei in this part of the periodic table as may be seen from Table II. The individual values of the intrinsic, the collective, and the single-particle quadrupole moment are listed separately. The table exemplifies the necessity of using the complete formula (4) rather than expression (5) for $K=I$. The projection factor in the expression (5) for the collective quadrupole moment not only reduces the magnitude of the collective contribution, by as much as a factor of 10 in the case of vanadium isotopes; but may also cause a change in the sign of the quadrupole moment. Thus the proper treatment of the band mixing problem is essential to obtaining the observed quadrupole moments of these nuclei. The usual practice of determining the deformation from the observed quadrupole moment, using expressions (6) and (5) for $K=I$ only, can clearly

[9] B. F. Bayman, J. D. McCullen, and L. Zamick, Phys. Rev. Letters **11**, 215 (1963).
[10] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.).
[11] D. W. Padgett, J. G. Brennan, and W. M. Frank, Nucl. Phys. **73**, 424 (1966).
[12] F. Villars, Helv. Phys. Acta **20**, 476 (1947).
[13] R. D. Lawson, Phys. Rev. **124**, 1500 (1961).
[14] I. Y. Krause, Phys. Rev. **129**, 1330 (1963).

TABLE II. The computed and observed quadrupole moments (Q.M.) of odd nuclei in the $1f_{7/2}$ shell. The experimental data are taken from Ref. 10 unless mentioned otherwise.

	Nucleus	Ground-state spin	Deformation β	Intrinsic Q.M. in 10^{-24} cm^2	Collective Q.M. in 10^{-24} cm^2	Single-particle Q.M. in 10^{-24} cm^2	Computed Q.M. in 10^{-24} cm^2	Measured Q.M. in 10^{-24} cm^2
Odd-proton	Sc^{43}	$\frac{7}{2}$	+0.18	+0.520	−0.112	−0.069	−0.181	−0.26±0.06[a]
	Sc^{45}	$\frac{7}{2}$	+0.20	+0.598	−0.146	−0.067	−0.213	−0.22[b]
	Sc^{47}	$\frac{7}{2}$	+0.14	+0.427	−0.087	−0.071	−0.159	−0.22±0.03[a]
	V^{47}	$\frac{7}{2}$	−0.39	−1.194	+0.143	−0.047	+0.095	
	V^{49}	$\frac{7}{2}$	−0.39	−1.194	+0.143	−0.047	+0.095	
	V^{51}	$\frac{7}{2}$	−0.30	−0.985	+0.074	−0.054	+0.020	±0.007[c]
	Mn^{51}	$\frac{5}{2}$	+0.27	+1.056	+0.315	−0.005	+0.310	
	Mn^{53}	$\frac{7}{2}$	+0.20	0.794	+0.211	−0.005	+0.206	
Odd-neutron	Ca^{43}	$\frac{7}{2}$	−0.36	−0.908	+0.092	...	+0.092	
	Ti^{45}	$\frac{7}{2}$	−0.20	−0.587	−0.007	...	−0.007	±(0.015±0.015)[d]
	Ca^{45}	$\frac{7}{2}$	+0.22	+0.628	+0.175	...	+0.175	
	Ti^{47}	$\frac{5}{2}$	+0.27	+0.880	+0.261	...	+0.261	+0.291[e]
	Cr^{49}	$\frac{5}{2}$	+0.24	+0.873	+0.261	...	+0.261	
	Ti^{49}	$\frac{7}{2}$	+0.15	+0.493	+0.156	...	+0.156	+0.24[e]
	Cr^{51}	$\frac{7}{2}$	−0.12	−0.423	+0.073	...	+0.073	

[a] R. G. Cornwell, W. Happer, Jr., and J. D. McCullen, Phys. Rev. **141**, 1106 (1966).
[b] G. Fricke, H. Kopfermann, S. Penselin, and K. Schupmann, Z. Physik **156**, 416 (1959).
[c] H. Nagasawa, S. K. Takeshita, and Y. Tomono, J. Phys. Soc. (Japan) **19**, 764 (1964).
[d] R. G. Cornwell and J. D. McCullen, Phys. Rev. **148**, 1157 (1966).
[e] K. H. Channappa and J. M. Pendlebury, Proc. Phys. Soc. (London) **86**, 1145 (1965).

lead to erroneous values not only with regard to magnitude but also with respect to sign in these nuclei. The table also exhibits that the single-particle contribution to the quadrupole moment has important bearing on its final value for most of the odd-proton nuclei. For odd-proton nuclei of Group I (i.e., Sc isotopes), the single-particle contribution adds coherently to the collective part and contributes as much as 30% of the total quadrupole moment. For nuclei of Group II and Group III, the contribution adds destructively to the collective part. It may be even greater in magnitude than the total quadrupole moment for the nuclei in Group II. For nuclei in Group III it gives insignificant contributions.

For a ground-state spin of $\frac{7}{2}$, the projection factor in (5) is negative for $K=\frac{1}{2}$ and $K=\frac{3}{2}$ and positive for $K=\frac{5}{2}$ and $K=\frac{7}{2}$. The ground-state wave function of nuclei in Group I is primarily composed of contributions from $K=\frac{1}{2}$ and $K=\frac{3}{2}$ bands. As a result, the projection factor for nuclei in this group is negative. Consequently the observed quadrupole moments have the opposite sign from the intrinsic moment. The ground-state wave functions of nuclei in Group II have about equal contributions from four bands. As a result, contributions to the collective quadrupole moment from $K=\frac{1}{2}$ and $\frac{3}{2}$ bands cancel those from $K=\frac{5}{2}$ and $K=\frac{7}{2}$ bands. Consequently, although these nuclei are strongly deformed, their quadrupole moments are very small and sensitive to slight changes in the wave function. A good case in point is Ti^{45}; although the wave function changes only slightly from the case of core overlap of unity ($\beta=-0.41$) to the case of the core overlap of 0.75 ($\beta=-0.2$), it is sufficient to change the quadrupole moment from +0.132 b to −0.007 b. The ground-state wave function of nuclei with spin $\frac{5}{2}$ in Group III has large components of the $K=\frac{5}{2}$ band and those with the ground-state spin $\frac{7}{2}$ have large components of $K=\frac{5}{2}$ and $K=\frac{7}{2}$ bands. In consequence, the projection factor is positive and the observed quadrupole moments are large and their sign is the same as that of the intrinsic quadrupole moment. On the same grounds, Ti^{49} of Group IV shows a large positive quadrupole moment. The tabulated quadrupole moment for Cr^{51} is given for a negative deformation for which the projection factor is negative. If, however, the nucleus is prolate, the wave function and the quadrupole moment of Cr^{51} would be similar to those of Ti^{49}.

Except in Ti^{47} and V^{51} absolute transition rates have not yet been measured in odd nuclei in this part of the periodic table. As already noted,[1] the predicted magnetic dipole and electric quadrupole transition rates in V^{51} agree well with the observation. It is worth emphasizing that this agreement has been obtained using the free proton and neutron charges of unity and zero, respectively.

The calculated value of $B(E2;\frac{7}{2}\rightarrow\frac{5}{2})$ in Ti^{47} is 1.60 $\times10^{-50}e^2$ cm^4, whereas the measured value is $3.0\times10^{-50}e^2$ cm^4 (Ref. 15).

The major difficulty of the shell-model computation with residual interaction among nucleons in the $1f_{7/2}$ shell[4,5] is the large discrepancy between the theoretical calculation of the quadrupole moment and the reduced $B(E2)$ transition rates and the experimental observations. This difficulty is embodied in the necessity of using the unusually large effective charges between 1.9

[15] G. M. Temmer and N. P. Heidenburg, Phys. Rev. **104**, 967 (1956).

and 3 both for neutrons and for protons in the $1f_{7/2}$ shell depending on the type of radial wave function used. These large effective charges cannot be justified as consequences of the recoil effect, which introduces an effective proton and an effective neutron charge of the magnitude of $(1-2/A+Z/A^2)$ and $(-Z/A^2)$, respectively. Furthermore, as already noted by Kendall and Talmi,[16] it is not possible to account for all observed $B(E2)$ in some nuclei such as V^{51}, using a single set of effective charges. In addition, the vanishingly small quadrupole moment of V^{51} requires an effective charge of less than one for the proton, in contradiction with the large proton effective charge necessary to explain the enhanced $B(E2)$ values. It is worth noting that the use of an effective neutron charge has the important consequence of introducing additional matrix elements in the expressions for the quadrupole moment and the transition rates. Thus, the actual enhancement may be considerably more than would be given simply by the effective charge or by the square of the effective charge for the quadrupole moment and for the $B(E2)$ value, respectively, e.g., in the neutron rich nuclei the actual enhancement may be even more than a factor of 100.

Only two magnetic dipole partial lifetimes, one in V^{51} and the other in Ti^{47} as determined from their respective first excited to ground-state transition have been measured. In view of the very large cancellation among the four large terms in the expression for $B(M1)$, the computed value of $B(M1;\frac{5}{2}\rightarrow\frac{7}{2})=0.002$ nm^2 for V^{51} and of $B(M1;\frac{7}{2}\rightarrow\frac{5}{2})=0.02$ nm^2 for Ti^{47} may be considered to be in good agreement with the experimental result of 0.005 nm^2 (Ref. 17) and between (0.03 and 0.06) nm^2 (Ref. 18), respectively. A measure of the amount of the cancellation involved may be obtained by noting that the individual contributions of the four terms in the bracket of Eq. (7) for Ti^{47} are 0.000, -0.872, $+0.452$, and $+0.151$, respectively. Thus a slight change in the wave function can easily reproduce the exact observed value without changing, in any appreciable fashion, the computed level schemes, moments and $B(E2)$ values. Hence the predicted transition rates are in reasonable agreement with the observed values. Large inhibition of the $B(M1)$ which are allowed in the usual collective model without the band mixing is possible within the framework of this model because of the destructive interference between the direct and the cross terms in the transition rates.

[16] H. W. Kendall and I. Talmi, Phys. Rev. **128**, 792 (1962).

[17] Deduced from the observed total radiation width (Ref. 10) and the $E2/M1$ mixing ratio as measured by R. C. Ritter, P. H. Stelson, F. K. McGowan, and R. L. Robinson [Phys. Rev. **128**, 2320 (1962)].

[18] Deduced using the observed mean partial $M1$ lifetime of (0.326±0.10) nsec as given by R. E. Holland and F. J. Lynch [Phys. Rev. **121**, 1464 (1961)].

IV. CONCLUSIONS

In view of the agreement obtained between the theoretical predictions of the moments and transition rates and the few available experimental data, we conclude that the odd nuclei in the $1f_{7/2}$ shell allow a collective-model description in terms of a static deformation. The good agreement between theoretical computation of the magnetic moments and the observation nullifies the argument that the observed magnetic moments of odd nuclei are indicative of the validity of the spherical shell model alone. The earlier success of the Coriolis-coupling model in predicting the correct ground-state spins and reasonable level spectra for odd nuclei in this mass region together with the present success in accounting for the magnetic moments, transition rates, and large and small quadrupole moments suggests that a "*deformed-shell model*" may be closer to reality in this part of the nuclear-mass table. The contention of the importance of the Coriolis-coupling terms in describing various nuclear properties in light nuclei is clearly borne out in cases of moments and transition rates. Apparently contradictory features, such as the enhanced $B(E2)$ and vanishingly small quadrupole moment in a single nucleus, e.g., V^{51}, are adequately explained as consequences of strong band mixture owing to the Coriolis-coupling term. Furthermore, it has been demonstrated that a nucleus may possess a large deformation but a small quadrupole moment and that the observed sign of the quadrupole moment in these nuclei in many cases may not reflect the actual sign of the deformation. Because of the Coriolis coupling, strong inhibition of $M1$ transition rates is also possible for transitions which are allowed in a collective model without band mixing and the proper treatment of this coupling term is also important for understanding the magnetic moments in terms of the collective model.

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