PHYSICAL REVIEW VOLUME 153, NUMBER 4 20 JANUARY 1967

Investigation of the Effects of a Spin-Spin Interaction on the Elastic Scattering of Neutrons by Nuclei*

A. P. STAMP
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 26 May 1966)

The effects of an interaction between the spin of the incident neutron and the spin of the target nucleus are considered in the framework of the optical model. It is found that the differential elastic scattering is insensitive to the interaction. However, the interaction splits the single-particle states, and the elastic scattering shows structure as a function of energy when the absorption is small. The spin-spin interaction has negligible effect on the polarization. The rotation parameter $R(\theta)$ and the asymmetry parameter $A(\theta)$ are sensitive to the spin-spin interaction for $\theta=180°$ and $90°$, respectively. For angles near the minima in the differential elastic scattering, the depolarization is approximately zero.

I. INTRODUCTION

OVER the past decade various refinements have been made to the optical-model potential first proposed by Feshbach, Porter, and Weisskopf.[1] In the present work a term of the form $V_{I\sigma}F(r)\mathbf{I}\cdot\boldsymbol{\sigma}$, proposed by Feshbach,[2] is added to the usual optical-model potential, where $\mathbf{I}$ is the spin of the target nucleus, and $\boldsymbol{\sigma}=(2/\hbar)\mathbf{s}$, where $\mathbf{s}$ is the spin of the incident neutron and $F(r)$ is the form factor. The usual optical-model potential produces no depolarization, i.e., $D=1.0$. We find that the introduction of a term dependent on the target spin produces depolarization; however, the term has negligible effect on the polarization. If the spin-orbit interaction is zero the spin-spin interaction would produce no polarization when the target in unpolarized. The $V_{I\sigma}F(r)\mathbf{I}\cdot\boldsymbol{\sigma}$ term can modify the polarization only through the presence of the spin-orbit term. Thus, it has a small effect on the polarization.

The characteristic effect produced by the spin-spin term is the splitting of the single-particle resonances. The spin-orbit term splits a state of angular momentum l into two states: $j=l\pm\frac{1}{2}$. If one considers a central potential plus a spin-spin interaction, the single-particle state l is split into several states. For the channel spin, $S=I+\frac{1}{2}$ and, $l\geqslant S$, there are $2I+2$ resonances for $J=|l-I-\frac{1}{2}|$ to $J=l+I+\frac{1}{2}$. One therefore expects to see resonances in the differential elastic scattering, polarization, asymmetry, and depolarization. For example, for the scattering of neutrons by ^{27}Al ($I=\frac{5}{2}$) we took the central potential $V_1=50$ MeV and the spin-orbit potential $V_{l\sigma}=6$ MeV. The $f_{7/2}$ partial wave resonates at $E_n=4.2$ MeV. Upon introducing the spin-spin interaction we would expect to see resonances for $l=3$ in channels $J=0$ to 6. When the absorption potential W is zero, the calculated differential elastic-scattering cross section shows resonances as a function of energy. For any reasonable value of W the states are spread out and the calculations do not give any structure.

The optical model with a central-plus-spin-orbit potential gives zero for the spin-flip amplitude for scattering in the forward direction because of the conservation of the Z component of angular momentum. Hence the rotation $R(0^0)$ and the depolarization $D(0^0)$ are unity and the polarization $P(0^0)$ and asymmetry $A(0^0)$ are zero. The spin-spin interaction allows the incident neutron to flip its spin, $\mu=\frac{1}{2}\rightarrow\mu=-\frac{1}{2}$, when scattered in the forward direction; thus $R(0^0)$ and $D(0^0)$ are no longer identically unity. The simplest method of detecting a spin-spin interaction would be to measure R and D in the forward direction. A departure from unity would confirm the presence of a spin-spin interaction. The calculations predict the departure of R and D from unity to be small in the forward direction because only amplitudes dependent on $P_l^0(\cos\theta)$ are nonzero, and thus measurements in the forward direction may not be profitable.

The most dramatic effect on R due to the spin-spin interaction is for $\theta=180°$. The spin-spin interaction can cause R to become positive where as for $I=0$, $R(180°)=-1$. In the absence of a spin-spin interaction the depolarization is identically unity for all angles. The The spin-spin interaction causes D to approach zero for angles corresponding to the minima in the differential elastic scattering.

In the present work the channel-spin coupling representation is used and the spin-flip and nonspin-flip amplitudes are obstained by using vector addition coefficients to transform to the uncoupled representation. In Sec. II we give the mathematical formulation and in Sec. III we present the results.

II. MATHEMATICAL FORMULATION

We consider the elastic scattering of neutrons, spin $\frac{1}{2}$, by target nuclei, spin I. The interaction is defined by the complex potential

$$-V(r)=V_1f(r)+iWg(r)+V_{I\sigma}4a^2h(r)\mathbf{I}\cdot\boldsymbol{\sigma}+V_{l\sigma}\lambda_\pi^2h(r)\mathbf{l}\cdot\boldsymbol{\sigma},\quad(1)$$

* Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-2098.

[1] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448 (1954).

[2] H. Feshbach, in *Nuclear Spectroscopy Part B*, edited by Fay Ajzenberg-Selove (Academic Press Inc., New York, 1960).

where

$$f(r)=[1+\exp(r-R)/a]^{-1},$$

$$h(r)=-\frac{1}{r}\frac{df(r)}{dr},$$

$$g(r)=S_1f(r)+(1-S_1)4arh(r),$$

and $R=r_0A^{1/3}$. The parameter S_1 determines the ratio of volume to surface absorption.

The spin-orbit term is of the Thomas type where λ_π is the pion Compton wavelength, taken to be $\sqrt{2}$ F exactly. The form factor for the spin-spin interaction was taken to be $h(r)$ for convenience in computation. In this model the real part of the potential accounts for refraction of the incident wave, the imaginary part provides for absorption, the spin-orbit term generates polarization and the spin-spin interaction produces depolarization.

The wave function describing the scattering is written in the channel-spin representation. The neutron spin is coupled to the spin of the nucleus to give states of channel spin S. The usual separation of the radial and angular coordinates of the channel radius $\mathbf{r}$ is made giving states of relative orbital angular momentum lm. Finally the channel spin S is coupled to l to give states of good JM. Thus

$$\psi_{Sl}{}^{JM}=|(Is)SlJM\rangle(1/r)f_{Sl}{}^J(r),\qquad(2)$$

where

$$|(Is)SlJM\rangle=\sum(Ims\mu|Sm+\mu)(lM-m-\mu Sm+\mu|JM)\times\phi_I{}^m\chi_s{}^u\{i^lY_l{}^{M-m-\mu}(\theta\phi)\}.\qquad(3)$$

The total wave function is

$$\Psi=\sum_{JMSl}\psi_{Sl}{}^{JM}.\qquad(4)$$

With this choice of representation the spin-spin interaction is diagonal giving $-(I+1)$ when $S=I-\frac{1}{2}$ and I for $S=I+\frac{1}{2}$. Substituting Eqs. (2), (3), and (4) into the Schrödinger equation for scattering in the center-of-mass system, multiplying on the left by $\langle(Is)SlJM|$ and integrating over angular variables gives the following coupled equations:

$$\left[\frac{d^2}{dr^2}+\frac{2m}{\hbar^2}\left[E+V_1f(r)+iWg(r)+\begin{Bmatrix}-I-1\\ I\end{Bmatrix}V_{I\sigma}4a^2h(r)\right]-\frac{l(l+1)}{r^2}\right]f_{Sl}{}^J(r)=\sum\mathfrak{M}_{SS'}{}^{Jl}f_{S'l}{}^J(r),\qquad(5)$$

where S, S' take the values $I\pm\frac{1}{2}$. The coupling matrix element

$$\mathfrak{M}_{SS'}{}^{Jl}=V_{l\sigma}\lambda_\pi{}^2 2l(l+1)[(2S+1)(2S'+1)]^{1/2}(-1)^{S+S'}\times[W(ISl^+l|\tfrac{1}{2}J)W(IS'l^+l|\tfrac{1}{2}J)-W(ISl^-l|\tfrac{1}{2}J)W(IS'l^-l|\tfrac{1}{2}J)]h(r).\qquad(6)$$

In Eq. (6) l^+ and $l^-=l\pm\frac{1}{2}$, respectively, and the W's are Racah coefficients. For each J value there are $2I+2$ allowed values of $l(J\geq I+\frac{1}{2})$. For $l=J\pm(I+\frac{1}{2})$ the equations are uncoupled; however, for the remaining $2I$ values of l the functions $f_{I+\frac{1}{2}\,l}{}^J(r)$ and $f_{I-\frac{1}{2}\,l}{}^J(r)$ are coupled.

For $r<R_m$, where the interaction $V(r)$ is nonzero, Eqs. (5) were solved numerically. For an incident partial-wave channel $|JlS\rangle$ the boundary conditions which yield the scattering matrix elements $S_{SS}{}^{Jl}$ and $S_{SS'}{}^{Jl}$ are

$$f_{Sl}{}^J(R_m)=I_{Sl}{}^J(R_m)-S_{SS}{}^{Jl}O_{Sl}{}^J(R_m),$$
$$f_{S'l}{}^J(R_m)=-S_{SS'}{}^{Jl}O_{S'l}{}^J(R_m).\qquad(7)$$

The functions $I_{Sl}{}^J(r)$ and $O_{Sl}{}^J(r)$ are the usual ingoing and outgoing waves,

$$I_{Sl}{}^J(r)=r(n_l(r)-ij_l(r)),$$
$$O_{Sl}{}^J(r)=r(n_l(r)+ij_l(r)).\qquad(8)$$

To calculate the observables we must evaluate the scattering amplitudes $A_{\mu'\mu}{}^{nm}(\Omega)$. The amplitude $A_{\mu'\mu}{}^{nm}(\Omega)$ gives the probability that the neutron initially in state $|\frac{1}{2}\mu\rangle$ will be scattered to state $|\frac{1}{2}\mu'\rangle$ while the target changes from initial-state spin I, projection m to its final-state $|In\rangle$. The scattering amplitudes are obtained by transforming from the channel-spin representation in which the S matrix elements are calculated, to the uncoupled representation using the usual vector-addition coefficients. Hence

$$A_{\mu'\mu}{}^{nm}(\Omega)=\frac{1}{2ik}\sum_l(2l+1)\left[\frac{(l-|\lambda|)!}{(l+|\lambda|)!}\right]^{1/2}P_l{}^{|\lambda|}(\cos\theta)e^{i\lambda\phi}\times(-1)^{(\lambda+|\lambda|)/2}\sum_{JSS'}(S_{SS'}{}^{Jl}-\delta_{SS'})\times(In\tfrac{1}{2}\mu'|S'n+\mu')(Im\tfrac{1}{2}\mu|S\,m+\mu)\times(S\,m+\mu\,l0|J\,m+\mu)(S'\,n+\mu'\,l\lambda|J\,m+\mu),\qquad(9)$$

where

$$\lambda=m+\mu-n-\mu'\quad\text{and}\quad\Omega=(\theta,\phi).$$

It is convenient to write the non-spin-flip amplitudes as

$$A_{\frac{1}{2}\frac{1}{2}}{}^{nm}(\Omega)=A_{\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)e^{i(m-n)\phi}=\frac{1}{2ik}\sum_l(2l+1)\left[\frac{(l-|m-n|)!}{(l+|m-n|)!}\right]^{1/2}e^{i(m-n)\phi}P_l{}^{|m-n|}(\cos\theta)(-1)^{(m-n+|m-n|)/2}\sum_{JSS'}(S_{SS'}{}^{Jl}-\delta_{S'S})\times(In\tfrac{1}{2}\tfrac{1}{2}|S'\,n+\tfrac{1}{2})(Im\tfrac{1}{2}\tfrac{1}{2}|S\,m+\tfrac{1}{2})\times(S'\,n+\tfrac{1}{2}\,l\,m-n|J\,m+\tfrac{1}{2})\times(S\,m+\tfrac{1}{2}\,l0|J\,m+\tfrac{1}{2}),\qquad(10)$$

and the spin-flip amplitude

$$
\begin{aligned}
&A_{-\frac{1}{2}\,\frac{1}{2}}{}^{nm}(\Omega)\\
&=B_{-\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta)e^{i(m-n+1)\phi}\\
&=\frac{1}{2ik}\sum_{l}(2l+1)\left[\frac{(l-|m-n+1|)!}{(l+|m-n+1|)!}\right]^{1/2}P_l{}^{|m-n+1|}\\
&\quad\times(\cos\theta)e^{i(m-n+1)\phi}(-1)^{(m-n+1+|m-n+1|)/2}\\
&\quad\times\sum(S_{SS'}{}^{Jl}-\delta_{SS'})(In\tfrac{1}{2}-\tfrac{1}{2}|S'n-\tfrac{1}{2})(Im\tfrac{1}{2}\tfrac{1}{2}|Sm+\tfrac{1}{2})\\
&\quad\times(S'\,n-\tfrac{1}{2}\,l\,m-n+1|J\,m+\tfrac{1}{2})\\
&\qquad\qquad\times(S\,m+\tfrac{1}{2}\,l0|J\,m+\tfrac{1}{2}). \qquad (11)
\end{aligned}
$$

The amplitudes have the following symmetry properties:

$$A_{\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)=(-1)^{m-n}A_{-\frac{1}{2},-\frac{1}{2}}{}^{-n,-m}(\theta)\,, \qquad (12)$$

$$A_{\frac{1}{2},-\frac{1}{2}}{}^{nm}(\theta)=-(-1)^{m-n}A_{-\frac{1}{2},\frac{1}{2}}{}^{-n,-m}(\theta)\,. \qquad (13)$$

We write the scattering matrix M in terms of two-dimensional submatrices $\mathcal{A}^{nm}$. The diagonal terms of $\mathcal{A}^{nm}$ are the non-spin-flip amplitudes $A_{\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)$ and the off-diagonal terms are the spin-flip amplitudes $B_{-\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta)$

$$
\begin{aligned}
M&=\mathcal{A}^{nm}\\
&=e^{i(m-n)\phi}\begin{pmatrix} A_{\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta) & B_{\frac{1}{2},-\frac{1}{2}}{}^{nm}(\theta)e^{-i\phi}\\ B_{-\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta)e^{i\phi} & B_{-\frac{1}{2},-\frac{1}{2}}{}^{nm}(\theta)\end{pmatrix}. \qquad (14)
\end{aligned}
$$

Using Eqs. (12) and (13),

$$
\begin{aligned}
&\mathcal{A}^{nm}=(-1)^{m-n}e^{i(m-n)\phi}\\
&\times\begin{pmatrix} A_{\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta) & -B_{-\frac{1}{2},\frac{1}{2}}{}^{-n,-m}(\theta)e^{-i\phi}\\ B_{-\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)e^{i\phi} & A_{\frac{1}{2}\frac{1}{2}}{}^{-n,-m}(\theta)\end{pmatrix}. \qquad (15)
\end{aligned}
$$

To calculate the observables we use the density matrix formalism.[3] The expectation value of the operators S^η is

$$I_0\langle\bar{S}^\eta\rangle=\frac{1}{2(2I+1)}\sum_\nu\langle\bar{S}^\nu\rangle\mathrm{Tr}(MS^\nu M^\dagger S^\eta)\,. \qquad (16)$$

The initial state is described by specifying the average values of the operators S^ν. The bar denotes the statistical average and the quantum-mechanical average is denoted by the brackets. In Eq. (16), I_0 is the intensity and M is the scattering matrix.

In the uncoupled representation the operators S^ν are products of operators of the incident neutron $S_i{}^\nu$ and the target nucleus $S_t{}^\nu$,

$$S^\nu=S_i{}^\nu S_t{}^\nu\,. \qquad (17)$$

When neutrons are scattered by unpolarized targets, $S_t{}^\nu=1$, and if measurements are only made of the scattered neutrons, $S_t{}^\eta=1$. Combining Eqs. (14), (16), and (17) we obtain the following expression for the scattering of neutrons by an unpolarized target:

$$I_0\langle\bar{S}_i{}^\eta\rangle=\frac{1}{2(2I+1)}\sum_\nu\langle\bar{S}_i{}^\nu\rangle\sum_{mn}\mathrm{Tr}(\mathcal{A}^{nm}S_i{}^\nu\mathcal{A}^{nm\dagger}S_i{}^\eta)\,. \qquad (18)$$

The intensity of an unpolarized beam of neutrons is given by taking $\langle\bar{S}_i{}^\nu\rangle=1$ and $S_i{}^\eta=1$. Thus[4]

$$I_0=\frac{1}{2I+1}\sum_{mn}(|A^{nm}|^2+|B^{nm}|^2)\,. \qquad (19)$$

To obtain expressions for the polarization, rotation, asymmetry, and depolarization we specify the following axes. The direction normal to the scattering plane is denoted by $\mathbf{n}$:

$$\mathbf{n}=\frac{\mathbf{k}_i\times\mathbf{k}_f}{|\mathbf{k}_i\times\mathbf{k}_f|}\,.$$

Taking $\mathbf{k}_i$ along the z direction and $\mathbf{k}_f$ in the x-z plane, $\mathbf{n}$ is along the y direction. The direction perpendicular to the incident direction is defined by

$$\mathbf{s}_i=\frac{\mathbf{n}\times\mathbf{k}_i}{|\mathbf{n}\times\mathbf{k}_i|}\,.$$

The unit vector $\mathbf{s}_i$ is in the x direction. The unit vector $\mathbf{s}_f$, perpendicular to the outgoing direction is defined by

$$\mathbf{s}_f=\frac{\mathbf{n}\times\mathbf{k}_f}{|\mathbf{n}\times\mathbf{k}_f|}\,.$$

Thus,

$$\boldsymbol{\sigma}\cdot\mathbf{n}=\sigma_y\,,\quad \boldsymbol{\sigma}\cdot\mathbf{s}_i=\sigma_x\,,\quad \boldsymbol{\sigma}\cdot\mathbf{s}_f=\sigma_x\cos\theta-\sigma_z\sin\theta\,. \qquad (20)$$

The polarization is the expectation value of $\boldsymbol{\sigma}$ when the incident beam is unpolarized. Equations (18) and (20)

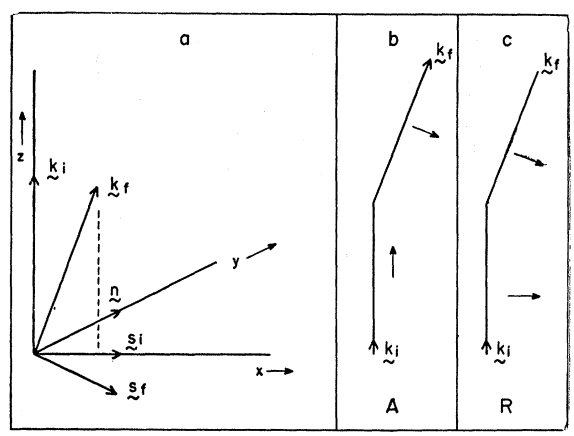


FIG. 1. The unit vectors used to specify the polarization, rotation, and asymmetry.

[3] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

[4] To simplify the notation we make the abbreviations

$$A^{nm}=A_{\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta)$$

and

$$B^{nm}=B_{-\frac{1}{2},\frac{1}{2}}{}^{nm}(\theta)\,.$$

give

$$I_0 P\mathbf{n} = \frac{1}{2I+1} \sum_{nm} 2\,\mathrm{Im} A^{nm*} B^{nm}. \quad (21)$$

The only component which is nonzero is perpendicular to the scattering plane.

The asymmetry is a measure of the component of the incident beam which is polarized along the incident direction $\mathbf{k}_i$ which after scattering points in the direction $\mathbf{n}\times\mathbf{k}_f$; see Fig. 1(b).

$$I_0 A\mathbf{s}_f = \frac{1}{2(2I+1)} \sum_{nm} \mathrm{Tr}\mathcal{A}^{nm}\boldsymbol{\sigma}\cdot\mathbf{k}_i\mathcal{A}^{nm\dagger}\boldsymbol{\sigma}\cdot\mathbf{n}\times\mathbf{k}_f$$

$$= \frac{1}{2I+1} \sum_{nm} [-\sin\theta(|A^{nm}|^2 - |B^{nm}|^2) + \cos\theta 2\,\mathrm{Re} A^{nm*}B^{nm}]. \quad (22)$$

The rotation measures the polarization in the direction $\mathbf{n}\times\mathbf{k}_f$ when the incident beam is polarized in the direction $\mathbf{n}\times\mathbf{k}_i$; see Fig. 1(c).

$$I_0 R\mathbf{s}_f = \frac{1}{2(2I+1)} \sum_{nm} \mathrm{Tr}\mathcal{A}^{nm}\boldsymbol{\sigma}\cdot\mathbf{n}\times\mathbf{k}_i\mathcal{A}^{nm\dagger}\boldsymbol{\sigma}\cdot\mathbf{n}\times\mathbf{k}_f$$

$$= \frac{2}{2I+1} \sum_{nm} \mathrm{Re}\{\sin\theta A^{nm}B^{-n-m*} + \cos\theta(A^{nm}A^{-n-m*} - B^{nm}B^{-n-m*})\}. \quad (23)$$

The remaining observable is the depolarization which gives the polarization along $\mathbf{n}$ after scattering when the incident beam is polarized along $\mathbf{n}$:

$$I_0 D\mathbf{n} = \frac{2}{2I+1} \sum_{nm} \mathrm{Re}(A^{nm}A^{-n-m*} + B^{nm}B^{-n-m*}). \quad (24)$$

From Eqs. (19), (21), (22), (23), and (24) we see that measurements of $I_0(\theta)$, $P(\theta)$, $A(\theta)$, $R(\theta)$, and $D(\theta)$ give the quantities,

$$\sum_{nm} (|A^{nm}|^2 \pm |B^{nm}|^2),$$

$$\mathrm{Re} \sum_{nm} A^{nm*}B^{nm},$$

$$\mathrm{Im} \sum_{nm} A^{nm*}B^{nm},$$

$$\mathrm{Re} \sum_{nm} A^{nm}B^{-n-m*},$$

and

$$\sum_{nm} (A^{nm}A^{-n-m*} - B^{nm}B^{-n-m*}).$$

The choice of the channel-spin representation provides a convenient check to the computer program written to calculate the scattering. The spin-spin interaction appears in the formalism [Eq. (5)] in a simple manner. It is diagonal and equals $-(I+1)4a^2 \times V_{I\sigma}h(r)$ or $I4a^2V_{I\sigma}h(r)$. The usual spin-orbit term couples the channels; $S=I\pm\frac{1}{2}$. If $V_{I\sigma}$ is zero, the potential is just the usual optical-model potential; however, the formulation is complicated and in order to calculate the non-spin-flip and spin-flip amplitudes $A_{\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)$ and $B_{-\frac{1}{2}\frac{1}{2}}{}^{nm}(\theta)$ we must sum over l, J, S, and S'.

TABLE I. Optical-model parameters (MeV).

V_1	W	$V_{l\sigma}$	$V_{I\sigma}$
48	5.75	5.5	7.58

The scattering by the usual optical-model potential is solved by coupling s to l to form j. Then, the spin-orbit term is diagonal and the non-spin-flip, $A(\theta)$, and the spin-flip, $B(\theta)$ amplitudes are obtained as a sum over l and j:

$$A(\theta) = \frac{1}{2ik} \sum_l [(l+1)(S_l^+ - 1) + l(S_l^- - 1)] \times P_l^0(\cos\theta), \quad (25)$$

$$B(\theta) = \frac{1}{2ik} \sum_l [S_l^+ - S_l^-] P_l^1(\cos\theta). \quad (26)$$

Equation (10) and (11) are the generalizations of (25) and (26). Since Eqs. (10) and (11) give sums over J, S, S', and l, we see that when $V_{I\sigma}=0$ there is a strong check on the amplitudes, as differences of many terms must give the same results as Eqs. (25) and (26). If the spin-orbit interaction is zero the scattering matrix elements are diagonal in S and independent of J, see Eq. (5). The only nonzero amplitudes then are A^{mm} and B^{m+1m}. Hence, the polarization is zero.

III. RESULTS OF NUMERICAL CALCULATIONS

The calculations were carried out for several nuclei in order to investigate the general effects of the spin-spin interaction. Light, medium, and heavy nuclei were chosen and nuclei of similar mass, but with different spins were compared. The values of the optical potentials were taken from Rosen *et al.*[5] and are given in Table I. The radius parameter r_0 was taken as 1.25 F and the diffuseness parameter $a=0.65$ F, except for the surface absorpition potential ($S_1=0$) when a was taken to be 0.7 F. An estimate of the spin-spin interaction strength was made by assuming that the target spin is due solely to the odd particle and taking the two-body force to be of the Rosenfeld mixture and separable,

$$V(r_1,r_2) = -V_0(a+b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)F(r_1)F(r_2). \quad (27)$$

The spin-spin interaction arises from the $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ term. The interaction potential,

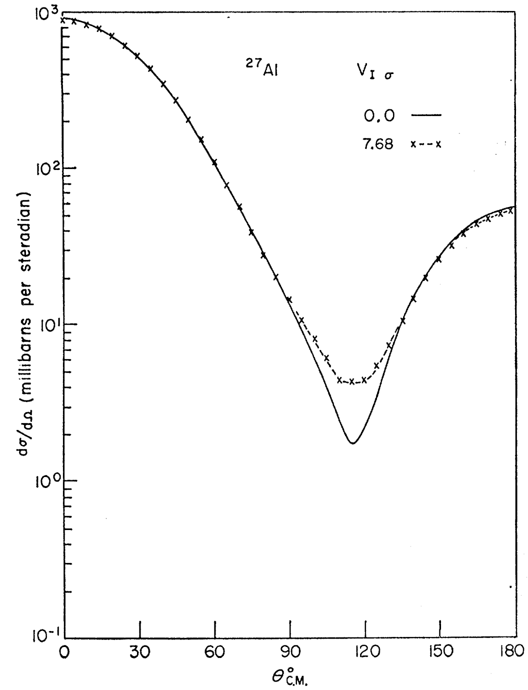
$$V = -V_0 bF(r)\boldsymbol{\sigma}\cdot\langle F(r)\boldsymbol{\sigma}\rangle, \quad (28)$$

[5] L. Rosen, J. G. Berry, A. S. Goldhaber, and E. H. Auerbach, Ann. Phys. (N. Y.) 34, 96 (1965).

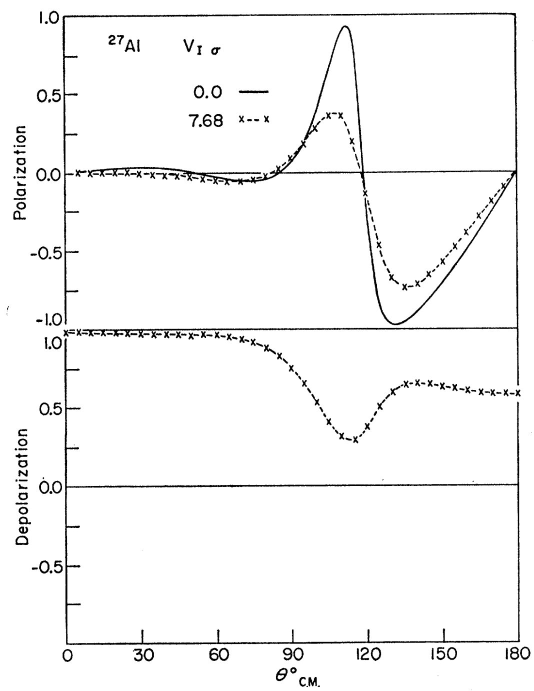
where

$$\langle F(r)\sigma\rangle = \sum \langle 0|F(r_i)\sigma_i|0\rangle. \tag{29}$$

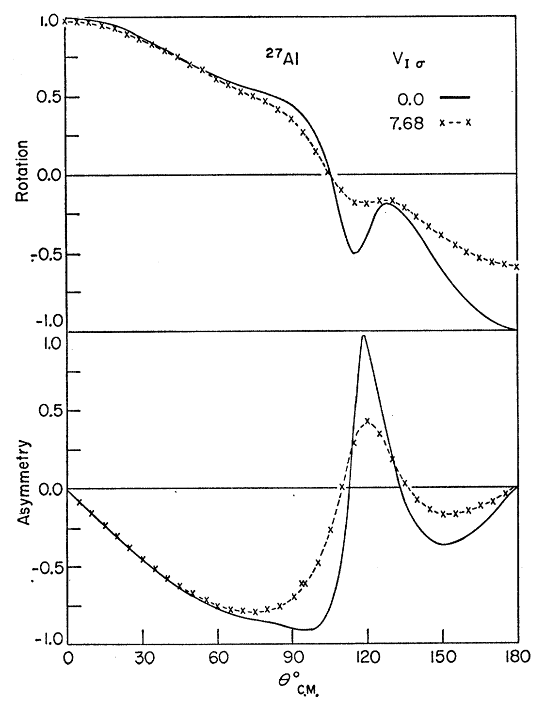
In Eq. (29), $|0\rangle$ is the ground-state wave function of the target and the sum is over the nucleons in the target. The expectation value of $\boldsymbol{\sigma}$ is just the polarization of the target; hence when the target nucleus is assumed to consist of a spherical core plus an odd particle in state $|nlI\rangle$, then only the odd particle contributes to



(a)



(b)



(c)

FIG. 2. (a) The differential elastic-scattering cross section of neutrons scattered by ^{27}Al, $I=\frac{5}{2}$, at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (b) The polarization and depolarization of neutrons scattered by ^{27}Al at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (c) The rotation and asymmetry of neutrons scattered by ^{27}Al at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV.

the sum in Eq. (29). Thus,

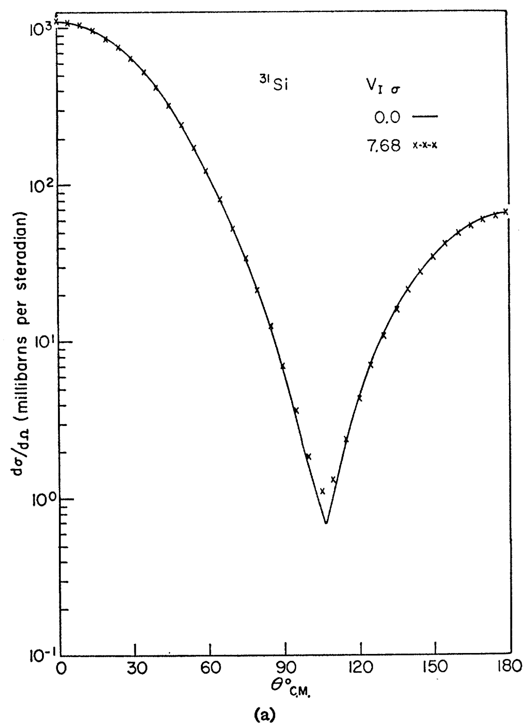
$$\langle F(r)\boldsymbol{\sigma}\rangle = \mathbf{I}\frac{\langle (ls)I\|\sigma\|(ls)I\rangle}{\langle (ls)I\|I\|(ls)I\rangle}R$$
$$= \mathbf{I}\frac{\sqrt{3}}{[I(I+1)]^{1/2}}(-1)^{l+1-s-I}$$
$$\times(2s+1)(2I+1)W(sIsI:l1)R, \quad (30)$$

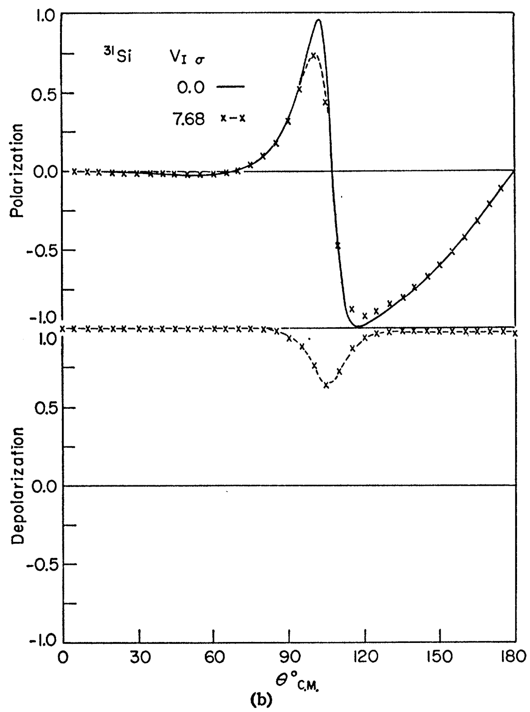
where $R=\int R_{nl}^2(r)F(r)r^2dr$, and $\mathbf{I}$ is the average value of the target spin. Substituting the analytical expression for the W coefficient in Eq. (30) we obtain

$$V = -V_{I\sigma}F(r)\mathbf{I}\cdot\boldsymbol{\sigma},$$

where

$$V_{I\sigma} = (-1)^{l+1}V_0bR\left[1-\frac{l(l+1)-0.75}{I(I+1)}\right].$$





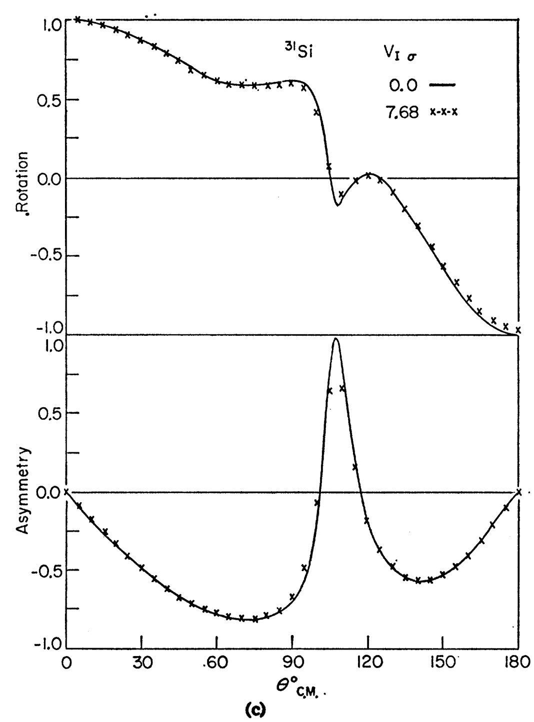


FIG. 3. (a) The differential elastic-scattering cross section of neutrons scattered by ^{31}Si, $I=\frac{1}{2}$ at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (b) The polarization and depolarization of neutrons scattered by ^{31}Si at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (c) The rotation and asymmetry of neutrons scattered by ^{31}Si at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV.

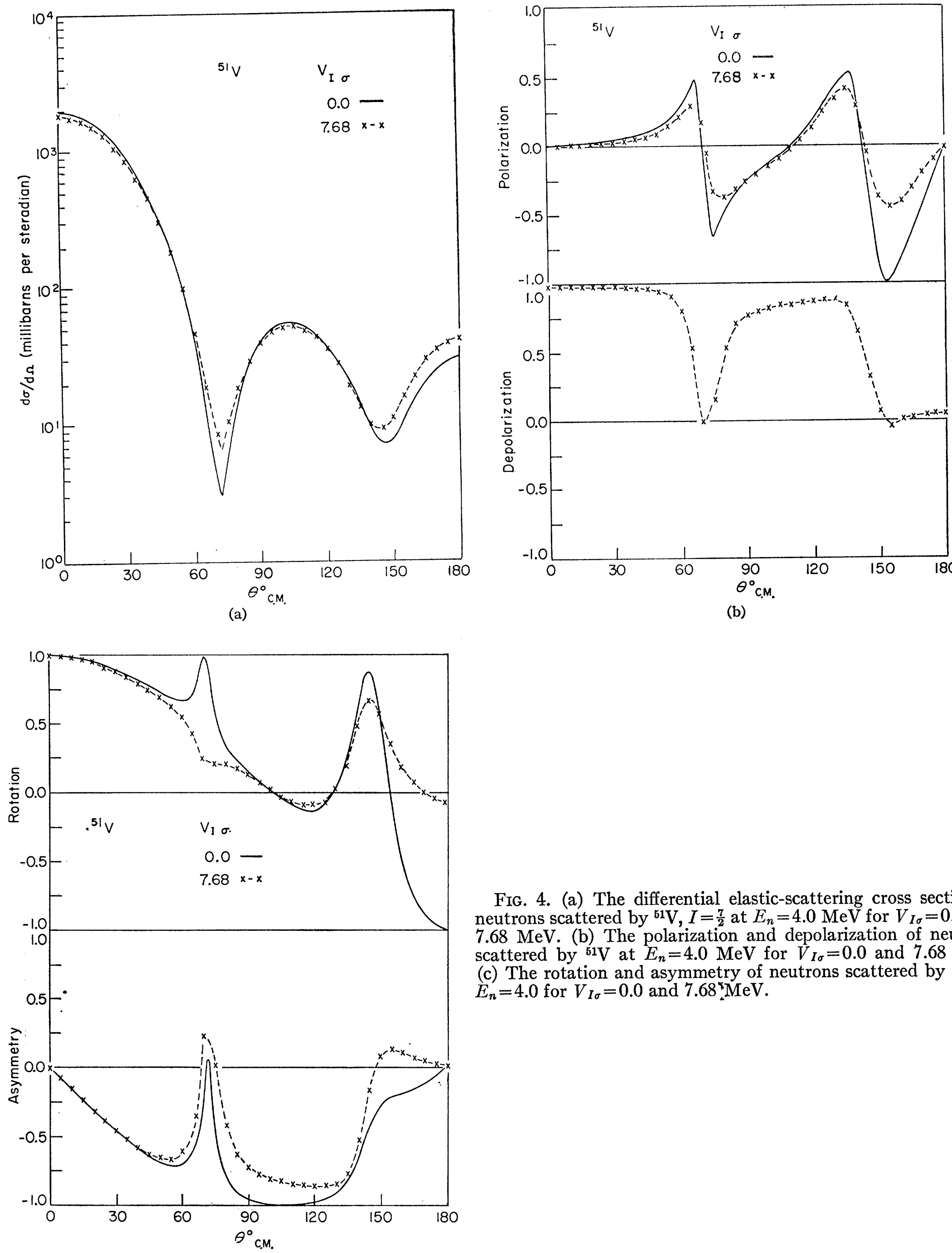


FIG. 4. (a) The differential elastic-scattering cross section of neutrons scattered by ^{51}V, $I=\frac{7}{2}$ at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (b) The polarization and depolarization of neutrons scattered by ^{51}V at $E_n=4.0$ MeV for $V_{I\sigma}=0.0$ and 7.68 MeV. (c) The rotation and asymmetry of neutrons scattered by ^{51}V at $E_n=4.0$ for $V_{I\sigma}=0.0$ and 7.68 MeV.

Taking $V_0=50$ MeV, $b=0.7$, $F(r)=4a^2h(r)$, $l=2$, and $j=\frac{5}{2}$, we obtain $R\approx 0.2$ and $V_{I\sigma}$ is of the order of 3 MeV. Collective efforts are likely to increase the value of $V_{I\sigma}$; hence we take $V_{I\sigma}=3.84$ and 7.68 MeV and assume the I dependence is small. The greater I is, the greater the core polarization, and hence, the greater the collective contribution to $V_{I\sigma}$; hence the assumption of I independence is reasonable.

In Figs. 2, 3, and 4, the results are presented for the scattering of 4-MeV neutrons elastically scattered by ^{27}Al$(I=\frac{5}{2})$, ^{31}Si$(I=\frac{1}{2})$, and ^{51}V$(I=\frac{7}{2})$ for $V_{I\sigma}=0.0$ and 7.68 MeV. The depolarization was calculated assuming the incident beam is 100% polarized along the direction $\mathbf{n}$, similarly the rotation and asymmetry were calculated for the incident beam 100% polarized in the direction $\mathbf{n}\times\mathbf{k}_i$. The minima in the differential elastic scattering occur when the real and imaginary parts of the non-spin-flip amplitudes are approximately zero. The characteristic structure in the polarization (i.e., rapid change in sign of P) and the rapid fluctuations in D, R, and A are all due to both the real and imaginary parts of $A_{\mu\mu}{}^{nm}(\theta)\approx$zero.

In Fig. 5 we illustrate D and R as a function of $V_{I\sigma}$ for the reaction $^{27}\mathrm{Al}(n,n)^{27}\mathrm{Al}$.

The effect of the spin-spin interaction is greatest for large values of I. However, changes induced in the differential elastic scattering and the polarization by $V_{I\sigma}$, would probably be reproduced by taking $V_{I\sigma}=0$ and varying the remaining parameters.

The most characteristic effect due to $V_{I\sigma}$ is seen in D and in R ($\theta=180$). For large angles the depolarization departs from unity and approaches zero for angles corresponding to the minima in the elastic scattering. For $\theta=180°$ and $V_{I\sigma}=0$ we can see from Eq. (23) that $R(180)=-1$. However, $V_{I\sigma}$ causes $R(\theta=180)$ to approach zero as I increases.

For the scattering of neutrons by ^{27}Al we investigated the energy dependence of $d\sigma/d\Omega$, P, D, R, and A. For $V_{I\sigma}=0.0$ the $f_{7/2}$ wave resonantes at $E_n=4.2$ MeV the functions $f_{S3}{}^{J}(r)$ for $J=0$ to 6 will all resonate for E_n near 4.0 MeV, and hence we expect to see structure in $d\sigma/d\Omega$, P, D, R, and A as a function of E_n.

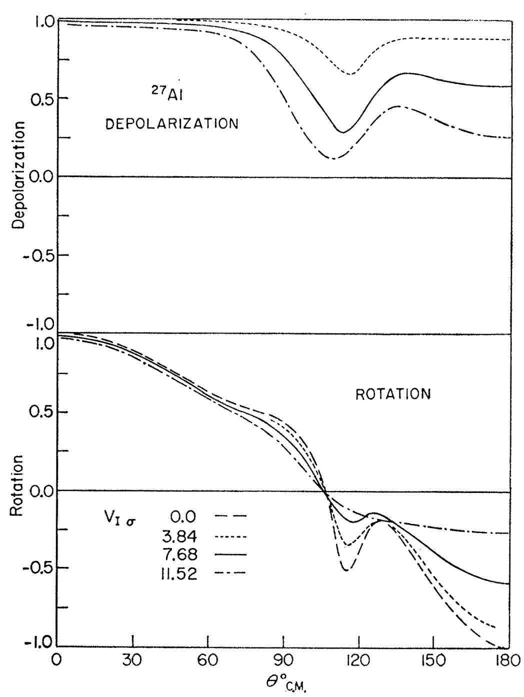


FIG. 5. The depolarization and rotation of neutrons scattered by ^{27}Al, $I=\frac{5}{2}$, as a function of $V_{I\sigma}$.

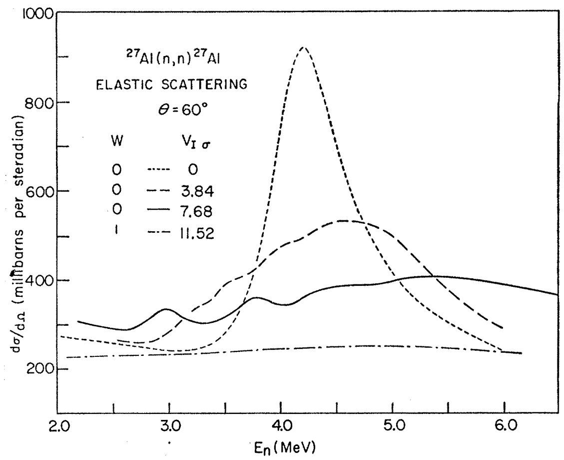


FIG. 6. The elastic-scattering cross section at $\theta=160°$ for neutrons scattered by ^{27}Al as a function of the neutron energy. For $W=1.0$ MeV, the diffuseness parameter of the absorption potential was taken to be 0.6 F.

In Figs. 6, 7, 8, and 9 we present the results for $V_1=50$ MeV, $W=0$, $V_{I\sigma}=6$ MeV, $r_0=1.3$ F, and $a=0.65$ F. The energy dependence of the elastic scattering is illustrated in Figs. 6 and 7, for $\theta=160°$ and 90°, respectively. For $W=1.0$ MeV we see (Fig. 6) that the single-particle resonances are spread over several hundred keV and no longer show up. For $V_{I\sigma}=7.68$ MeV, the resonances occur at $E_n=2.96$, 3.76, and 4.45 MeV when the imaginary part of $S_{33}{}^{Jl}$, for $l=3$, $J=1$, 2, and 3, is zero and the real part of $S_{33}{}^{Jl}$ is approximately -1. For $J=4$ and 5 the imaginary part of $S_{33}{}^{J3}$ is zero for $E_n=5.20$ and 5.56; however, the resonances are wide and do not show up. When $V_{I\sigma}=3.84$ MeV, the imaginary part of $S_{33}{}^{J3}$ is zero at 3.06, 3.26, 3.58, 4.00, 4.48, and 4.82 MeV for $J=0$, 1, 2, 3, 4, and 5, respectively. However, the resonances are not very distinct. As expected, the resonances which

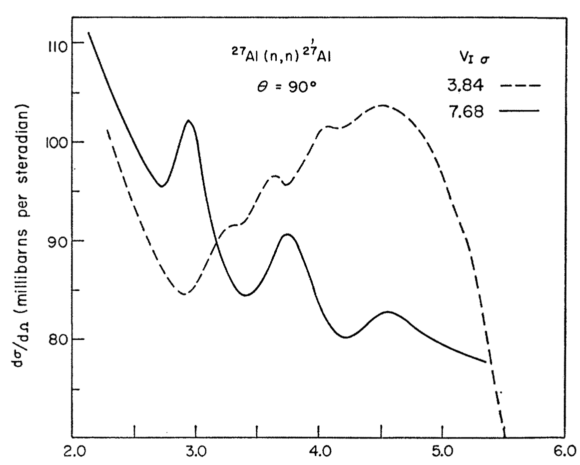


FIG. 7. The elastic-scattering cross section at $\theta=90°$ for neutrons scattered by ^{27}Al as a function of energy.

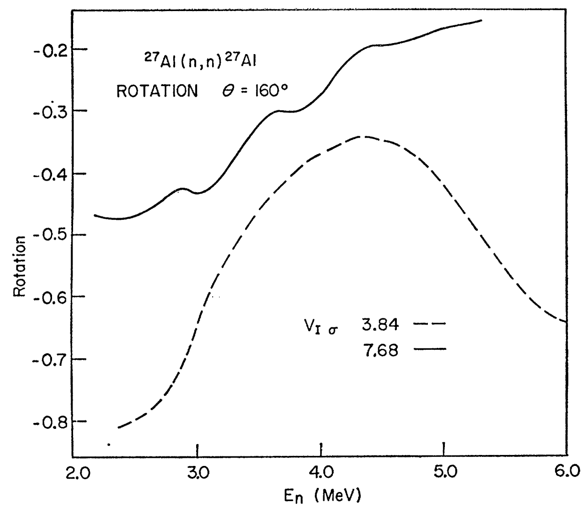


FIG. 8. The energy dependence of the rotation at $\theta=160°$ for neutrons scattered by ^{27}Al.

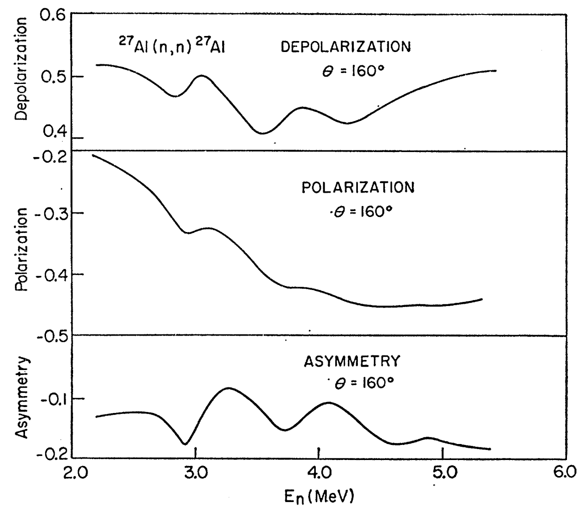


FIG. 9. The energy dependence of the depolarization, polarization, and asymmetry at $\theta=160°$ for neutrons scattered by ^{27}Al.

occur at the lowest energies (i.e., small J) are sharpest.

The energy dependence of the rotation is given in Fig. 8 and of the depolarization, polarization, and asymmetry in Fig. 9. When $W=1$ MeV, the structure in D, P, R, and A is lost. Elwyn *et al.*[6] have measured the polarization and the cross section for the reaction ^{27}Al(n,n) ^{27}Al for E_n below 2.2 MeV. They observe resonances at intervals of 300–400 keV, which have widths of 100–200 keV and point out that the structure is not inconsistent with an intermediate structure described by Block and Feshbach[7] and discussed qualitatively by Kerman *et al.*[8] We have shown that a spin-spin interaction generates resonances provided the absorption is small. If the central potential V_1 is increased, the resonances occur at lower energies and are narrower. Elwyn *et al.*[6] analyzed their results in terms of the optical model in order to account for the general features of the elastic scattering and polarization and took $W=16.5$ MeV. We have shown that for $W=1.0$ MeV, the structure due to the spin-spin interaction is lost, and therefore the resonances observed by Elwyn are unlikely to be due to a spin-spin interaction.

The simplest method of experimentally verifying the existence of a spin-spin interaction would be to measure the depolarization for angles corresponding to the minima in the elastic scattering or for $\theta=180°$. A value of D different from unity would show the existence of a spin-spin interaction.

ACKNOWLEDGMENT

I would like to thank Professor H. Feshbach for suggesting the investigation.

[6] A. J. Elwyn, J. E. Monahan, R. O. Lane, and A. Langdorf, Jr., Nucl. Phys. **59**, 113 (1964).
[7] B. Block and H. Feshbach, Ann. Phys. (N.Y.) **23**, 47 (1963).
[8] A. K. Kerman, L. S. Rodberg, and J. E. Young, Phys. Rev. Letters **11**, 422 (1963).