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Possible Role of ΛNN Forces in Hypernuclei

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The effect of a repulsive three-body ΛNN force on the Λ-N scattering lengths and the ${}_{\Lambda}He^4-{}_{\Lambda}H^4$ binding-energy difference is considered. It is found that a suitable three-body force strength needed to adjust the Λ-N scattering lengths to the measured Λ-p ones reduces substantially the additional Coulomb repulsion in ${}_{\Lambda}He^4$. Some remarks are made concerning the charge-symmetry-breaking components of the Λ-N interaction.

I. INTRODUCTION

RECENTLY the $\Lambda-p$ scattering lengths were determined experimentally[1,2]: $a_s=-2.46$ F, $a_t=-2.07$ F (Ref. 1), and $a_s=-2.20$ F, $a_t=-1.91$ F (Ref. 2). The errors are indicated by the equal-likelihood contours in the (a_s,a_t) plane shown in Fig. 3 of Ref. 1. Also indicated there are the corresponding points obtained from hypernuclear analysis.[3–5] These points, derived from binding energies of the s-shell hypernuclei are characterized by $a_s/a_t\geq 4$, in clear disagreement with the results of Λ-p scattering. The singlet scattering length, which is mainly determined by calculations on ${}_{\Lambda}H^3$, seems to agree with Refs. 1 and 2. The trouble appears with the triplet scattering length, which comes out in hypernuclear calculations lower (~-0.6 F) than the experimental one. The triplet scattering length is mainly determined in hypernuclear calculations by ${}_{\Lambda}He^5$. All calculations assume a smooth, central, spin-dependent Λ-N interaction of an intrinsic range corresponding to two-pion exchange or K-meson exchange, with or without a hard core. A review of the uncertainties encountered in hypernuclear calculations and the possible ways of dealing with them is to be found in Ref. 6. In particular, the two modifications in hypernuclear calculations on ${}_{\Lambda}H^3$ and ${}_{\Lambda}He^5$, suggested by Bodmer,[7] tend to correct for a_t in the desired direction. Quantitatively, however, they are insufficient, since they raise $|a_t|$ only to about 0.9 F.

Another effect which has to be considered is that of a charge-symmetry-breaking (CSB) component in the $\Lambda-N$ interaction which shows up in the ${}_{\Lambda}He^4-{}_{\Lambda}H^4$ binding-energy difference.[8–10] Since the hypernuclei ${}_{\Lambda}H^3$ and ${}_{\Lambda}He^5$, from which the scattering lengths are calculated, constitute scalar isomultiplets, the CSB component of the $\Lambda-N$ interaction in these hypernuclei is cancelled out. The scattering lengths derived in such a way need not therefore be equal to the measured $\Lambda-p$ ones. Indeed, the calculations of $\Lambda-N$ scattering lengths carried out with CSB potential, which fits the ${}_{\Lambda}He^4-{}_{\Lambda}H^4$ binding-energy difference,[10] yield pronounced differences in the $\Lambda-n$ and $\Lambda-p$ scattering lengths. But still, these cannot account for the experimental results of $\Lambda-p$ scattering. (See Table I of Ref. 10., in particular, cases a_2 and b_2 which correspond to the more theoretically established[9] form of the CSB potential.) The difficulty is again the low value obtained for $|a_t|$.

There is still a more "conventional" possibility for understanding the experimental $\Lambda-p$ scattering lengths on the basis of hypernuclear data, namely, contributions of ΛNN three-obdy forces.[11–13] Figure 1 is a diagram illustrating how three-body forces arise. The range of this force is not short at all compared to the $\Lambda-N$ two-body force range. Its common form used in most hypernuclear calculations is

$$V_3=\sum_{i<j}^{N}-\tfrac{1}{3}W(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\frac{e^{-\mu r_{i\Lambda}}}{\mu r_{i\Lambda}}\frac{e^{-\mu r_{j\Lambda}}}{\mu r_{j\Lambda}},\qquad(1)$$

where i and j stand for the two nucleons and μ^{-1} is the pion Compton wavelength. For the s-shell hypernuclei $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)=-3$. This form of V_3 appears to rep-

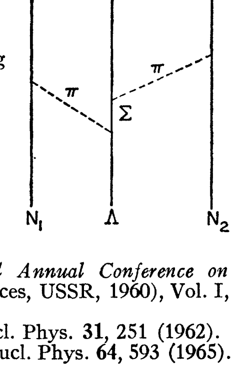

FIG. 1. A long-range diagram contributing to ΛNN force.

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resent only its asymptotic shape, while noncentral, nonseparable components are very likely to dominate at short distances.[14] It is quite obvious that a short-range three-body force will not seriously affect the very loosely bound system $_\Lambda H^3$, but may affect $_\Lambda He^5$ because of its tighter binding and large number of bonds (six). The presence of a strong repulsive ($W>0$) three-body force would therefore be accompanied by a larger average two-body $\Lambda-N$ interaction in $_\Lambda He^5$ than in the usual treatment. Since the average $\Lambda-N$ interaction in $_\Lambda He^5$ is dominated by the triplet $\Lambda-N$ interaction, one may increase $|a_t|$ in this way.

In Sec. II we briefly review the way a_s and a_t are obtained from hypernuclei in the presence of a three-body interaction. The strength of this interaction is then fixed subject to the experimental values for a_s and a_t. Section III is devoted to calculating the combined effect of two-body and three-body interactions on the radial mode of compression of the nuclear core in the $A=4$ hypernuclei. Likewise, the problem of charge-symmetry breaking in the $\Lambda-N$ interaction is examined within the framework of the quark model. In the concluding section we discuss our results and their relevance to the $_\Lambda He^4-{}_\Lambda H^4$ binding energy difference, as well as to other related hypernuclear problems.

II. THREE-BODY FORCES AND Λ-*N* SCATTERING LENGTHS

Most hypernuclear calculations,[4,5,8,12,13] with only two-body forces, assume some definite central $\Lambda-N$ interaction of intrinsic range corresponding to two-pion or K-meson exchange. The volume integrals of the interaction in the singlet state and in the triplet state, U_s and U_t, respectively, are then found by using the binding energies of some hypernuclei, usually $_\Lambda H^3$ and $_\Lambda He^5$. The values of U_s and U_t may be easily translated[3] into the scattering lengths a_s and a_t. The inclusion of a hard core in the $\Lambda-N$ potential poses no new problem in deriving the scattering lengths. Here we shall restrict ourselves to the case in which the intrinsic range corresponds to two-pion exchange. Reference 15 gives $U_s=-380\pm20$ MeV, $U_t=-180\pm20$ MeV (without hard core). The corresponding scattering lengths are[3]: $a_s=-(2.4_{-0.6}^{+1.2})$ F, $a_t=-(0.52\pm0.12)$ F. For comparison we also quote the scattering lengths obtained[5] with a hard core of 0.4 F[5]: $a_s=-(2.89_{-0.41}^{+0.59})$ F, $a_t=-(0.71\pm0.06)$ F. As mentioned eariler $|a_t|$ is significantly lower than the experimental value.

It was shown by Dalitz[16] that for a three-body force of the type

$$V_3(\Lambda,i,j)=-\tfrac{1}{3}W(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)v(r_{i\Lambda})v(r_{j\Lambda}) \qquad (2)$$

the volume integral U_N of a hypernucleus containing N nucleons (which is a linear combination of U_s and U_t) is modified by adding to it the three-body contribution

$$W_N=\tfrac{1}{2}WN(N-1)\int d^3r_{12}d^3r_\Lambda\rho(r_{12})v(r_{1\Lambda})v(r_{2\Lambda}), \quad (3)$$

where $\rho(r_{12})$ is the correlation function between pairs of nucleons normalized to unity. It has been pointed out[12,16] that it is sufficient to compute the integral of (3) with $\rho(r_{12})$ appropriate to He^4. (We shall denote this integral by w.) The value of W_N is then given by

$$W_N=\tfrac{1}{2}WN(N-1)\eta w,$$

where η is a correction factor. For $_\Lambda H^3$ one has $\eta=0.6$. The two-body results for U_s and U_t, as derived from U_2 and U_4, are to be replaced by

$$U_s+\tfrac{3}{4}(1-\eta)wW, \quad U_t-[2+\tfrac{1}{4}(1-\eta)]wW. \quad (4)$$

It is evident that U_s, and therefore a_s, is only slightly affected, while U_t, and therefore a_t, might change appreciably. The presence of a strong repulsive three-body force ($W>0$) would increase the absolute value of U_t, thereby also increasing $|a_t|$.

We now treat W as a parameter and seek the value which yields the experimental results for a_s and a_t. The correlation function is taken to be

$$\rho(r)=(\epsilon/2\pi)^{3/2}e^{-\frac{1}{2}\epsilon r^2},$$

where $\epsilon=3/2R^2$ and $R=1.44$ F is the rms radius of He^4. This form is obtained from harmonic-oscillator shell-model wave functions. For convenience in calculations we take $v(r)$ of (2) to have the Gaussian form $v(r)=e^{-\beta r^2}$. The choice of β is not self-evident and we shall assume a series of values for it. For the present let us fix it by requiring that $e^{-\beta r^2}$ and $e^{-\mu r}/\mu r$ (where μ^{-1} is the pion Compton wavelength) have the same intrinsic range. This leads to $\beta=0.234$ F^{-2}. The result for w turns out to be

$$w=(2\pi/\beta)^{3/2}\int\rho(r)e^{-\frac{1}{2}\beta r^2}d^3r=\left[\frac{\pi\epsilon}{2\beta(\beta+\epsilon)}\right]^{3/2}=11.4\text{ F}^3. \quad (5)$$

From (4) one then finds that $W\sim8$ MeV would shift U_t to the vicintity of U_s and modify the scattering lengths, yielding $a_t\sim a_s\sim-2.0$ F. In the presence of a hard core the results are essentially the same: $|a_s|$ turns out to be slightly larger. In the next section we examine whether a three-body force with the above value of W can influence significantly the binding-energy difference between $_\Lambda He^4$ and $_\Lambda H^4$.

III. THREE-BODY FORCES AND $_\Lambda He^4-{}_\Lambda H^4$ BINDING-ENERGY DIFFERENCE

A. Charge-Symmetry-Breaking Effects

The problem of $_\Lambda He^4-{}_\Lambda H^4$ binding-energy difference was experimentally dealt with by Raymund,[17] who

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concluded that $\Delta B_\Lambda({}_\Lambda He^4 - {}_\Lambda H^4) = 0.30 \pm 0.14$ MeV. A recent investigation,[18] using more restrictive criteria of acceptable data, yields $\Delta B_\Lambda = 0.12 \pm 0.17$ MeV, which is consistent with zero. Dalitz and Von Hippel[9] concluded that CSB effects may lead to $\Delta B_\Lambda = 0.25 \pm 0.05$ MeV. This should be corrected to $\Delta B_\Lambda \sim 0.20$ MeV since more accurate values for the Σ^+-Σ^0 mass difference are available now. One assumes in their derivation that the charge-symmetry-violating $\Lambda\Lambda\pi$ vertex is generated through the electromagnetic mixing of Σ^0, Λ and η, π^0:

$$g_{\Lambda\Lambda\pi} = \left\{ -2\frac{(\Sigma^0|\delta M|\Lambda)}{M(\Sigma)-M(\Lambda)} + \frac{(\pi^0|\delta m^2|\eta)}{m^2(\eta)-m^2(\pi)} \right\} g_{\Sigma\Lambda\pi}. \quad (6)$$

(This is formula (10) of Ref. 9.) Use has been made of the $SU(3)$ relation $g_{\Sigma\Lambda\pi} = -g_{\Lambda\Lambda\eta}$. The two off-diagonal matrix elements are then calculated by postulating certain transformation properties for the electromagnetic mass operator. The results read

$$\begin{aligned} &\frac{(\Sigma^0|\delta M|\Lambda)}{M(\Sigma)-M(\Lambda)} = 0.013 \pm 0.002\,, \quad \text{(Refs. 9 and 19)} \\ &\frac{(\pi^0|\delta m^2|\eta)}{m^2(\eta)-m^2(\pi)} = -0.0105 \pm 0.0013\,, \quad \text{(Ref. 9)} \end{aligned} \quad (7)$$

and are large compared to the usual electromagnetic parameter of $\alpha/\pi = 0.0023$.

The results are not the same, however, in the quark model. Using quark wave functions which are assumed to be symmetric in their spin-isospin components, one obtains quite generally[20] that

$$(\Sigma^0|\delta M|\Lambda) = (1/2\sqrt{3})\{[M(\Xi^{*-})-M(\Xi^{*0})] - [M(\Xi^-)-M(\Xi^0)]\}.$$

Putting in the experimental values of the M's one gets

$$\begin{aligned} (\Sigma^0|\delta M|\Lambda) &= (1/2\sqrt{3})[5.7 \pm 3.0 - (6.5 \pm 1.0)] \\ &= -0.23 \pm 0.94 \text{ MeV}. \end{aligned}$$

(See Refs. 21 and 19, respectively.) (8)

Hence,

$$(\Sigma^0|\delta M|\Lambda)/[M(\Sigma)-M(\Lambda)] = -0.003 \pm 0.012.$$

The significance of this number is of course very doubtful because of the large experimental error. The same procedure applies to η, π^0 mixing. Here the further assumption is made that the nonstrange quark-antiquark electromagnetic interaction is proportional to the product of their charges. The mixing turns out to be given by

$$(\pi^0|\delta m|\eta) = \frac{\cos\beta + \sqrt{2}\sin\beta}{\sqrt{3}} \{[\delta m_n - \delta m_p] + \tfrac{1}{3}[m(\pi^+) - m(\pi^0)]\}, \quad (9)$$

where $\delta m_n - \delta m_p \sim 1.9$ MeV is known[20] from the baryon case and β is the mixing angle of η and X^0: $\eta = \eta_8 \cos\beta - \eta_1 \sin\beta$.[22] For $|\beta| < 34^0$ [$\beta = \pm 10^0$ or $\pm 23^0$ is implied by SU(3) square mass relations; for a further discussion of this question see Ref. 22], the resulting $(\pi^0|\delta m|\eta)/[m(\eta) - m(\pi)]$ is less in magnitude than before and also has the opposite sign. Thus in the quark model, the CSB contribution to the ${}_\Lambda He^4 - {}_\Lambda H^4$ binding-energy difference might be smaller than previously believed and even of opposite sign.

B. Nuclear-Structure Effects

It is well known[8] that the Λ, occupying an s orbit in the $A=4$ hypernuclei, compresses the nuclear core by interacting via two-body attractive forces with the nucleons. Thus, allowing only for a radial mode of compression, Dalitz and Downs[8] found a compression of about 11% (for nuclear stiffness of $K=60$ MeV) in the $A=4$ hypernuclei. A more realistic calculation by Herndon, Tang, and Schmid,[5] including a hard core of 0.4 F in the $\Lambda - N$ interaction, yields about the same amount of compression. The additional Coulomb repulsion between the two protons of ${}_\Lambda He^4$, when brought nearer on the average, contributes about -0.1 MeV to ΔB_Λ. This is in the opposite direction that of the experimental values. On the other hand a repulsive three-body ΛNN interaction induces repulsive two-body forces between the nucleons, which tend to expand the nuclear core. On the whole, the nuclear core may retain its previous dimensions and the additional Coulomb interaction may drop out. We shall now investigate this effect with the asymptotic form of the three-body force discussed earlier. We allow for variations of the nuclear core only through the radial mode. The neglect of other modes does not seem to constitute a serious deficiency, since the additional Coulomb interaction is determined mostly by this mode. However, a drawback of our calculation is that we have to assume some form of the three-nucleon wave function and some value for the nuclear stiffness. We also note that the noncentral short-range components of the three-body force are neglected here. It is felt that only the long-range part of this force may further correlate the nucleons in addition to the correlation caused by the hard core of the nucleon-nucleon interaction.

Let H_0 stand for the nuclear three-body Hamiltonian (in the c.m. system) and ψ_0 its eigenfunction corresponding to the ground state: $H_0\psi_0 = E_0\psi_0$. H is the total Hamiltonian of the $A=4$ hypernucleus and we

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shall assume that to a good approximation the total wavefunction may be written as a product $\psi\varphi(r)$, where r is the Λ coordinate relative to the center of mass of the three nucleons. ψ is taken to differ from ψ_0 only by its radial extension, while $\varphi(r)$ is taken as the best trial wave function of the form

$$\varphi(r)=L(e^{-\nu r^2}+ze^{-xr^2})$$

given by Dalitz[23] using only central attractive two-body Λ-N interactions of a Gaussian type. L is a normalization constant and the other parameters are $z=0.336$, $\nu=0.277$ F^{-2}, $x=0.045$ F^{-2}. Now the total hypernuclear energy is approximately given by

$$E=(\psi\varphi,H\psi\varphi)=(\psi\varphi,H_0\psi\varphi)+(\psi\varphi,(H-H_0)\psi\varphi),$$

which for small radial variations around ψ_0 may be expanded as

$$E=E_0+\tfrac{1}{2}K\delta^2+(\psi\varphi,(H-H_0)\psi\varphi). \quad (10)$$

Here $\delta=R/R_0-1$, where R_0 and R are the rms radii of the free nuclear core and the compressed one, respectively. K is the nuclear-stiffness coefficient estimated as $K=60$ MeV for He^3.[8] The matrix element on the right-hand side of (10) consists of the Λ kinetic energy relative to the nuclear center of mass and the expectation values of the two-body and the three-body Λ-N interaction. The kinetic energy is independent of δ, while the interaction terms depend on δ through the δ dependence of ψ. Expanding the right-hand side of (10) in powers of δ and neglecting higher powers than the second, we shall find the value of δ which minimizes the energy E. We shall not attempt to reproduce E in this manner, since the ψ's we shall use are known to give poor agreement with binding energies. However, we shall choose ψ's so as to reproduce quite well the electromagnetic form factors and Coulomb energy of the nuclear three-body system. Following Schiff[24] we take $\psi_0=Ae^{-\frac{1}{2}\alpha^2(r_{12}^2+r_{23}^2+r_{31}^2)}$, $\alpha=0.384$ F^{-1}. Our ψ's are then of the form $\psi=Ce^{-\frac{1}{2}\gamma^2(r_{12}^2+r_{23}^2+r_{31}^2)}$, where $\gamma(1+\delta)=\alpha$. A and C are normalization constants. The two-body Λ-N interaction, averaged with the spin functions of ${}_{\Lambda}He^4$, is taken to be of a Gaussian form

$$V_2(i,\Lambda)=\tfrac{1}{3}U_3(b/\pi)^{3/2}e^{-br_{i\Lambda}^2}.$$

The range parameter $b=0.935$ F^{-2} corresponds to two-pion exchange. The $v(r)$ of (2) is also taken in a similar form: $v(r)=e^{-\beta r^2}$, $\beta=\beta_0\equiv0.234$ F^{-2} for one-pion exchange. All calculations are performed analytically in triangular coordinates. With the values thus chosen for the parameters we get

$$\begin{aligned}E-E_0=7.73\text{ MeV}&+(0.01038\,U_3+0.292\,W)\\&-(0.00851\,U_3+0.3634\,W)\delta\\&+(\tfrac{1}{2}K-0.0044\,U_3-0.054\,W)\delta^2. \quad (11)\end{aligned}$$

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TABLE I. The equilibrium δ and the corresponding additional Coulomb repulsion in ${}_{\Lambda}He^4$. β is the three-body-force range parameter. $\beta_0=0.234$ F^{-2}.

β	$-\delta$ $K=40$ MeV	$-\delta$ $K=60$ MeV	$-\delta$ $K=80$ MeV	ΔE_c (MeV) $K=40$ MeV	ΔE_c (MeV) $K=60$ MeV	ΔE_c (MeV) $K=80$ MeV
No three-body force	0.186	0.132	0.103	0.169	0.114	0.087
β_0	0.129	0.092	0.071	0.112	0.076	0.058
$2\beta_0$	0.073	0.054	0.043	0.060	0.043	0.034
$4\beta_0$	0.031	0.024	0.020	0.024	0.019	0.015

K and W should be given in MeV while U_3 in MeV F^3. The equilibrium δ is therefore determined by the relation

$$\delta=\frac{0.00851\,U_3+0.3634\,W}{K+2(-0.0044\,U_3-0.054\,W)}. \quad (12)$$

For U_3 and W we take the values discussed in Sec. II in connection with the scattering-length problem, namely $W\sim8$ MeV, $U_3=\frac{3}{2}(U_s+U_t)\approx-3\times360=-1080$ MeV F^3. Calculations were performed for $K=40$, 60, and 80 MeV. The results are shown in Table I. Also indicated there are the results for different three-body range parameters, $\beta=2\beta_0$ and $\beta=4\beta_0$, which may correspond to the average of the inner part of the three-body interaction. By increasing β, the value of the integral w [see Eq. (5)] decreases. Since it is the product wW which appears in the scattering-length problem [see Eq. (4)], the calculations are now performed taking also different values for W: $W=30$ MeV for $\beta=2\beta_0$ and $W=136$ MeV for $\beta=4\beta_0$.

The results of Table I show clearly that the three-body interaction can reduce considerably the compression of the nuclear core. Thus, the additional Coulomb interaction in ${}_{\Lambda}He^4$ may contribute to ΔB_Λ only about -0.05 MeV and even less. It is interesting to point out in this connection a qualitative difference between the two-body interaction and the three-body one. The contribution of the two-body interaction to the binding energy is dominated by its δ-independent part, while the contribution to the compression is determined by its term linear in δ. The same applies of course to the three-body interaction. Now, the ratio of the δ-independent term to the coefficient of the linear term in δ is [from (11)] 1.22 for the two-body interaction, while it is only 0.80 for the three-body interaction. For $\beta=2\beta_0$ and $\beta=4\beta_0$ the difference is even more pronounced, the ratios being 0.55 and 0.43, respectively. This means that three-body ΛNN interactions which induce two-body interactions among nucleons are more capable of correlating the nucleons than two-body ΛN interactions. The latter induce only a single-nucleon potential which affects the energy strongly. Unlike the results of calculating the equilibrium δ (for two-body Λ-N forces alone) in Ref. 8, we find that the results are quite sensitive to the value assumed for the stiffness coefficient. In particular, the value $K=40$ MeV seems to yield appreciably larger values for δ than those of other treatments.[5,8] The other two values, $K=60$ MeV and $K=80$ MeV are in a good agreement with Refs. 5 and 8.

IV. DISCUSSION

From the results of the previous section it is clear that a repulsive three-body ΛNN interaction, when taken strong enough to give the correct Λ-N scattering lengths, might considerably diminish the compression of the nuclear core due to the two-body Λ-N force. The ${}_\Lambda He^4-{}_\Lambda H^4$ binding-energy difference due to the additional Coulomb repulsion between the two protons of ${}_\Lambda He^4$ is thereby reduced to only about -0.05 MeV. We also remark that if initially He^3 is more diffuse than H^3, then the repulsive ΛNN three-body interaction in the $A=4$ hypernuclei results, contrary to the two-body interaction, in $\Delta B_\Lambda < 0$ (in the right direction). However, unless definite conclusions about the radii of the nuclear cores He^3 and H^3 are arrived at, we prefer to deal with charge-independent wave functions.[24] The value of W used in our calculation [8 MeV for a Gaussian $v(r)$ of range parameter β_0 and therefore about twice that for a Yukawa-type $v(r)$] is quite large compared to the theoretical estimate[14] of $W\sim 2$ MeV. It was, however, pointed out by Dalitz[16] that higher order diagrams, in particular the one in which the intermediate Σ in Fig. 1 is replaced by Y^*, may modify considerably the value of W.

A large-three body repulsion is not supported by the analysis of ${}_\Lambda He^5$ and ${}_\Lambda C^{13}$ made in Ref. 13. It is concluded there that only a small three-body force is compatible with the hypernuclear binding energies. However, it seems to us desirable at the present time to extract as much information as possible from the s-shell hypernuclei alone. The p-shell ones still pose many unsolved questions.[6] We would like to mention the ${}_\Lambda Li^9$-${}_\Lambda Be^9$ large binding-energy difference of about 2 MeV.[18] Bodmer and Murphy[13] have already noted that this large value may perhaps be connected with a strong (of the same order of magnitude as ours) three-body repulsion.

The effect of the ΛNN three-body force on the nuclear extensions is not likely to show up quantitatively in heavier hypernuclei. The Λ cannot compress the p-shell nucleons to the same extent as the s-shell nucleons. Also, the relative efficiency of the three-body interaction is reduced in this case. To illustrate, we have taken two p nucleons to be represented by a shell-model harmonic-oscillator wave function

$$u_p(r) \sim (r/a_p) e^{-r^2/2a_p^2}.$$

The two nucleons are L-S coupled to $S=0$, $I=1$. (This might be the case with ${}_\Lambda He^7$ and ${}_\Lambda Be^7$). The radial extension a_p of the p nucleons is assumed to vary around 2F.[25] The linear terms in δa_p, which result from the expectation values of V_2 and V_3 are then -0.00269 U_4 and $-0.0470\,W$, respectively. $U_4=3U_t+U_s$ is expressed in MeV F^3 and W in MeV, β and b are the same as in (11), while the Λ wave function is slightly more concentrated. In (11) the ratio of two-body to three-body linear terms is 3.16, while here it is (for the same W) 10.30, indicating that mainly the two-body Λ-N interactions are responsible for the nuclear extensions.

Looking for p-shell hypernuclei belonging to the same isomultiplet, we point out that the slight ${}_\Lambda Be^7$-${}_\Lambda He^7$ binding-energy difference ($B_\Lambda({}_\Lambda Be^7)=5.94\pm0.77$ MeV, $B_\Lambda({}_\Lambda He^7)=5.06\pm0.39$ MeV)[18] is more likely to be explained as being due to the different core sizes.[25] This follows from the weak binding of the p nucleons in these core nuclei. (It should be remembered that Be^6 is particle-unstable.) On the other hand, the binding energies of ${}_\Lambda Li^8$ and ${}_\Lambda Be^8$ ($I=\frac{1}{2}$) are equal to each other[18]: 6.60 ± 0.13 MeV, compared to 6.57 ± 0.20 MeV, respectively.

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