a value of $\lambda \approx 1.0$ for Serber forces, which thus is in good agreement with the corresponding $\lambda \approx 1.45$ derived in the present study. Of course the numerical value assigned to λ should not be taken too seriously, since it is primarily decided by fitting the 0^+ state from the $s_{1/2}^2$ dominant configuration, the variation of the negative-parity energy levels with λ being too small to make any significant difference. It is also dependent on the choice of the 2^-–3^- doublet separation to calculate the potential depth and other details of the assumed interaction.

It may be noted that in the present study we have adopted rather an extreme model, in the form of the jj-coupling assumption, which, as is well known, is not strictly applicable to p-shell nuclei. Here we have specifically taken C^{12} to constitute an inert core, although the importance of the core-excited states for obtaining agreement with the ground-state energy of C^{14} is pointed out. The choice of the single-particle energies as derived from the C^{13} spectrum is also based on this consideration. One simple way of including, in part, the contributions from the core would be to treat these single-particle energies as free parameters, but it is clear that, with the limited experimental information thus far available on the levels in C^{14}, the input parameters, with such a flexible approach, would outnumber the output. It will be necessary to identify experimentally the other positive-parity states before further details of the effective interaction can be fruitfully investigated.

ACKNOWLEDGMENTS

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Deuteron Optical-Model Analysis with Spin-Orbit Potential*

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An optical-model analysis of 19 deuteron elastic-scattering angular distributions in the energy range from 12 to 26 MeV was performed both with and without a spin-orbit term included in the potential. The spin-orbit potential was of the Thomas form which had been found satisfactory for fitting the measured deuteron elastic polarization. Including the spin-orbit term always resulted in an improvement in the quality of the fits. Without the spin-orbit term, various families of potentials fitted the data equally well, but when the spin-orbit potential was added the various families were no longer equivalent in many cases. The deuteron optical-model potential which most closely approximates the sum of the neutron and proton optical-model potentials was found to give the most consistently satisfactory results for all the angular distributions.

INTRODUCTION

In a previous paper,[1] hereinafter referred to as Paper I, we reported on an optical-model analysis of data on the elastic scattering of 11- to 27-MeV deuterons from many nuclei. In general, excellent fits to the data could be obtained with an optical-model potential that did not contain any spin-orbit or tensor potential terms.[2] Upon completion of that work, some measurements on deuteron-nucleus elastic-scattering polarization were reported by Beurtey[3] and these measurements have since been successfully analyzed by Raynal[4] in terms of an optical-model potential that does include a spin-orbit potential. Raynal also analyzed the data with various tensor terms in the optical-model potential, but since the fits were not improved, he suggested that there does not at the moment seem to be any justification for the inclusion of such terms in the deuteron optical-model potential.

A few angular distributions which we reported in Paper I could not be fitted well at back angles. This was particularly true for elastic scattering from Ca at 21.6 MeV, and the availability of polarization data on this element at this energy[3] prompted a subsequent analysis of the data with an optical-model potential that includes a spin-orbit term. The result was a much improved fit to the data. The parameters so obtained were very close to those obtained by Raynal,[4] and they predicted polarizations in good agreement with the measured ones,[3]

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† Consultant.
[1] C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).
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In view of this improvement, it seemed worthwhile, even though polarization data for the other angular distributions analyzed in I were not available, to perform a new analysis with an optical-model potential that includes a spin-orbit term, at least for those cases which had not been fitted well at back angles. Of course, this type of analysis is unlikely to yield as much information as it would if polarization data were available. However, in view of the recently observed j dependence of nuclear reactions[5] and lack of success in completely reproducing the effects with a distorted-wave analysis when deuterons are involved,[6] we felt that such an analysis would give some useful information on the spin-orbit potential in the deuteron optical-model potential. At least, if such analyses were performed for a number of cases, the consistency of the parameters, or lack of consistency, would give an idea of the reliability of the technique. The basic assumption in this reasoning is that the parameters should not vary drastically from one nucleus to the next or as a function of the energy. There is no *a priori* reason for expecting such variations in the case of deuteron elastic scattering, although they are known to occur in proton elastic scattering.

The data chosen for re-analysis were the 11.8-MeV data[7] for Mg and Al and the 21.6-MeV data[8] for Mg, Ca, Ni, Cu, and Zn. In the case of Mg and Al at 11.8 MeV, it was shown in Paper I that these were the only elements for which very good fits could not be obtained at this energy when all the parameters were allowed to vary. Assuming that this is due to the fact that these elements are the lighest nuclei, we have included in the current analysis new data[9] for the light nuclei Si, S, and Ca. The 21.6-MeV data for the nuclei Mg, Ca, Ni, Cu, and Zn were selected because of the structure in their angular distributions. The fits at back angles could be improved, particularly for Mg and Ca. In fact, as was pointed out above, the 21.6-MeV Ca data were the first to be re-analyzed and led to the current analysis on the basis of Raynal's work.

For those cases in which the angular distributions were well fitted by an optical potential that does not include a spin-orbit term, it seems unlikely that in the absence of polarization data a re-analysis of this type would give reliable results concerning the spin-orbit potential; therefore, none of these data were re-analyzed. However, new data at 25.9 MeV plus a few data at 25.3 MeV have become available[10] and an analysis of these data, both with and without spin-orbit coupling, has been included. In addition, the problem of the various families of potentials discussed in I was studied further, and some sensitivity of the 26-MeV data was found for the various families.

METHOD OF ANALYSIS

The definition of the optical-model potential used is identical to the one given in Paper I, with the exception of the spin-orbit part, and is given as the sum of the following potentials:

$$\text{Real part}, -V_S f(r,r_{0S},a_S);$$

$$\text{Imaginary part}, 4a_I W_D (d/dr) f(r,r_{0I},a_I).$$

We also have a Columb potential of

$$(ze^2/2R_c)[3-(r^2/R_c^2)] \quad \text{for} \quad r \leq R_c,$$

and

$$ze^2/r \quad \text{for} \quad r > R_c.$$

The function $f(r,r_0,a)$ is the usual Woods-Saxon form factor,

$$f(r,r_0,a) = \{1+\exp[(r-r_0A^{1/3})/a)\}^{-1},$$

where A is the atomic mass of the nucleus in atomic mass units.

The imaginary potential is a surface one, with the factor $4a_I$ being introduced so that the surface form factor $4a_I\, df/dr$ has unity for its maximum value. The Coulomb potential written would be that produced by a uniform charge distribution of radius R_c. Since the results of the calculations are not sensitive to the value of R_c, during this analysis it has been set equal to $1.3A^{1/3}$.

The spin-orbit potential selected has the Thomas form:

$$\boldsymbol{\sigma}\cdot\mathbf{l}\frac{V_{So}}{r}\frac{d}{dr}f(r,r_{0S},a_S).$$

This form of spin-orbit potential is the one conventionally used in optical-model analysis. Since it was used successfully in analyzing the Ca polarization data, there was no justification for trying a different one. Although Raynal had found that slightly better fits were obtained if the spin-orbit potential radius parameter were made smaller than the one for the real potential, in accordance with what is now found[11] for proton scattering, we felt that for this analysis it was preferable to vary only the depth, V_{So}.

Several families of potential were used successfully to fit the data in Paper I, the essential difference between the families being their well depths. As in Paper I, the analysis presented here uses two of the families, referred to as families a and b, and all the parameters are varied in the automatic search code to obtain a best fit, in the χ^2 sense, to the data. Experimental errors were 5% for all data points at 11.8 and 21.6 MeV,

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[10] H. R. E. Tjin A Djie, F. Udo, and L. A. C. Koerts, Nucl. Phys. **53**, 625 (1964).
[11] F. G. Perey, in International Conference on Polarization Phenomena, Karlsruhe, September, 1965 (unpublished).

whereas the experimental errors for the 26-MeV data varied from 1 to 15%, depending upon the angle. It is important to remember that the errors are not uniform for the 26-MeV data, a point which will be discussed later.

RESULTS AND DISCUSSION

Optical-model fits were obtained for a total of 19 angular distributions, using both families of potentials *a* and *b* with and without spin-orbit coupling. The numerical values obtained for the parameters and for χ^2 are given in Tables I and II. In the case of the family of potentials *b*, Figs. 1, 2, and 3 show the results.

Before discussing some of the cases, we shall make some general remarks concerning the over-all result on the basis of the χ^2 values alone. Although the χ^2 criterion has been used for some time in optical-model analyses, it is very difficult to understand the exact meaning of the numerical value of the χ^2 since experimentalists usually do not include systematic errors in such a way as to allow the errors to be used meaningfully in a χ^2 analysis. Furthermore, very little work has been done to understand how various systematic errors might affect the fit to the angular distribution and what the effects will be on the optical-model parameters, with the exception of the absolute normalization.[12] We shall return to this point later, but we would like at this time to emphasize that we have seldom found it useful to compare numerical values of χ^2 for two angular distributions which varied somewhat differently as a function of angle. What we have found most useful is the comparison of the quality of fits made to the same angular distribution as a function of some change in the optical-model potential on the basis of the χ^2 values. The following remarks are based only on comparisons of the numerical values of χ^2 for the individual angular distributions and do not refer to their magnitude.

Let us compare first the results for the families *a* and *b* without spin-orbit coupling. Out of the 19 cases, χ^2 is smaller for 12 angular distributions when fitted with the type-*b* potential. However, we prefer to consider that a definite improvement in the quality of the fit is made only if the value of χ^2 is decreased by 30%. On this basis, eight angular distributions are fitted better with type-*b* potentials, as opposed to only one with type-*a* potentials. The remaining distributions, 50% of the cases, are somewhat indifferent to the particular set used.

With the spin-orbit coupling, the proportion is slightly changed: 14 of the angular distributions have a

TABLE I. Comparison of results of search codes on fitting data with set-*a* parameters with and without a spin-orbit potential. (All parameters, including those of the spin-orbit potential, were adjusted for a minimum χ^2.)

Energy (MeV)	Element	V_S (MeV)	r_{0S} (F)	a_S (F)	V_{So} (MeV)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
11.8	Al	46.90	1.212	0.909	4.86	18.02	1.556	0.475	1196	7.1
		42.7	1.300[a]	0.868		20.11	1.555	0.475	1210	8.5
	Si	65.95	0.954	1.110	5.21	19.41	1.516	0.542	1317	11
		55.17	1.111	1.031		18.78	1.526	0.538	1319	12
	S	63.12	1.000[a]	1.007	0.39	16.01	1.563	0.473	1202	7.3
		62.19	1.000[a]	1.009		14.85	1.582	0.502	1253	7.6
	Ca	68.42	0.966	0.897	6.51	7.25	1.576	0.643	1306	1.6
		62.52	1.041	0.945		13.90	1.498	0.495	1173	5.9
21.4	^{40}Ca	66.39	0.917	0.849	5.38	6.08	1.579	0.837	1641	5.7
		45.0	1.287	0.590		5.71	1.195	1.075	1462	17
21.6	Ni	60.99	1.044	0.939	4.39	12.25	1.382	0.710	1637	5.4
		56.3	1.093	0.923		14.37	1.377	0.689	1629	6.9
	Cu	70.78	0.970	0.965	5.89	10.58	1.399	0.773	1772	4.2
		68.3	0.984	1.049		17.15	1.373	0.668	1709	9.3
	Zn	72.08	0.965	0.998	5.65	12.22	1.370	0.768	1784	5.5
		61.6	1.071	0.973		16.42	1.345	0.709	1732	11
25.9	Fe[b]	67.37	0.927	1.046	5.39	9.70	1.394	0.825	1790	12
		69.00	0.890	1.156		15.35	1.419	0.676	1740	16
	Ni	68.60	0.912	1.110	5.00[a]	14.88	1.383	0.667	1667	20.3
		62.86	0.970	1.108	0.38	17.55	1.372	0.640	1650	19.9
		68.48	0.915	1.126		16.54	1.394	0.665	1711	29
	Zn	69.29	0.960	1.019	3.13	13.62	1.344	0.754	1786	12
		65.66	0.996	1.013		14.29	1.343	0.745	1781	19
	Zr	68.80	1.033	0.986	5.34	11.00	1.415	0.716	1963	1.5
		64.94	1.075	0.986		16.02	1.385	0.652	1909	5.6
25.3	Ag[c]	65.92	1.057	1.015	2.35	19.81	1.321	0.672	1952	8.2
		63.49	1.087	0.995		20.24	1.323	0.666	1954	8.2
25.9	Ce	66.09	1.125	0.874	4.60	11.95	1.346	0.737	2093	1.0
		64.65	1.146	0.871		14.50	1.323	0.711	2044	3.6
	Ta[c]	55.75	1.258	0.858	2.28	20.65	1.316	0.724	2320	7.4
		54.50	1.282	0.832		20.97	1.318	0.712	2311	7.3
25.3	Au	78.35	1.066	0.988	5.46	12.46	1.365	0.781	2335	2.8
		74.63	1.105	0.978		14.49	1.351	0.763	2302	5.1

[a] Value of parameter not adjusted by code. [b] Data renormalized by 1.2. [c] Data renormalized by 1.1.

[12] J. K. Dickens, Phys. Rev. **143**, 758 (1966).

TABLE II. Comparison of results of search codes on fitting data with set-*b* parameters with and without a spin-orbit potential. (All parameters, including those of the spin-orbit potential, were adjusted for a minimum χ^2.)

Energy (MeV)	Element	V_S (MeV)	r_{0S} (F)	a_S (F)	V_{So} (MeV)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
11.8	Mg	143.6	1.083	0.738	15.53	17.1	1.426	0.652	1301	2.5
		122.7	1.219	0.710		40.0	1.362	0.550	1254	3.9
	Al	101.1	1.115	0.823	9.78	20.6	1.545	0.465	1209	5.2
		120.4	0.955	0.920		28.0	1.524	0.478	1241	8.1
	Si	188.5	0.651	1.047	11.21	16.6	1.518	0.564	1331	10
		179.7	0.656	1.086		26.3	1.488	0.535	1344	11
	S	154.6	0.785	0.953	5.91	14.6	1.567	0.534	1281	4.2
		156.9	0.755	1.010		18.2	1.577	0.505	1296	6.3
	Ca	109.7	1.031	0.809	8.16	10.3	1.508	0.596	1266	1.6
		104.5	1.077	0.820		20.8	1.471	0.423	1124	6.1
21.6	Mg	73.98	1.145	0.753	11.92	11.95	1.227	0.861	1344	5.9
		61.2	1.416	0.571		17.40	1.088	0.847	1291	45
21.4	^{40}Ca	104.8	1.063	0.721	6.51	7.81	1.416	0.857	1566	2.7
		98.5	1.070	0.931		20.27	1.570	0.483	1486	19
21.6	Ni	96.31	1.119	0.735	6.37	12.63	1.261	0.842	1689	2.6
		101.4	1.064	0.856		19.52	1.330	0.683	1640	5.0
	Cu	107.8	1.039	0.819	7.25	13.00	1.321	0.799	1770	2.9
		102.8	1.055	0.892		21.06	1.348	0.676	1735	9.2
	Zn	109.7	1.028	0.840	7.14	14.14	1.297	0.815	1804	3.7
		106.5	1.038	0.886		20.32	1.307	0.736	1765	6.6
25.9	Mg[a]	79.94	1.103	0.769	8.33	11.60	1.203	0.897	1341	8.2
		76.63	1.151	0.745		11.94	1.156	0.930	1346	31
	Fe[b]	97.90	1.096	0.795	5.58	10.95	1.223	0.957	1811	4.7
		99.26	1.088	0.796		10.82	1.215	0.971	1816	7.4
	Ni	103.6	1.048	0.825	6.62	11.88	1.232	0.888	1749	16
		103.5	1.048	0.915		17.54	1.373	0.669	1719	37
	Zn	114.9	0.986	0.883	6.90	14.60	1.283	0.802	1805	9.3
		116.1	0.979	0.891		16.65	1.285	0.770	1791	13
	Zr	100.8	1.099	0.835	6.53	13.14	1.344	0.747	1956	2.0
		95.49	1.131	0.874		20.27	1.376	0.616	1911	7.5
25.3	Ag[c]	109.9	1.011	0.953	7.75	21.8	1.307	0.662	1943	9.2
		111.3	1.002	0.961		25.4	1.283	0.660	1932	9.6
25.9	Ce	95.10	1.143	0.800	6.6	13.88	1.312	0.756	2125	1.3
		94.26	1.148	0.810		17.85	1.300	0.717	2106	4.6
	Ta[c]	85.83	1.194	0.858	10.72	21.4	1.304	0.721	2302	7.8
		110.4	1.254	0.758		36.6	1.259	0.678	2297	8.5
25.3	Au	103.2	1.104	0.903	6.92	14.40	1.342	0.771	2324	2.9
		101.4	1.117	0.893		16.42	1.326	0.766	2320	3.4

[a] Data for angles less than 25° not used. [b] Data renormalized by 1.2. [c] Data renormalized by 1.1.

lower χ^2 value for the set-*b* potentials. For 10 of the angular distributions, i.e., 50% of the cases, the χ^2 values for set-*b* potentials are at least 30% lower than the corresponding values for set-*a* potentials and in no case is the χ^2 value for set *a* lower by more than 30%.

On the basis of the particular set of angular distributions chosen and with the type of spin-orbit coupling used, the set-*b* potential, which comes closer to being the sum of the standard neutron and proton optical-model potential, is definitely favored. As we shall see, for some particular cases the improvement of selecting this potential (set *b*) is very great once spin-orbit coupling is used. This is somewhat different from the previous analysis in Paper I where a large set of angular distributions and the lack of spin-orbit coupling tended to favor the set-*a* potentials slightly.

The lighest element for which we have analyzed angular distributions is Mg, and we have experimental data at all three energies. It is necessary to consider this element separately because we did not succeed in finding a potential of the type-*a* family to fit the data at all three energies. When spin-orbit coupling was not included, we did find a potential at both 11.8 and 21.6 MeV, but at 26 MeV we failed to find a minimum in the χ^2 surface in the region of the parameters of set *a*. When spin-orbit coupling was introduced at all three energies, we failed to find a minimum in the χ^2 surface corresponding to set *a* if we requested a nonzero value for the spin-orbit potential strength. With type-*b* potentials, a good improvement in the quality of the fits was found when the spin-orbit term was added, particularly at 21.6 MeV, where the χ^2 value decreased by a factor of more than 7, the improvement being mostly at back angles where the oscillations are greatly damped. Since for Mg large fluctuations in the best-fit parameters are found, it is not possible to be absolutely certain that the potentials given for Mg in Table II belong to the same family of potentials. It may be more appropriate to say that with spin-orbit terms we have failed to find potentials which would belong to a family of well depths smaller than those listed in Table II.

At 11.8 MeV spin-orbit coupling improves the fits for the set-*b* potentials by decreasing the χ^2 values by almost a factor of 2, whereas the fits for the set-*a*

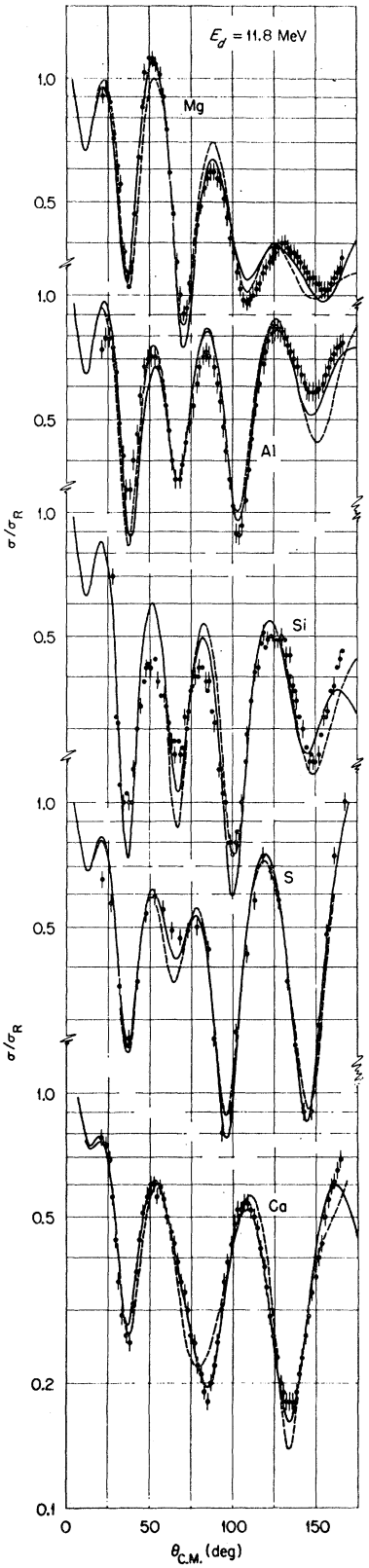

FIG. 1. Comparison of 11.8-MeV deuteron elastic-scattering angular distributions with curves from set-*b* parameters given in Table II. The dashed curves were obtained without a spin-orbit potential and allowing all six parameters of the optical model to be adjusted for a minimum χ^2. The solid curves were obtained by introducing, in addition to the other parameters, a spin-orbit potential and allowing its parameters to also be adjusted for a minimum χ^2.

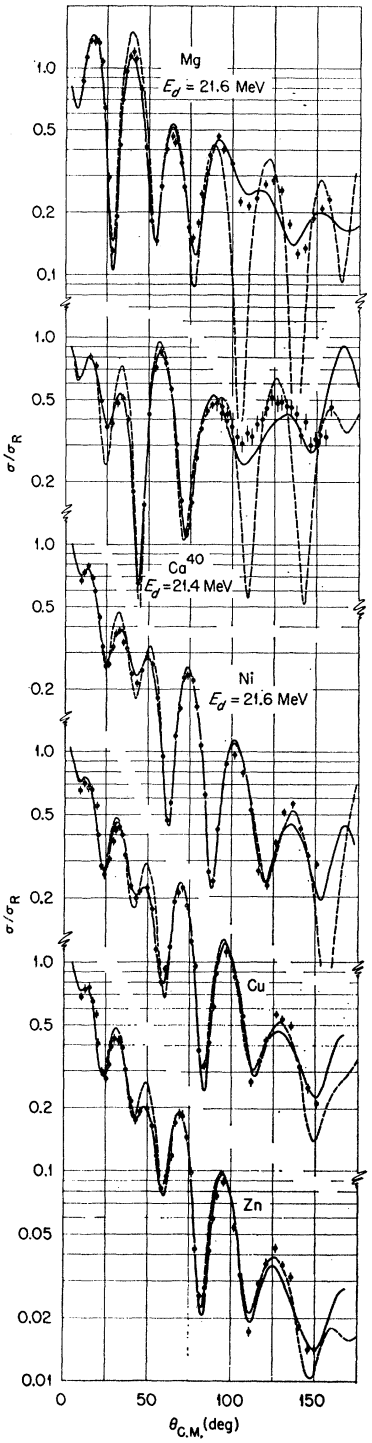

FIG. 2. Comparison of 21.4- and 21.6-MeV deuteron elastic-scattering angular distributions with curves from set-*b* parameters given in Table II. The dashed curves were obtained without a spin-orbit potential and allowing all six parameters of the optical model to be adjusted for a minimum χ^2. The solid curves were obtained by introducing, in addition to the other parameters, a spin-orbit potential and allowing its parameters to also be adjusted for a minimum χ^2.

potentials are not improved very much, with the exception of the fit for Ca, where for both potentials the χ^2 values decreased by a factor of 4. The lower χ^2 values for both sets of potentials are brought about by better fits in the angular range 50 to 100 deg. The fits are very good except for Si in the angular range 50 to 100 deg.

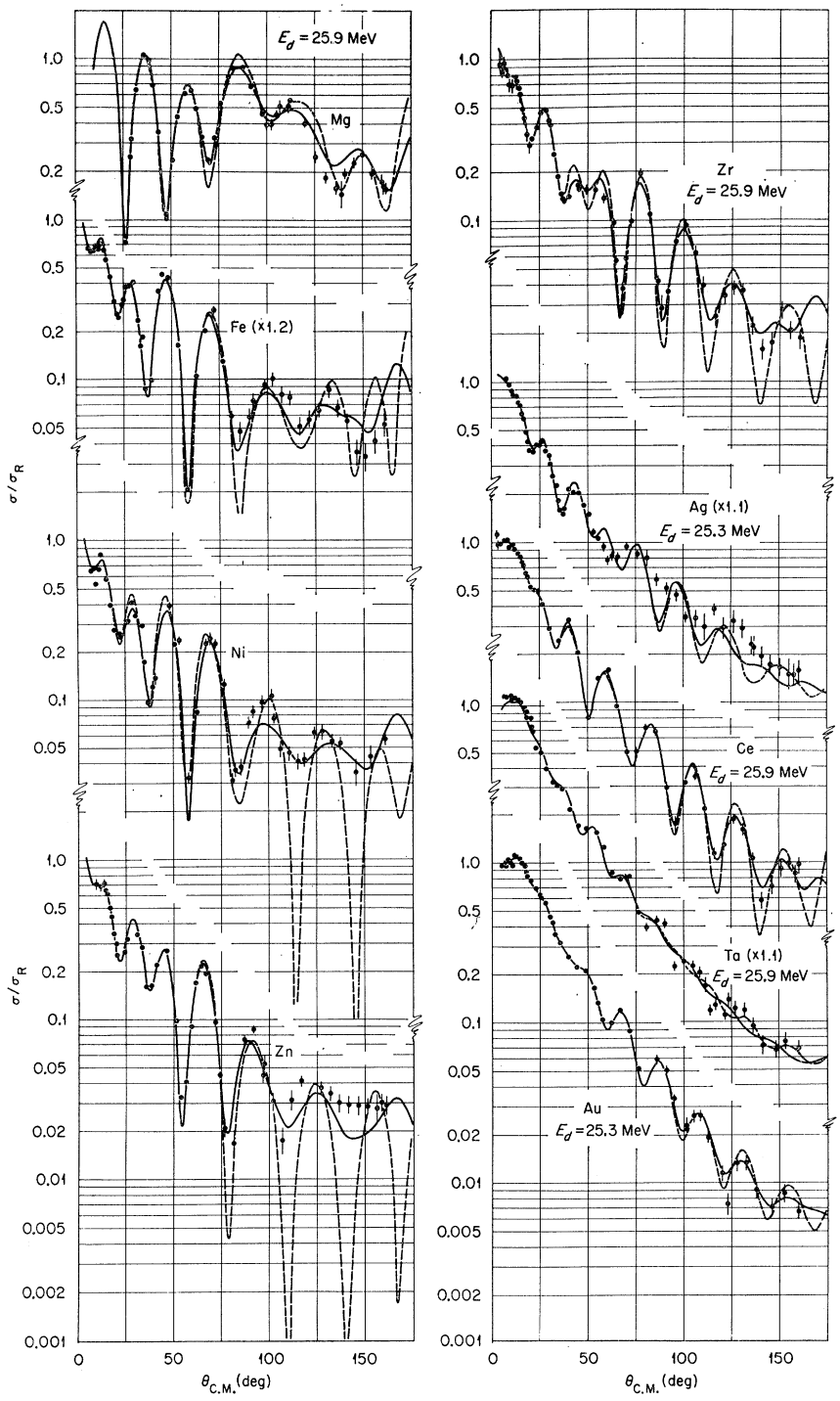

FIG. 3. Comparison of 26-MeV deuteron elastic-scattering angular distributions with curves from the set-*b* parameters given in Table II. The dashed curves were obtained without a spin-orbit potential and allowing all six parameters of the optical model to be adjusted for minimum χ^2. The solid curves were obtained by introducing, in addition to the other parameters, a spin-orbit potential and allowing its parameters to also be adjusted for a minimum χ^2. The data for Fe are renormalized by 1.2 and the data for Ag and Ta by 1.1.

At 21.6 MeV spin-orbit coupling results in very large improvements in the quality of the fits, particularly for Mg and Ca, where χ^2 decreases by more than a factor of 7 owing to a very good fit at back angles. For other distributions the χ^2 values decrease by about a factor of 2, most of which is due to a better fit in the range of 25 to 50 deg. This also occurs in the cases of Mg and Ca.

The data at 26 MeV are different in one important aspect. The errors as a function of angle vary from 1% at forward angles to 15% at back angles. At the other two energies, the errors were assumed to be 5% at

TABLE III. Results of search code on fitting some data at 26 MeV when experimental error on each point is fixed at 5%. (Parameters are from set b and are adjusted by the code for a minimum χ^2).

Energy (MeV)	Element	V_S (MeV)	r_{0S} (F)	a_S (F)	V_{So} (MeV)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
25.9	Fe[a]	101.0	1.059	0.868	6.70	13.33	1.341	0.768	1750	5.8
		101.3	1.070	0.769		10.82	1.210	0.987	1815	26.0
	Ni	96.50	1.100	0.793	6.30	11.70	1.232	0.889	1746	14.0
		93.96	1.116	0.859		26.90	1.330	0.547	1560	22.0
	Zn	124.9	0.934	0.876	5.72	12.95	1.265	0.885	1865	11.0
		125.4	0.922	0.953		19.08	1.297	0.710	1754	33.0
	Zr	102.2	1.088	0.851	6.44	15.12	1.353	0.686	1906	5.95
		98.42	1.095	0.919		23.16	1.364	0.614	1937	10.0

[a] Data renormalized by 1.2.

all angles; i.e., there was uniform weighting at all angles. We shall investigate later the effect of the uniform weighting on the fits but will first discuss the fits with the nonuniform weighting.

The experimental angular distributions for Ag and Ta are lacking completely in structure for angles greater than 75 deg. The disappearance of structure in the angular distribution in the region of Ag was noted[13] at 11.8 MeV and attributed to the excitation of collective levels. However, if such an explanation adequately explained a similar effect observed in proton scattering at 17 MeV, one would have to admit that the effect is much stronger in the case of deuteron scattering in view of the large differences between the Ag and Ta data. In the absence of more complete data to clarify the question, we shall ignore the results for At and Ta as unlikely to shed light on the question of the spin-orbit potential in the deuteron optical model. We shall only note that the fits at forward angles are very satisfactory but that we had to renormalize the data in order to obtain such good fits. While on the subject of normalization, we should state that we also found it necessary to renormalize the data for Fe by 20% in order to obtain good fits at forward angles.

Without spin-orbit coupling, the curves in general show too much oscillation at backward angles, particularly for Ni and Zn. The data for Fe and Ni are qualitatively the same, but it was impossible to find a set of parameters to fit the Ni data which would not have the large oscillations shown. The introduction of the spin-orbit term in the potential improved the fit greatly at back angles. With the exception of the Zr

TABLE IV. Results of search code on fitting data at 26 MeV by allowing all the parameters to be adjusted for a minimum χ^2 with three consecutive sets of parameters.

Energy (MeV)	Element	Type	V_S (MeV)	r_{0S} (F)	a_S (F)	V_{So} (MeV)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
25.9	Fe[a]	a	67.37	0.927	1.046	5.39	9.70	1.394	0.825	1790	12
		b	97.90	1.096	0.795	5.58	10.95	1.223	0.957	1811	4.7
		c	142.4	1.138	0.707	8.51	14.39	1.096	1.004	1832	9.8
	Ni	a	68.60	0.912	1.110	5.00[c]	14.88	1.383	0.667	1667	20
		b	103.6	1.048	0.825	6.62	11.88	1.232	0.888	1749	15.7
		c	143.6	1.128	0.695	8.72	15.25	1.045	0.999	1775	16.5
	Zn	a	69.29	0.960	1.019	3.13	13.62	1.344	0.754	1786	12
		b	114.9	0.986	0.883	6.90	14.60	1.283	0.802	1805	9.3
		c	163.1	1.029	0.778	8.53	17.09	1.192	0.851	1814	9.6
	Zr	a	68.80	1.033	0.986	5.34	11.00	1.415	0.716	1963	1.5
		b	100.8	1.099	0.835	6.53	13.14	1.344	0.747	1956	2.0
		c	138.9	1.134	0.743	7.98	15.58	1.266	0.793	1969	2.8
25.3	Ag[b]	a	65.92	1.057	1.015	2.35	19.81	1.321	0.672	1952	8.2
		b	109.9	1.011	0.953	7.75	21.8	1.307	0.662	1943	9.2
		c	171.3	0.940	0.932	8.50[c]	22.5	1.296	0.668	1944	9.9
			171.3[c]	0.940[c]	0.932[c]	12.74	19.46	1.313	0.683	1966	9.2
25.9	Ce	a	66.09	1.125	0.874	4.60	11.95	1.346	0.737	2093	1.0
		b	95.10	1.143	0.800	6.60	13.88	1.312	0.756	2125	1.3
		c	128.1	1.162	0.719	8.40	15.29	1.256	0.824	2173	2.0
	Ta[b]	a	55.75	1.258	0.858	2.28	20.65	1.316	0.724	2320	7.4
		b	85.83	1.194	0.858	10.72	21.4	1.304	0.721	2302	7.8
		c	113.8	1.221	0.790	11.58	27.6	1.284	0.691	2297	8.2
25.3	Au	a	78.35	1.066	0.988	5.46	12.46	1.365	0.781	2335	2.8
		b	103.2	1.104	0.903	6.92	14.40	1.342	0.771	2324	2.9
		c	134.8	1.111	0.851	8.81	15.87	1.324	0.773	2325	3.0

[a] Data renormalized by 1.2. [b] Data renormalized by 1.1. [c] Value of parameter not adjusted by code.

[13] A. E. Forest, Phys. Letters 7, 130 (1963).

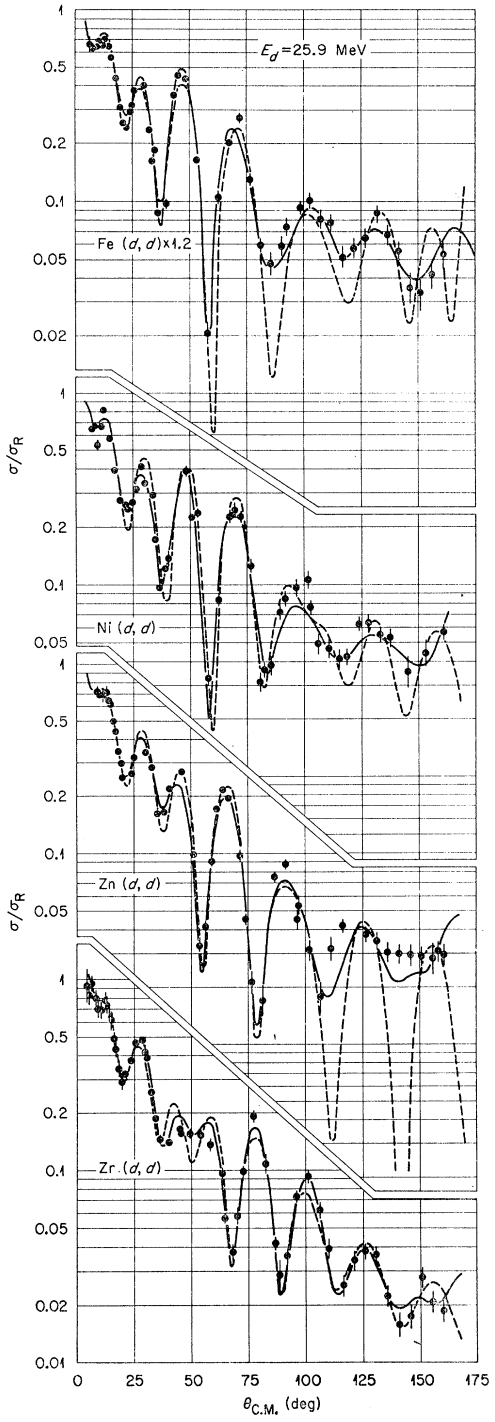

FIG. 4. Comparison of some 26-MeV deuteron elastic-scattering angular distributions with curves obtained by setting, in the search program, a 5% experimental error on each data point. The parameters obtained are from set *b* and are given in Table III. The dashed curves were obtained without a spin-orbit potential and allowing all six parameters of the optical model to be adjusted for a minimum χ^2. The solid curves were obtained by introducing, in addition to the other parameters, a spin-orbit potential and allowing its parameters to also be adjusted for a minimum χ^2.

data, the fits up to at least 75 deg are identical with and without spin-orbit coupling, probably due to the very small errors associated with points at angles less than 75 deg. For Zr the data around 50 deg are better fitted with spin-orbit coupling.

An attempt was made to investigate the effect of the variation of the weighting as a function of angle and a new search for optical-model parameters was made on Fe, Ni, Zn, and Zr with a 5% error assigned at all angles. The parameters so found in the case of set-*b* potentials are given in Table III, and the comparisons with the data are shown in Fig. 4. As might have been expected, the fits at forward angles are not as good but they are better at back angles. In the case of Zn, the weighting at back angles is not sufficient to prevent

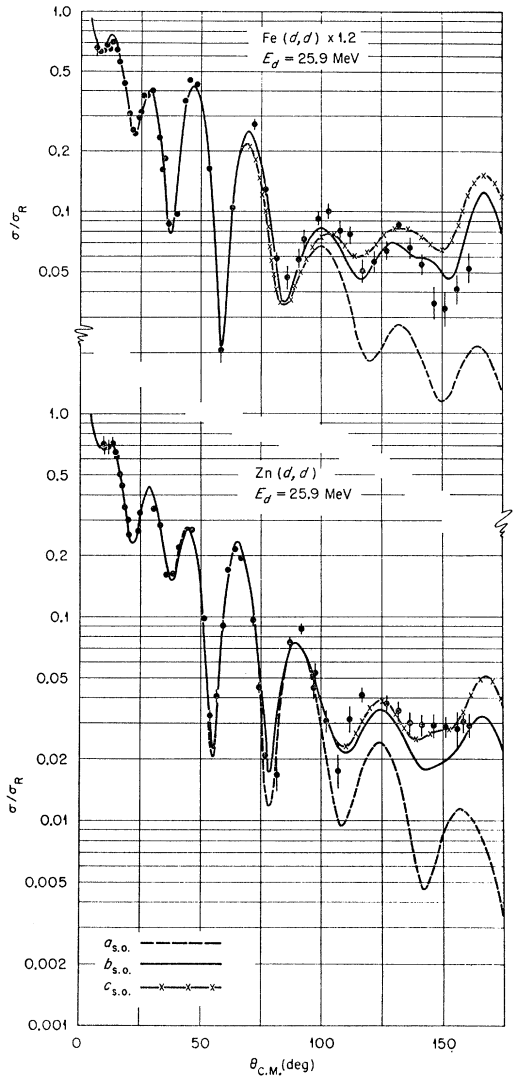

FIG. 5. Comparison of 26-MeV deuteron elastic-scattering angular distributions for Fe and Zn with curves obtained from three sets of parameters given in Table IV. All the parameters of the potentials were adjusted by the code for a minimum χ^2.

large oscillations without spin-orbit coupling. The spin-orbit coupling not only improves the fit at back angles but also at angles in the range of 25 to 50 deg. The ratios of the χ^2 values with and without spin-orbit coupling are about the same under the two different weighting conditions.

Since the analysis as reported so far seemed to favor type-*b* potentials, we decided to determine now what the next deeper well, which we shall call the type-*c* potential, would do. Searches were made for all the 26-MeV data. The parameters and the χ^2 values obtained for all three types of potentials are shown in Table IV. In no case did the type-*c* potential give as good a fit as the type-*a* or type-*b* potential. One remarkable feature of the fits obtained by the three types of potentials is shown in Fig. 5 and concerns the back-angle behavior of the curves: the deeper the potential wells the larger the cross sections at back angles. Judging from the back-angle cross-section data for elements up to Zr, the calculated cross sections are always too small for the set-*a* potentials, and, except for Zn, they are slightly too large for the set-*c* potentials. For the heavier elements, the differences in fits at back angles are not as large, as reflected by very similar χ^2 values. The predictions of the vector polarization corresponding to the three types of potentials for Fe and Zn are shown in Fig. 6. Differences in the predictions are found only at back angles.

CONCLUSIONS

The introduction of a spin-orbit term of the Thomas form in the optical-model potential improves significantly the fit to the deuteron elastic-scattering angular distributions. The addition of the spin-orbit term largely affects the imaginary part of the optical potential by reducing its strength appreciably. It is not clear what the exact meaning of this reduction in well depth means since it is accompanied by a bigger diffusivity of the imaginary potential which will offset some of the effect of the decreased well depth, as evidenced by the same reaction cross section found with and without spin-orbit potential. On the basis of the particular set of angular distributions analyzed, the type-*b* potential definitely gives better fits, particularly when a spin-orbit potential is used. It appears that the sensitivity of the data to the various potential depths is greater the lighter the nuclei and the higher the deuteron energy. The potential which most closely approximates the sum of the neutron and proton optical-model

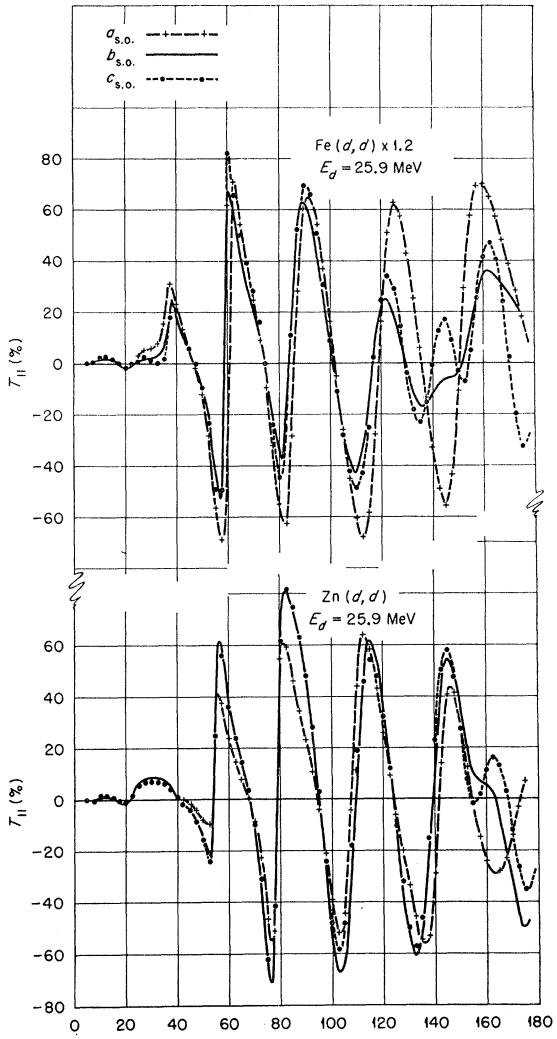

FIG. 6. Comparison of the vector polarization for Fe and Zn corresponding to the potentials given in Table IV. The differential cross sections obtained with the same parameters are shown in Fig. 5.

potentials is the only one which gives satisfactory results in all cases. The other potentials probably have no particular physical significance and result from the fact that at sufficiently low energy and for sufficiently heavy nuclei it is possible, owing to the strong absorption, to obtain for most of the low partial waves another half-wavelength in the radial wave function inside the potential and still have a phase shift close to the correct one.