consistent with the results of Lieb,[5] while those for the O^{15} 6.86-MeV level give a rough estimate of the mean lifetime: $\tau=0.10\pm0.06$ psec. It was also determined that the intensities of the gamma-ray branches from the N^{15} 7.15- and 7.56-MeV levels to the $\frac{1}{2}^+$ 5.30-MeV level were both less than 4% of their respective branches to the $\frac{5}{2}^+$ 5.27-MeV level.

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Effective Interactions in C^{14}

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Conventional shell-model calculations on the assumption of *jj* coupling have been made for the level spectrum of C^{14} considering C^{12} as the core. The three different choices of the exchange mixture considered are (a) *s*-state interaction only, (b) singlet-even interaction only, and (c) singlet-even and triplet-odd interaction. The configuration-mixing effects are also studied and are found to be insignificant. The energy levels are satisfactorily explained with an attractive but weak singlet-even (Serber-type) interaction operating only in *s* and *d* states, and of a fairly long Gaussian range. The importance of the *d*-state forces is pointed out and the configuration assignments to various levels are made.

1. INTRODUCTION

THE calculation of energy levels in *p*-shell nuclei has generally been carried out within the framework of the *LS*-coupling or the intermediate-coupling models. It was even remarked[1] that conventional shell-model calculations are not likely to be made for mass-14 nuclei. However, such calculations were made for N^{14} by True[2] using a central two-body interaction of the Gaussian type. The odd-parity states of C^{14} have been discussed on the basis of the usual *jj*-coupling model by several authors.[3–8] Nagarajan[9] suggested an intermediate-coupling configuration calculation for the nuclear spectrum of C^{14}. However, no detailed shell-model calculations are available so far for this nucleus. The aim of the present study is to calculate the level spectrum of C^{14} using a central two-body interaction of the Gaussian type in an effort to determine the nature (exchange character) and the parameters of the effective interaction.

The calculations are made using the method of relative coordinates outlined in Sec. 2. The experimental information available on the level scheme is presented in Sec. 3, followed by a discussion of the relevant shell-model configurations. The results of our calculations for various types of assumed interaction are given in Sec. 5 and a discussion of these results is presented.

2. METHOD OF CALCULATIONS

The most general form for the central two-body interaction can be written as

$$H_{12}=[A_0+A_1M+A_2B+A_3MB]V(r_{12}), \quad (1)$$

where M and B are, respectively, space and spin-exchange operators, (Majorana and Bartlett), and the A_k's are constants. The constants are so normalized that

$$A_0+A_1+A_2+A_3=1. \quad (2)$$

The radial dependence has been chosen to be of Gaussian shape $V_0\exp[-(r/r_0)^2]$ as a matter of convenience in computations; V_0 and r_0 being the strength and range of the potential. Since in the evaluation of the Hamiltonian matrix in a given configuration space, we shall consider states with definite isotopic spin T,

* Work supported in part by the National Research Council of Canada.

[1] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960).
[2] W. W. True, Phys. Rev. **130**, 1530 (1963).
[3] P. C. Sood and Y. R. Waghmare, Nucl. Phys. **46**, 181 (1963).
[4] F. C. Barker, Phys. Rev. **122**, 572 (1961).
[5] E. K. Warburton, H. J. Rose, and E. N. Hatch, Phys. Rev. **114**, 214 (1959).
[6] I. Unna and I. Talmi, Phys. Rev. **112**, 452 (1958).
[7] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[8] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948).
[9] M. A. Nagarajan, Nucl. Phys. **42**, 454 (1963).

so that $MB=\pm 1$, expression (1) giving the exchange character can be directly written in the form

$$H_{12}=[a+b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]V_0\exp(-r^2/r_0^2), \quad (3)$$

where

$$a=A_0\pm A_3 \quad \text{and} \quad b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=(A_2\pm A_1)B. \quad (4)$$

Putting in terms of triplet ($S=1$) and singlet ($S=0$) strengths, we get

$$H_{12}=[V_t\pi_t+V_s\pi_s]V(r_{12}), \quad (5)$$

with

$$\pi_t=\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2), \quad \pi_s=\tfrac{1}{4}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \quad (6)$$

and

$$\begin{aligned} V_t&=a+b, \quad S=1,\\ V_s&=a-3b, \quad S=0. \end{aligned} \quad (7)$$

The parameters V_t and V_s in expression (5), giving the triplet and singlet strengths of the potential, are to be evaluated from experimental data.

The matrix elements of the two-body interaction Hamiltonian can be written as

$$\langle j_1j_2:JM|H_{12}|j_1'j_2':JM\rangle. \quad (8)$$

The corresponding energy expression involving the diagonal matrix elements only is then given by

$$E(j_1j_2:J)=E(j_1)+E(j_2)+\langle j_1j_2:JM|H_{12}|j_1j_2:JM\rangle, \quad (9)$$

where the first two terms on the right are the single-particle energies. The single-particle energies (rather the relative spacings of the levels involved) appropriate for the nucleus under study, are taken from the experimental data on nuclei having closed shells (or subshells) plus one nucleon outside the closed shells (or subshells). For nuclei whose single-particle energy levels are not known it can be taken as a free parameter.[10]

Two methods have been developed to evaluate the two-body interaction Hamiltonian matrix. The method of calculations for the low-lying energy levels in terms of relative coordinates is developed by Moshinsky,[11] Lawson and Mayer,[12] Arima and Terasawa,[13] and some others.[14] The other method of Slater integrals (F^k) is due to Talmi,[15] supplemented by Thieberger's tables[16] of the necessary coefficients of expansion. The calculations in the present study are made in terms of the relative coordinates. This method has been illustrated in the various recent papers[17,18] and is briefly outlined below.

[10] Y. R. Waghmare, R. K. Gupta, and N. Kumar, Progr. Theoret. Phys. (Kyoto) **31**, 765 (1964).
[11] M. Moshinsky, Nucl. Phys. **13**, 104 (1959).
[12] R. D. Lawson and M. G. Mayer, Phys. Rev. **117**, 174 (1960).
[13] A. Arima and T. Terasawa, Progr. Theoret. Phys. (Kyoto) **23**, 115 (1960).
[14] A. N. Mitra and S. P. Pandya, Nucl. Phys. **20**, 455 (1960).
[15] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).
[16] R. Thieberger, Nucl. Phys. **2**, 533 (1956).
[17] S. K. Shah and S. P. Pandya, Nucl. Phys. **38**, 420 (1962).
[18] S. P. Pandya, Nucl. Phys. **43**, 636 (1963); Y. R. Waghmare, Phys. Rev. **136**, B1261 (1964).

TABLE I. The matrix elements $I_{nl}=\langle nl|e^{-(r/r_0)^2}|nl\rangle$.

I_{nl} \ λ	1.0	1.2	1.4	1.6
I_{0s}	0.3536	0.4535	0.5388	0.6098
I_{1s}	0.2210	0.2725	0.3285	0.3875
I_{2s}	0.1721	0.2127	0.2511	0.2938
I_{3s}	0.1500	0.1803	0.2117	0.2458
I_{0p}	0.1768	0.2676	0.3568	0.4385
I_{1p}	0.1547	0.2052	0.2582	0.3133
I_{2p}	0.1359	0.1743	0.2139	0.2544
I_{0d}	0.0884	0.1580	0.2363	0.3153
I_{1d}	0.0994	0.1481	0.1980	0.2501
I_{2d}	0.0988	0.1367	0.1761	0.2156
I_{0f}	0.0422	0.0932	0.1564	0.2267
I_{1f}	0.0608	0.1032	0.1489	0.1978
I_{0g}	0.0221	0.0550	0.1036	0.1631
I_{1g}	0.0359	0.0698	0.1104	0.1551
I_{0h}	0.0110	0.0325	0.0686	0.1173
I_{0i}	0.0055	0.0192	0.0454	0.0843

As we are interested in the relative angular-momentum states, we first transform the jj-coupling wave functions in expression (8) into the LS coupling and then transform the space part into the relative and center-of-mass coordinates. The two-body Hamiltonian matrix elements for the central forces (i.e., $l=l'$ and $S=S'$) then can be written as

$$\begin{aligned}\langle j_1j_2:JM|H_{12}{}^c|j_1'j_2':JM\rangle &= \sum_{LL'SN\Lambda nln'} A\begin{bmatrix} l_1 & s_1 & j_1\\ l_2 & s_2 & j_2\\ L & S & J\end{bmatrix} A\begin{bmatrix} l_1' & s_1' & j_1'\\ l_2' & s_2' & j_2'\\ L' & S & J\end{bmatrix}\\ &\times B_{N\Lambda nl}{}^{n_1l_1n_2l_2}(L)B_{N\Lambda n'l'}{}^{n_1'l_1'n_2'l_2'}(L')\delta_{LL'}\\ &\times\langle nl,S|H_{12}{}^c|n'l,S\rangle, \end{aligned} \quad (10)$$

where the A's are the LS-jj transformation coefficients[19] (9-j symbols) and the B's are the Moshinsky brackets.[20] The radial matrix element, denoted by I_{nl}, is given by

$$I_{nl}=\langle nl,S|H_{12}{}^c|nl,S\rangle=\int_0^\infty R_{nl}{}^2(r)V(r)r^2dr. \quad (11)$$

If we introduce a range parameter λ, defined as $\lambda=r_0/r_l$, r_0 being the range of the Gaussian potential and r_l that of nucleon wave function, the radial matrix elements I_{nl} can be calculated[16,21] as a function of λ. This is the only parameter coming in our present calculations. Table I gives the values of I_{nl} for λ varying from 1.0 to 1.6 required for the present calculations. This is in supplement to the table given in our previous publication.[10] This makes the evaluation of the Hamiltonian matrix elements quite simple as the tables for all the quantities now become available.

[19] J. M. Kennedy and M. J. Cliff, Chalk River Report No. C.R.T.-609, Chalk River, Ontario, 1955 (unpublished).
[20] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Instituto de Fesica, Mexico City, 1960).
[21] S. K. Shah, Ph.D. thesis, Gujrat University, Baroda, India, 1962 (unpublished).

Next we consider the simplification of the expression (10) for various types of the two-body forces. Taking odd or even values for l and S, the nuclear interactions for central forces can be further divided into four categories: singlet-even ($S=0$, $l=$even), singlet-odd ($S=0$, $l=$odd), triplet-even ($S=1$, $l=$even), and triplet-odd ($S=1$, $l=$odd). Various possible mixtures of these strengths are used for different calculations and we discuss a few useful cases for our purpose.

For singlet forces, the A coefficients give terms of the type δ_{LJ} and $\delta_{L'J}$. If the two particles are equivalent (i.e., both protons or both neutrons), then one obtains only l-even forces, i.e., singlet-even forces (Serber-type forces). In a similar way for triplet forces ($S=1$), one would obtain only odd-l states. For light nuclei ($A\leqslant 50$) the isotopic spin T also becomes a good quantum number.[22] If we consider the two-nucleon states with $T=1$ only, as in our case, then the matrix elements of the interaction need be considered only in singlet-even and triplet-odd states.

Still further simplification to only the s-state ($l=0$) interaction would give the term δ_{AJ}, as is easily seen from expression (10).

3. EXPERIMENTAL LEVEL SCHEME

The experimental energy levels for this nucleus with excitation energy up to 8.32 MeV are shown in Fig. 1(a). There are several known levels[23] above 9.8-MeV excitation energy without any definite spin-parity assignment which we have not included in our discussion. In the following we comment briefly on the spin-parity assignments to the levels shown in Fig. 1(a) which have been added or modified since the 1962 compilation of Lauritsen and Ajzenberg-Selove.[23]

The 6.58-MeV level has been definitely shown to have the spin-parity $J^\pi=0^+$ from the pair-spectrometer measurements.[24,25]

The recent proton-gamma angular-correlation measurements[26] provide practically conclusive evidence for the assignments $J^\pi=3^-$ for the 6.72-MeV level and $J^\pi=2^-$ for the 7.34-MeV level. This assignment for the 6.72-MeV level is also favored in the pair-spectrometer studies.[25]

The parity of the 6.89-MeV level has recently been confirmed[27] to be the same as that of the 6.09-MeV level by establishing the $M1$ character of the 0.81-MeV cascade transition through measurement of its plane polarization.

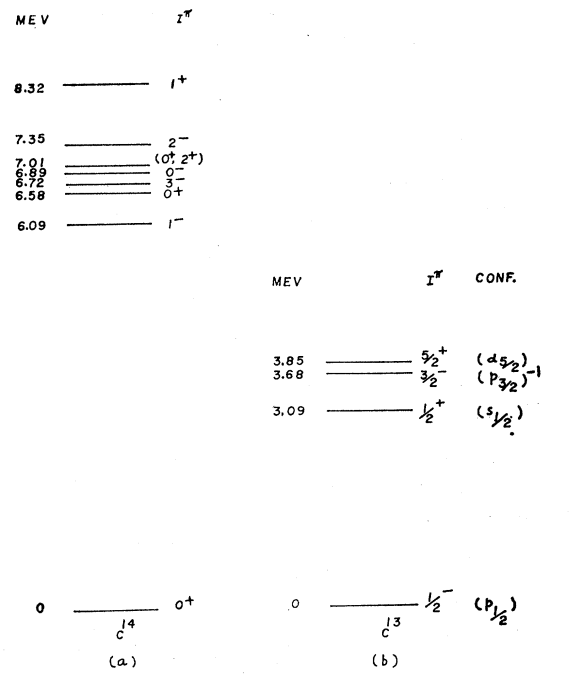

FIG. 1. The experimental energy levels of C^{14} and C^{13} nuclei. (G.S.≡ground state; conf.≡configuration).

Alburger and Warburton[24] believe that the assignment 0^+ to the 7.01-MeV level made from a fit to the (t,p) angular distribution[28] should not be taken as definite, and they present "strong indirect evidence" in favor of the assignment $J^\pi=2^+$.

Mention may be made of the recent proton angular-distribution studies from (t,p) reaction[29] which give several L-value assignments in conflict with known spins and parities for C^{14}.

4. SHELL-MODEL CONFIGURATIONS

For our calculations we consider this nucleus as having a C^{12} inert core. Then the single-particle energies are simply obtained from the low-lying levels of C^{13} as shown in Fig. 1(b). In terms of pure configurations, the ground state of C^{14} arises from two neutrons in $p_{1/2}$ orbit, and the excited levels result from the excitation of one or both of the $p_{1/2}$ neutrons to the d-s shell. Leaving out the $d_{3/2}$ level which lies about 5 MeV above the $d_{5/2}$ level, we get the following configurations:

$$(p_{1/2}s_{1/2})_{1^-,0^-};\quad (p_{1/2}d_{5/2})_{3^-,2^-};$$
$$(s_{1/2})^2_{0^+};\quad (s_{1/2}d_{5/2})_{2^+,3^+};\quad (d_{5/2})^2_{0^+,2^+,4^+}.$$

Further, the core itself may be considered as soft, giving

[22] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962).
[23] T. Lauritsen and F. Ajzenberg-Selove, in *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1962).
[24] D. E. Alburger and E. K. Warburton, Phys. Rev. **132**, 790 (1963).
[25] E. K. Warburton, D. E. Alburger, A. Gallmann, P. Wagner, and L. F. Chase, Jr., Phys. Rev. **133**, B42 (1964).
[26] J. M. Lacambra, D. R. Tilley, N. R. Roberson, and R. M. Williamson, Nucl. Phys. **68**, 273 (1965).
[27] F. Riess and W. Trost, Nucl. Phys. **78**, 385 (1966).
[28] A. A. Jaffe, F. De Barros, P. D. Forsyth, J. Muto, I. J. Taylor, and S. Ramavataram, Proc. Phys. Soc. (London) **76**, 914 (1960).
[29] R. Middleton and D. J. Pullen, Nucl. Phys. **51**, 63 (1964).

TABLE II. Shell model configuration assignments to various energy levels in C^{14}.

Energy (MeV)	J^π experimental	Dominant configuration
0	0^+	$(p_{1/2}^2)$
6.09	1^-	$(p_{1/2}s_{1/2})$
6.58	0^+	$(s_{1/2}^2)$
6.72	3^-	$(p_{1/2}d_{5/2})$
6.89	0^-	$(p_{1/2}s_{1/2})$
7.01	$(0^+, 2^+)$	core-excited
7.34	2^-	$(p_{1/2}d_{5/2})$
8.32	1^+	core-excited

rise to the so-called core-excited states; in our case such states result from the excitation of one or two $p_{3/2}$ protons to the $p_{1/2}$ orbit giving the configurations

$$(p_{3/2}^{-1}p_{1/2})_{2^+,1^+}; \quad (p_{3/2}^{-2}p_{1/2}^2)_{0^+,2^+}.$$

The configuration assignments to the various experimental levels in C^{14} have usually been made[1] by comparison with the analog $T=1$ states in the spectrum of N^{14} which has been very widely studied and analyzed. In his study of N^{14}, True[2] concluded that the eigenfunctions for practically all the states are quite pure jj two-particle wave functions, and this is borne out in our study as well. There does not seem to be any ambiguity in assigning the $p_{1/2}$ doublet configuration to the four negative-parity states in C^{14}. The 2.31-MeV 0^+, $T=1$ state in N^{14} which is the analog of C^{14} ground state has been found to be almost pure $p_{1/2}^2$ configuration in all the studies.[1,2,7] The 8.63-MeV 0^+, $T=1$ state in N^{14}, which is the analog of the 6.58-MeV 0^+ level in C^{14}, has been shown to have the configuration $s_{1/2}^2$ by True.[2] Unna and Talmi[6] obtained the binding energy of this level to an accuracy of 140 keV assuming this configuration. Alburger and Warburton[24] have suggested a more complicated wave function with possible contributions from the core.

The 7.01-MeV state in C^{14} has been suggested[24] to be the analog of the 9.17-MeV, 2^+, $T=1$ state in N^{14} which has been recognized as the core-excited state.[2] The relative position of the other member of the $(p_{3/2}^{-1}p_{1/2})$ doublet may be approximately calculated from Eqs. (51) and (52) given by Talmi and Unna[30]; the doublet separation is estimated to be about 1.1 MeV with the 1^+ state lying higher than the 2^+ state. Thus the observed 8.32-MeV state may be the corresponding 1^+ core-excited state, leaving its analog in N^{14} around 10.4 MeV still unobserved. On the other hand, the assignment 0^+ to the 7.01-MeV level would suggest the core-excited configuration $p_{3/2}^{-2}$. This state is formed by $C^{12}(t,p)C^{14}$ double stripping and it is unlikely that the configuration $p_{3/2}^{-2}$ would be strongly formed by such a reaction.[28] Further, an intermediate-coupling calculation due to Wilmore[31] suggests that the lowest 0^+ state of C^{14} arising from such a configuration would have an excitation of at least 12 MeV.

The shell-model dominant configurations for the levels in C^{14} are summarized in Table II; as shown in the next section, these assignments are in agreement with our calculations.

Keeping the core as inert, we have also investigated the effect of the configuration mixing on the 0^+ state considering the admixture of $s_{1/2}^2$ and $d_{5/2}^2$ configurations. The effect on the energy of this level is found to be unimportant.

TABLE III. Variation of the potential strength V given in MeV with the range parameter λ: Case (a) refers to the S-state ($l=0$) interaction V_0, case (b) to the singlet-even (Serber-type) interaction V_s, and case (c) to the singlet-even and triplet-odd (p-state) interactions V_s and V_t.

Case	V (MeV) \ λ	1.0	1.2	1.4	1.6
a	$-V_0$	14.3	11.1	9.4	8.3
b	$-V_s$	15.0	12.0	10.3	9.2
c	$-V_s$	58.9	27.0	18.3	13.3
	$-V_t$	42.6	11.9	6.0	2.8

5. RESULTS AND DISCUSSION

Since we are dealing with $T=1$ states, we need consider only the singlet-even ($S=0$, even l) and triplet-odd ($S=1$, odd l) forces. We have carried out our calculations for several exchange mixtures and will

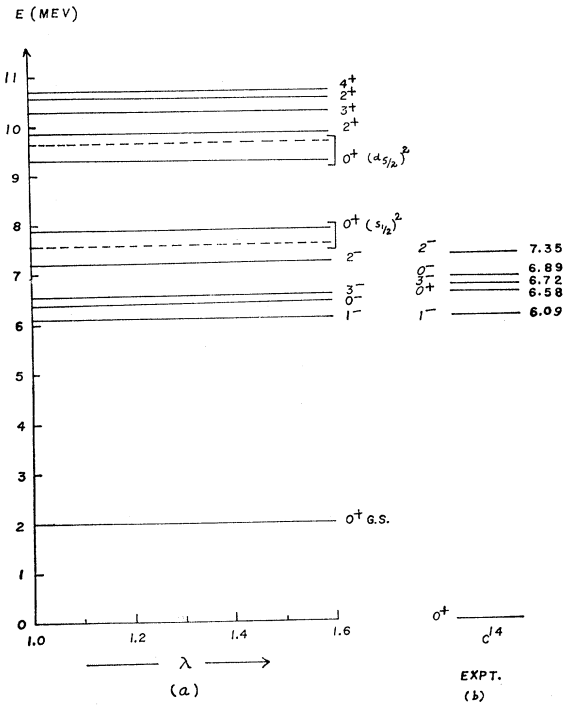

FIG. 2. (a) The calculated energy levels for the s-state interactions and (b) the experimental level spectrum of C^{14}. The dashed lines show the configuration-mixing effects.

[30] I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. **10**, 353 (1960).

[31] Wilmore (private communication to Jaffe *et al.*, Ref. 28).

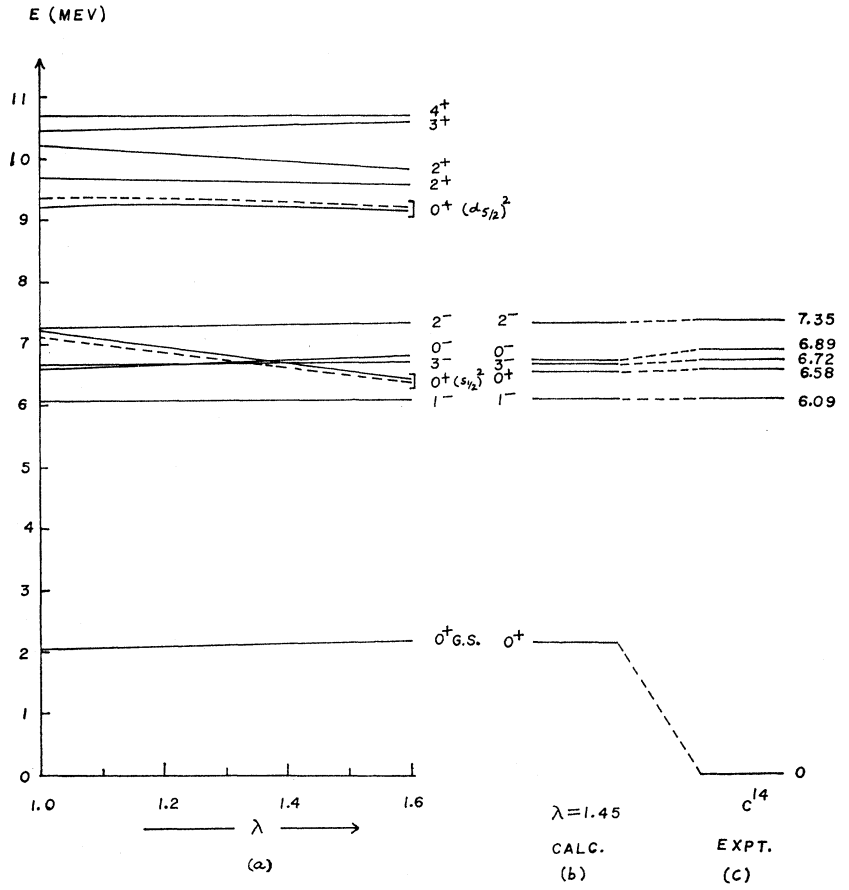

FIG. 3. The energy levels of C^{14}. The calculated energy levels for the singlet-even (Serber-type) forces are shown in (a). The dashed lines show the configuration-mixing effects. The experimental level spectrum is shown in (c) in comparison with the calculated energy levels for $\lambda \approx 1.45$ shown in (b), which gives the best fit to the experimental data.

present our results in the order of increasing complexity of the residual interaction as follows:

(a) The s-state interaction may describe the level structure of a shell-model nucleus, as suggested by Moszkowski[32] and found[10] satisfactory in the case of Ni^{58}.

(b) We include Serber-type forces in which the triplet force is assumed to be zero. This is based on the study by Shah and Pandya[17] of $T=1$ states of various configurations in light nuclei wherein they found the triplet forces to be negligible.

(c) We consider the general case wherein we include singlet-even and triplet-odd forces.

Considering C^{12} as the inert core, the energy levels are calculated as a function of the range parameter λ defined in Sec. 2. The potential depth, denoted as V_0 for case (a) and V_s for case (b), can be calculated by using the experimental separation of either of the doublets $(p_{1/2},s_{1/2})_{1^-,0^-}$ and $(p_{1/2}d_{5/2})_{2^-,3^-}$. The latter choice was found to be somewhat more favorable and has been adopted for the results quoted below. For the case (c) the potential depths V_s and V_t can be calculated by using the experimental splittings for both these doublets. The potential depths thus calculated for each case are given in Table III for the values of λ of interest in our case.

Thus, the only free parameter left at our disposal is λ which is chosen to give the best fit to the experimental level scheme. The calculated energy levels for the three cases are shown in Figs. 2(a)–4(a) as a function of λ for comparison with the experimental results, and will be discussed individually for each case later in this section. First we make some general comments.

(i) Regarding the relative position of the ground state we find the situation as described by Sood and Waghmare[3] that it lies too high by about 2 MeV. It was suggested[3] that the situation may be possibly remedied by assuming an additional pairing force operating only in the $p_{1/2}^2$ state to depress it by the required amount. However, the recent calculations of Waghmare and Majumdar[33] do not find the sufficient contribution coming from this factor alone. True[2] also found a similar discrepancy in the case of the location of $p_{1/2}^2$

[32] S. A. Moszkowski, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada* (North-Holland Publishing Company, Amsterdam, 1960), p. 502.

[33] Y. R. Waghmare and C. K. Majumdar, Phys. Letters 14, 144 (1965).

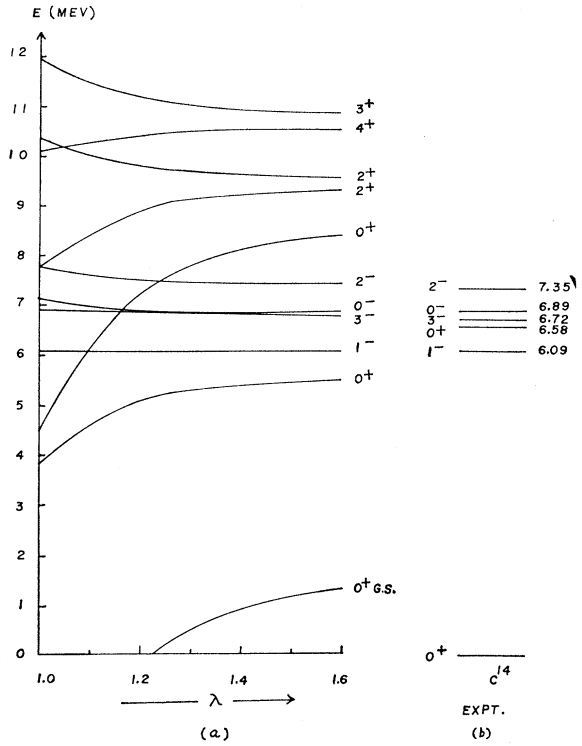

FIG. 4. (a) The calculated energy levels for the singlet-even, triplet p-state interactions and (b) the experimental level spectrum of C^{14}.

states in N^{14}. As suggested by him,[2] one expects a greater amount of admixture with the core-excited states for the p-shell states than for the other states and the above discrepancy could quite reasonably be attributed to this factor. In view of this discrepancy we normalize our calculated energy levels with respect to the observed 6.09-MeV, 1^- state.

(ii) In shell-model calculations it is usual to mix "nearly degenerate" configurations in the low-lying energy levels. In practice one defines this near degeneracy quite arbitrarily and it is hoped within the framework of the Brueckner theory that the higher two-particle excitations are all included in the R matrix. In actual calculations the importance or unimportance of the configuration mixing is established by the ratio of the nondiagonal matrix elements to the separation of the corresponding perturbing levels. Out of the states of interest to us, configuration mixing can be a factor for only the 0^+ states from the $s_{1/2}{}^2$ and $d_{5/2}{}^2$ configurations and is investigated in our study. The contribution is found to be quite small, thus verifying the conclusion of True[2] that "in some manner which is not completely clear, the effective potential seems to include some of the more important aspects of configuration mixing."

(iii) In Figs. 2–4 for comparison with the calculated energy levels we have left out of the experimental level scheme the "core-excited" states as defined in Sec. 4, since they are outside the scope of the present model.

Now we discuss the comparison of the calculated energy levels with the experimental results for each case separately.

For case (a), i.e., S-state forces, the results are shown in Fig. 2. The dashed lines show the configuration-mixing effects allowed in the 0^+ states. It is seen that the energies of all the states are insensitive to the variations of λ and hence the relative spacing of the levels does not provide a preference for any particular value of λ. The ordering of 3^- and 0^- levels is reversed and also the 0^+ level lies too high. Thus, the s-state interaction is not found to be satisfactory.

Next we investigate case (b), i.e., Serber-type forces operating in singlet-even states only. The calculated energy levels are shown in Fig. 3(a) as a function of λ. The relative positions of 0^+, 3^-, and 0^- levels point to a value of $\lambda \approx 1.45$ as giving the best fit. The calculated energies for this value of λ are shown separately in Fig. 3(b) in comparison with the experimental results in Fig. 3(c). The agreement is seen to be very good. It may be mentioned that except for the $(d_{5/2}{}^2)_{4^+}$ level, which contains an additional g-state force, the singlet-even interaction contains only s- and d-state forces. The vastly improved agreement from case (a) points to the importance of the d-state forces for the analysis of the C^{14} spectrum.

We have also investigated the effect of including the p-state forces and the results are shown in Fig. 4. The observed splitting of the negative-parity doublets is taken as an input to yield the singlet and the triplet well depths given in Table III. It is seen that the strengths of the singlet and the triplet forces are approximately equal for $\lambda < 1.2$. The change in energies of these states is quite small within that region, but the $(s_{1/2}{}^2)_{0^+}$ state always lies well below the 1^- state. Although the p-state forces are shown to be attractive in agreement with the conclusion of Shah and Pandya,[17] the agreement is not satisfactory.

Further, we calculated the energy levels with the inclusion of f-state forces as well. The potentials are found to be much deeper, and the singlet and the triplet forces are approximately equal for the range of λ discussed. The positive-parity states are further pushed down from Fig. 4, thus leading us farther away from the experimental levels.

We conclude that the level spectrum in C^{14} can be satisfactorily explained with the assumption of singlet-even (Serber-type) forces and the configuration assignments are in agreement with those listed in Table II. The forces are quite weak but of quite long range. The range r_0 of the potential can be calculated for a given value of λ accepting the value of $r_l = 1.64$ F[4] for $A = 14$. Thus, for $\lambda = 1.45$ we get the range $r_0 = 2.38$ F. While discussing p-shell nuclei, Waghmare[34] concluded that λ for p-shell nuclei is about 1.4 times that for nuclei around Ni^{58}. For the latter, agreement was found[10] with

[34] Y. R. Waghmare, Phys. Rev. 134, B1185 (1964).

a value of $\lambda \approx 1.0$ for Serber forces, which thus is in good agreement with the corresponding $\lambda \approx 1.45$ derived in the present study. Of course the numerical value assigned to λ should not be taken too seriously, since it is primarily decided by fitting the 0^+ state from the $s_{1/2}^2$ dominant configuration, the variation of the negative-parity energy levels with λ being too small to make any significant difference. It is also dependent on the choice of the 2^-–3^- doublet separation to calculate the potential depth and other details of the assumed interaction.

It may be noted that in the present study we have adopted rather an extreme model, in the form of the jj-coupling assumption, which, as is well known, is not strictly applicable to p-shell nuclei. Here we have specifically taken C^{12} to constitute an inert core, although the importance of the core-excited states for obtaining agreement with the ground-state energy of C^{14} is pointed out. The choice of the single-particle energies as derived from the C^{13} spectrum is also based on this consideration. One simple way of including, in part, the contributions from the core would be to treat these single-particle energies as free parameters, but it is clear that, with the limited experimental information thus far available on the levels in C^{14}, the input parameters, with such a flexible approach, would outnumber the output. It will be necessary to identify experimentally the other positive-parity states before further details of the effective interaction can be fruitfully investigated.

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Deuteron Optical-Model Analysis with Spin-Orbit Potential*

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An optical-model analysis of 19 deuteron elastic-scattering angular distributions in the energy range from 12 to 26 MeV was performed both with and without a spin-orbit term included in the potential. The spin-orbit potential was of the Thomas form which had been found satisfactory for fitting the measured deuteron elastic polarization. Including the spin-orbit term always resulted in an improvement in the quality of the fits. Without the spin-orbit term, various families of potentials fitted the data equally well, but when the spin-orbit potential was added the various families were no longer equivalent in many cases. The deuteron optical-model potential which most closely approximates the sum of the neutron and proton optical-model potentials was found to give the most consistently satisfactory results for all the angular distributions.

INTRODUCTION

In a previous paper,[1] hereinafter referred to as Paper I, we reported on an optical-model analysis of data on the elastic scattering of 11- to 27-MeV deuterons from many nuclei. In general, excellent fits to the data could be obtained with an optical-model potential that did not contain any spin-orbit or tensor potential terms.[2] Upon completion of that work, some measurements on deuteron-nucleus elastic-scattering polarization were reported by Beurtey[3] and these measurements have since been successfully analyzed by Raynal[4] in terms of an optical-model potential that does include a spin-orbit potential. Raynal also analyzed the data with various tensor terms in the optical-model potential, but since the fits were not improved, he suggested that there does not at the moment seem to be any justification for the inclusion of such terms in the deuteron optical-model potential.

A few angular distributions which we reported in Paper I could not be fitted well at back angles. This was particularly true for elastic scattering from Ca at 21.6 MeV, and the availability of polarization data on this element at this energy[3] prompted a subsequent analysis of the data with an optical-model potential that includes a spin-orbit term. The result was a much improved fit to the data. The parameters so obtained were very close to those obtained by Raynal,[4] and they predicted polarizations in good agreement with the measured ones,[3]

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