

esses may also contribute to certain of the weaker unassigned bands. A LiNbO<sub>3</sub> crystal cut in another orientation, which from predicted scattering efficiencies could lead to an unambiguous determination of the longitudinal-branch frequencies and/or allow one to measure the frequency and intensity angular dependences, would obviously be desirable but unfortunately was not available. *Note added in proof.* The frequencies and strengths of infrared-active modes of

LiNbO<sub>3</sub> have recently been determined from reflectivity measurements by J. D. Axe and D. F. O'Kane, *Appl. Phys. Letters* **9**, 58 (1966).

#### ACKNOWLEDGMENTS

We are grateful to M. Holland for lending us the LiNbO<sub>3</sub> crystal used in this study, and to M. Lichtensteiger for the x-ray crystal orientation.

## Reformulation of the Dielectric Constant and of Ohm's Law in an Absorbing Medium\*

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(Received 29 December 1965; revised manuscript received 9 May 1966)

It is proposed that a fundamental revision of the formulation of the dielectric constant and of the conductivity of an absorbing and dispersive medium be considered. In the conventional formulation the dielectric constant  $\epsilon$  of such a medium depends on the mechanism controlling the energy absorption, and therefore  $\epsilon$  has the form  $\epsilon = \epsilon(\omega, \gamma)$ , where  $\omega$  is the frequency and  $\gamma$  is a "frictional" parameter representing the absorption process. Such a formulation, although widely accepted in the current literature and in the textbooks on electromagnetism, is not consistent with Maxwell's theory, and therefore is not adapted to the analysis of the energetic behavior of absorbing media. A different formulation is proposed in which the inconsistency with Maxwell's theory is removed. In the proposed formulation the dielectric constant of an absorbing medium is independent of the mechanism which controls the absorption process. The dielectric constant is of the type  $\epsilon = \epsilon(\omega)$  and not  $\epsilon = \epsilon(\omega, \gamma)$ , i.e., it is the same as if the absorption mechanism did not exist at all. The mechanism of energy absorption is used to determine the conductivity of the medium but has no effect on the formulation of the dielectric constant. The novelty of the proposed formulation is based on the assumption which is used to arrive at a suitable definition of the driving force involved in the motion of a harmonic oscillator in the molecular model of an absorbing medium. In the conventional formulation the driving force  $e\mathfrak{F}$  is assumed to be equal to  $e\mathfrak{E}$  (where  $\mathfrak{E}$  is the electric field intensity and  $e$  is the charge of an electron), whereas in the proposed formulation the assumption  $e\mathfrak{F} = e\mathfrak{E}$  is replaced by the assumption  $e\mathfrak{F} = e\mathfrak{E} + e\mathfrak{G}^{(\text{extr})}$ , where  $e\mathfrak{G}^{(\text{extr})}$  is an "extraneous" force. The extraneous force results from the conversion of the electrical field energy stored in the medium into "extraneous" energy (such as heat) which is removed from the medium during the interaction with an electric field. Therefore,  $e\mathfrak{G}^{(\text{extr})}$  acts as an energy sink. The distinction between the proposed formulation and the one used in the standard theory is based on a behavior of dispersive media which is physically different from the one customarily assumed, and therefore it does not represent merely a question of formalism. Using the concepts introduced in this analysis, Ohm's law is reformulated as representing a mechanism in which the rate of production of the dissipated energy is proportional to the mean energy stored in the medium. Such an interpretation of Ohm's law cannot be made in the framework of the standard theory, since the standard theory does not provide a procedure for determining the mean energy stored in an absorbing medium.

### I. INTRODUCTION

WITHIN the last several decades and particularly in the first quarter of this century great advances have been made in the theory of dispersive media. As a result of the pioneer work of Sommerfeld and his associates<sup>1</sup> (particularly Brillouin), such concepts as the density energy stored by an alternating field in a dispersive medium, the velocity of energy transfer in an electric field interacting with a dispersive medium, have been clarified and formulated.

These pioneer investigations dealt, however, with dispersive media which are nonabsorbing. The results obtained have, therefore, a restricted applicability. No successful efforts have been made to date in extending these results to media in which a portion of the field energy is absorbed during the interaction process. Thus, some of the energetic aspects of the wave-matter interaction are very inadequately described at the present time. For instance, there is no adequate description of the process of building up of an electric field energy in an absorbing medium by means of a gradually increasing oscillating field, and therefore the concept of the energy stored by an oscillating field in an absorbing medium has not been formulated to date.

\* Research sponsored by the U. S. Atomic Energy Commission under contract to Union Carbide Corporation.

<sup>1</sup> L. Brillouin, *Wave Propagation and Group Velocity* (Academic Press Inc., New York, 1960).

These unsuccessful efforts are due, apparently, to certain assumptions which have been made in formulating the macroscopic parameters of absorbing media. The purpose of this analysis is to discuss the necessity for reformulating these parameters and to suggest a new formulation. Of particular concern here are the standard interpretations of the dielectric constant  $\epsilon$  and the conductivity  $\sigma$  of an absorbing medium.

The dielectric constant is usually represented by a complex number:

$$\epsilon = \epsilon_1 + i\epsilon_2, \quad (1.1)$$

where  $\epsilon_1$  and  $\epsilon_2$  are real and can be expressed as

$$\epsilon_1 \equiv \epsilon_1(\omega, \gamma) \quad \text{and} \quad \epsilon_2 \equiv \epsilon_2(\omega, \gamma), \quad (1.2)$$

$\omega$  is the frequency, and  $\gamma$  is a "frictional" parameter representing the absorption process.

The formulation (1.1) and (1.2), although widely accepted in the current literature and in the textbooks on electromagnetic theory, appears to be inconsistent with a postulate of Maxwell's theory, the postulate of conservation of energy. This is due to some basic assumptions which have been made and which are not considered by us as acceptable in a physically consistent theory. These assumptions have been discarded in this analysis and a new formulation is proposed in which the inconsistencies with Maxwell's theory do not occur. In the proposed formulation both the dielectric constant and the conductivity of an absorbing medium have a different form from the one used in the standard theory.

The distinction between the proposed formulation and the one used in the standard theory is based on a behavior of dispersive media which is physically different from the one customarily assumed, and, therefore, it does not represent merely a question of formalism. Thus, when interpreting some experimental data such as selective absorption of light by a thin slab of dielectrical substance, we obtain different quantitative results from those based on the standard theory.

It is shown in this analysis that in order to be consistent with the postulates of Maxwell's theory, the dielectric constant of an absorbing medium must be independent of the mechanism which controls the absorption process. Thus, the frictional parameter  $\gamma$  does not enter in the formulation of the dielectric constant even if absorption is present. The proposed expression for the dielectric constant is of the type  $\epsilon \equiv \epsilon(\omega)$  and not  $\epsilon \equiv \epsilon(\omega, \gamma)$ , i.e., it is the same as if the absorption mechanism did not exist at all. The mechanism of energy absorption determines the conductivity of the medium but has no effect on the formulation of the dielectric constant.

A portion of this analysis is devoted to the interpretation of the most common dissipative mechanism which is encountered in a medium interacting with an electromagnetic field, i.e., the mechanism described by Ohm's law. The standard theory does not provide an adequate energetic description of the most common dissipative

mechanism. It would be, for instance, desirable to compare the rate at which the energy is removed from a medium interacting with an alternating field with the amount of energy stored in the medium and to establish a relationship between these two quantities. Such a relationship cannot be provided in the framework of the standard theory because there is no procedure in the standard theory for determining the energy stored in a dispersive and absorbing medium by an alternating electrical field. The proposed formulation removes this difficulty and suggests a reformulation of Ohm's law in order to describe it more adequately from the energetic point of view. It is shown that in those absorbing media in which the dissipational mechanism is controlled by Ohm's law there must be a proportionality between the rate at which the dissipated energy is produced and the mean energy stored in the medium.

The novelty of the proposed formulation is based on the assumption that there are two types of forces which are effective in the interaction between an electrical field and an absorbing medium. These are "electrical" forces and "extraneous" forces. The electrical forces are involved in the formulation of the "electrical energy" per se and are used to determine the dielectric constant of the medium. The extraneous forces represent the energy which is dissipated during the interaction of the field with the medium and are used to determine the conductivity of the medium.

The concept of an extraneous force apparently has not been used in the analysis of the absorption process. This concept, however, not only is consistent with but is implied by Maxwell's theory. An extraneous force represents an energy sink and is analogous to the "impressed" force which is used in the standard theory to represent an energy source.<sup>2</sup> An electromagnetic field cannot exist per se, and, therefore, the presence of a source and of a sink of electrical energy is necessary.

The proposed formulation is based on a drastic revision of some concepts which are strongly entrenched in the current periodical literature and in the textbooks on electromagnetic theory. It provides, however, a basis for a logically consistent and physically acceptable theory. If the proposed formulation is not accepted, we are faced with the necessity of revisions which are considerably more drastic than the ones suggested here since such revisions would involve abandoning Maxwell's theory.

## II. INCONSISTENCIES IN THE CURRENT FORMULATION

### A. Principle of Conservation of Energy

The inconsistency of the conventional theory becomes apparent if one tries to apply the standard expression

<sup>2</sup> R. Becker, *Electromagnetic Fields and Interactions, Electromagnetic Theory and Relativity* (Blaisdell Publishing Company, New York, 1964), Vol. I, p. 120.

for the dielectric constant to Maxwell's formulation of the principle of conservation of energy.

Assume a nonmagnetic dispersive and absorbing medium interacting with an electromagnetic field. Then

$$\text{curl}\mathfrak{G} = -\frac{1}{c} \frac{\partial \mathfrak{B}}{\partial t}, \quad (2.1)$$

and

$$\text{curl}\mathfrak{B} = -\frac{1}{c} \frac{\partial \mathfrak{D}}{\partial t} + \frac{4\pi}{c} \mathfrak{J}, \quad (2.2)$$

where  $\mathfrak{G}$  and  $\mathfrak{B}$  designate, respectively, the intensity of the electric and magnetic fields (more specifically,  $\mathfrak{B}$  is the magnetic induction which is equal to the magnetic field intensity),  $\mathfrak{D}$  is the electric displacement, and  $\mathfrak{J}$  is the current density. To complete the description of this interaction one needs a "constitutive" relation between the macroscopic quantity  $\mathbf{D}$  and the corresponding microscopic quantity  $\mathbf{E}$ . Such a relation is

$$\mathbf{D} = \epsilon \mathbf{E}. \quad (2.3)$$

The German symbols  $\mathfrak{G}$ ,  $\mathfrak{D}$ ,  $\mathfrak{B}$ , and  $\mathfrak{J}$  represent, respectively, the electric intensity, electric displacement, electric polarization, and current density as functions of time, whereas the corresponding Roman symbols  $\mathbf{E}$ ,  $\mathbf{D}$ ,  $\mathbf{P}$ , and  $\mathbf{J}$  are the spectral components of  $\mathfrak{G}$ ,  $\mathfrak{D}$ ,  $\mathfrak{B}$ , and  $\mathfrak{J}$ . Thus  $\mathfrak{G} \equiv \mathfrak{G}(t)$ ,  $\mathfrak{D} \equiv \mathfrak{D}(t)$ , whereas  $\mathbf{E} \equiv \mathbf{E}(\omega)$  and  $\mathbf{D} \equiv \mathbf{D}(\omega)$ .

Writing the scalar product of  $\mathfrak{G}$  with (2.1) and  $\mathfrak{B}$  with (2.2) and subtracting one product from the other we obtain a relation

$$\mathfrak{G} \cdot \mathfrak{J} = -\frac{c}{4\pi} \text{div}(\mathfrak{G} \times \mathfrak{B}) - \frac{1}{4\pi} \left( \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} + \mathfrak{B} \cdot \frac{\partial \mathfrak{B}}{\partial t} \right), \quad (2.4)$$

representing the principle of conservation of energy. The form of this relation suggests that the left side represents the rate of dissipation of energy per unit volume of the medium; the second term on the right has to do with the rate of energy stored by the field, and the first term on the right gives the expression for the energy flow. Our particular attention will be directed to the second term on the right and particularly to the expression

$$\frac{dW_e}{\partial t} = \frac{1}{4\pi} \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t}, \quad (2.5)$$

representing the rate of storage of the electrical energy.

In order to ascertain whether or not the standard expression  $\epsilon \equiv \epsilon(\omega, \gamma)$  is valid from the standpoint of the principle of energy conservation, we need to substitute  $\mathbf{D} = \epsilon(\omega, \gamma) \mathbf{E}$  in  $(1/4\pi)(\mathfrak{G} \cdot d\mathfrak{D}/dt)$  and to determine whether or not as a result of such substitution the expression  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$  retains the physical meaning which is attributed to it in Maxwell's theory. It will be shown that this is not the case, i.e., the expression  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$  thus obtained does not represent the rate of storage of the electrical energy. On the other

hand, if we assume that the medium is nonabsorbing ( $\gamma=0$ ) and substitute  $\mathbf{D} = \epsilon(\omega) \mathbf{E}$  in  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$ , the above inconsistency with Maxwell's theory does not occur.

In substituting  $\mathbf{D} = \epsilon(\omega, \gamma) \mathbf{E}$  or  $\mathbf{D} = \epsilon(\omega) \mathbf{E}$  in  $(1/4\pi) \times (\mathfrak{G} \cdot d\mathfrak{D}/dt)$ , we should note that  $\mathbf{E}$  and  $\mathbf{D}$  are functions of frequency, whereas  $(1/4\pi)(\mathfrak{G} \cdot d\mathfrak{D}/dt)$  contains the corresponding quantities  $\mathfrak{G}$  and  $\mathfrak{D}$  which vary with time. It is convenient in that connection to select a particular expression for the electric field intensity which is well adapted both to the formulation in which the field quantities appear as a function of frequency and to the formulation in which they appear as a function of time. We select, therefore, an electric field intensity which has its frequency spectrum centered in the neighborhood of a dominant frequency  $\omega_1$ . Then the corresponding expression for  $\mathfrak{G}$  has the form of an almost monochromatic wave,

$$\mathfrak{G}(t) = \mathbf{E}_0(t) \sin \omega_1 t, \quad (2.6)$$

having an amplitude  $\mathbf{E}_0(t)$  slowly varying during the period  $1/\omega_1$ . It is assumed that the amplitude  $\mathbf{E}_0(t)$  slowly increases with time and that  $\mathbf{E}_0(t) = 0$  for  $t = -\infty$ . The usefulness of the formulation (2.6) is based on the fact that the dominant frequency  $\omega_1$  appears explicitly both in  $\mathfrak{G} = \mathfrak{G}(t)$  and  $\mathbf{E} = \mathbf{E}(\omega)$ .

It is assumed that the dielectric constant of the medium varies slowly with frequency in the neighborhood of  $\omega_1$ . This makes it necessary for the frequency spectrum of (9.6) not to be close to any of the "resonant" frequencies of the medium. The "resonant" frequencies are the binding frequencies of oscillating dipoles which are used to represent the atomic or molecular structure of the medium. Then assuming that  $\omega = \omega_1 + \alpha$ , where  $\alpha \ll \omega_1$ , the dielectric constant can be approximated by the first two terms of a Taylor's expansion about  $\omega_1$ . If there is no absorption, then

$$\epsilon(\omega_1 + \alpha) = \epsilon(\omega_1) + \alpha [d\epsilon(\omega)/d\omega]_{\omega=\omega_1}, \quad (2.7)$$

and if absorption is present, then

$$\epsilon(\omega_1 + \alpha, \gamma) = \epsilon(\omega_1, \gamma) + \alpha [d\epsilon(\omega, \gamma)/d\omega]_{\omega=\omega_1}. \quad (2.8)$$

Consider now the expression  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$  under the assumption that  $\mathfrak{G}(t)$  is represented by (2.6) and the medium can be either nonabsorbing or absorbing. To simplify the notation, we replace  $\omega_1$  by  $\omega$  in (2.6) to (2.8).

### B. Calculation of $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$ for a Nonabsorbing Medium

Applying (2.7) to  $\mathbf{D} = \epsilon(\omega) \mathbf{E}$ , where  $\mathfrak{G}$  is represented by (2.6) and using (2.5), we obtain<sup>3</sup>

$$\left\langle \frac{dW_e}{dt} \right\rangle = \frac{1}{4\pi} \left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle = \frac{1}{16\pi} \frac{d}{d\omega} (\omega \epsilon) \frac{d}{dt} [E_0(t)]^2. \quad (2.9)$$

<sup>3</sup> L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Inc., New York, 1960), pp. 254-255.

The angular brackets enclosing  $dW_e/dt$  and  $\mathfrak{G} \partial \mathfrak{D}/\partial t$  designate a process of averaging over a time interval which is large when compared to  $(1/\omega)$ . The expression (2.9) is a total derivative with respect to time of a quantity  $\langle W_e \rangle$  which should be interpreted as representing the average energy density stored in the medium. Thus, by integrating (2.9) with respect to time from  $-\infty$  to  $t$  we obtain

$$\langle W_e \rangle = \frac{1}{4\pi} \int_{-\infty}^t \left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle dt = \frac{1}{16\pi} \frac{d}{d\omega} (\omega \epsilon) [E_0(t)]^2. \quad (2.10)$$

In the limiting case when  $dE_0(t)/dt$  tends to zero the field (2.6) becomes monochromatic and then

$$\langle W_e \rangle = \frac{1}{16\pi} \frac{d}{d\omega} (\omega \epsilon) E^2. \quad (2.11)$$

It will be shown, subsequently, that a result equivalent to (2.9) and (2.10) cannot be obtained for an absorbing medium.

### C. Calculation of $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$ for an Absorbing Medium

The formulation (2.10) or (2.11) is valid when  $\epsilon(\omega)$  is a slowly varying function of  $\omega$ , i.e., when  $\omega$  is not in the neighborhood of any of the resonance frequencies of the medium. In calculating  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$  for a non-absorbing medium,  $\epsilon$  is always real. However, if we take into account energy absorption, then, in accordance with the standard theory,  $\epsilon$  must be complex ( $\epsilon = \epsilon_1 + i\epsilon_2$ ). Then applying (2.8) to  $\mathbf{D} = [\epsilon_1(\omega, \gamma) + i\epsilon_2(\omega, \gamma)]\mathbf{E}$ , where  $\mathfrak{G}$  is represented by (2.6), and using (2.5), we obtain

$$\frac{1}{4\pi} \left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle = \frac{1}{16\pi} \frac{d}{d\omega} (\omega \epsilon_1) \frac{d}{dt} [E_0(t)]^2 + \frac{1}{8\pi} \omega \epsilon_2 [E_0(t)]^2. \quad (2.12)$$

(See Appendix.) This expression is not a derivative with respect to time of a quantity which depends on the electric field intensity. Consequently, as pointed out by Ginzburg,<sup>4</sup> the integral

$$\frac{1}{4\pi} \int_{-\infty}^t \left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle dt \quad (2.13)$$

depends on the previous history of  $\mathbf{E}_0(t)$ , i.e., on the manner in which the electric field was established. Such an expression cannot be interpreted as representing the energy stored in the medium. Ginzburg has not indicated any procedure which would help to remove this difficulty.

<sup>4</sup> V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon Press, Ltd., London, 1964), pp. 244-245, 477-495.

There is, therefore, an inconsistency. In accordance with Maxwell's theory, the expression  $(1/4\pi)(\mathfrak{G} \cdot \partial \mathfrak{D}/\partial t)$  is interpreted as representing the rate of storage of the electrical energy in the medium. Such an interpretation cannot be applied to (2.12).

Attempts have been made to use various approaches in order to adapt the formulation (2.5) to an absorbing medium. It has been assumed that the dielectric constant can be represented by the real component in the expression (1.1) for  $\epsilon$ , i.e., that  $\epsilon = \epsilon_1$ . Then it was conjectured that the energy density can be obtained from (2.11) by a mere replacement of  $\epsilon$  by  $\epsilon_1$ . Thus, the energy density would have been expressed as

$$\langle W_e \rangle = \frac{1}{16\pi} \frac{d}{d\omega} (\omega \epsilon_1) E^2. \quad (2.14)$$

The expression (2.14) is used sometimes in the current literature under the assumption that it does represent the average energy density stored by an alternating field in a dissipating medium. This interpretation of (2.14) is not permissible since it is not based on any valid physical reasons which could be implied directly or indirectly from Maxwell's theory. As shown by Ginzburg,<sup>4</sup> (2.14) gives in some instances even paradoxical results. Thus, for a plasma with  $\omega \ll \nu_{\text{eff}}$  (where  $\nu_{\text{eff}}$  is the effective electron-ion or electron-molecule collision frequency), the energy density as formulated in (2.14) is negative.

## III. GENERALIZED FORMULATION OF THE DIELECTRIC CONSTANT

### A. Standard Derivation of the Expression for the Dielectric Constant

In order to resolve the above difficulties we shall analyze the basic assumptions in the standard formulation of the dielectric constant.

Consider an idealized structure of an atomic medium in which each atom is represented by a single oscillating dipole having binding frequency  $\omega_e$ . It is assumed that each electron in the dipole experiences a frictional force proportional to its velocity and oppositely directed. Then the motion of the electron can be expressed as

$$m(\ddot{\mathbf{r}} + \gamma \dot{\mathbf{r}} + \omega_e^2 \mathbf{r}) = e \mathfrak{E}, \quad (3.1)$$

where  $\dot{\mathbf{r}} = d\mathbf{r}/dt$  and  $\ddot{\mathbf{r}} = d^2\mathbf{r}/dt^2$ ;  $m$  and  $e$  are the mass and charge of the electron, respectively;  $\mathbf{r}$  is the perturbed position of the electron due to the driving force  $e\mathfrak{E}$ ;  $m\gamma\dot{\mathbf{r}}$  is the frictional force proportional to the velocity and oppositely directed. The frictional parameter  $\gamma$  comprises two terms<sup>5</sup>:

$$\gamma = \gamma^{(P)} + \gamma^{(L)}, \quad (3.2)$$

<sup>5</sup> K. L. Wolf and K. F. Hertzfeld, in *Handbuch der Physik*, edited by S. Flügge (Julius Springer, Berlin, 1928), Vol. XX, pp. 520-525.

where  $\gamma^{(P)}$  is the "Planck term" representing the energy loss due to the waves radiated by an oscillating electron, and  $\gamma^{(L)}$  is the "Lorentz term" due to the collision of an excited dipole with an atom in the ground state in which the excitation energy of the dipole appears as the translational energy of the atom and thus accounts for the generation of thermal energy (Joule's heat). If we deal with an electronic plasma, then the binding frequency  $\omega_b$  is zero and  $\gamma^{(L)}$  results from collisions of an oscillating dipole with stationary ions.<sup>6</sup>

In order to determine the motion of the electron in an oscillating dipole, we need to define the driving force  $e\mathfrak{F}$ . The departure of the proposed formulation from the one conventionally used is based on an assumption which is made in order to arrive at a suitable definition of the driving force.

In the standard formulation it is assumed that the driving force is due entirely to the intensity of the electrical field, i.e., that

$$e\mathfrak{F} = e\mathfrak{G}. \quad (3.3)$$

Then substituting (3.3) in (3.1), one obtains

$$d^2\mathfrak{P}/dt^2 + \gamma d\mathfrak{P}/dt + \omega_b^2\mathfrak{P} = \omega_e^2\mathfrak{G}, \quad (3.4)$$

where  $\mathfrak{P} = Ner$  is the polarization density of the medium,  $\omega_e^2 = 4\pi Ne^2/m$ , and  $N$  is the number of atoms per unit volume. Consequently, the polarization

$$\mathbf{P} = \frac{1}{4\pi} \frac{\omega_e^2}{\omega_b^2 - \omega^2 + i\gamma\omega} \mathbf{E} \quad (3.5)$$

is represented as a complex quantity. Applying the relationship  $\mathbf{D} = \epsilon\mathbf{E} = \mathbf{E} + 4\pi\mathbf{P}$ , one obtains the well-known expression for the dielectric constant,

$$\epsilon = 1 + \frac{\omega_e^2}{\omega_b^2 - \omega^2 + i\gamma\omega}. \quad (3.6)$$

The expression (3.6) has the complex form (1.1) in which

$$\epsilon_1 = 1 + \frac{\omega_e^2(\omega_b^2 - \omega^2)}{(\omega_b^2 - \omega^2)^2 + \omega^2\gamma^2}$$

and

$$\epsilon_2 = -\frac{\omega_e^2\omega\gamma}{(\omega_b^2 - \omega^2)^2 + \omega^2\gamma^2}. \quad (3.7)$$

### B. Proposed Formulation of the Dielectric Constant

The novelty of the proposed formulation is based on the definition of the driving force  $e\mathfrak{F}$ . We believe that the assumption (3.3) is not physically acceptable, i.e., that the force which drives an oscillating dipole in a dissipating medium does not depend solely on the electrical field. The assumption (3.3) is valid only if there are

no frictional effects, and, therefore, (3.3) cannot be combined with (3.1) when  $\gamma \neq 0$ .

There are two different processes described in the left side of (3.1). The term  $m\gamma\dot{\mathbf{r}}$  involves frictional effect and represents, therefore, the process of energy dissipation. The terms  $m\ddot{\mathbf{r}}$  and  $m\omega_b^2\mathbf{r}$  involve, respectively, inertia and elasticity effects and represent, therefore, the process of energy storage. In our subsequent analysis a clear distinction will be made between these processes, and a more general formulation will be used in order to describe the process of energy dissipation. The assumption that the dissipational force is of the type  $m\gamma\dot{\mathbf{r}}$  (analogous to that of a viscous fluid) is based on a relatively crude model which does not always give an adequate representation of the actual physical processes which do occur. Therefore, using a formulation which does not depend on any particular model, the dissipational force will be expressed as  $e\mathfrak{G} \equiv e\mathfrak{G}(\dot{\mathbf{r}}, \omega_b, \lambda_1, \lambda_2, \dots, \lambda_n)$ , where  $\mathfrak{G}$  represents a function of  $\dot{\mathbf{r}}$ ,  $\omega_b$ , and other parameters  $\lambda_1, \lambda_2, \dots, \lambda_n$  which are necessary for the complete description of the dissipation mechanism.

The energy storage and dissipative effects will be described in terms of "electrical" forces  $e\mathfrak{G}$  and "extraneous" forces  $e\mathfrak{G}^{(\text{extr})}$ . The electrical forces will be used to represent the storage of the energy of the electrical field per se and more particularly the mechanism which is effective in storing the polarization energy. The extraneous forces will be involved in the dissipative processes in which the polarization energy is transformed into other forms of energy such as heat. The driving force  $e\mathfrak{F}$  should account both for electrical and dissipative effects. We propose, therefore, to replace the assumption (3.3) by

$$e\mathfrak{F} = e\mathfrak{G} + e\mathfrak{G}^{(\text{extr})}. \quad (3.8)$$

The above two processes can be formally separated one from the other and expressed as

$$m(\ddot{\mathbf{r}} + \omega_b^2\mathbf{r}) = e\mathfrak{G}, \quad (3.9)$$

and

$$e\mathfrak{G}(\dot{\mathbf{r}}, \omega_b, \lambda_1, \lambda_2, \dots, \lambda_n) = e\mathfrak{G}^{(\text{extr})}. \quad (3.10)$$

The concept of an extraneous force is related to the concept of an "impressed" force  $e\mathfrak{G}^{(\text{impr})}$  used in the customary formulation of Ohm's law<sup>2</sup> in a medium comprising distributed energy sources. Thus,

$$\mathfrak{S} = \sigma(\mathfrak{G} + \mathfrak{G}^{(\text{impr})}), \quad (3.11)$$

where  $\sigma$  is the conductivity. An impressed force represents a source of an electrical field and can occur, for instance, in a medium in which there is a variation of concentration within a dilute aqueous solution of a strong electrolyte such as HCl. In such a medium an electric current sets in which is due to the difference of mobility and, therefore, to the difference of the diffusion velocity of  $\text{H}^+$  and  $\text{Cl}^-$  ions. An impressed force can also occur whenever there is a contact of a metal with an electrolyte. Thus, when a copper bar is immersed

<sup>6</sup> V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon Press, Ltd., London, 1964), pp. 39-53.

in dilute copper sulfate, a small electric current is produced which is due to the  $\text{Cu}^{++}$  ions entering into solution.

The underlying basis of (3.8) is the same as that of (3.11), i.e., an "extraneous" force or an "impressed" force occurs whenever there is a transformation of the electrical field energy into another form of energy or vice versa. Thus, in the process described by (3.9) and (3.10), electrical polarization energy is transformed into dissipated energy (such as heat), whereas in (3.11) chemical energy is transformed into electrical energy.

Using (3.9) and  $\mathfrak{B} = Ner$ , we obtain

$$\frac{d\mathbf{P}}{dt} + \omega_b^2 \mathfrak{B} = \omega_e^2 \mathfrak{C}, \quad (3.12)$$

and, therefore,

$$\mathbf{P} = \frac{1}{4\pi} \frac{\omega_e^2}{\omega_b^2 - \omega^2} \mathbf{E} = \chi_1 \mathbf{E}, \quad (3.13)$$

where  $\chi_1$  is the electric susceptibility.

The quantities  $\mathfrak{C}$  and  $\mathfrak{B}$  vary with time and are, therefore, more directly descriptive of observed phenomena than the corresponding quantities  $\mathbf{E}$  and  $\mathbf{P}$  which are functions of frequency. Both  $\mathbf{E}$  and  $\mathbf{P}$  are amplitudes of harmonic functions and do not represent directly the physical entities which actually do occur since these harmonic functions "existed" at  $t = -\infty$  and will continue to exist at  $t = \infty$ . Thus, in any direct description of a medium interacting with an electrical field,  $\mathbf{E}$ ,  $\mathbf{P}$ , and  $\chi$  appear in the integrands in the expressions such as

$$\mathfrak{C} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{E} e^{i\omega t} d\omega, \quad (3.14)$$

and

$$\mathfrak{B} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{P} e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \chi \mathbf{E} e^{i\omega t} d\omega, \quad (3.15)$$

in which the integration is applied to the entire real  $\omega$  axis.

The electric susceptibility has been represented in (3.15) as  $\chi$ , i.e., by a different symbol than the electric susceptibility  $\chi_1$  which appears in (3.13). The reason for this is as follows.

In order to perform the integration as shown in (3.15), the electric susceptibility  $\chi$  has to be known for all values of  $\omega$  along the real axis including  $\omega = \omega_b$ . On the other hand, (3.13) defines the electric susceptibility  $\chi_1$  for all values of  $\chi_1$  except  $\omega = \omega_b$ . ( $\chi = \chi_1$  when  $\omega \neq \omega_b$ .) In order to determine  $\chi$  for  $\omega = \omega_b$ , we need to consider the causal relationship between the force represented by  $\mathfrak{C}$  and the response to that force represented by  $\mathfrak{B}$ . Thus, if an electric force appears in the medium as a sudden impulse applied at a time  $t = t_0$ , i.e., if  $\mathfrak{C} = \mathfrak{C}_0 \delta(t - t_0)$ , where  $\mathfrak{C}_0$  is a constant and  $\delta(t)$  is the

Dirac  $\delta$  function,<sup>7</sup> then  $\mathbf{P}$  should be zero for all times  $t < t_0$  before the impulse occurred.

The causal relationship between  $\mathfrak{C}$  and  $\mathfrak{B}$  will be established if the electric susceptibility has the form  $\chi = \chi_1 + i\chi_{11}$  in which<sup>8</sup>

$$\chi_1 = -P \int_{-\infty}^{\infty} \frac{\chi_{11}(x)}{x - \omega} d\omega, \quad (3.16)$$

and

$$\chi_{11} = -P \int_{-\infty}^{\infty} \frac{\chi_1(x)}{x - \omega} d\omega, \quad (3.17)$$

where  $P$  designates the principal-value integration along the real axis (not to be confused with  $\mathbf{P}$  representing the Fourier transform of  $\mathfrak{B}$ ).

Substituting  $\chi_1$  as expressed by (3.13) in (3.17), we obtain<sup>9</sup>

$$\chi_{11} = (\omega_e^2 \omega_b / 4\omega) \delta(\omega_b^2 - \omega^2). \quad (3.18)$$

Consequently, the dielectric constant can be expressed as

$$\epsilon = 1 + \omega_e^2 \left[ P \frac{1}{\omega_b^2 - \omega^2} + i\pi \frac{\omega_b}{\omega} \delta(\omega_b^2 - \omega^2) \right]. \quad (3.19)$$

The expression (3.19) is valid for all frequencies along the real  $\omega$  axis.

### C. Energy Stored in an Absorbing Medium

In order to determine the mean energy density stored by an alternating field in an absorbing medium, we apply the same procedure which has been used for a nonabsorbing medium, i.e., a procedure based on the assumption that the stored energy is due to an accumulative effect of successive energy increments produced by a very slowly increasing almost monochromatic wave. We do not encounter here the difficulties which exist in the standard theory since the dielectric constant for the frequency range under consideration does not have the form  $\epsilon = \epsilon_1 + i\epsilon_2$ . Since we are confining ourselves to the frequency region in which  $\epsilon$  varies slowly with  $\omega$ , then (3.19) can be replaced by

$$\epsilon = 1 + \frac{\omega_e^2}{\omega_b^2 - \omega^2}, \quad (3.20)$$

which is real. This expression, when substituted in (2.11), provides a formulation for the energy density

<sup>7</sup> The properties of the Dirac delta function are described, for instance, in W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, England, 1957), 3rd ed., pp. 66-76.

<sup>8</sup> H. A. Kramers, *Atti Congr. Intern. Fis. Como* 2, 545 (1927); R. deL. Kronig, *J. Opt. Soc. Am.* 12, 547 (1926). See also L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Inc., New York, 1960), pp. 258-260.

<sup>9</sup> See in that connection tabulated Hilbert transforms published in *Tables of Integral Transforms*, compiled by Staff of Bateman Manuscript Project (McGraw-Hill Book Company, Inc., New York, 1954), Vol. II, pp. 243-262.

which is valid for both absorbing and nonabsorbing media.

There is an implied assumption in our analysis that the dielectric constant is independent of the temperature of the medium. Such an assumption is justified since the heat generated during the polarization or depolarization is negligible.<sup>10</sup> Consequently, in the framework of this analysis, there is no distinction between internal energy and free energy, and in both cases the energy density is expressed by (2.11).

To illustrate the applicability of (2.11), consider a cold uniform and extended plasma comprising electrons which are charge-equilibrated by heavy positive ions and assume that such a plasma interacts with an oscillating electric field having frequency  $\omega$ . As a result of the interaction, a portion of the energy of the field is continuously dissipated in the plasma. The dissipation is due to the collisional and radiative effects of electrons which acquire an organized oscillatory motion induced by the field. In accordance with our formulation, the dielectric constant of such a plasma is independent of the dissipational mechanism and is obtained from (3.20) by assuming  $\omega_b=0$  and  $\omega_e=4\pi N_e^2/m$ , where  $N_e$  is the number of electrons per unit volume. The dielectric constant has the form,

$$\epsilon = 1 - \frac{\omega_e^2}{\omega^2} \quad (3.21)$$

(The singular point  $\omega=0$  is excluded.) The expression (3.21) is different from the standard expression  $\epsilon=1-\omega_e^2/\omega(\omega-i\gamma)$  which depends on the frictional parameter  $\gamma$ .

Substituting (3.21) in (2.11), we obtain

$$\langle W_e \rangle = \frac{1}{16\pi} E^2 + \frac{1}{16\pi} \frac{\omega_e^2}{\omega^2} E^2. \quad (3.22)$$

The first term on the right side of (3.22) is the vacuum energy. The second term can be expressed as

$$N_e(m\langle v^2 \rangle/2), \quad (3.23)$$

where  $\langle v^2 \rangle$  is the mean square of the velocity of an oscillating plasma electron. Thus, (3.23) represents the mean kinetic-energy density due to the organized oscillatory electron motion induced by the alternating electrical field in plasma. The expression (3.22) is the same whether or not the dissipative processes occur.

#### IV. REFORMULATION OF OHM'S LAW

##### A. Conductivity

The most common mechanism of energy dissipation is described by an empirical relation

$$\langle Q \rangle = E^2 \sigma z \quad (4.1)$$

where  $\langle Q \rangle$  is the average rate of energy dissipation per unit of volume and  $\sigma$  may depend on  $\omega$  but does not depend on  $\mathbf{E}$ . The relationship (4.1) deals with the behavior of continuous media. It is, however, analogous to the relationship represented by Ohm's law in the electric circuit theory. Hence, the name "Ohm's law" is also applied to (4.1) and  $\sigma$  is designated as the "conductivity" of the medium. There are, however, significant differences between the processes described by (4.1) and the processes described by Ohm's law in the electric-circuit theory. Ohm's law in the electric-circuit theory deals exclusively with collisional effects which lead to thermalization and it therefore defines  $\langle Q \rangle$  as the quantity representing the rate of generation of heat. On the other hand,  $\langle Q \rangle$  which appears in (4.1) represents the rate of generation of the total energy which is dissipated in the medium. This includes not only heat produced by the collisional effects but also the energy of the waves which are emitted by oscillating electrons due to their organized motion induced by the field.

It is seen from (2.11) and (4.1) that both  $\langle W_e \rangle$  and  $\langle Q \rangle$  are proportional to  $E^2$ . Therefore, whenever Ohm's law is applicable, there must be a proportional relationship between  $\langle W_e \rangle$  and  $\langle Q \rangle$ . The coefficient of proportionality depends on the frequency of the applied oscillating field and is

$$16\pi \frac{\sigma(\omega)}{d/d\omega(\omega\epsilon)}. \quad (4.2)$$

Consequently, in dealing with dispersive media and using the energetic point of view, Ohm's law can be reformulated as representing a mechanism in which the mean rate of production of the dissipated energy is proportional to the mean energy stored. Such an interpretation of Ohm's law cannot be made in the framework of the standard theory since the standard theory does not provide a procedure for determining the mean energy in the presence of dissipation.

In order to obtain Ohm's law from our general formalism, we postulate that the extraneous force is proportional in magnitude to the force produced by the electric field, i.e., that

$$\mathbf{E}^{(\text{extr})} = iK(\omega)\mathbf{E}, \quad (4.3)$$

where  $iK(\omega)$  is a coefficient of proportionality which may depend on frequency.

Consider again the simplified model of matter composed of oscillating dipoles. The rate of dissipation of the energy of an oscillating dipole is  $[e\mathbf{E}^{(\text{extr})} \cdot \dot{\mathbf{r}}]$ , and, consequently, the corresponding average rate of energy dissipation per unit volume of the medium is

$$\langle Q \rangle = Ne \langle \mathbf{E}^{(\text{extr})} \cdot \dot{\mathbf{r}} \rangle = \left\langle \mathbf{E}^{(\text{extr})} \cdot \frac{d\mathcal{P}}{dt} \right\rangle. \quad (4.4)$$

In order to calculate  $\langle Q \rangle$  for an alternating field having frequency  $\omega$  we apply the relationship (4.3) and

<sup>10</sup> I. E. Tamm, *Osnovy Teorii Elektricheskva (Foundations of the Theory of Electricity)* (Gosudarstvennoe Izdatel'stvo Tekhnicheskoe Teoreticheskii Literatura, Moscow, 1949), 4th ed., p. 419.

use a formalism based on a complex representation of field quantities.<sup>3</sup> Thus, replacing  $\mathbf{E}^{(\text{extr})}$  with  $0.5(\mathbf{E}^{(\text{extr})} + \mathbf{E}^{(\text{extr})*})$  and  $\mathbf{P}$  with  $0.5(\mathbf{P} + \mathbf{P}^*)$ , where the asterisk designates a complex conjugate, we obtain

$$\langle Q \rangle = 0.5(\mathbf{E}^{(\text{extr})*} \cdot \mathbf{P}) = \frac{1}{8\pi} K(\omega) \frac{\omega_e^2 \omega}{\omega_b^2 - \omega^2} E^2. \quad (4.5)$$

Taking into account (4.1), we obtain an expression for conductivity which is

$$\sigma_1 = \frac{1}{8\pi} K(\omega) \frac{\omega_e^2 \omega}{\omega_b^2 - \omega^2}. \quad (4.6)$$

The relationship (4.6) has a general form. No specific dissipational model has been postulated in order to describe the behavior of  $K(\omega)$ .

When  $\omega$  approaches  $\omega_b$  the conductivity  $\sigma_1$  increases indefinitely and it is not defined for  $\omega = \omega_b$ . In order to determine the conductivity for all values of  $\omega$  along the real frequency axis including  $\omega = \omega_b$ , we need to apply the causality principle.<sup>8</sup> Then the conductivity assumes the form

$$\sigma = \sigma_1 + i\sigma_{11}, \quad (4.7)$$

where

$$\sigma_1 = \frac{1}{8\pi} P \left( K(\omega) \frac{\omega_e^2 \omega}{\omega_b^2 - \omega^2} \right), \quad (4.8)$$

and

$$\sigma_{11} = \frac{1}{8} K(\omega) \omega_e^2 \omega_b \delta(\omega_b^2 - \omega^2). \quad (4.9)$$

Consider now a particular case of the dissipational mechanism. Assume that the relationship (4.3) is not frequency-dependent, i.e., that  $K(\omega) = K_1$ , where  $K_1$  is a constant. Then the conductivity is

$$\sigma = \frac{\omega_e^2 K_1}{8\pi} \left[ P \frac{\omega}{\omega_b^2 - \omega^2} + \pi i \omega_b \delta(\omega_b^2 - \omega^2) \right]. \quad (4.10)$$

The expressions (4.9) and (4.10) are valid for all frequencies along the real  $\omega$  axis.

## V. THEORY AND EXPERIMENT

We wish to recall the distinction between the dielectric constant  $\epsilon$  which appears in the constitutive equation  $\mathbf{D} = \epsilon \mathbf{E}$  and the index of refraction  $n = ck/\omega$  (where  $k$  is the wave number and  $c$  is the velocity of light). The dielectric constant is a primary concept since it enters in the formulation of Maxwell's equations. On the other hand, the index of refraction is a derived concept since it is based on Maxwell's equations and is obtained from the dispersion equation for a medium interacting with a wave.

The index of refraction can be expressed as

$$n^2 = \frac{c^2 k^2}{\omega^2} = \epsilon + i4\pi \frac{\sigma}{\omega}, \quad (5.1)$$

where  $\epsilon$  is given by (3.19) and  $\sigma$  is expressed by (4.7) to (4.9).

There is extensive literature on measurements of the index of refraction for various substances interacting with electromagnetic waves, particularly in the optical frequency range. These measurements are not always in a quantitative agreement with the standard theory. The difficulty is based, apparently, on the theoretical estimates of  $\gamma = \gamma^{(P)} + \gamma^{(L)}$  which are by far too small to account for experimental results. The radiative effect represented by  $\gamma^{(P)}$  is not significant, particularly in a condensed substance interacting with a light beam since the damping of each oscillator due to radiation can be more or less entirely compensated by radiation from the surrounding oscillators. Thus, the surrounding oscillators restore to the original light beam the portion of energy emitted by each oscillator in the nonforward direction.<sup>11</sup> It appears also that  $\gamma^{(L)}$  plays a relatively minor role in the absorption processes, i.e., that the mechanism based on the transfer of the energy of excitation of a dipole into the translational energy of a struck atom is not significant.<sup>12</sup> These considerations led Landsberg<sup>12</sup> to the conclusion that "we are compelled to recognize that these (absorption) processes exist but we are not able to form a comprehensive representation of these (processes)."

The standard theory has been, however, successful in establishing a qualitative agreement between theory and experiments. Thus, using the considerations based on assumption (3.3), one can verify the general qualitative behavior of the absorption mechanism and indicate the characteristic maximum absorption at resonance.

The lack of quantitative agreement between the standard theory and experiment is probably due to the fact that the assumption (3.3) is incorrect in a sense that it is not consistent with the principle of conservation of energy. Therefore, a verification of the standard theoretical assumptions by means of an experiment can hardly be expected. Even if one obtains a satisfactory quantitative agreement between the standard theory and the experiment, such an agreement should be considered as fortuitous. There are also other theoretical inconsistencies in the standard formulation of the absorption mechanism and these concern the term  $\gamma^{(P)}$ . The best expression for  $\gamma^{(P)}$  which can be obtained from the classical theory is  $\gamma^{(P)} = 2e^2 \omega^2 / 3mc^2$ . This expression violates, however, the principle of causality.<sup>13</sup> In order to have a causal dissipative mechanism, one makes a rather arbitrary assumption that  $\gamma^{(P)}$  is independent of frequency and has a value  $2e^2 \omega_b^2 / 3mc^2$ .

A quantitative discussion of the absorption processes in the framework of the proposed theory is beyond the

<sup>11</sup> This has been pointed out by L. I. Mandelstam. See Ref. 12, p. 475.

<sup>12</sup> G. S. Landsberg, *Optika (Optics)* (Gosudarstvenno Izdatel'stvo Tekhnicheskoe Teoreticheskii Literatura, Moscow, 1952), pp. 475-476.

<sup>13</sup> J. Hamilton, in *Progress in Nuclear Physics*, edited by O. R. Frisch (Pergamon Press, Inc., New York, 1960), Vol. 8, p. 156.

scope of this analysis. A few remarks will be made, however, concerning the interaction of an electromagnetic wave with dielectric substances at optical frequencies, particularly in the neighborhood of resonance. We shall consider in that connection a beam of light within the spectral range,

$$\omega_b - \Delta\omega_b < \omega < \omega_b + \Delta\omega_b, \quad (5.2)$$

where  $\Delta\omega_b \ll \omega_b$ .

It is apparent from our formulation that the refractive index in the frequency range (5.2) is very large, and it increases indefinitely when  $\omega$  approaches  $\omega_b$ . Consequently, the corresponding wavelengths  $\lambda$  are very small and tend to zero when  $\omega$  approaches  $\omega_b$  since  $\lambda^2 = c^2/n^2\omega$ . Therefore, we deal here with the propagation of waves in an extended medium ( $\lambda \ll L$ , where  $L$  represents the linear dimensions of the medium). The phase velocity of these waves is very small and it tends to zero when  $\omega$  approaches  $\omega_b$ .

Assume now that a light beam having a frequency spectrum within the range (5.2) is perpendicularly incident on a relatively thin slab of a dielectric substance. Because of the very large value of the index of refraction a great portion of the incident energy is reflected at the surface of the slab and this accounts for the relatively small amount of energy which can be measured after the beam has traversed the slab. For the particular frequency component  $\omega = \omega_b$ , the reflection is total, i.e., a wave cannot be propagated through the dielectric substance at resonance frequency.

In the standard theory the frictional term  $\gamma$  limits the magnitude of the index of refraction at resonance ( $\omega = \omega_b$ ) and accounts for relatively smaller values of the index in the neighborhood of resonance. Therefore, the amount of light reflected by the slab as determined on the basis of the standard theory is smaller and, consequently, the amount of light transmitted through the slab is larger than the corresponding amount which could be obtained from the proposed formulation.

A detailed theoretical analysis of selective absorption in the range (5.2) is of some complexity and may involve considerations based on the proximity of other oscillators having frequencies in the neighborhood of  $\omega_b$ .

## VI. CONCLUDING REMARKS

There are inconsistencies and some confusion in the standard terminology which is used to describe the properties of both stationary and moving dielectric media. Some of these inconsistencies have been discussed elsewhere<sup>14</sup> in an analysis of the dispersive properties of media comprising matter in motion, such as plasma and media having plasma-like behavior, i.e., electrons or ion beams interacting with stationary matter, multibeam systems, etc. It has been shown that

the concept of the dielectric constant which is used to describe these media is misleading.<sup>14</sup> Thus, the term "dielectric constant" when applied to a plasma at nonzero temperature depends not only on the electric properties but also on the magnetic properties of plasma. It does not represent, therefore, the "dielectric constant" as interpreted in the framework of the Maxwell-Lorentz theory.

The media considered in this analysis are stationary and, therefore, the above inconsistencies do not occur since there is here a clear separation between the electric and the magnetic forces which are involved in the wave-matter interaction. The inconsistency which one encounters here is of a different nature since it involves the principle of conservation of energy. If we retain the standard theory, which is based on assumption (3.3), then, as pointed out by Ginzburg,<sup>15</sup> "the familiar expression

$$\langle W_e \rangle = \frac{1}{4\pi} \int_{-\infty}^t \langle \mathcal{E} \cdot \partial \mathcal{D} / \partial t \rangle dt \quad (6.1)$$

even when reduced to the form (constant  $\times E^2$ ) cannot be regarded as the total energy density when absorption is present if only because the value of  $\langle W_e \rangle$  may be negative."

Obviously, the expression (6.1) must represent the energy density, under *any* condition, whether or not absorption is present. This is essential in order to attribute to (2.4) the physical meaning given in Maxwell's theory. There is a clear separation in (2.4) between the term  $(1/4\pi)(\mathcal{E} \cdot \partial \mathcal{D} / \partial t)$  representing the rate of energy storage and the term  $(\mathcal{E} \cdot \mathcal{J})$  representing the energy dissipation. If the medium is nonabsorbing ( $\mathcal{E} \cdot \mathcal{J}$  is zero), then the rate of the total storage, i.e., the term  $(1/4\pi)(\mathcal{E} \cdot \partial \mathcal{D} / \partial t + \mathcal{J} \cdot \mathcal{B} / \partial t)$  is equal to the divergence of incoming energy flow. If the medium is absorbing, then the divergence of the incoming energy flow exceeds the rate of total energy storage. The term  $(1/4\pi) \times (\mathcal{E} \cdot \partial \mathcal{D} / \partial t)$  retains, however, the same physical meaning, i.e., it represents the rate of energy storage *whether or not* there is dissipation. The concept represented by this term is believed to be more basic than the concept of the electric field intensity. Thus, according to Planck,<sup>16</sup> "It is not easy to define the absolute value ( $\mathcal{E}$ ) of the electric intensity of the field. To arrive at one we start out from the concept of the energy of the field. The conclusion that an electric field contains a certain supply of energy emerges from the fact that the field can set a body into motion."

The essential feature of this analysis is based on an equivalence between the extraneous force  $e\mathcal{E}^{(\text{extr})}$  which represents a sink of an electrical field and the impressed force  $e\mathcal{E}^{(\text{impr})}$  which represents a source of an electrical field. An extraneous force is associated with a mechanism

<sup>15</sup> V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon Press, Ltd., London, 1964), p. 477.

<sup>16</sup> Max Planck, *Theory of Electricity and Magnetism* (Macmillan and Company, Ltd., London, 1932), p. 8.

<sup>14</sup> J. Neufeld, Phys. Rev. 123, 1 (1961); J. Appl. Phys. 34, 2549 (1963).

in which the polarization energy is destroyed and the resulting energy, which generally includes heat, is removed from the substance interacting with the field.

The distinction between the electrical field forces and the extraneous or impressed forces is based on two fundamental concepts in Maxwell's theory. These are the concept of an electromagnetic field per se, which serves as a transmitting medium, and the concept of "observable" forms of energy which are transmitted by means of an electromagnetic field from one region to another, more or less distant, region of space. The electromagnetic field per se is described in terms of "electrical" forces, whereas the "observable" forms of energy are described in terms of "extraneous" and "impressed" forces.

At the transmitting end of an electromagnetic system the energy of the electromagnetic field is created at the expense of an "observable" form of energy, and this accounts for the occurrence of "extraneous" forces. There is a complete equivalence in the process of creation and destruction of an electromagnetic field. A similar equivalence exists between the concepts of a source and of a sink of electrical energy. There is some logical inconsistency in the formalism used in the standard theory. The standard formalism includes the concept  $\mathbf{E}^{(\text{impr})}$  but does not provide the equivalent concept  $\mathbf{E}^{(\text{extr})}$ . Such an inconsistency does not exist in the proposed formalism.

In some instances in the standard theory the dielectric constant is replaced by dynamic conductivity. The procedure used to formulate the dynamic conductivity<sup>17</sup> is similar to the one used in determining the dielectric constant. It involves a description of electron motion similar to (3.1) under the assumption that  $\omega_b = 0$  and under an assumption based on the equality (3.3). By replacing (3.3) by (3.8), it can be shown that the dynamic conductivity can be reformulated in accordance with the principles outlined in the proposed theory.

#### ACKNOWLEDGMENTS

The author wishes to acknowledge valuable discussions on the subject of this paper with H. Grad, N. Marcuvitz, R. H. Ritchie, H. C. Schweinler and H. A.

<sup>17</sup> Reference 2, pp. 237-239.

Wright. However, the responsibility for the formulation presented in this paper rests entirely with the author.

#### APPENDIX: CALCULATION OF $(1/4\pi)\langle \mathfrak{G} \cdot \partial \mathfrak{D} / \partial t \rangle$ FOR AN ABSORBING MEDIUM

Consider an almost monochromatic wave  $\mathfrak{G}(t) = \mathbf{E}_0(t) \times \sin \omega_1 t$  having amplitude  $\mathbf{E}_0(t)$  slowly increasing during the period  $1/\omega_1$ . Assume that  $E_0(t) = E e^{\alpha t}$ , where  $\alpha \ll \omega$ . The wave has, therefore, the form  $\mathfrak{G}(t) = \text{Re} E e^{-i\omega t}$ , where  $\omega = \omega_1 + i\alpha$ . Thus,

$$\mathfrak{G} = 0.5(\mathbf{E} e^{i\omega_1 t} + \text{c.c.}), \quad (\text{A1})$$

and

$$\partial \mathfrak{D} / \partial t = 0.5(-i\omega_1 \epsilon(\omega_1) \mathbf{E} e^{i\omega_1 t} + \text{c.c.}), \quad (\text{A2})$$

where c.c. designates a complex conjugate.

Assuming that  $\omega \epsilon(\omega)$  can be approximated by two terms of its Taylor's expansion about  $\omega$  and taking into account that  $\epsilon = \epsilon_1 + i\epsilon_2$ , we obtain

$$-i\omega \epsilon(\omega) = \alpha \frac{d}{d\omega} [\omega \epsilon(\omega)]_{\omega=\omega_1} + [\omega \epsilon_1(\omega)]_{\omega=\omega_1} - i \left\{ \omega \epsilon_1(\omega) - \alpha \frac{d}{d\omega} [\omega \epsilon_2(\omega)] \right\}_{\omega=\omega_1}. \quad (\text{A3})$$

The average energy stored in the medium is

$$\left\langle \frac{dW_e}{dt} \right\rangle = \frac{1}{16\pi} \left( \mathfrak{G} \cdot \frac{\partial \mathfrak{D}^*}{\partial t} + \text{c.c.} \right), \quad (\text{A4})$$

where  $\mathfrak{D}^*$  designates a complex conjugate of  $\mathfrak{D}$ . Combining (A1) with (A4), and replacing  $\omega_1$  by  $\omega$ , we obtain

$$\left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle = \frac{1}{8\pi} \left\{ \alpha \frac{d}{d\omega} [\omega \epsilon_1(\omega)] + \omega \epsilon_2(\omega) \right\} E e^{2\alpha t}. \quad (\text{A5})$$

Then substituting in (A5),  $\mathbf{E}_0(t) = E e^{\alpha t}$ , we obtain

$$\frac{1}{4\pi} \left\langle \mathfrak{G} \cdot \frac{\partial \mathfrak{D}}{\partial t} \right\rangle = \frac{1}{16\pi} \frac{d}{d\omega} [\omega \epsilon_1(\omega)] \frac{d}{dt} [E_0(t)]^2 + \frac{1}{8\pi} \omega \epsilon_2(\omega) [E_0(t)]^2. \quad (\text{A6})$$