# Magnetoresistance and Coupled Orbits in Tin*

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We have observed large quantum oscillations in the magnetoresistance of pure white tin at liquid-helium temperatures and in fields up to 150 kG. The oscillations are best developed when **H** is approximately 5° from the $c$ axis and perpendicular to [110], and are interpreted as a magnetic breakdown effect which gives rise to a linear chain of coupled orbits. A calculation of the resistance, based on Pippard's suggestion that phase coherence is important only around small orbits, is in good agreement with the experimental results when fitted with a relaxation time for phase coherence of $2.3\times10^{-13}$ sec.

THE quantization of the coupled orbits produced by magnetic breakdown is able to cause large quantum oscillations in the magnetoresistance of a metal. In this paper we describe an experimental and theoretical investigation of this effect in metallic tin. When the magnetic field is 5–10° off the $c$ axis, the network of coupled orbits produced by magnetic breakdown in tin[1] is infinite only in one dimension, i.e., a linear chain. This linear chain is much more readily analyzed than the two-dimensional networks appropriate to zinc,[2–4] and we will show that our experimental results are well explained by a simple analysis similar to Pippard's.[4]

The oscillations shown in Fig. 1. were observed for **H** at 5° to the $c$ axis and approximately perpendicular to [110]. They were taken in a 150-kG steady field magnet at the National Magnet Laboratory. It can be seen that the relative amplitudes of the oscillatory and steady parts of the resistance are approximately equal at the highest field while both are still increasing with field. The line shapes are clearly nonsinusoidal. The oscillations are periodic in $H^{-1}$ and the period $5.8\times10^{-7}$ $G^{-1}$ is in reasonable agreement with the de Haas–van Alphen period attributed by Gold and Priestley[5] to the hole orbit $3\delta$ on the 3rd zone of their Fermi surface of tin. Since we have previously shown that magnetic breakdown links this orbit to the electron orbit $4a\zeta$ of the 4th-zone surface, it seems likely that the oscillations are due to magnetic breakdown.

We will now examine the linear network of coupled orbits produced in tin when breakdown allows the orbits $3\delta$ and $4a\zeta$ to couple along [110], but the field is tilted too far from the $c$ axis for any coupling to occur in the perpendicular direction. This linear net is shown in Fig. 2(a), with breakdown changing the chain of large electron and small hole orbits into an open orbit parallel to [110]. We will follow Pippard in postulating that the phase coherence over the large orbit may be neglected, so that the probability that an electron will tunnel from one large orbit to the next will depend only on a phase coherence analysis of the small network circled in Fig. 2(a) and enlarged in 2(b). This small network is the same as that analyzed by Pippard.[3] However, since scattering is sufficient to randomize the phase around a large orbit, we modify Pippard's treatment to take account of the appreciable scattering to be expected even on a small orbit by introducing a relaxation time $\tau_p$ for the preservation of phase coherence, thus converting Pippard's loss-free network into a more physically realistic lossy network. If the cyclotron frequency on the small orbit is $\omega_c$, while $P$ is the probability of breakdown at one energy gap ($Q=1-P$), and $\phi$ is the phase change around the small orbit, the expressions for $A$ and $B$, the reflection and

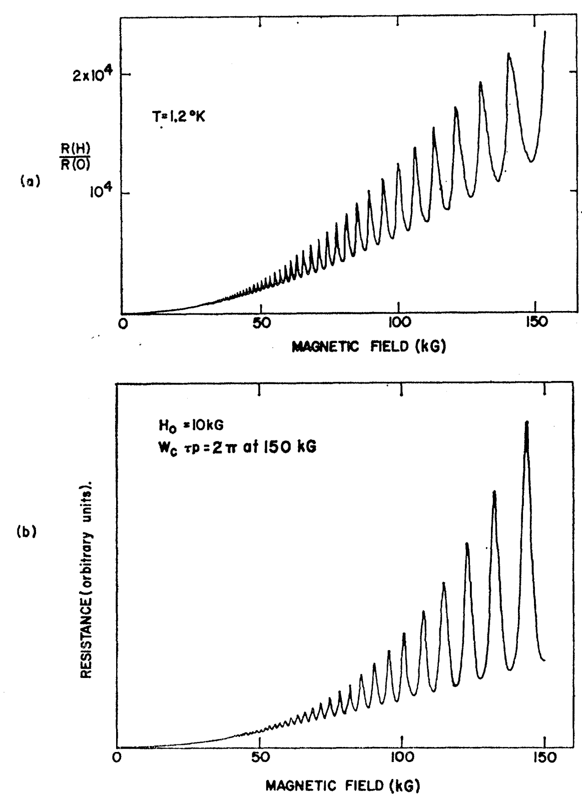


FIG. 1 (a) Recorder tracing of resistance versus magnetic field. (b) Plot of Eq. (4) for $H_0=10$ kG and $\omega_c\tau_p=2\pi$ at 150 kG.

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[1] R. C. Young, Phys. Rev. Letters 15, 262 (1965).
[2] R. W. Stark, Phys. Rev. Letters 9, 482 (1962).
[3] A. B. Pippard, Phil. Trans. Roy. Soc. London 256, 317 (1964).
[4] A. B. Pippard, Proc. Roy. Soc. (London) A287, 165 (1965).
[5] A. V. Gold and M. G. Priestley, Phil. Mag. 5, 1089 (1960).

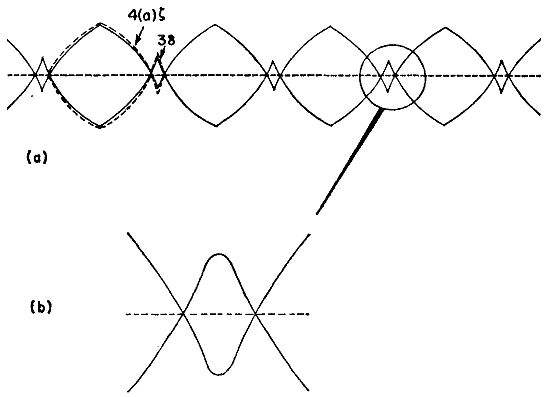


FIG. 2 (a) Linear network of coupled orbits produced when 3δ and $4a\zeta$ become linked by the breakdown which can occur whenever the electron intersects the dashed line. (b) Small network for determining the transmission and reflection coefficients $B$ and $A$.

transmission coefficients for the small network, become

$$A=Q(1+b^2-2b\cos\phi)/(1+Q^2b^2-2Qb\cos\phi), \quad (1)$$

$$B=b^2(1+Q)^2/(1+Q^2b^2-2Qb\cos\phi), \quad (2)$$

where $b=\exp(-2\pi/\omega_c\tau_p)$.

It is worth noting that the behavior of the small network has exact analogs in the microwave-resonance resonant transmission cavity or the Fabry-Perot interferometer. As $H$ increases, $P[=\exp(-H_0/H)]$ increases, so that the resonant value of $B$ increases while the shape of $B$ away from resonance resembles the "$Q$ curve" of a resonant cavity.

In order to analyze the effect that the oscillations in $A$ and $B$ have on the resistance, we will calculate an effective relaxation time $\tau_1$.[6] An electron on the open orbit has a probability $A$ of being removed from the open orbit at each major junction, so that it has a probability $2\omega A$ of being removed per sec, and $\tau_1$ is given by

$$1/\tau_1=1/\tau_0+2\omega A. \quad (3)$$

The third relaxation time $\tau_0$ is the regular relaxation time, while $\omega$ is the cyclotron frequency of the combined large and small orbits. By analysis of the magnetoresistance of a simple model of a compensated metal with open orbits, similar to that carried out by Fawcett,[7] except that the open orbits are on convoluted cylinders and have a relaxation time $\tau_1$, it can be shown that the resistance $R\sim\omega^2\tau_1$. If $A\omega\tau_0\gg1$, we may substitute $\tau_1$ from (3) to give (omitting factors not dependent on the magnetic field)

$$R\sim\frac{\omega}{2A}=\frac{\omega(1+Q^2b^2-2Qb\cos\phi)}{2Q(1+b^2-2b\cos\phi)}. \quad (4)$$

Writing $\phi=(A_\delta\hbar/eH)$ we see that (4) gives a resistance which is periodic in $H^{-1}$ with a period depending on the area $A_\delta$ of the orbit 3δ. In Fig. 1(b) we show a plot of (4) for $H_0=10$ kG and $\omega_c\tau_p=2\pi$. The nonsinusoidal line shape is well reproduced at high fields; in fact this shape is expected from (4) since at high fields ($Q$ small), the equation resembles the transmission characteristic of the Fabry-Perot interferometer.

The discrepancy in amplitude, in high fields, between Figs. 1(a) and 1(b) probably occurs because we have neglected to consider contributions from planes over a range of values of $k_H$. Both $A_\delta$ and $H_0$ vary with $k_H$ and this will have two effects: (1) adding the contributions to the conductivity on a "Cornu spiral" will reduce the amplitude of the oscillations although the extremal value of $A_\delta$ will still dominate, and (2) at high fields more electrons will contribute to the conductivity, raising it and so reducing the resistance from the value expected for a constant number of electrons [Eq. (4)].

We have thus shown that our network analysis is in reasonable agreement with our experimental observations, certainly of the correct magnitude, the oscillations being due to the coupled orbits produced by magnetic breakdown. The value of $H_0=10$ kG agrees with studies of the magnetoresistance of tin.[8] From Khaikin's[9] mass of 0.1 m for 3δ, we estimate $\tau_p=2.3\times10^{-13}$ sec, comparable to estimates from the de Haas–van Alphen effect.[10]

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[6] L. M. Falicov and P. R. Sievert, Phys. Rev. 138, A88 (1965).

[7] E. Fawcett, Advan. Phys. 13, 139 (1964).

[8] J. G. Anderson and R. C. Young (to be published).

[9] M. S. Khaikin, Zh. Eksperim. i Teor. Fiz. 42, 27 (1962) [English transl.: Soviet Phys.—JETP 15, 18 (1962)].