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Hall Effect and Transverse Magnetoresistance in Some Ferromagnetic Iron-Chromium Alloys*

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The two Hall coefficients, R_0^* and R_s^*, and the transverse magnetoresistance were measured for five alloys containing nominally 0.75, 2.3, 5.1, 12.7, and 25.1 at.% chromium in iron, at temperatures ranging from that of boiling helium to room temperature. For all compositions and temperatures both R_0^* and R_s^* were positive. R_0^* increases smoothly as the electron concentration passes through that of pure iron into lower concentrations, and reaches a maximum near 12.7% Cr (25.7 e/atom). No sharp maximum in R_0^* was found at pure iron as had previously been found for pure Ni and pure Co. This behavior is shown to be consistent with a simple band picture, although the detailed interpretation of the data is uncertain because of the complicated multiband structure in the pure-iron region. The observed sign and magnitudes of the ordinary constant do show the importance of the hole conduction in the $3d$ sub-band relative to the electronic conduction in the $4s$ band. The behavior of R_s^* as a function of resistivity ρ is found to agree with the theories that predict a ρ^2 dependence in the more concentrated alloys and also in the dilute alloys in the higher temperature region (above ~100°K). Determinations of the high-field slopes of the transverse-magnetoresistance curves were made. The data appeared linear within the accuracy of the observations.

INTRODUCTION

IT has been shown that in ferromagnetic materials the Hall effect is described by the equation:

$$\rho_H = R_0 B + R_s M\,, \tag{1}$$

where ρ_H is the Hall electric field per unit current density, which is usually referred to as the Hall resistivity, B is the applied magnetic induction, M is the magnetization and R_0 is the ordinary Hall coefficient. R_0 gives an indication of the number of conduction carriers per atom, as does the ordinary Hall coefficient for nonferromagnetic materials. R_s is called the extraordinary Hall coefficient and the presence of this term in the Hall resistivity is characteristic of magnetic materials. Usually R_s is much larger and is much more temperature-dependent than R_0. At the present time the theory describing the ordinary Hall effect is much more complete than that of the extraordinary effect.

In some alloys of the ferromagnetic metals of the first transition series, Hall measurements indicated a smaller number of conduction electrons in the $4s$ band, than was indicated by the values directly obtained from saturation-magnetization data. A four-band model based on an idea by Mott[1] was employed by one of us[2] to explain successfully the observed values of R_0, and their temperature dependence. The four bands involved are the $4s$ and the $3d$ bands, both considered split in the sense that those electrons in the same band, but of opposite spin may have different transport properties. On the energy scale the usual model is adhered to, namely that the $4s$ band for electrons of opposite spins is not split, and the $3d$ band for opposite spins is. It was then shown by one of us (E.M.P.) that, if one can assume a Hall coefficient R_{0j} and a conductivity for each such sub-band, then one can write for the resultant ordinary Hall coefficient:

$$R_0 = \textstyle\sum_j (\sigma_j/\sigma)^2 R_{0j}\,, \tag{2}$$

where σ is the algebraic sum of the conductivities of all the j sub-bands and the summation is over all sub-bands contributing to conduction. The boundary condition that the net current in the transverse direction is zero is used.

For a single spherical conduction band that is either nearly full or nearly empty the Hall constant is given by

$$R_0 = (\mp)/nNe \tag{3}$$

in mks units, where n is the average number of electrons per atom in a nearly empty band or the number of holes (vacancies) in a nearly full band, N is the number of atoms per cubic meter and e is the magnitude of the electronic charge. The minus sign is for electron conduction and the plus sign is for hole conduction. It has been pointed out[2] that R_{0j}, for a single band in a cubic structure, is very large for a spherical band, which is either nearly empty or nearly full, but becomes small when the Fermi surface becomes more complicated. Therefore, if one of the $\sigma_j^2 R_{0j}$'s is derived from a spherical sub-band and is large compared to those of the other sub-bands this term in Eq. (2) will dominate. Only if this is the case, then it is meaningful to speak of the effective number of charge carriers n^* being defined as

$$n^* = 1/NeR_0\,. \tag{4}$$

Then, except for smaller contributions to n^* of the remaining terms in Eq. (2), this number gives at once an indication of the number of charge carriers of the spherical sub-band. This was found to be the situation in the more concentrated alloys of nickel, where n^* gave the number of electrons in the parallel $4s$ band. How-

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[1] N. F. Mott, Proc. Roy. Soc. (London) A153, 699 (1936).

[2] E. M. Pugh, Phys. Rev. 97, 647 (1955).

ever, in the dilute alloys of iron, such as the iron-chromium alloys studied here, this cannot be the case. Here the Hall effect may indicate only the relative importance of the various terms in Eq. (2). In pure iron R_0 is found to be very small (at room temperature a value of $+2.28\times10^{-11}$ m^3/C was measured by Soffer *et al.*,[3] which merely indicates approximate cancellation.

By measuring high-field magnetoresistance of very pure iron at low temperature, Fawcett and Reed[4] found that in this metal the number of holes equals the number of electrons (compensation). The fact that it has a nonzero Hall coefficient may be due to the four separate bands described above. For example, in the case of all spherical sub-bands Eq. (2) becomes

$$R_0=\frac{1}{Ne}\sum_j \frac{(\mp)}{n_j}\left(\frac{\sigma_j}{\sigma}\right)^2. \tag{5}$$

Now there may be equal electrons and holes in two sub-bands, but because of different scattering probabilities, we may have $\sigma_e\neq\sigma_h$, and the two terms involving these bands do not cancel one another.

The ordinary Hall coefficient has been found to become negative upon alloying small amounts of Ni or Co with iron and it remains negative for alloys of Fe-Co,[5] Fe-Ni,[3] Ni-Co,[6] Ni-Cu,[7] Fe-Ni-Cu,[8,9] up to electron concentrations of that of copper. The constant seems to rise very sharply for pure nickel and somewhat less sharply for pure cobalt. Near iron, upon lowering the electron concentration, the rise is very sharp again. The assumption that for alloys having electron concentrations less than that of iron the value of R_0 drops again into the negative region did not seem unreasonable. It has been indicated, however, that the density-of-states function of the 3*d* band near 26 *e*/atom (Fe) is very small for the spin antiparallel sub-band[10] and the other half of the 3*d* band just begins to empty near iron and continues to do so as the average number of electrons per atom is decreased. The ordinary Hall coefficient, according to this picture if the rigid-band model is applied, should continue to increase for lower concentrations, if the concentration in the 4*s* band does not change much. Fe-Cr alloys were used to investigate this variation of R_0 with electron concentration in the region of 26 to 25.5 *e*/atom.

The first reasonably successful theory concerning the extraordinary Hall coefficient involved spin-orbit coupling and was given by Karplus and Luttinger.[11] The theory indicates that R_s is proportional to the square of the resistivity. Since then the theory has been developed further,[12–15] but where calculations were possible the results are not very different. Since some, though by no means all, observed values of R_s have shown agreement with this theory, there is an indication that the effect is due to a spin-orbit–interaction term. A calculation has been made by Berger[16] giving more evidence that this term is responsible for the R_sM term in Eq. (1). It involves a calculation based on the spin-orbit coupling which at the electron concentration of 27.7 *e*/atom provides an explanation for the observed sign change of R_s. Berger's theory has also been able to account for an anomalously large ferromagnetic anisotropy at that concentration, also indicating that here *LS* coupling is very important.

The definition of the ferromagnetic anisotropy is (for B such that the material is magnetically saturated):

$$[\rho_{\parallel}(B)-\rho_{\perp}(B)]/\rho_0, \tag{6}$$

the difference in the resistivities of the material when the magnetic induction is parallel to the sample current and when it is perpendicular, divided by the resistivity of the material in the technically demagnetized state.

Using the physical arrangement, that was used for the Hall measurements, it was possible to measure the transverse magnetoresistance, $(\rho_{\perp}-\rho_0)/\rho_0$, but not the longitudinal magnetoresistance. In addition, the determination of ρ_0 was not very accurate due to problems encountered in demagnetizing the material using the available equipment. Thus, $\rho_{\perp}(B)-\rho_0$ was not determined with sufficient accuracy to use the approximate relation $\rho_0=\frac{1}{3}\rho_{\parallel}+\frac{2}{3}\rho_{\perp}$ for finding the anisotropy. However, using the same methods as were used to determine the high-field slopes of the Hall curves, the transverse magnetoresistance in the high-field region could be inspected closely. High-field data are of interest because they are related to the type of scattering of the electrons. For ferromagnetic materials the resistivity usually varies linearly with applied field, above magnetic saturation of the sample. The slopes of the lines may be positive but are usually negative. A small curvature has been observed[9] in some materials, but none was observed in the Fe-Cr alloys studied here. When the mean free path of the charge carriers is large (low temperature, low impurity content) the effect of the Lorentz force may become apparent in that the slope of the magnetoresistance curve is positive. (Lorentz force causes positive slope in nonferromagnetic materials under those conditions.) A negative slope can be attributed to magnon scattering: The number of magnons decreases with increasing applied field, thus decreasing

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[14] C. Strachan and A. M. Murray, Proc. Phys. Soc. (London) **73**, 433 (1959).

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[16] L. Berger, Physica **30**, 1141 (1964); Phys. Rev. **138**, A1083 (1965).

the number of scattering centers and reducing the resistivity. Galkina and Chernikova,[17] and Kondorskii, Galkina and Chernikova[18] were able to conclude from their magnetoresistance and saturation-magnetization data of three Ni-Cu alloys that magnon scattering dominates at low temperatures (20°K and lower). It is likely that at higher temperatures this effect also is the cause of the negative magnetoresistance slopes, the number of phonons being independent of the applied magnetic field. Smit[19] discusses another possible explanation of the negative slope. He employs an idea due to Mott, of shifting bands with applied magnetic field. Such a contribution may in some cases also be present, but should vanish for low temperatures, where dM/dB goes to zero.

EXPERIMENT

Samples of five different compositions were used. The samples contained 0.75, 2.3, 5.1, and 25.1% (wt) Cr in Fe. The sample materials have been prepared for metallurgical studies, done by Horne, Roy, and Paxton.[20] The zone-melted iron was made by Battelle Memorial Institute and the U. S. Steel Corporation, Fundamental Research provided the other materials. Analysis[20] of the alloys before annealing found the impurity content (C, N, H, and O) to be small. The materials were first cold-rolled and then machined to make samples of 10 cm×1 cm×0.1 cm. Two such samples were made of each composition. Probes and wires were also made of the same materials. The samples were cleaned in trichlorethylene before they were sealed in quartz tubes in $\frac{2}{3}$ atm helium. The annealing temperature was chosen above the Curie point, at 820°C for the alloy of 5.1% Cr in Fe, and 850°C for all other compositions. This allowed the material to remain in the bcc α phase during annealing. All samples were annealed for three hours. They were then slowly cooled to room temperature. Two extra samples that had been made of the alloy containing 2.3% Cr in Fe were annealed in the same way and then quenched to room temperature.

Density measurements were done on one of the two samples of each composition, after which they were cut crosswise at the location where the Hall measurements are made. These cross sections were then polished, etched and checked for grain size. In all cases the grain size was found to be small so that the measured coefficients are not a function of the sample orientation.[21]

The one sample of each composition that remained was then mounted in the usual way.[8] The Hall voltage and the transverse magnetoresistance curves then were determined as functions of B for certain fixed temperatures. The incremental method[6] was used in order to eliminate all drifts that are linear in time.

The samples were in direct contact with the constant temperature baths. The following boiling liquids were used (at atmospheric pressure): helium, hydrogen, nitrogen, methane (112°K), ethylene (169°K), and propane (231°K). A method was devised to liquefy the latter three gases at a rather rapid rate directly into the Dewar in which the measurements were made. For room temperature, a bath of a low-viscosity oil (Dow-Corning 200 fluid) was used. The oil was circulated through a system, setting it in thermal contact with a heat reservoir at room temperature. This was done in order to eliminate a steady temperature rise as well as a temperature gradient parallel to the current along the longest dimension of the sample.

All the measurements were made using an A. D. Little Company magnet with $5\frac{1}{2}$ in. fact diameter pole pieces, using a $\frac{7}{8}$-in. pole gap for the helium and hydrogen baths, and a $\frac{5}{8}$-in. gap for all other temperatures. A maximum field of 32.5 kG was obtained in the latter case.

In order to improve the sensitivity of the measurements a new current-control system had been purchased[22] and installed. The magnitude of the magnetic fields produced were determined with a Rawson gaussmeter which was calibrated using standardized NMR equipment. Field settings were reproduced by bringing the magnet current to the desired value, which was done with a Wenner potentiometer, hooked up to a Leeds and Northrup dc null detector. This gave a reproducibility to within one tenth of a percent at 28.5 kG. The field was uniform to within the same amount in the region where all the measurements were made.

The data were analyzed by averaging all four of the observed values of incremental Hall resistivities for the two field directions and sample current directions, thus eliminating all effects that reverse with sample current and not magnetic field and vice versa. Those resulting data points, that were found to lie in the linear region which occurs well above magnetic saturation, were fitted to a straight line, using the method of least squares. Both R_0^* and R_s^* were determined from this straight line. The coefficients are starred because they represent experimentally determined values which may include additional contributions. The coefficient R_s^* can be determined from the high-field data if the saturation magnetization of the materials at the measured temperatures is known.

The magnetoresistance data were obtained, and processed in the same manner. In this case the voltage

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[17] O. S. Galkina and L. A. Chernikova, Zh. Eksperim. i Teor. Fiz. **38**, 3 (1960) [English transl.: Soviet Phys.—JETP **11**, 1 (1960)].

[18] E. I. Kondorskii, O. S. Galkina, and L. A. Chernikova, Zh. Eksperim. i Teor. Fiz. **35**, 1070 (1958) [English transl.: Soviet Phys.—JETP **7**, 741 (1958)].

[19] B. J. Smit, Physica **17**, 612 (1951).

[20] G. T. Horne, R. B. Roy, and H. W. Paxton, J. Iron Steel Inst. (London) **201**, 161 (1963), contains a table of these impurities.

[21] Recently an anisotropy in the ordinary Hall coefficient has been measured in some materials by E. Tatsumoto and T. Okamoto, J. Phys. Soc. Japan **14**, 226 (1959).

[22] Funds supplied by the NSF were used to provide a new power supply using solid-state components for the accurate control of the magnet current.

TABLE I. The measured values of R_0^*, R_s^*, n^*, the resistivity for $B=0$, the density and the saturation magnetization as deduced from data by Fallot.

Nominal chromium (at.%)	Temp. (°K)	$R_0^* \times 10^{10}$ (m³/C)	M_s (weber/m²)	$R_s^* M_s \times 10^{10}$ (V-m/A)	$R_s^* \times 10^{10}$ (m³/C)	n^*	$\rho_0 \times 10^7$ (Ω-m)
0.75	309	0.414	2.109	24.1	11.6	1.80	1.44 (304°K)
	231	0.924	2.119	10.9	5.16	0.806	1.06
	169	0.371	2.127	6.14	2.89	2.01	0.755
	112	0.789	2.132	2.01	0.942	0.944	0.482
	77	1.33	2.134	0.958	0.449	0.559	0.330
	20	1.83	2.137	4.94	2.31	0.406	0.204
	4.2	1.71	2.137	5.63	2.63	0.436	0.199
2.3	316	0.599	2.064	45.8	22.2	1.42	2.11 (307°K)
	314	0.614	2.064	45.4	22.0	1.21	
	231	0.402	2.078	25.5	12.3	1.85	1.58
	231	0.625	2.078	23.5	11.3	1.19	
	169	0.649	2.089	13.0	6.22	1.15	1.19
	112	1.18	2.095	5.82	2.78	0.630	0.829
	77	2.15	2.098	3.72	1.77	0.345	0.616
	20	2.26	2.101	13.8	6.57	0.329	0.451
	4.2	1.75	2.101	16.6	7.88	0.426	...
5.1	313	0.626	1.988	85.1	42.8	1.19	3.04 (307°K)
	305	0.783	1.988	73.5	37.0	0.950	
	231	0.814	2.005	44.7	22.3	0.914	2.35
	169	0.920	2.019	25.5	12.7	0.809	1.85
	112	1.55	2.027	12.5	6.19	0.480	1.38
	77	2.39	2.031	8.54	4.20	0.316	1.14
	20	2.87	2.034	21.5	10.6	0.259	0.922
	20	2.76	2.034	21.9	10.7	0.270	
	4.2	2.23	2.034	24.3	11.9	0.342	0.913
12.7	308	0.671	1.736	111.0	64.2	1.11	4.23 (306°K)
	231	0.671	1.756	69.5	39.6	1.11	3.30
	231	0.887	1.756	67.8	38.6	0.839	
	169	1.08	1.773	40.2	22.6	0.691	2.56
	112	1.64	1.783	21.1	11.8	0.455	1.91
	77	2.60	1.788	14.6	8.18	0.286	1.59
	20	4.48	1.791	12.7	7.08	0.162	1.37
	4.2	4.08	1.792	14.2	7.90	0.182	1.37
25.1	307	0.614	1.417	104	73.3	1.21	4.74 (306°K)
	231	0.808	1.445	65.6	45.4	0.920	3.67
	169	0.680	1.467	40.6	27.7	1.09	2.80
	112	1.38	1.482	21.6	14.6	0.537	2.04
	77	1.50	1.488	14.6	9.78	0.497	1.63
	20	1.87	1.494	9.87	6.61	0.397	1.35
	4.2	2.53	1.494	7.54	5.05	0.235	1.34

drop along the sample for a fixed distance between two probes was measured as a function of applied magnetic field. This voltage was measured with the Wenner potentiometer and each reading was immediately followed by a sample current reading.

RESULTS

Upon inspection of the results it was found that data points of ρ_H versus B obtained below 28.5 kG were usually below the straight line observed at higher fields. These and lower field points should therefore not be included in the least-squares calculations of these straight lines, because apparently for these alloys complete saturation of the sample only occurs above such fields. Ordinary Hall coefficients calculated from lower field data will therefore give results that are too large. The results are given in Table I. Found values of $R_s^* M_s$ are given as well as R_s^* using M_s values calculated from data by Fallot.[23]

Some Hall measurements had previously been made on a few alloys of low chromium content by Jellinghause and de Anderes[24] (up to 5.6% Cr in Fe), all at room temperature. On an alloy containing 0.9% Cr, Okamoto[25] measured the Hall coefficients as a function of temperature, from 77°K to temperatures above the Curie point. The values of R_s^* given in Table I for alloys of such compositions agree reasonably well with those found by these authors. The temperature dependence of R_0^* found by Okamoto is also in good agreement with our results shown in Table I. The values of R_0^* found by Jellinghause and de Anderes are somewhat larger than

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[23] M. Fallot, Ann. Phys. (Paris) 6, 305 (1936).

[24] W. Jellinghause and M. P. de Anderes, Ann. Physik 7, 189 (1961).

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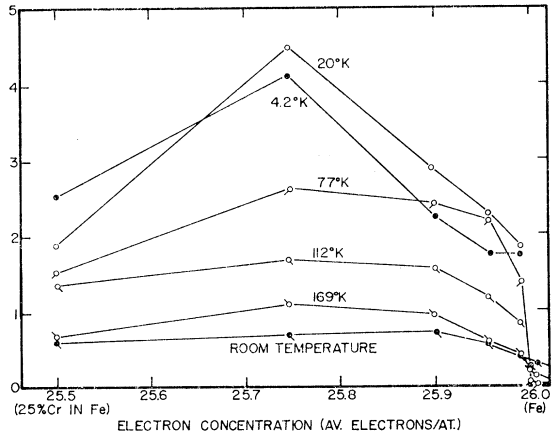

FIG. 1. Ordinary Hall coefficients R_0^* at different temperatures for alloys having different concentrations of Cr in Fe.

were obtained here, possibly because they did not have a sufficiently strong magnetic field available.

In Fig. 1 is shown the dependence of R_0^* on composition and temperature. Some data on Fe-Co by Beitel[5] *et al.*, and Fe-Ni by Soffer[3] *et al.*, are included here. Figure 1 shows the important result that no peak is found in R_0^* as was found in the cases of both pure Ni and pure Co. Instead a smooth increase of R_0^* was found as the electron concentration decreased through that of pure iron.

All measurements of the ordinary Hall coefficient in pure iron have shown a small positive effect. As has been mentioned, Soffer *et al.*,[4] obtained 2.28×10^{-11} m^3/C as compared with $\sim25\times10^{-11}$ m^3/C in the nickel rich Ni-Cu alloys. From the Hall data alone this can be interpreted either as being primarily due to a single band more than half filled or to two bands, one nearly full and the other nearly empty, that nearly cancel each other. Any interpretations from such small magnitude coefficients are ambiguous. Data from other experiments suggest that the second of these two interpretations is the most likely.

While the interpretations from small magnitude R_0^*'s are ambiguous, those from large magnitude R_0^*'s are not necessarily so. For example, the $R_0^*=+44.8\times10^{-11}$ m^3/C observed in the alloy (Cr 12.7%—Fe 87.3%) is probably due to a single nearly filled band. Of course, it is not mathematically impossible to obtain such a magnitude from two competing bands, one practically empty and the other practically full. However, to obtain the observed conductivity of 7.3×10^6 Ω^{-1} cm^{-1} the mobilities in each of these bands would need to be comparable to those found in pure metals at 20°K. This seems highly unlikely in an alloy of this concentration. This suggests then that the parallel $3d$ is the primary contributor to the R_0^* and that it contains approximately 0.16 holes/atom. Such a conclusion is consistent with the saturation-magnetization measurements and other evidence as to the structure of these bands.

In Fig. 2 is plotted R_s^* versus ρ_0 on a log-log scale. Observed values of R_s^* for given compositions at various temperatures are connected. The figure shows that in the high-temperature regions there are nearly ρ_0^2 dependences for the values of R_s^*. The slopes found, using the higher temperature results are: 2.16, 2.21, 2.14, 2.08, and 1.85 for the samples containing 0.75, 2.3, 5.1, 12.7, and 25.1% Cr in Fe, respectively. If in Fig. 2 the values of R_s^* at a given temperature, for the various compositions, were connected, then straight lines of somewhat smaller slopes would be found at the higher temperatures. Again at the low temperatures the spin-orbit theories available at the present time do not explain the observed behavior.

In Fig. 3 the extraordinary Hall conductivity, defined as

$$\gamma_{\rm HS}^*\equiv R_s^*M_s/\rho_0^2 \qquad (7)$$

is plotted as a function of electron concentration at the temperatures 4.2, 20, 77°K and at room temperature. Figure 4 shows a relatively smooth variation of $\gamma_{\rm HS}^*$ for temperatures down to about 169°K, in the pure-iron region. For lower temperatures this Hall conductivity clearly becomes dependent on the composition especially near 26 e/atom. This means that in this region R_s^* is not such a simple function of resistivity alone.

The effect of quenching of the alloy containing 2.3% Cr in Fe did not have a significant effect on the Hall

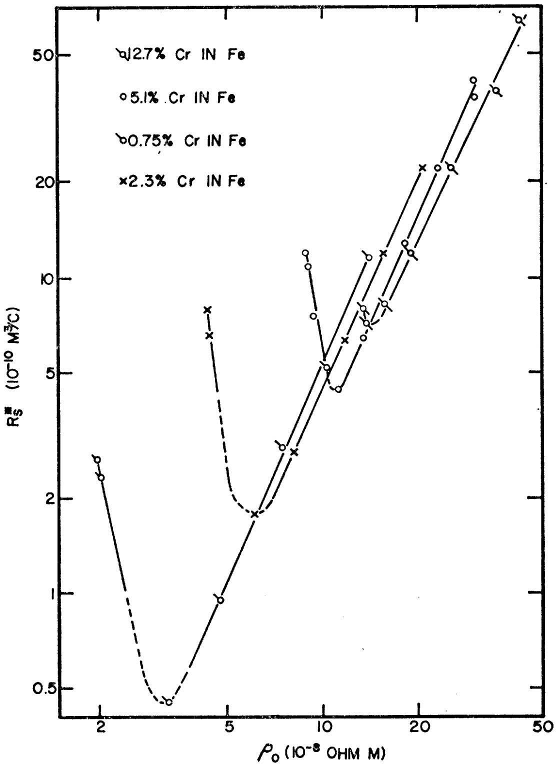

FIG. 2. Extraordinary Hall coefficients R_s^* versus ρ_0 for the different Fe-Cr alloys using a log-log plot.

coefficients, the variations being at most slightly larger than the estimated error of the determined Hall coefficients. This error, for the ordinary Hall coefficient, is estimated to be near 3%, and is mainly due to the following three factors: (a) sample thickness determination, (b) the Nernst effect, and (c) a possible consistent variation of the sample current with the applied magnetic field. Sample thickness was determined with a Pratt and Whitney Electrolimit Gauge, which should keep this error below 1.5%. The Nernst effect was reduced below one percent by using long samples in thermal but not electrical contact with copper plates. A calculation[26] shows that an error is introduced in these measurements if the voltage responsible for the main sample current, rather than the current itself is held constant. Such a variation can be due to the magnetoresistance of the material, which gives an error equal to the slope of the magnetoresistance curve multiplied by (R_0B+R_sM). In these experiments this magnetoresistance error was negligible because the total resistance of the sample was small compared to the rest of the resistance in the sample-current circuit. However, the changes in the magnetic field may introduce slight movements that cause consistent changes in the resistances at the current contacts. Such errors can be reduced by careful mounting.

The error should not exceed 3% for R_0^*, but may be a little larger for R_s^*. For the temperatures at which these measurements were made, errors introduced due to the paramagnetic increase of the saturation magnetization giving rise to a finite $R_s dM/dB$ term in the high-field slope, are found to be negligible.[25]

The slopes of the transverse magnetoresistance curves are given in Table II. There was no consistent, measureable curvature in the high-field data. The resistivities at zero field that are listed in Table I were measured by lifting the entire Dewar out from between the poles of the magnet.

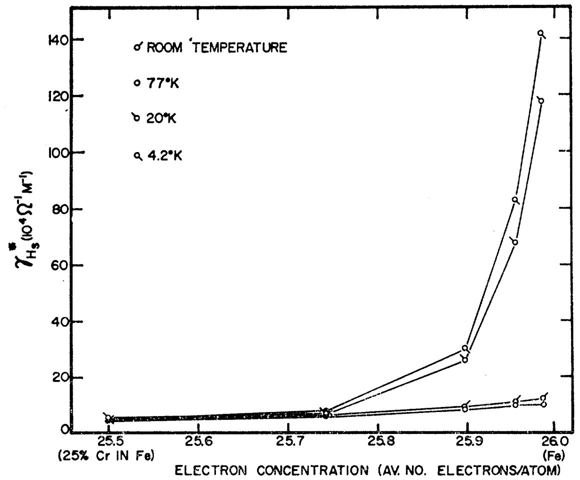

FIG. 3. Extraordinary Hall conductivity $\gamma_{HS}^*=R_s^*M_s/\rho_0^2$ for 25% Cr in Fe at different temperatures.

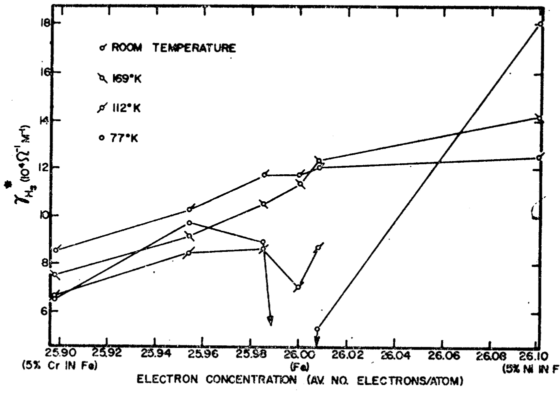

FIG. 4. Extraordinary Hall conductivity $\gamma_{HS}^*=R_s^*M_s/\rho_0^2$ for 5% Cr in Fe at different temperatures.

DISCUSSION

It is known that the Fermi surface of iron and dilute alloys of iron is a rather complicated one. Therefore, the results of the Hall measurements cannot be used to make definite conclusions concerning the band structure. The fact that the Hall coefficients are all positive for these Fe-Cr alloys down to an electron concentration of 25.2 *e*/atom is of importance. It is a direct indication that the 3*d* electrons are not tightly bound in the alloys studied, particularly not in the region of 25.7 *e*/atom. The lack of a cusp or peak in R_0^* at pure iron is also of interest. The continuous increase of R_0^* as electron concentration is decreased can most simply be interpreted using a rigid band model as described in a previous section, namely that in which the 3*d*-antiparallel sub-band is thought to have only very few electronic states at the Fermi surface, and the other 3*d* sub-band just begins to empty. The continuing increase in R_0^* then may be a direct result of the continuing depletion of this latter sub-band, provided its σ_j^2 increases more rapidly with increasing holes than R_{0j} decreases. The number of holes at the Fermi surface in that sub-band seems to be increasing steadily and suggests that no major band shift occurs at iron. It is assumed that a change in the 4*s* band would have no major effect on the variation of R_0^* in this region. The fact that the Hall effect shows the conduction by holes in the 3*d* parallel sub-band to dominate the conduction by the electrons of both 4*s* sub-bands, somewhat justifies this conclusion. The Hall effect in pure iron is expected to be small because the metal is compensated,[4] so that any band shift within the limits of compensation is not detected by the Hall effect.

The results of the R_s^* measurements have shown, as have several R_s^* values found for other materials, that the theory at the present time is still inadequate to explain all the results. Particularly at the lower temper-

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[26] G. C. Carter, thesis, Carnegie Institute of Technology, 1965 (unpublished).

TABLE II. High-field slopes G_0, of the equation $(\rho_{BL}-\rho_0)/\rho_0=G_s(M_s)+G_0B$.

Nominal Cr content	Temp. (°K)	$G_0(10^{-5}\ \mathrm{m^2/Wb})$	Nominal Cr content	Temp. (°K)	$G_0(10^{-5}\ \mathrm{m^2/Wb})$
0.75%	304	35.9	2.3%	313	56.4
	231	4.58		231	7.44
	169	15.8		169	102
	112	−246		112	11.8
	77	−548		77	−102
	20	−2700		20	−443
	4.2	−4000		4.2	...
2.3% (quenched)	302	37.4	5.1%	307	148
				231	63.3
				169	1.42
				112	−18.3
	77	−210		77	−93.7
	20	−1330		20	−984
	4.2	−1325		4.2	−1280
12.7%	306	98.0	25.1%	304	67.0[a]
	231	37.0		308	78.7
	169	42.6		231	52.4
	112	28.7		169	92.5
	77	−34.4		112	109
	20	−213		77	71.5[a]
	4.2	−3.6		77	75.9[b]
				20	0.397[a]
				20	1.15[b]
				4.2	−8.75

[a] Data taken at 3.5 A.

[b] Data taken at 12.5 A.

atures and with small amounts of impurities there seems to be disagreement between theory and experiment. To date no theories employing spin-orbit have led to any dependence other than the ρ_0^2 dependence.

The resulting plots of ρ_H versus B were inspected for curvature for the high-field data. Unfortunately with the materials studied here noise was so large that a more precise determination of curvature was not possible.

Because in the magnetoresistance an effect is present which saturates with the magnetization, the magnetoresistance in Table II has been written in a form analogous to that of the Hall effect:

$$(\rho_{BL}-\rho_0)/\rho_0=G_0B+G_s(M). \tag{8}$$

This second term, which is not linear in M, is thought to be due to spin-orbit coupling. The constant G_0 is the high-field slope. A possible term in B^2 is not included here even though in some pseudonickel alloys a curvature in the high-field region has been observed.[9] The results listed that show negative slopes at high fields indicate that the effects of ferromagnetic origin dominate. Cases where the Lorentz force cause the slopes to become positive seem to occur in general under the conditions where the extraordinary Hall coefficients deviate from the ρ_0^2 dependence.

Results of the resistivity at zero-applied field, as a function of the temperature showed that for these materials Mathiessen's rule is not obeyed, that is one cannot separate the resistivity into two parts, one being due to impurity scattering only and the other being due to thermal vibrations of the atoms and containing all the temperature dependence.

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