1%, which seems to be attainable in this range of energies.

Although the relationship between superfluidity and macroscopic occupation of one mode is generally accepted, its experimental demonstration has been rather indirect. The proposed experiment, if even qualitatively successful, would yield information on the wave function of He II which would confirm the basic postulate of Fritz London on the relationship between Bose-Einstein condensation and the superfluidity of He^4.

ACKNOWLEDGMENTS

One of us (P.H.) wishes to thank Professor P. Nozières for an early discussion on this subject. We have also benefited from a discussion with Professor Michael Cohen.

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Density, Coefficient of Thermal Expansion, and Entropy of Compression of Liquid He^4 under Pressure Below 1.4°K†

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The density of liquid He^4 at pressures up to 24.5 atm and at temperatures between 0.5 and 1.4°K has been measured by the dielectric-constant method. From the density data we derive the coefficient of thermal expansion, the entropy of compression, and the compressibility as a function of temperature and pressure. The results are discussed in terms of the Landau theory for liquid He^4, and the parameters Δ, μ, and p_0 of the roton excitation spectrum are determined at several pressures and compared with previous determinations.

I. INTRODUCTION

In this paper we describe measurements of the density ρ, the isobaric coefficient of thermal expansion α_p and the deduction of related thermodynamic properties for liquid He^4 under pressure between 0.5 and 1.4°K. The principle of the method is the determination of the dielectric constant ϵ and the use of the Clausius-Mosotti relation

$$\rho=\frac{3M}{4\pi A}\frac{\epsilon-(1)}{(\epsilon+2)}$$

to derive the density ρ for a given pressure and temperature. Here M is the molecular weight and A is the polarizability. Until last year, only the early results of the PVT studies of Keesom and Keesom[1] were available down to 1.2°K, as well as measurements on the density and coefficient of thermal expansion for the liquid at saturated vapor pressure[2,3] extending down to 0.9°K. Recently, Mills and Sydoriak[4] presented measurements of α_p by the method of adiabatic expansion between 0.5 and 1.5°K. They also calculated the entropy of compression and the change in compressibility as a function of temperature. Mills[5] then interpreted the results in terms of the Landau model of liquid helium.[6–8]

After a short description of the experiment in Sec. 2, the results obtained by the dielectric constant method will be presented and tabulated. Relevant thermodynamic properties such as the entropy of compression are tabulated also and compared with previous results. In Sec. IV, we will derive the Landau parameters Δ, μ and p_0 as a function of density and compare these results with those obtained by previous authors.

II. APPARATUS AND EXPERIMENTAL PROCEDURE

The apparatus and the principle of measurement are the same as for the work on liquid He^3, described in another paper.[9] The discussion[9] on the validity of the Clausius-Mosotti relation is also relevant here. The measurements had to be limited to temperatures above 0.5°K because of the large heat influx into the cryostat resulting from the superfluidity of He II. At temperatures below 0.65°K, the value of the coefficient of thermal expansion became rather small. Its absolute value

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† Research supported by a grant of the National Science Foundation and of the U. S. Army Research Office (Durham). Data of this research were recently presented with tabulations in the Ph.D. thesis by C. Boghosian, Duke University, August 1965. A preliminary report was given by C. Boghosian and H. Meyer, Bull. Am. Phys. Soc. **10**, 258 (1965).

[1] W. H. Keesom and A. P. Keesom, Commun. Kamerlingh Onnes Univ. Leiden Lab., 224e (1933). W. H. Keesom, *Helium* (Elsevier Publishing Company, Amsterdam, 1942).

[2] K. R. Atkins and M. H. Edwards, Phys. Rev. **97**, 1429 (1955).

[3] E. C. Kerr and R. D. Taylor, Ann. Phys. (N.Y.) **26**, 292 (1964).

[4] R. L. Mills and S. G. Sydoriak, Ann. Phys. (N.Y.) **34**, 277 (1965).

[5] R. L. Mills, Ann. Phys. (N.Y.) **35**, 411 (1965).

[6] L. D. Landau, J. Phys. (USSR) **5**, 71 (1941).

[7] L. D. Landau, J. Phys. (USSR) **11**, 91 (1947).

[8] See also K. R. Atkins, *Liquid Helium* (Cambridge University Press, New York, 1959).

[9] C. Boghosian, H. Meyer, and J. E. Rives, Phys. Rev. **146**, 110 (1966).

TABLE I. The density $\rho(0,P)$ (in g/cm^3) of liquid He^4 extrapolated to $T=0$°K.

P(atm)	0.16	0.49	1.1	5.38	7.77	10.9	15.2	20.0	245·
ρ	0.1454	0.1459	0.1469	0.1533	0.1564	0.1600	0.1644	0.1686	0.1721

The temperature dependence of the density expressed in terms of $\left[\frac{\rho(T,P)}{\rho(0,P)}-1\right]\times 10^5$.

T (°K) \ P(atm)	0.16	0.49	1.1	5.38	7.77	10.9	15.2	20.0	24.5
0.5	−1	−1	−1	−1	−1	0	0	0	0
0.6	−2	−2	−1	−1	−1	−1	−1	−1	0
0.7	−3	−4	−3	−1	−2	−1	−1	0	0
0.8	−6	−6	−6	−2	−2	−1	0	2	+2
0.9	−9	−9	−7	−1	−1	+2	+4	6	8
1.0	−11	−10	−7	+3	+5	9	12	17	22
1.1	−11	−10	−5	10	17	22	28	27	48
1.2	−9	−6	0	24	35	45	56	69	91
1.3	−4	+1	10	44	61	80	100	123	...

at elevated pressures at 0.5°K was comparable with the accuracy in the measurements, ± 0.0003°K^{-1}.

At the beginning of a series of experiments, the frequency ω_0 of the oscillator when the density cell was empty, was measured as a function of T by means of the magnetic susceptibility of a paramagnetic salt, chromium methylammonium alum. This magnetic thermometer was in turn calibrated against the temperature derived from vapor-pressure measurements of liquid He^3 in the sample pot above 0.8°K. Simultaneously several carbon resistors placed at various parts of the sample holder were accurately calibrated versus the magnetic susceptibility and the He^3 vapor pressure. Because there was only negligible heat influx, they were all at the same temperature.

Helium-4 was then introduced into the density cell via the capillary and the frequency ω of the oscillator was measured. For attaining temperatures below 1.2°K (which was the lowest temperature reached by the main liquid-He^4 bath) the temperature of the shield was lowered by pumping on the liquid He^3 in the shield. This established a temperature gradient in the capillary between the shield and the mean helium bath. Because of the large heat conductivity of the liquid, the temperature of the density cell very quickly became equal to the lowest temperature of the He II in the capillary. Since there was no heat leak from other parts of the cryostat, the He^4 inside the cell was then in thermal equilibrium with the copper walls, the various carbon resistors and the paramagnetic salt. Frequency measurements could be made in this way down to about 0.7°K. Still lower temperatures of the cell were obtained by pumping in addition on the He^3 in the sample pot. A gradient in temperature therefore existed between the He II near the heat shield and the He II in the density cell. A rough calculation was made to estimate the temperature gradient between the He in the cell and the walls of the cell. This calculation took into account the thermal conductivity of the liquid along the capillary and the temperature gradient due to the Kapitza boundary resistance.[10] It was estimated that at 0.5°K, the temperature inhomogeneity within the cell was of the order of a few mdeg. The possible error in our results arising from this effect was found to be negligible in comparison with the other uncertainties in this temperature range.

The derivation of the density and α_p from the measurement of ω_0 and of ω at various pressures and as a function of T was carried out as described before.[9] The constants K and $B(P)$ for the oscillator circuit, determined from calibration with liquid He^3, were used for the present results. Also the same polarizability constant $A=0.1230$ was used. [*Note added in proof.* We must point out that a small variation of the polarizability A with density cannot be excluded. When the density cell is calibrated, the function $B(p)$ may well include implicitly such a variation. This function was found[9] to be a linear function of pressure, and was thought to be caused mostly by elastic effects in the density cell.]

III. RESULTS

Measurements were made at pressures of 0.16, 0.5, 1.1, 3.1, 5.4, 7.8, 10.9, 15.2, 20.0 and 24.5 atm. The results of the density ratio $[(\rho(T,P)/\rho(T=0,P))-1]$, as a function of temperature and pressure are presented in Table I. The density versus pressure at 1.25°K is plotted in

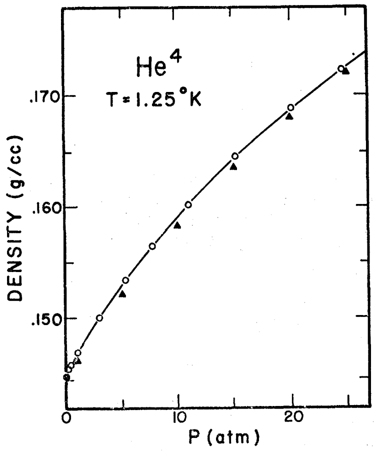

FIG. 1. The density of liquid He^4 as a function of pressure at 1.25°K. Solid line and open circles: present research, triangles: Keesom and Keesom (Ref. 1), solid circle: Kerr and Taylor (Ref. 3).

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[10] S. G. Sydoriak and R. H. Sherman, J. Res. Natl. Bur. Std. A **68**, (1964).

TABLE II. Coefficient of thermal expansion α_p of liquid He^4 (10^{-3} deg^{-1}). The values at $P=0$ have been obtained by extrapolation. Since α_p changes rapidly at low pressures, we estimate the possible systematic uncertainty at $P=0$ to be $\pm 5\times 10^{-5}$ or 5%, which ever is larger. For the other pressures, we estimate α_p to be correct within $2\text{–}3\times 10^{-5}$ or 5%, whichever is larger.

T (°K) \ P(atm)	0	1	2	5	10	15	20	25
0.60	0.19	0.090	0.080	0.060	0.040	+0.023	+0.006	−0.040
0.70	0.30	0.185	0.150	0.075	+0.006	−0.035	−0.095	0.181
0.80	0.42	0.22	0.15	+0.00	−0.15	0.22	0.30	0.45
0.90	0.44	+0.14	+0.02	−0.21	0.44	0.58	0.72	0.97
1.00	0.34	−0.05	−0.20	0.54	0.92	1.17	1.45	1.94
1.10	+0.14	0.34	0.55	1.04	1.66	2.09	2.55	3.33
1.20	−0.20	0.70	1.00	1.65	2.65	3.45	4.14	5.50
1.30	−0.69	1.15	1.50	2.50	4.00	5.35	6.85	8.9
1.40	−1.3	1.80	2.2	3.5	5.9	7.3	9.5	12.1

Fig. 1 where it is compared with the data of Keesom and Keesom.[1] It is particularly satisfying that the extrapolated density at zero pressure agrees within about 0.1% with that found by Kerr and Taylor.[3] This is a proof that the polarizability is the same within 0.1% for both He^3 and He^4, assuming that the Clausius Mosotti law holds for both liquids.[11]

The coefficient of expansion is presented in Table II for several standard pressures as a function of T. Comparison with the results of Mills and Sydoriak[4] is given in Figs. 2 and 3 where the data are plotted against pressure for several relevant temperatures. The agreement is satisfactory for the temperature range between 0.7 and 1.0°K, except for a systematic deviation at high pressures. This can best be seen at $T=1.3$°K, as shown in Fig. 3. According to Mills and Sydoriak, the coefficient of thermal expansion changes less rapidly with pressure for $P \gtrsim 20$ atm than do our data. However the data of Grilly and Mills[12] also indicate a rapid change of α_p at high pressures. We have not found it possible to express our data by means of the relationship used by Mills and Sydoriak[4] [Eq. (7) of their paper]. An independent check of the data was made between 1.2 and 1.5°K in another cryostat which also used the dielectric method. This apparatus was calibrated at 1.75°K with density data of Lounasmaa.[13] In the over-

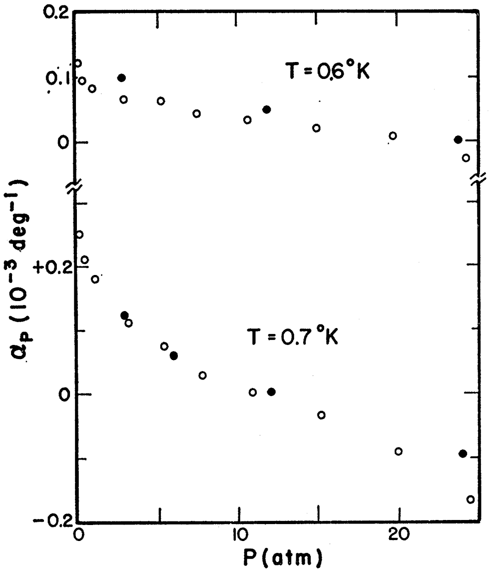

FIG. 2. The coefficient of thermal expansion of liquid He^4 as a function of pressure at 0.6 and 0.7°K. Open circles: present results, Solid circles: Sydoriak and Mills (Ref. 4) obtained from their Table I by interpolation.

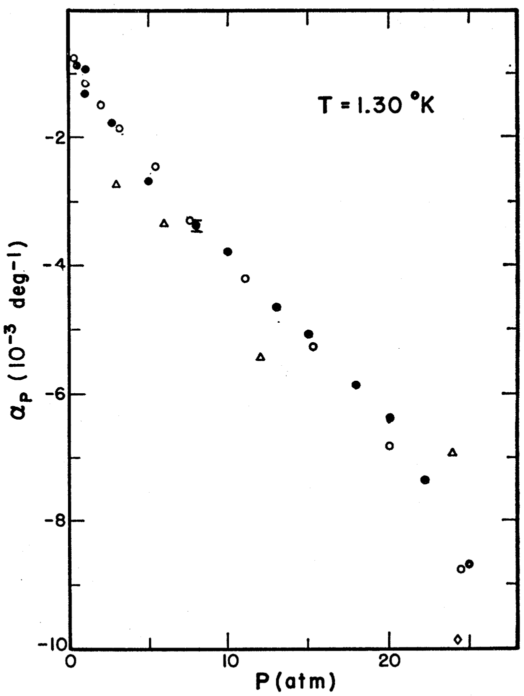

FIG. 3. The coefficient of thermal expansion of liquid He^4 as a function of pressure at 1.3°K. Open circles: present results, open triangles: Sydoriak and Mills (Ref. 4) obtained from their Table I by interpolation, closed circles: Elwell and Meyer (Ref. 14), open diamond: Grilly and Mills (Ref. 12).

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[11] This, however, does not mean that the absolute value of the polarizability is exactly 0.1230. In our experiments with He^3 (Ref. 9) this value taken from the paper by C. E. Chase, E. Maxwell and W. E. Millett [Physica 27, 1129 (1961)] and was used to calibrate the capacitance circuit. If a slightly different value of A (say 0.1235) had been taken, slightly different constants K and $B(P)$ would have been obtained, but then with these different constants, practically the same density of He^4 would have been obtained as those we present here.

[12] R. Grilly and R. L. Mills, Ann. Phys. (N.Y.) 18, 250 (1962).

[13] O. V. Lounasmaa, Cryogenics 1, 1 (1961).

TABLE III. Entropy of compression of liquid He^4 (mJ/g deg). The error in the entropy can be deduced from that associated with the coefficient of thermal expansion. For instance at 0.6°K and 20 atm, the uncertainty is probably ±0.1 mJ/g deg.

P (atm) \ T (°K)	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
0	0	0	0	0	0	0	0	0
1	−0.08	−0.16	−0.219	−0.195	−0.098	+0.079	+0.312	+0.638
3	−0.19	0.36	0.428	−0.235	+0.174	+0.821	1.64	+2.67
5	0.28	0.49	0.486	−0.048	+0.765	2.02	3.56	5.57
10	0.44	0.62	−0.230	+1.02	3.15	6.41	10.6	16.1
15	0.53	0.57	+0.348	2.62	6.41	12.3	20.2	30.6
20	0.58	0.38	1.13	4.58	10.4	19.3	31.7	49.1
24	−0.56	−0.08	+1.96	+6.52	14.2	26.0	42.5	+67.2

lapping temperature region, these new results[14] are in excellent agreement with those presented here.

Figure 4 shows the experimental relation between the temperature where $\alpha_p=0$ and the pressure. There is good agreement with the results of Mills and Sydoriak. In Figure 5 our data for α_p extrapolated to $P=0$ are compared with those of Atkins and Edwards[2] and Kerr and Taylor[3] at saturated vapor pressure, taking account of the correction

$$\alpha_p=\alpha_s+\frac{\gamma}{\rho u_1^2}\left(\frac{dP}{dT}\right)_S \tag{1}$$

where α_s is the coefficient of thermal expansion at saturated vapor pressure, γ is the ratio of the specific heats at constant pressure and constant volume, u_1 is the velocity of first sound and $(dP/dT)_S$ is the slope of the vapor-pressure curve.

The temperature for $\alpha_p=0$ was found to be 1.14±0.01°K, in excellent agreement with the results of Atkins and Edwards, who found $T=1.15$°K. The results of Kerr and Taylor[3] show this temperature to be 1.185°K.

The entropy of compression has been calculated from the thermodynamic relation

$$\left(\frac{\partial S}{\partial P}\right)_T=-M\alpha_p\rho^{-1} \tag{2}$$

using smoothed values of α_P and the new density data. The values of $S(T,P)-S(T,0)$ are presented in Table III. The absolute entropy of liquid He was calculated using new data of Wiebes and Kramers[15] at saturated vapor pressure. The values in Table III have been compared with those calculated by Mills and Sydoriak.[4] The agreement is good near 1.1°K, but above and below this temperature, there is a systematic discrepancy. Below 1°K, this comes to a great extent from the way the authors have analyzed their α_p data using their relation (7). At low pressures, this analysis gives values of α_p that are algebraically larger than ours and can shift them outside the experimental error. Yet the agreement of the experimental results in Ref. 4 and in our work is close, particularly between 0.7 and 1.1°K.

Our results of the entropy of compression at 1.3 and 1.2°K are compared in Fig. 6 with those of Van den

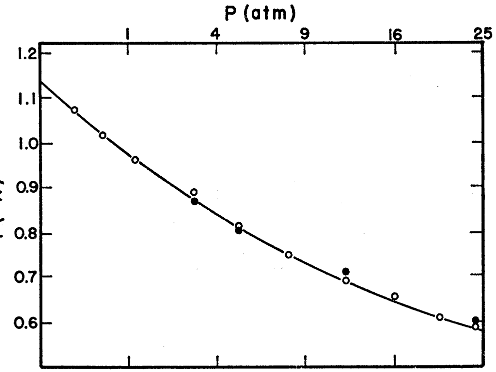

FIG. 4. The locus ($\alpha_p=0$) as a function of pressure plotted versus p^2 to give a less steep slope near zero pressure. Open circles: present results, closed circles: Sydoriak and Mills (Ref. 4).

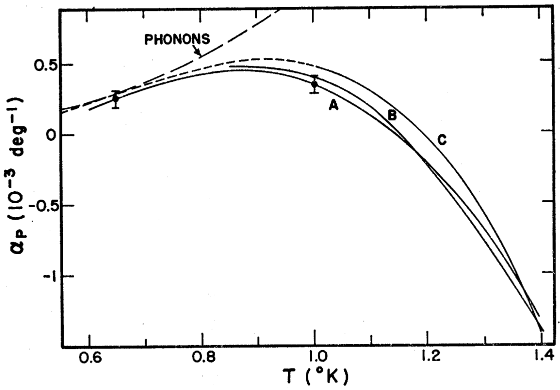

FIG. 5. The coefficient of expansion extrapolated to zero pressure as a function of temperature. Solid line A: present results; curve B: Atkins and Edwards (Ref. 2); curve C: Kerr and Taylor (Ref. 3).

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[14] D. Elwell and H. Meyer (to be published).

[15] J. Weibes and Kramers (private communication). These values are only slightly different from published ones e.g. *Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio, 1964* (Plenum Press, Inc., New York, 1965), p. 258.

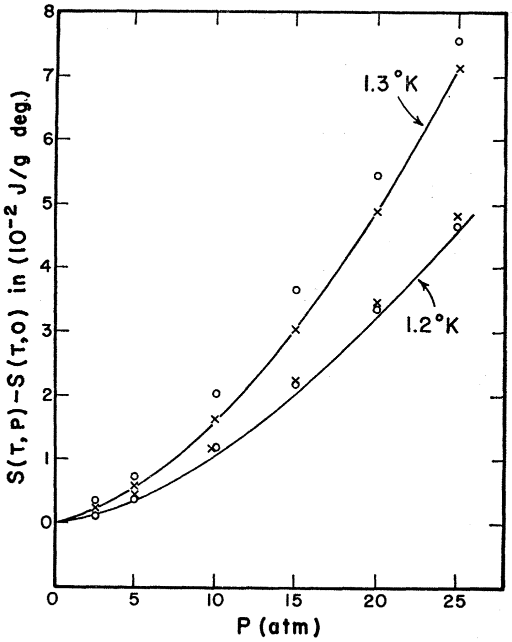

FIG. 6. The entropy of compression of liquid He^4 versus pressure at 1.3 and 1.2°K. Solid curve: present results, crosses: Van den Meijdenberg *et al.* (Ref. 16), open circles: Mills and Sydoriak (Ref. 4).

Meijdenberg *et al.*[16] from measurements of the fountain effect. At 1.3°K, the agreement is excellent, and at 1.2°K, there is a discrepancy of about 5%.

In Table IV, we present a calculation of the density ρ_n obtained from the relation[6]

$$\frac{\rho_n}{\rho}=\left(1+\frac{u_2^2(\partial S/\partial T)_V}{S^2}\right)^{-1}, \tag{3}$$

where ρ_n is the normal-density fluid and u_2 is the second-sound velocity. The values of u_2 are those of Maurer and Herlin,[17] tabulated and extrapolated to 0.9°K by Mills.[18] The values of S and $(\partial S/\partial T)_V \approx (\partial S/\partial T)_P$ were obtained from our own data, using again the entropy at saturated vapor pressure of Wiebes and Kramers.[15]

TABLE IV. The density ρ_n (10^{-3} g cm^{-3}) calculated from the relation (3) in the text.

T(°K) \ P(atm)	0	2.5	5	10	15	20	25
1.3	7.05	7.79	8.76	11.3	14.5	18.7	22.7
1.2	4.08	4.55	5.21	7.06	9.02	11.3	14.16
1.1	2.176	2.48	2.89	4.02	5.37	6.95	9.04
1.0	1.034	1.18	1.39	1.95	2.71	3.63	4.81
0.9	0.44_8	0.49_8	0.60	0.87_4	1.28	1.74	2.33

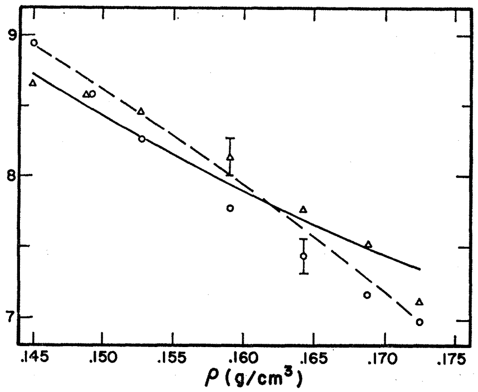

FIG. 7. The eneggy gap Δ determined from the entropy (open circles) and from the density of the normal component, ρ_n (open triangles). Dashed curve: Van den Meijdenberg *et al.* Solid line: Mills and Sydoriak.

The compressibility at $T=0$°K has been derived from the extrapolated slope of the density-versus-pressure curve at $T=0$°K. The change of the compressibility with temperature $k_T(0,P)-k_T(T,P)$ is calculated from the thermodynamic relation

$$\left(\frac{\partial k_T}{\partial T}\right)_P=-\left(\frac{\partial \alpha_P}{\partial P}\right)_T \tag{4}$$

and is presented in Table V. Extrapolation to zero pressure becomes increasingly difficult as the temperature increases because of the large change of compressibility in this region. Up to about 15 atm there is qualitative agreement of our calculations with those of Mills and Sydoriak. However since our plot of α_p versus P shows an inflection, the derivative $(\partial k_T/\partial T)_P$ becomes larger at higher pressures, in disagreement with the data of these authors.

IV. DETERMINATION OF THE PARAMETERS OF LANDAU'S THEORY

There have been several experimental determinations,[19] of the Landau parameters $\Delta, \mu/m_{\mathrm{He}}$ and p_0, where

TABLE V. The compressibility $k_T(T=0)$ (in 10^{-2} atm^{-1}) extrapolated to $T=0$

P(atm)	0	1	2	5	10	15	20	24
$k_T(0)$	1.18	1.12	1.06	0.90	0.719	0.581	0.473	0.402

The temperature dependence of the compressibility $[k_T(T,p)-k_T(0,P)]$ in 10^{-7} atm^{-1} as a function of pressure.

T (°K) \ P(atm)	0	1	2	5	10	15	20	24
0.6	8^{a}	6	6	3	2	1	2	3
0.7	12^{a}	10	8	5	3	2	4	5
0.8	20^{a}	16	14	7	5	3	5	7
0.9	36^{a}	28	22	12	7	5	8	12
1.0	70^{a}	46	34	19	12	9	14	19
1.1	···	69	51	30	20	15	21	35
1.2	···	99	75	47	34	25	37	62
1.25	···	116	90	59	44	33	47	83

[a] Extrapolated values.

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[16] C. J. N. Van den Meijdenberg, K. W. Taconis, and R. De-Bruyn Ouboter, Physica **27**, 197 (1961).

[17] R. D. Mauer and M. A. Herlin, Phys. Rev. **81**, 444 (1951).

[18] We wish to express our indebtness to Dr. R. L. Mills for providing us with the values u_1 and u_2, smoothed and extrapolated, which he used for his own calculations of the Landau parameters.

[19] A tabulation of all the previous determinations of these parameters is given by R. L. Mills (Ref. 5).

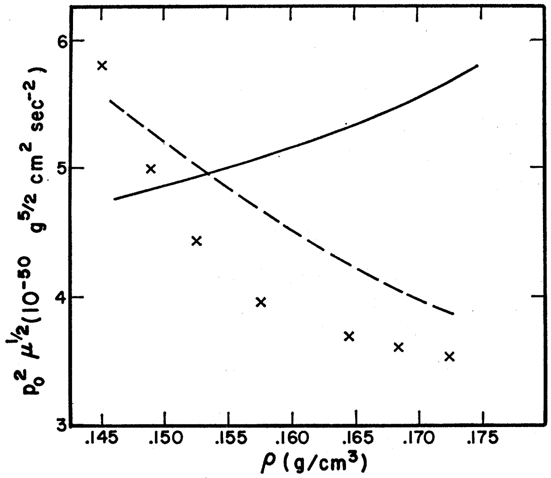

FIG. 8. The product $p_0^2\mu^{1/2}$ versus density. Dashed curve: Van den Meijdenberg *et al.* Solid line: Mills and Sydoriak. Crosses: present results.

Δ is the energy gap, μ/m_{He} is the ratio of the effective mass μ to the actual mass m_{He} of the He atom, and p_0 is the momentum at the minimum of the dispersion curve. Recent determinations of these parameters for liquid He under pressure were made from neutron scattering,[20] fountain effect,[16] specific heat[15] and adiabatic expansion.[4,5] Detailed discussions on the uncertainties of these parameters and tabulation of previously obtained values have been presented Van den Meijdenberg *et al.*[16] and by Mills,[5] and will not be repeated here. Mills has also used the plot of thermal-expansion data, suggested by Atkins and Edwards[2] to test Landau's theory.

In what follows, we will derive the gap Δ from the experimental entropy of compression and also from the normal component of the density. This determination will be made from data below 1.2°K where the value of Δ can be assumed to be constant with temperature. Combination of the theoretical expressions for S_r and ρ_n will then give the parameters μ/m and p_0. According to Landau's theory[6–8] the entropy of liquid He⁴ at sufficiently low temperatures can be written as the sum of two terms

$$S=S_{\mathrm{r}}+S_{\mathrm{ph}} \tag{5}$$

the roton and phonon contributions, where

$$S_r=\frac{2\mu^{1/2}k^{1/2}p_0^2\Delta}{(2\pi)^{3/2}T^{1/2}\hbar^3}\left(1+\frac{3kT}{\Delta}\right)\exp(-\Delta/kT). \tag{6}$$

Here k and $\hbar$ have their usual meaning, Δ, p_0 and μ are parameters of the energy-versus-momentum curve for liquid He⁴. Assuming the temperature to be low enough so that the T^3 region is reached, the entropy due to the phonons is given by

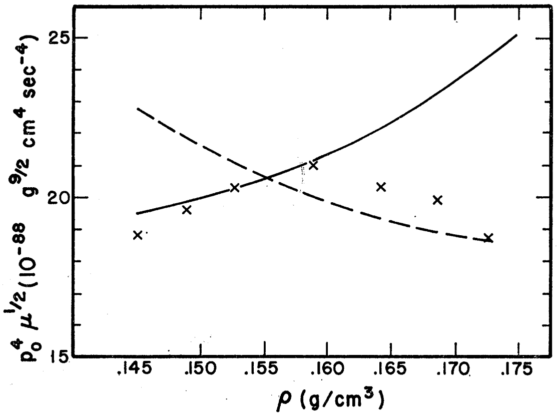

FIG. 9. The product $p_0^4\mu^{1/2}$ versus density. Crosses: present data, dashed curve: Van den Meijdenberg *et al.*, solid line: Mills and Sydoriak.

$$S_{ph}=\frac{2\pi^2k^4T^3}{45\hbar^3u_1^3\rho}, \tag{7}$$

where u_1 is the velocity of first sound at 0°K. The entropy S_{ph} was calculated using the extrapolated and smoothed data of u_1 obtained by Atkins and Stasior.[21,18] From Eq. (6), the values of Δ and the product $\mu^{1/2}p_0^2$ were obtained and are presented in Fig. 7 and 8. The small but not negligible discrepancy between the value of Δ determined from entropy and that determined from neutron data[20,22,23] at low densities has already been discussed by several authors.[5,15,16] For the gap, there is good consistency with previous workers,[5,16,20] although the results of Sydoriak and Mills[4,5] at high densities are slightly higher. For the product $\mu^{1/2}p_0^2$, the present results are consistent within the experimental uncertainty with those of Van den Meijdenberg,[16] but disagree with those of Mills and Sydoriak.[4,5,24]

The normal fluid density is related to the Landau parameters through the relations[6–8]

$$\rho_n=\rho_{n(ph)}+\rho_{n(r)}, \tag{8}$$

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[20] D. G. Henshaw and A. D. B. Woods, in *Proceedings of the Seventh International Conference on Low Temperature Physics, 1960* (University of Toronto Press, Toronto, Canada, 1961), p. 539.

[21] K. R. Atkins and R. A. Stasior, Can. J. Phys. **31**, 1156 (1953).

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[24] The difference in the value of $p^2\mu^{1/2}$ between our results and those of Mills at saturated vapor pressure is due to his choice of a lower Δ, which strongly affects the value of this product. Mills and Sydoriak used the data of Kramers, Wasscher, and Gorter [Physica **18**, 329 (1952)] which give higher values of the entropy below 0.9°K than the more recent results of Wiebes and Kramers. Using the earlier results on the entropy one finds a smaller energy gap and smaller values of the product $p_0^2\mu^{1/2}$.

where

$$\rho_{n(ph)}=\frac{2\pi^2k^4T^4}{45\hbar^3u_1^5}. \qquad (9)$$

and

$$\rho_{n(r)}=\frac{2\mu^{1/2}p_0^4}{3(2\pi)^{3/2}k^{1/2}\hbar^3T^{1/2}}\left(1+\frac{6\mu kT}{p_0^2}\right)\exp\left(\frac{-\Delta}{kT}\right). \qquad (10)$$

The normal density due to the phonons $\rho_{n(ph)}$ was calculated using the values of u_1 from Atkins and Stasior.[21] It was then subtracted from ρ_n obtained from Eq. (3). The parameter Δ is somewhat different from that determined from S_r and is shown in Fig. 7. Its estimated accuracy is approximately the same as for the gap determined from S_r data. The agreement between both determinations is within the combined accuracy exept at intermediate densities and at zero pressure. The values of $p_0^4\mu^{1/2}$ obtained from Eq. (10) using the gap Δ (from Eq. 10) were combined with those of p_0^2. $\mu^{1/2}$ to obtain $p_0/\hbar$ and μ/m_{He}. These parameters are found to agree best with those of Van den Meijdenberg *et al.* within their rather large combined error. A reasonable estimate of the uncertainty is probably 30% for μ/m_{He} and 8% for $p_0/\hbar$. Table VI shows these parameters as a function of pressure.

The influence of the choice of Δ on the other parameters cannot be overemphasized. At $P=0$, if an average value of $\Delta=8.8°\mathrm{K}$ is taken, μ/m and $p_0/\hbar$ would, respectively, become ~0.3 and ~1.9 Å^{-1}, in better agreement with neutron diffraction data, where $p_0/\hbar=1.92$ and $\mu/m_{\mathrm{He}}=0.16$. Determination of these parameters from thermodynamic data is therefore not very satisfactory, except for the gap Δ. However here, the simple dispersion spectrum used by Landau may lead to slightly different "effective" gaps Δ, depending on the property which is being considered.

The determination of momentum and mass, imperfect as it is in comparison with neutron data, shows nevertheless trends that can be compared with previous experiments. The momentum p_0 increases with density which is in agreement with previous work, and the parameter μ is shown to decrease strongly with increasing density. This is consistent with an analysis of the neutron-scattering data[20] at a pressure of 25.3 atm near the minimum of the dispersion curve, where a ratio μ/m_{He} of 0.1 at most was estimated. Our conclusion is consistent with the results of Van den Meijdenberg *et al.* but in contradiction with the results of Mills who found this ratio to increase slightly with increasing density.

TABLE VI. The parameters μ and p_0 as a function of pressure.

P(atm)	0	2.5	5	10	15	20	25
μ/m_{He}	0.5	0.25	0.15	0.08	0.065	0.06	0.06
$p_0/\hbar$	1.7	1.9	2.0	2.2	2.2	2.2	2.2

CONCLUSION

Density data of liquid He II under pressure are presented between 0.5 and 1.4°K. When extrapolated to zero pressure, the density and the thermal expansion coefficient are in good agreement with dilatometric data[2,3] carried out above 0.9°K. This justifies the assumption of the Clausius-Mosotti relation and there is evidence from our measurements that the polarizability is the same for liquid He^3 and He^4 within about 0.1% at least at low pressures. Therefore it is possible to measure the density of He^3-He^4 solutions by this convenient dielectric method. Such experiments on dilute solutions of He^3 in He^4 are being carried out presently in this laboratory between 0.35 and 2°K.

The parameters of the Landau theory on liquid He^4 have been deduced from data at temperatures below 1.2°K and compared with the values obtained by different authors from other experiments. Obviously these data cannot yield the information a neutron-scattering experiment can give and further neutron experiments at elevated pressures are very much needed. Within the uncertainties of our calculation, the parameters are consistent with those from neutron data and from fountain effect. In another cryostat, accurate measurements of the density of He^4 under pressure have been carried out[14] between 1.2 and 4.2°K using again the dielectric constant method, in order to test the Clausius-Mosotti relation under elevated pressures at still higher temperatures, and have been reported elsewhere.[25]

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