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Partially Conserved Axial-Vector Current Restrictions on Pion Photoproduction and Electroproduction Amplitudes

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We discuss numerically the restrictions imposed by the partially conserved axial-vector current (PCAC) on the pion photoproduction amplitude $V_1^{(+)}(0)$ and on the pion electroproduction amplitude $V_6^{(-)}(0)$. We find that the magnetic-dipole dominance and the narrow-resonance approximations are unreliable. The nonresonant s waves make an important contribution to $V_1^{(+)}(0)$, and we find that the PCAC prediction for this amplitude is reasonably well satisfied. The electric and longitudinal multipoles appear to make a much bigger contribution to $V_6^{(-)}(0)$ than does the magnetic dipole M_{1+}, which is strongly suppressed by the kinematics.

I. INTRODUCTION AND CONCLUSIONS

As has been much emphasized recently,[1] the partially conserved axial-vector current (PCAC) hypothesis, supplemented by current commutation relations, relates any weak or electromagnetic process in which a zero four-momentum pion is emitted to the same process in the absence of the pion. In particular, when applied to pion electroproduction, PCAC implies the relations[2]

$$(g_r(0)/M_N)F_2^V(k^2) = V_1^{(+)}(\nu=\nu_B=(M_\pi^f)^2=0,\, k^2)\,, \quad (1a)$$

$$(g_r(0)/M_N)F_2^S(k^2) = V_1^{(0)}(\nu=\nu_B=(M_\pi^f)^2=0,\, k^2)\,, \quad (1b)$$

$$\frac{g_r(0)}{M_N}\left[\frac{g_A(k^2)}{g_A(0)}-F_1^V(k^2)\right](k^2)^{-1} = V_6^{(-)}(\nu=\nu_B=(M_\pi^f)^2=0,\, k^2)\,. \quad (1c)$$

Here $F_1^V(k^2)$ is the isovector nucleon Dirac form factor; $F_2^V(k^2)$ and $F_2^S(k^2)$ are, respectively, the isovector and isoscalar nucleon Pauli form factors; $g_A(k^2)$ is the nucleon axial-vector form factor [$g_A(0)=1.18$]; and $g_r(0)$ is the pion-off-mass-shell pion-nucleon coupling constant [$g_r\equiv g_r(-M_\pi^2)$, $g_r^2/4\pi\approx 14.6$]. The pion photoproduction amplitudes $V_1^{(+,0)}$ and the pion electroproduction amplitude $V_6^{(-)}$ will be specified more precisely below. When $k^2=0$, Eqs. (1a) and (1b) become the photoproduction relations of Fubini, Furlan, and Rossetti[3]; and Eq. (1c) becomes a relation between the axial-vector and charge radii of the nucleon.

The main purpose of this paper is to give a careful numerical analysis of Eqs. (1a) and (1c) at $k^2=0$. In the dispersion integrals for $V_1^{(+)}$ and $V_6^{(-)}$ we keep only the multipoles which resonate around the $N^*(1238)$ and the $N^{**}(1520)$, and the nonresonant s waves. As a preliminary, in Sec. II we state the needed kinematics and briefly derive Eqs. (1). In Sec. III we give the numerical discussion, using the photoproduction analyses of Schmidt and Höhler[4] and of Walker[5] in the region of the first two pion-nucleon resonances.

We reach the following conclusions:

1. The magnetic-dipole (M_{1+}) contribution to $V_1^{(+)}(0)$ from the neighborhood of the $N^*(1238)$ equals only about 0.75 times the left-hand side of Eq. (1a). Estimates based on the narrow-resonance approximation indicate a larger M_{1+} contribution, but we find that the narrow-resonance approximation for the $N^*(1238)$ overestimates integrals over the resonance by about 60%. When the resonant E_{1+}, M_{2-}, and E_{2-} multipoles are included, the value of $V_1^{(+)}(0)$ is reduced to about 0.6 times the left-hand side of Eq. (1a). However, the nonresonant s waves make a large contribution to the integral,[6] making the total integral for $V_1^{(+)}(0)$ equal to about 0.85 of the value predicted by PCAC.

2. The dispersion integral for $V_6^{(-)}(0)$ is *not* magnetic-dipole-dominated, because the M_{1+} contribution is kinematically suppressed. For instance, the multipole E_{1+} (electric quadrupole) in the $N^*(1238)$ region makes a contribution three times as big as the multipole M_{1+} to $V_6^{(-)}(0)$, even though the E_{1+} multipole is much

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[1] Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962); Y. Nambu and E. Shrauner, *ibid.* **128**, 862 (1962); S. L. Adler, *ibid.* **139**, B1638 (1965); M. Suzuki, Phys. Rev. Letters **15**, 986 (1965); C. G. Callan and S. B. Treiman, *ibid.* **16**, 153 (1966).
[2] These relations are contained implicitly in the weak pion production results of Nambu and Shrauner (Ref. 1). The covariant forms have been derived by a number of authors: S. L. Adler, in Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, 1965, p. 291 (unpublished); Riazuddin and B. W. Lee, Phys. Rev. **146**, B1202 (1966); G. Furlan, R. Jengo, and E. Remiddi, Nuovo Cimento **44**, 427 (1966).
[3] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965).
[4] W. Schmidt and G. Höhler, Ann. Phys. (N. Y.) **28**, 34 (1964); W. Schmidt, Z. Physik **182**, 76 (1964).
[5] R. L. Walker (private communication).
[6] The nonresonant s wave also makes an important contribution to the sum rule relating the isovector nucleon magnetic moment and charge radius to photoproduction cross sections—see F. J. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966).

smaller than the M_{1+}. The value of $V_6^{(-)}(0)$ depends sensitively on the hard-to-measure longitudinal multipoles. Under the dubious assumption that the known proportionality of longitudinal and electric multipoles for zero photon momentum holds unchanged for large photon momenta as well, Eq. (1c) predicts an axial-vector form factor which falls off somewhat more slowly with k^2 than does $F_1^V(k^2)$.

The results of this paper should not be regarded as final, since the input multipole data may change as better analyses of photoproduction become available. What is definitely indicated, however, is that a comparison of Eqs. (1) with experiment must avoid unreliable narrow-resonance and M_{1+}-dominance approximations.

II. KINEMATICS AND DERIVATION OF PCAC RELATIONS

A. Kinematics

Let us consider the reaction

$$\gamma(k)+N(p_1)\rightarrow\pi(q)+N(p_2)\,, \tag{2}$$

where the initial gamma may be real or virtual. The external particle masses are, respectively,

$$-k^2,\quad -p_1^2=M_N^2,\quad -q^2=(M_\pi{}^f)^2,\quad -p_2^2=M_N^2. \tag{3}$$

We define invariant-energy and momentum-transfer variables ν and ν_B by

$$\nu=-(p_1+p_2)\cdot k/(2M_N)\,,\quad \nu_B=q\cdot k/(2M_N)\,; \tag{4}$$

these are related to W, the invariant mass of the final pion-nucleon system, by

$$\nu-\nu_B=(W^2-M_N^2)/(2M_N)\,. \tag{5}$$

All noninvariant quantities used in this paper refer to the reaction center-of-mass frame, in which $\mathbf{k}+\mathbf{p}_1=\mathbf{q}+\mathbf{p}_2=0$. We denote by y the cosine of the angle between the photon and pion directions:

$$y=\hat{q}\cdot\hat{k}\,, \tag{6}$$

and by $|\mathbf{k}|=(k_0^2+k^2)^{1/2}$ and $|\mathbf{q}|=(q_0^2-(M_\pi{}^f)^2)^{1/2}$ the photon and pion momenta. The photon and pion energies are given by

$$k_0=\frac{W^2-M_N^2-k^2}{2W}\,,\quad q_0=\frac{W^2-M_N^2+(M_\pi{}^f)^2}{2W}\,. \tag{7}$$

The matrix element for the electroproduction reaction of Eq. (2) takes the form

$$m=e_r\epsilon_\lambda\,{}_{\text{out}}\langle\pi(q)N(p_2)\,|\,J_\lambda{}^{I3}+J_\lambda{}^Y\,|\,N(p_1)\rangle\,, \tag{8}$$

with e_r the electric charge, ϵ_λ the virtual photon polarization vector (which satisfies $k\cdot\epsilon=0$), and with $J_\lambda{}^{I3}$ and $J_\lambda{}^Y$, respectively, the third component of the isospin current and the hypercharge current. The

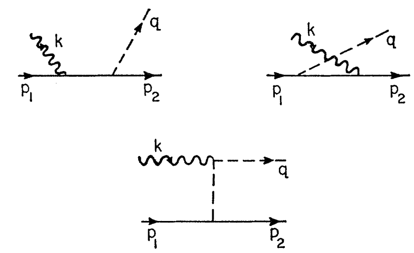

FIG. 1. Born approximation diagrams.

isospin structure of the matrix element is given by

$$\begin{aligned}{}_{\text{out}}\langle\pi N\,|\,J_\lambda{}^{I3}\,|\,N\rangle&=a^{(+)}V_\lambda{}^{(+)}+a^{(-)}V_\lambda{}^{(-)}\,,\\ {}_{\text{out}}\langle\pi N\,|\,J_\lambda{}^{Y}\,|\,N\rangle&=a^{(0)}V_\lambda{}^{(0)}\,,\end{aligned} \tag{9}$$

with[7]

$$\begin{aligned}a^{(\pm)}&=\chi_f{}^{I*}\psi_c{}^*\tfrac{1}{4}(\tau_c\tau_3\pm\tau_3\tau_c)\chi_i{}^I\,,\\ a^{(0)}&=\chi_f{}^{I*}\psi_c{}^*\tfrac{1}{2}\tau_c\chi_i{}^I\,.\end{aligned} \tag{10}$$

In Eq. (10), ψ_c, $\chi_f{}^I$, and $\chi_i{}^I$ are, respectively, the isospinors of the final pion, the final nucleon, and the initial nucleon. The space-spin structure of the matrix element is given by

$$\epsilon_\lambda V_\lambda{}^{(\pm,0)}=\sum_{j=1}^{6}V_j{}^{(\pm,0)}(\nu,\nu_B,(M_\pi{}^f)^2,k^2)\times\bar{u}(p_2)O(V_j)u(p_1)\,. \tag{11}$$

Defining $\{a,b\}=a\cdot\epsilon b\cdot k-a\cdot kb\cdot\epsilon$, we may take the $O(V_j)$ as

$$\begin{aligned}&O(V_1)=\tfrac{1}{2}i\gamma_5\{\gamma,\gamma\}\,, && \eta_1{}^V=1\,;\\ &O(V_2)=i\gamma_5\{p_1+p_2,\,q\}\,, && \eta_2{}^V=1\,;\\ &O(V_3)=\gamma_5\{\gamma,q\}\,, && \eta_3{}^V=-1\,;\\ &O(V_4)=\gamma_5\{\gamma,\,p_1+p_2\}-iM_N\gamma_5\{\gamma,\gamma\}\,, && \eta_4{}^V=1\,;\\ &O(V_5)=i\gamma_5\{k,q\}\,, && \eta_5{}^V=-1\,;\\ &O(V_6)=\gamma_5\{k,\gamma\}\,, && \eta_6{}^V=-1\,.\end{aligned} \tag{12}$$

The numbers $\eta_j{}^V$ specify the crossing properties of the invariant amplitudes:

$$V_j{}^{(\pm,0)}(\nu,\nu_B,(M_\pi{}^f)^2,k^2)=(\pm,+)\eta_j{}^VV_j{}^{(\pm,0)}(-\nu,\,\nu_B,\,(M_\pi{}^f)^2,\,k^2)\,. \tag{13}$$

To make the normalization precise, we state the contribution of the Born approximation diagrams of Fig. 1 to the invariant amplitudes. [In the following equations we take the external pion to be physical

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[7] Our notation follows that of a review article on pion electro- and weak production in preparation by one of the authors (S.L.A.). Our amplitudes are related to those of CGLN [G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. **106**, 1345 (1957)] as follows:

covariant amplitudes—$[V_1,V_2,V_3,V_4]^{(\pm,0)}{}_{\text{this paper}}=2[A,B,C,D]^{(\pm,0)}{}_{\text{CGLN}}$,

center-of-mass amplitudes—$[\mathfrak{F}_j{}^V]^{(\pm,0)}{}_{\text{this paper}}=(8\pi W/M_N)[\mathfrak{F}_j{}^V]^{(\pm,0)}{}_{\text{CGLN}}$,

multipoles—$[M_{l+},\text{ etc.}]^{(\pm,0)}{}_{\text{this paper}}=(8\pi W/M_N)[M_{l+},\text{ etc.}]^{(\pm,0)}{}_{\text{CGLN}}$.

$(M_\pi{}^f=M_\pi)$; $F_\pi(k^2)$ is the pion charge form factor.]

$$
\begin{aligned}
V_1{}^{(\pm)B}&=-\frac{g_rF_1{}^V(k^2)}{2M_N}\left(\frac{1}{\nu_B-\nu}\pm\frac{1}{\nu_B+\nu}\right),\\
V_2{}^{(\pm)B}&=\frac{g_rF_1{}^V(k^2)}{4M_N{}^2\nu_B}\left(\frac{1}{\nu_B-\nu}\pm\frac{1}{\nu_B+\nu}\right),\\
V_3{}^{(\pm)B}&=\frac{g_rF_2{}^V(k^2)}{2M_N}\left(\frac{1}{\nu_B-\nu}\mp\frac{1}{\nu_B+\nu}\right),\\
V_4{}^{(\pm)B}&=\frac{g_rF_2{}^V(k^2)}{2M_N}\left(\frac{1}{\nu_B-\nu}\pm\frac{1}{\nu_B+\nu}\right),\\
V_5{}^{(+)B}&=0,\quad V_5{}^{(-)B}=\frac{-2g_r}{k^2}\left(\frac{F_1{}^V(k^2)}{2M_N\nu_B}-\frac{2F_\pi(k^2)}{4M_N\nu_B-k^2}\right),\\
V_6{}^{(\pm)B}&=0.
\end{aligned}
\tag{14}
$$

While the consequences of PCAC are most simply expressed in terms of the invariant amplitudes V_j, pion photoproduction and electroproduction experiments are most easily analyzed in terms of the center-of-mass frame amplitudes $\mathfrak{F}_j{}^V$, defined by[7]

$$\epsilon_\lambda V_\lambda{}^{(\pm,0)}=\sum_{j=1}^{6}\mathfrak{F}_j{}^{V(\pm,0)}\chi_f{}^*\Sigma_j{}^V\chi_i. \tag{15}$$

Here χ_f and χ_i are the nucleon Pauli spinors, and the Σ's are chosen as follows:

$$
\begin{aligned}
&\Sigma_1{}^V=i(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\hat{k}\hat{k}\cdot\boldsymbol{\varepsilon}), && \Sigma_4{}^V=i\boldsymbol{\sigma}\cdot\hat{q}(\hat{q}\cdot\boldsymbol{\varepsilon}-\hat{q}\cdot\hat{k}\hat{k}\cdot\boldsymbol{\varepsilon}),\\
&\Sigma_2{}^V=\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot(\hat{k}\times\boldsymbol{\varepsilon}), && \Sigma_5{}^V=-ik^2\boldsymbol{\sigma}\cdot\hat{k}\hat{k}\cdot\boldsymbol{\varepsilon}/k_0,\\
&\Sigma_3{}^V=i\boldsymbol{\sigma}\cdot\hat{k}(\hat{q}\cdot\boldsymbol{\varepsilon}-\hat{q}\cdot\hat{k}\hat{k}\cdot\boldsymbol{\varepsilon}), && \Sigma_6{}^V=-ik^2\boldsymbol{\sigma}\cdot\hat{q}\hat{k}\cdot\boldsymbol{\varepsilon}/k_0.
\end{aligned}
\tag{16}
$$

The amplitudes $\mathfrak{F}_j{}^V$ have simple multipole expansions[7]:

$$
\begin{aligned}
\mathfrak{F}_1{}^V&=\sum_{l=0}^{\infty}(lM_{l+}+E_{l+})P_{l+1}{}'(y)+\sum_{l=2}^{\infty}[(l+1)M_{l-}+E_{l-}]P_{l-1}{}'(y),\\
\mathfrak{F}_2{}^V&=\sum_{l=1}^{\infty}[(l+1)M_{l+}+lM_{l-}]P_l{}'(y),\\
\mathfrak{F}_3{}^V&=\sum_{l=1}^{\infty}(-M_{l+}+E_{l+})P_{l+1}{}''(y)+\sum_{l=3}^{\infty}(M_{l-}+E_{l-})P_{l-1}{}''(y),\\
\mathfrak{F}_4{}^V&=\sum_{l=2}^{\infty}(M_{l+}-M_{l-}-E_{l+}-E_{l-})P_l{}''(y),\\
k_0\mathfrak{F}_5{}^V&=\sum_{l=0}^{\infty}(l+1)L_{l+}P_{l+1}{}'(y)-\sum_{l=2}^{\infty}lL_{l-}P_{l-1}{}'(y),\\
k_0\mathfrak{F}_6{}^V&=\sum_{l=1}^{\infty}[lL_{l-}-(l+1)L_{l+}]P_l{}'(y).
\end{aligned}
\tag{17}
$$

The index $l\pm$ of the multipole specifies the orbital angular momentum (l) and the total angular momentum ($J=l\pm\frac{1}{2}$) of the final pion-nucleon system. It is straightforward, but tedious, to calculate the linear transformations connecting the amplitudes V_j and $\mathfrak{F}_j{}^V$.[8]

B. Derivation

The PCAC relations of Eq. (1) come from the identity

$$
\begin{aligned}
i\int d^4x\,e^{-iq\cdot x}\psi_c{}^*(-\Box_x+M_\pi{}^2)\langle N(p_2)|T[\partial_\sigma J_\sigma{}^{Ac}(x),(J_\lambda{}^{I3}(0)+J_\lambda{}^Y(0))]|N(p_1)\rangle\epsilon_\lambda\\
=-i\int d^4x\,e^{-iq\cdot x}\psi_c{}^*(-\Box_x+M_\pi{}^2)\delta(x_0)\langle N(p_2)|[J_0{}^{Ac}(x),J_\lambda{}^{I3}(0)+J_\lambda{}^Y(0)]|N(p_1)\rangle\epsilon_\lambda\\
-q_\sigma\int d^4x\,e^{-iq\cdot x}\psi_c{}^*(-\Box_x+M_\pi{}^2)\langle N(P_2)|T[J_\sigma{}^{Ac}(x),(J_\lambda{}^{I3}(0)+J_\lambda{}^Y(0))]|N(p_1)\rangle\epsilon_\lambda,
\end{aligned}
\tag{18}
$$

which is obtained by integration by parts. Using the partially conserved axial-vector current hypothesis,[9]

$$\partial_\sigma J_\sigma{}^{Ac}(x)=\frac{M_NM_\pi{}^2g_A}{g_r(0)}\varphi_\pi{}^c(x), \tag{19}$$

we see that the left-hand side of Eq. (18) is just

$$\frac{M_NM_\pi{}^2g_A}{g_r(0)}\sum_{j=1}^{6}\bar{u}(p_2)O(V_j)u(p_1)[a^{(+)}\bar{V}_j{}^{(+)}+a^{(-)}\bar{V}_j{}^{(-)}+a^{(0)}\bar{V}_j{}^{(0)}]+\text{Born terms}, \tag{20}$$

where $\bar{V}_j$ denotes the non-Born part of the amplitude V_j.

[8] See, for example, R. Blankenbecler, S. Gartenhaus, R. Huff, and Y. Nambu, Nuovo Cimento **17**, 775 (1960); P. Dennery, Phys. Rev. **124**, 2000 (1961).

[9] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

Let us evaluate the two terms on the right-hand side of Eq. (18) in the limit as $q\rightarrow 0$. The equal-time commutator term approaches

$$-iM_\pi{}^2\psi_c{}^*\langle N(p_2)|\left[\left[\int d^3xJ_0{}^{Ac}(x),\,J_\lambda{}^{I3}(0)+J_\lambda{}^Y(0)\right]\right|_{x_0=0}|N(p_1)\rangle\epsilon_\lambda. \tag{21}$$

Because of the integration over all space, possible gradient terms in the commutator do not contribute, and we find for this term

$$(M_\pi{}^2g_A(k^2)/k^2)a^{(-)}\bar{u}(p_2)O(V_6)u(p_1). \tag{22}$$

(To simplify the algebra we have dropped terms proportional to $k\cdot\epsilon=0$.) The term proportional to q_σ, in the limit as $q_\sigma\rightarrow 0$, can be evaluated by keeping only the one-nucleon-pole terms.[10] This gives

$$-M_\pi{}^2\psi_c{}^*\bar{u}(p_2)\Bigg\{ig_A\gamma\cdot q\gamma_5\frac{\tau_c}{2}\frac{\gamma\cdot p_2+iM_N}{-2p_2\cdot q}\tfrac{1}{2}i[\gamma_\lambda(F_1{}^V(k^2)\tau_3+F_1{}^S(k^2))-\sigma_{\lambda\mu}k_\mu(F_2{}^V(k^2)\tau_3+F_2{}^S(k^2))]$$

$$+\tfrac{1}{2}i[\gamma_\lambda(F_1{}^V(k^2)\tau_3+F_1{}^S(k^2))-\sigma_{\lambda\mu}k_\mu(F_2{}^V(k^2)\tau_3+F_2{}^S(k^2))]\frac{\gamma\cdot p_1+iM_N}{2p_1\cdot q}ig_A\gamma\cdot q\gamma_5\frac{\tau_c}{2}\Bigg\}u(p_1)\epsilon_\lambda$$

$$=M_\pi{}^2g_A\Bigg\{-a^{(-)}\frac{F_1{}^V(k^2)}{k^2}\bar{u}(p_2)O(V_6)u(p_1)+(a^{(+)}F_2{}^V(k^2)+a^{(0)}F_2{}^S(k^2))\bar{u}(p_2)O(V_1)u(p_1)$$

$$-\frac{F_1{}^V(k^2)}{2}\left[a^{(+)}\left(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right)+a^{(-)}\left(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right)\right]\bar{u}(p_2)O(V_1)u(p_1)+\text{the other Born terms}\Bigg\}. \tag{23}$$

Comparing Eq. (20) with Eqs. (22) and (23), we get the relations

$$\begin{aligned}(g_r(0)/M_N)F_2{}^V(k^2)&=\bar{V}_1{}^{(+)}(\nu=\nu_B=(M_\pi{}^f)^2=0,\,k^2),\\ (g_r(0)/M_N)F_2{}^S(k^2)&=\bar{V}_1{}^{(0)}(\nu=\nu_B=(M_\pi{}^f)^2=0,\,k^2),\\ \frac{g_r(0)}{M_N}\left[\frac{g_A(k^2)}{g_A(0)}-F_1{}^V(k^2)\right](k^2)^{-1}&=\bar{V}_6{}^{(-)}(\nu=\nu_B=(M_\pi{}^f)^2=0,\,k^2).\end{aligned} \tag{24}$$

If we take $\nu=\nu_B=0$ to mean "first set $\nu_B=0$, *then* set $\nu=0$" the bars in Eq. (24) may be dropped, since the Born approximation to V_1 vanishes at $\nu_B=0$ (for all $\nu\neq 0$). This completes the deviation of Eqs. (1).

III. NUMERICAL ANALYSIS

We now proceed to a numerical analysis of Eqs. (1a) and (1c) at $k^2=0$. Introducing the abbreviations $V_1{}^{(+)}(0)\equiv V_1{}^{(+)}(\nu=\nu_B=(M_\pi{}^f)^2=k^2=0)$, $V_6{}^{(-)}(0)\equiv V_6{}^{(-)}(\nu=\nu_B=(M_\pi{}^f)^2=k^2=0)$, we write the equations in the form

$$\frac{g_r}{M_N}F_2{}^V(0)=\frac{g_r}{g_r(0)}V_1{}^{(+)}(0),\quad \frac{g_r}{M_N}\left[\frac{g_A'(0)}{g_A(0)}-F_1{}^{V\prime}(0)\right]=\frac{g_r}{g_r(0)}V_6{}^{(-)}(0). \tag{25}$$

In order to calculate $V_1{}^{(+)}$ and $V_6{}^{(-)}$ from experimental photoproduction data, we assume that $V_1{}^{(+)}$ and $V_6{}^{(-)}$ both satisfy unsubtracted fixed-momentum-transfer dispersion relations in the energy variable ν[11]:

$$V_1{}^{(+)}(\nu,\nu_B,(M_\pi{}^f)^2,k^2)=\frac{-g_r(-(M_\pi{}^f)^2)F_1{}^V(k^2)}{2M_N}\left(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right)$$
$$+\frac{1}{\pi}\int_{\nu_B+M_\pi+M_\pi{}^2/(2M_N)}^{\infty}d\nu'\,\mathrm{Im}V_1{}^{(+)}(\nu',\nu_B,(M_\pi{}^f)^2,k^2)\left(\frac{1}{\nu'-\nu}+\frac{1}{\nu'+\nu}\right), \tag{26}$$

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[10] See S. L. Adler, Ref. 1, where the rules for calculating the "pole insertions" are discussed. In Sec. II B we have ignored questions of gauge invariance. It is easily shown [S. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966), and M. Nauenberg, Phys. Letters **22**, 201 (1966)] that when the final pion is off mass shell, the photoproduction or electroproduction amplitude is not divergenceless, but has a divergence proportional to $(q^2+M_\pi{}^2)g_r[(q-k)^2]/[(q-k)^2+M_\pi{}^2]$. In order to maintain the correct divergence, additional terms must be added to the Born approximation calculated from the diagrams of Fig. 1. However, these additional terms vanish when $q=k\cdot\epsilon=0$, and thus do not affect the results of this paper. See also S. Fubini, Y. Nambu, and A. Wataghin, Phys. Rev. **111**, 329 (1958).

[11] Validity of the unsubtracted dispersion relation for $V_1{}^{(+)}(0)$ is indicated by the Regge-pole analysis of photoproduction given by G. Zweig, Nuovo Cimento **32**, 689 (1964).

$$V_6^{(-)}(\nu,\nu_B,(M_\pi^f)^2,k^2)=\frac{1}{\pi}\int_{\nu_B+M_\pi+M_\pi^2/(2M_N)}^{\infty} d\nu'\,\mathrm{Im}V_6^{(-)}(\nu',\nu_B,(M_\pi^f)^2,k^2)\left(\frac{1}{\nu'-\nu}+\frac{1}{\nu'+\nu}\right),$$

which imply that

$$\frac{g_r}{g_r(0)}\begin{Bmatrix}V_1^{(+)}(0)\\V_6^{(-)}(0)\end{Bmatrix}=\frac{2}{\pi}\int_{M_\pi+M_\pi^2/2M_N}^{\infty}\frac{d\nu'}{\nu'}\frac{g_r}{g_r(0)}\mathrm{Im}\begin{Bmatrix}V_1^{(+)}(\nu',0,0,0)\\V_6^{(-)}(\nu',0,0,0)\end{Bmatrix}. \tag{27}$$

In order to calculate the integrand of Eq. (27), we make a multipole expansion, keeping only those multipoles which can at present be determined from experiment. These are: (i) the nonresonant s-wave multipoles E_{0+} and L_{0+}. The E_{0+} makes a large contribution to charged pion photoproduction; (ii) the multipoles $M_{1+}^{(3/2)}$, $E_{1+}^{(3/2)}$, and $L_{1+}^{(3/2)}$, which are important around the $I=\frac{3}{2}N^*(1238)$; (iii) the multipoles $M_{2-}^{(1/2)}$, $E_{2-}^{(1/2)}$, and $L_{2-}^{(1/2)}$ which are important around the $I=\frac{1}{2}N^{**}(1520)$.

Doing the necessary arithmetic, we find

$$\frac{g_r}{g_r(0)}V_1^{(+)}(\nu,0,0,0)=\frac{2M_N}{W^2-M_N^2}\frac{g_r}{g_r(0)}\left[\tfrac{1}{3}E_{0+}^{(1/2)}+\tfrac{2}{3}E_{0+}^{(3/2)}+\tfrac{2}{3}M_{1+}^{(3/2)}+2E_{1+}^{(3/2)}-M_{2-}^{(1/2)}+\tfrac{1}{3}E_{2-}^{(1/2)}\right]\Big|_{M_\pi^f=0}, \tag{28a}$$

$$\frac{g_r}{g_r(0)}V_6^{(-)}(\nu,0,0,0)=\frac{2M_N}{W^2-M_N^2}\frac{1}{3}\frac{g_r}{g_r(0)}\left[\frac{E_{0+}^{(1/2)}-E_{0+}^{(3/2)}}{W-M_N}-\frac{2W(L_{0+}^{(1/2)}-L_{0+}^{(3/2)})}{W^2-M_N^2}+\frac{M_{1+}^{(3/2)}}{W+M_N}\right.$$
$$\left.-\frac{3(3W+M_N)E_{1+}^{(3/2)}}{W^2-M_N^2}+\frac{8WL_{1+}^{(3/2)}}{W^2-M_N^2}+\frac{3M_{2-}^{(1/2)}}{W+M_N}+\frac{(M_N-5W)E_{2-}^{(1/2)}}{W^2-M_N^2}-\frac{8WL_{2-}^{(1/2)}}{W^2-M_N^2}\right]\Big|_{M_\pi^f=0}. \tag{28b}$$

The multipoles appearing in Eq. (28) are not actually the physical multipoles, since they refer to zero final pion mass ($M_\pi^f=0$). We relate them to the physical multipoles by the prescription

$$\begin{Bmatrix}M\\E\\L\end{Bmatrix}_{l\pm}^{(I)}\Bigg|_{M_\pi^f=0}=\frac{g_r(0)}{g_r}\begin{Bmatrix}M\\E\\L\end{Bmatrix}_{l\pm}^{(I)}\Bigg|_{M_\pi^f=M_\pi}\left(\frac{|\mathbf{q}|_{M_\pi^f=0}}{|\mathbf{q}|_{M_\pi^f=M_\pi}}\right)^l, \tag{29}$$

where the subscripts on $|\mathbf{q}|$ indicate that $|\mathbf{q}|$ is to be computed from W with $M_\pi^f=0$ or M_π, respectively. The prescription of Eq. (29) gives the unphysical multipoles the correct threshold behavior and, approximately, the correct nearby left-hand singularities.[12] Using Eq. (29) eliminates the obnoxious factor $g_r/g_r(0)$ in Eq. (28) and leaves us with simple integrals over the physically measurable multipoles.

From pion-photoproduction experiments, the electric and magnetic multipoles can be measured. However, the longitudinal multipoles can only be measured in pion electroproduction experiments; so far little data is available. Consequently, we will have to make a guess as to the magnitude of the longitudinal multipoles. When the photon momentum $|\mathbf{k}|$ approaches zero, the longitudinal and electric multipoles become proportional with known coefficients,[13]

$$\begin{aligned}L_{l+}/E_{l+}&\to 1\,, & l&\geq 0\,,\\ L_{l-}/E_{l-}&\to -(l-1)/l\,, & l&\geq 2.\end{aligned} \tag{30}$$

For want of a better estimate, we will assume that these proportionalities hold for nonzero $|\mathbf{k}|$ as well. In other words, we take

$$L_{0+}\approx E_{0+}\,,\quad L_{1+}\approx E_{1+}\,,\quad L_{2-}\approx -\tfrac{1}{2}E_{2-} \tag{31}$$

in the numerical work described below.

A. Narrow-Resonance Approximation

We begin by discussing the narrow-resonance approximation for the magnetic dipole ($M_{1+}^{(3/2)}$) contribution. It is convenient here to use the CGLN model[14] for $M_{1+}^{(3/2)}$, which, as Schmidt and Höhler[4] and Schmidt[4] have shown, is in good agreement with photoproduction experiments. According to this model

$$M_{1+}^{(3/2)}=\frac{4.70g_r}{3}\frac{W}{M_N}\frac{|\mathbf{q}||\mathbf{k}|}{M_N^2}\frac{\exp(i\delta_{3,3})\sin\delta_{3,3}}{\frac{4}{3}f^2|\mathbf{q}|^3/M_\pi^2}, \tag{32}$$

with $f^2=0.08$, $\delta_{3,3}$ the pion-nucleon scattering phase shift in the (3,3) partial wave, and $|\mathbf{q}|$ evaluated with

[12] For a more detailed discussion see S. L. Adler, Phys. Rev. **140**, B736 (1965).

[13] J. D. Bjorken and J. D. Walecka, Ann. Phys. (N. Y.) **38**, 35 (1966); Y. Dothan and R. P. Feynman (private communciation).

[14] G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

TABLE I. Parameters for multipoles.

Resonance	W_R (units of M_π)	$\lvert\mathbf{q}\rvert_R$ (units of M_π)	Γ_R (units of M_π)	Multipole $\mathfrak{M}$	A (units of M_π^{-1})
$N^*(1238)$	8.85	1.65	0.860	$M_{1+}^{(3/2)}$	+0.112
				$E_{1+}^{(3/2)}$	−0.0080
$N^{**}(1520)$	10.80	3.20	0.860	$M_{2-}^{(1/2)}$	+0.0155
				$E_{2-}^{(1/2)}$	+0.0628

$M_\pi^f = M_\pi$. Substituting Eq. (32) into Eq. (27), we find for the magnetic-dipole contribution to $V_1^{(+)}(0)$

$$V_1^{(+)}(0)\big|_{\text{magnetic dipole}} = \frac{g_r}{M_N}\frac{8}{9}\frac{4.70}{2M_N}I\,,$$

$$I = \frac{1}{\pi}\int_{M_N+M_\pi}^{\infty} dW \frac{\sin^2\delta_{3,3}}{[\lvert\mathbf{q}\rvert_{M_\pi^f=M_\pi}]^3 \frac{4}{3} f^2/M_\pi^2}\,. \qquad (33)$$

According to the narrow-resonance approximation,[15] $I=1$, giving

$$V_1^{(+)}(0)\big|_{\text{magnetic dipole (narrow resonance)}} \approx 0.62/M_\pi^2\,, \qquad (34)$$

to be compared with the value predicted by PCAC [the left-hand side of Eq. (25)],

$$(g_r/M_N)F_2^V(0) \approx 0.55/M_\pi^2\,. \qquad (35)$$

Actually, the narrow-resonance approximation is very misleading. Direct numerical evaluation of I, using the experimental (3,3) phase shift,[16] gives $I=0.63$, so that actually

$$V_1^{(+)}(0)\big|_{\text{magnetic dipole}} \approx 0.39/M_\pi^2\,. \qquad (36)$$

In other words, the narrow-resonance approximation overestimates the integral I by 60%. [The narrow-resonance approximation is also misleading when used to evaluate the g_A sum rule. If only the (3,3) contribution to this sum rule is kept, one gets $g_A \approx 1.4$ when the integral is evaluated using the experimental πN cross section,[17] and $g_A = 3$ when one uses the narrow-resonance approximation.] To sum up, the narrow-resonance approximation for the $N^*(1238)$ is useful for making order-of-magnitude estimates, but should be avoided in quantitative tests of sum rules.

B. Resonant Contributions

We turn next to the evaluation of the resonant contributions to $V_1^{(+)}(0)$ and $V_6^{(-)}(0)$, using Walker's photoproduction analysis. Walker[5] has parametrized each resonant multipole $\mathfrak{M}$ around the $N^*(1238)$ and the $N^{**}(1520)$ in the form[18]

$$\mathfrak{M} = \frac{8\pi W}{M_N}\frac{A(\lvert\mathbf{q}\rvert_R/\lvert\mathbf{q}\rvert)^2\Gamma/2}{W_R - W - i\Gamma/2}\,,$$

$$\Gamma = \Gamma_R\left(\frac{\lvert\mathbf{q}\rvert}{\lvert\mathbf{q}\rvert_R}\right)^3 \frac{1+0.7735\lvert\mathbf{q}\rvert_R^2/M_\pi^2}{1+0.7735\lvert\mathbf{q}\rvert^2/M_\pi^2}\,. \qquad (37)$$

The parameters A, Γ_R, W_R, and $\lvert\mathbf{q}\rvert_R$ are listed in Table I. Using Eqs. (37) and (31) we have calculated the integrals for $V_1^{(+)}(0)$ and $V_6^{(-)}(0)$, obtaining the results listed in Table II.

TABLE II. Multipole contributions.

Multipole	Contribution to $[g_r/g_r(0)]V_1^{(+)}(0)$ (units of M_π^{-2})	Contribution to $[g_r/g_r(0)]V_6^{(-)}(0)$ (units of M_π^{-3})
$E_{0+}^{(1/2)}$	+0.055	+0.0329
$E_{0+}^{(3/2)}$	+0.081	−0.0212
$L_{0+}^{(1/2)}$		−0.0365
$L_{0+}^{(3/2)}$		+0.0238
$M_{1+}^{(3/2)}$	+0.413	+0.0133
$E_{1+}^{(3/2)}$	−0.088	+0.0471
$L_{1+}^{(3/2)}$		−0.0333
$M_{2-}^{(1/2)}$	−0.031	+0.0018
$E_{2-}^{(1/2)}$	+0.042	−0.0305
$L_{2-}^{(1/2)}$		+0.0281
Total	+0.472	+0.0255
PCAC prediction	+0.550	$\frac{g_r}{M_N}\left[\frac{g_A'(0)}{g_A(0)} - F_1^{V\prime}(0)\right]M_\pi^3 \approx 2M_\pi^2\left[\frac{g_A'(0)}{g_A(0)} + \frac{0.045}{M_\pi^2}\right]$

We note, first of all, that according to Table II the $M_{1+}^{(3/2)}$ contribution to $V_1^{(+)}(0)$ is

$$V_1^{(+)}(0)\big|_{\text{magnetic dipole}} \approx 0.41/M_\pi^2\,, \qquad (38)$$

in good agreement with the value of $0.39/M_\pi^2$ obtained above from the CGLN–Schmidt-Höhler work. The multipole $E_{1+}^{(3/2)}$ makes a significant contribution to the sum rule because it appears in Eq. (28a) with a coefficient three times as large as the coefficient of $M_{1+}^{(3/2)}$. Walker's $\mathrm{Im}E_{1+}^{(3/2)}$ has a constant negative sign across the $N^*(1238)$. If the suggestion of the CGLN model[14] [that $\mathrm{Im}E_{1+}^{(3/2)}$ changes sign from negative to positive around the (3,3) resonance peak] should prove to be correct, then the value for the $E_{1+}^{(3/2)}$ contribution given in Table II may be an overestimate.

Looking at the contributions to $V_6^{(-)}(0)$, it may at first seem surprising that the small $E_{1+}^{(3/2)}$ multipole makes a much bigger contribution than the large $M_{1+}^{(3/2)}$ multipole. But a glance at Eq. (28b) shows the reason why—the ratio of the coefficients of $M_{1+}^{(3/2)}$

[15] G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

[16] We obtained the same numerical result using the (3,3) phase-shift parametrizations of Schmidt (Ref. 4) and of L. D. Roper, University of California Report No. UCRL-7846 (unpublished). For an independent evaluation of this integral, see D. Lyth, Phys. Letters **21**, 338 (1966).

[17] See W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); S. L. Adler, *ibid.* **14**, 1051 (1965).

[18] Equation (37) does not give the multipoles M_{2-}, E_{2-} the correct threshold behavior, but the $N^{**}(1520)$ is far enough from threshold so that this is not too important. To make the off-mass-shell correction we have multiplied each $\mathfrak{M}$ by $[\lvert\mathbf{q}\rvert_{M_\pi^f=0}/\lvert\mathbf{q}\rvert_{M_\pi^f=M_\pi}]$, so that the off-mass-shell multipoles all have the correct threshold behavior.

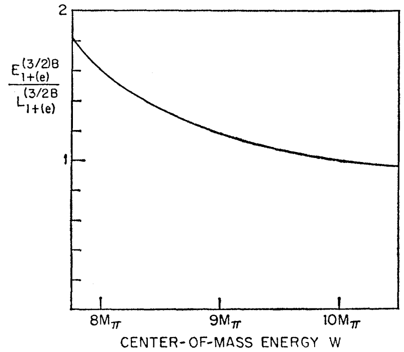

FIG. 2. Ratio of electric to longitudinal multipoles, for $k^2=0$.

and $E_{1+}^{(3/2)}$ in the integral for $V_6^{(-)}(0)$ is

$$\frac{1}{W+M_N}\left[\frac{-3(3W+M_N)}{W^2-M_N^2}\right]^{-1}=-\frac{(W-M_N)}{3(3W+M_N)},\quad(39)$$

which is numerically ≈ -0.02 at the peak of the $N^*(1238)$. In other words, the $M_{1+}^{(3/2)}$ contribution is very strongly kinematically suppressed. The longitudinal multipoles contribute with strength comparable to the electric multipoles. To emphasize the dubious nature of the approximation of Eq. (31) for the longitudinal multipoles, we have computed the Born approximations $E_{1+}^{(3/2)}$ and $L_{1+}^{(3/2)}$ from the diagrams of Fig. 1, splitting them into parts proportional to the electric charge e and the difference of the nucleon total magnetic moments $\mu_p-\mu_n$:

$$\begin{aligned}E_{1+}^{(3/2)B}&=eE_{1+(e)}^{(3/2)B}+(\mu_p-\mu_n)E_{1+(\mu)}^{(3/2)B},\\ L_{1+}^{(3/2)B}&=eL_{1+(e)}^{(3/2)B}+(\mu_p-\mu_n)L_{1+(\mu)}^{(3/2)B}.\end{aligned}\quad(40)$$

(Numerically, the e terms, which come largely from the pion-exchange graph, are much larger than the μ terms, which come from the crossed nucleon graph.) One can verify, by direct calculation, that at $|\mathbf{k}|=0$ (for all $k^2\neq0$),

$$\frac{E_{1+(e)}^{(3/2)B}}{L_{1+(e)}^{(3/2)B}}=\frac{E_{1+(\mu)}^{(3/2)B}}{L_{1+(\mu)}^{(3/2)B}}=1.\quad(41)$$

But at the physical threshold $|\mathbf{q}|=0$ we find for real photons that

$$\begin{aligned}E_{1+(e)}^{(3/2)B}/L_{1+(e)}^{(3/2)B}&=1.88,\\ E_{1+(\mu)}^{(3/2)B}/L_{1+(\mu)}^{(3/2)B}&=0.\end{aligned}\quad(42)$$

In Fig. 2 we have plotted the ratio of the numerically dominant e terms as a function of energy. Clearly, in determining the longitudinal multipole contributions to $V_6^{(-)}(0)$, assumptions such as Eq. (31) are unreliable and there will be no substitute for measurement of the longitudinal multipoles in electroproduction experiments.

C. Nonresonant S Wave

It is well known that there is an important s-wave contribution to charged-pion photoproduction. Since the s-wave pion-nucleon phase shifts are of order 15°–20° in the low-energy region, the imaginary parts of the s-wave amplitudes will make an important contribution to the integrals for $V_1^{(+)}(0)$ and $V_6^{(-)}(0)$. We estimate this contribution as follows. The Born approximations for the s-wave multipoles $E_{0+}^{(\pm,0)}$ are[14]

$$E_{0+}^{(+)B}\approx-\frac{W-M_N}{M_N}\frac{4.70}{M_\pi},\quad E_{0+}^{(0)B}\approx-\frac{W-M_N}{M_N}\frac{0.88}{M_\pi},$$
$$E_{0+}^{(-)B}\approx\frac{1}{M_\pi}\left[1+\frac{1-V^2}{2V}\ln\left(\frac{1+V}{1-V}\right)\right],\quad V=|\mathbf{q}|/q_0.\quad(43)$$

Pion-photoproduction experiments, as analyzed by Schmidt,[4] indicate that (i) in charged-pion photoproduction, the multipole E_{0+} is equal to the Born approximation; (ii) in neutral-pion photoproduction, $(M_N/W)E_{0+}$ is independent of energy, and at threshold is roughly one-half of the Born approximation. The charged and neutral pion amplitudes are related to $E_{0+}^{(\pm,0)}$ by

$$\begin{aligned}E_{0+}^{(\pi^+)}&=(1/\sqrt{2})(E_{0+}^{(-)}+E_{0+}^{(0)}),\\ E_{0+}^{(\pi^0)}&=\tfrac{1}{2}[E_{0+}^{(+)}+E_{0+}^{(0)}].\end{aligned}\quad(44)$$

If we assume that the isoscalar amplitude (which is small anyway) is not much different from its Born approximation,[19] then the experimental results imply

$$\begin{aligned}\mathrm{Re}E_{0+}^{(+)}&\approx-\frac{W}{M_N+M_\pi}0.4\frac{4.70}{M_N},\\ \mathrm{Re}E_{0+}^{(-)}&\approx E_{0+}^{(-)B}.\end{aligned}\quad(45)$$

We get the imaginary parts of the multipoles $E_{0+}^{(1/2,3/2)}$ by using the Fermi-Watson theorem, which tells us that

$$\begin{aligned}\mathrm{Im}E_{0+}^{(1/2)}&\approx\sin\delta_1\,\mathrm{Re}E_{0+}^{(1/2)}\\ &=\sin\delta_1[\mathrm{Re}E_{0+}^{(+)}+2\,\mathrm{Re}E_{0+}^{(-)}],\\ \mathrm{Im}E_{0+}^{(3/2)}&\approx\sin\delta_3\,\mathrm{Re}E_{0+}^{(3/2)}\\ &=\sin\delta_3[\mathrm{Re}E_{0+}^{(+)}-\mathrm{Re}E_{0+}^{(-)}],\end{aligned}\quad(46)$$

with δ_1, δ_3 the $I=\frac{1}{2}$, $\frac{3}{2}$ s-wave pion-nucleon phase shifts.

The numbers given in Table II have been obtained by using Eqs. (45) and (46), integrating from threshold to a center-of-mass energy $W=10\ M_\pi$, and taking the the pion-nucleon phase shifts from Roper's $l_m=4$ analysis.[20] Adding the s-wave result to the other

[19] This is suggested by the CGLN model, in which the isoscalar amplitude is given by the Born approximation, but the isovector amplitude differs appreciably from the Born approximation due to the presence of the dispersion integral over the $N^*(1238)$.

[20] L. D. Roper, Ref. 16.

contributions to $V_1^{(+)}(0)$ raises the total to about 0.85 of the value predicted by PCAC.[21] If we assume Eq. (31) for L_{0+}, the result for $V_6^{(-)}(0)$ obtained from the resonant multipoles is changed very little.

[21] If $\mathrm{Re}E_{0+}^{(+)}$ is taken to be zero, the $E_{0+}^{(1/2)}$ and $E_{0+}^{(3/2)}$ contributions to $V_1^{(+)}(0)$ become $0.062/M_\pi^2$ and $0.066/M_\pi^2$, respectively. Thus, as expected, the *s*-wave contribution comes mainly from the charged-pion photoproduction amplitude $E_{0+}^{(-)}$. The only multipole significant in the low-energy region which we have omitted from our analysis is M_{1-}. While $\mathrm{Re}M_{1-}$ is known, the P_{11} pion-nucleon phase shift becomes large only when the inelasticity in this channel is also large. This means that $\mathrm{Im}M_{1-}$ cannot then be reliably determined by the Fermi-Watson theorem.

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Feedback Mechanism for the *n*-*p* Mass Difference*

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A feedback mechanism which can yield the correct sign for the *n*-*p* mass difference is described in detail, and is shown to have its origin in the inelastic part of the Compton-amplitude contribution to the mass shift.

I. INTRODUCTION

RECENTLY, a new derivation of the *n*-*p* mass difference has been given[1,2] which provides a possible mechanism for reversing the sign of previous estimates of this effect. The basic assumptions made in Ref. 1 were:

(i) The electromagnetic interaction may be treated as a small perturbation compared to the strong interaction.

(ii) The self-energy operator of the nucleon, due to both strong and electromagnetic interactions, satisfies an unsubtracted dispersion relation.

(iii) The bare mass m_0 associated with the nucleon field is independent of these interactions and hence is the same for both neutron and proton.

These assumptions have the following consequences. Because of isotopic-spin symmetry in the absence of electromagnetic interactions, the neutron and proton masses are the same. When the electromagnetic interaction is turned on, these masses will be different; for example, because of that unitarity contribution to the nucleon's self-energy function whose intermediate state consists of a photon and a nucleon. In addition to and because of this electromagnetic effect, the masses of the intermediate nucleon and pions, which enter into the strong-interaction contribution to the nucleon self-energy unitarity integral, are also shifted. This shift can provide a feedback mechanism to reverse the sign of previous estimates,[3] which are based essentially on the nucleon-plus-photon contribution.

Applying this idea to the question of the $\pi^\pm$-π^0 mass difference, one can see that if the dominant unitarity contribution to the pion self-energy is due to the $N\bar{N}$ intermediate state, as suggested by the large π-N coupling constant and also by the success of the original derivation of the Goldberger-Treiman relation for the $\pi^\pm$ lifetime, then the feedback mechanism is not effective. Hence the $\pi^\pm$-π^0 mass difference can be understood qualitatively by the elementary classical argument, or more quantitatively by the calculation of Bose and Marshak.[4]

In this paper we make the same assumptions (i), (ii), and (iii), but we try to give some insight into the feedback mechanism by contrasting the result of Ref. 1 with that of the conventional level-shift formula, the latter written in terms of an integral over the difference between the neutron and proton Compton scattering amplitudes.[5] We emphasize that these methods and formulas are not in disagreement; rather, our formula attempts to extract and rearrange a portion of the so-called inelastic contributions to the absorptive part of the Compton amplitudes. In Sec. II we shall briefly discuss the conventional mass-shift formula and the passage to the description in terms of physical masses, unitarity integrals, etc., and then derive a new, exact formula for $\Delta m = m_p - m_n$. This formula is only an

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[5] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963). Extensive references to the recent literature may be found in G. Barton and D. Dare, Phys. Rev. **150**, 1220 (1966).