Determination of Cabibbo Parameters from Leptonic Baryon Decay*

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The parameters D, F, and θ appearing in Cabibbo's theory for the weak baryon current have been re-evaluated, following Willis *et al.*, using the latest available data on the leptonic decays of baryons. The momentum dependence of the form factors has been included. Seven data were used and three parameters fitted. The best least-squares fit to the data gives $D=-0.766\pm0.037$, $F=-0.415\pm0.035$, $\theta=0.245\pm0.010$ [probability=93% ($\chi^2=0.78$)].

I. INTRODUCTION

THE octet current hypothesis[1] for the nonleptonic weak current is successful in describing the decays of baryons into leptons. The weak current of the baryons can be expressed in terms of three free parameters, commonly denoted θ, D, and F; using these parameters, the experiments can be fitted quite closely.

The values of these parameters which best fit the experiments have been determined by Willis *et al.*[2] and others.[3] Since their work, new data on the leptonic decay of the baryons have become available. For this reason, the parameters θ, D, and F have been re-evaluated. Also, the weak magnetism terms and the momentum dependence of the form factors are included in this calculation. These two small corrections had been left out by Willis *et al.*[4]

II. OUTLINE OF THE THEORY

Weak leptonic decays[5] of the baryons are described by a phenomenological current-current Hamiltonian:

$$H=J_\lambda(x)j^\lambda(x)\,, \tag{1}$$

where J_λ is the baryon current and j^λ is the lepton current. The form of the lepton current is well known:

$$j^\lambda=\sum_{l=\mu,e}\psi_l^\dagger\gamma^0\gamma^\lambda(1+\gamma^5)\psi_{\nu_l}$$

[The notation used throughout is as follows: The metric is $g^{\mu\nu}=(1,-1,-1,-1)$; γ^0 is Hermitian, the $\gamma^i(i=1,2,3)$ are anti-Hermitian; the scalar product of two four-vectors is $A_\alpha B^\alpha$, where the repeated index is summed from 0 to 3, and $A_\alpha=g_{\alpha\beta}A^\beta$. $\gamma^5=\gamma_5=-i\gamma_0\gamma_1\gamma_2\gamma_3$.]

The baryon current is known to have two parts obeying different selection rules. The strangeness-conserving part of the current obeys the rules $\Delta S=0$ and $|\Delta\mathbf{I}|=1$, while the strangeness-changing past of the current obeys the rules $\Delta S=\Delta Q_h$ and $|\Delta\mathbf{I}|=\frac{1}{2}$.

The octet current hypothesis[1] proposes that these nonleptonic weak currents transform as menbers of an $SU(3)$ octet. In particular, following Cabibbo's suggestion, the strangeness-conserving part of the current equals $(G_V/\sqrt{2})[O_\lambda(x)]_2{}^1$, and the strangeness changing part of the current equals $(G_S/\sqrt{2})[O_\lambda(x)]_3{}^1$. The octet is normalized so that G_V is the Fermi constant for β decay, $G_V=1.011\times10^{-5}/M^2$ (where M is the proton's mass), and G_S and G_V are related by $G_S/G_V=\tan\theta$, where θ is the Cabibbo angle.

Hence, writing the current as $J_\lambda(x)_b{}^j$, where λ is a Lorentz index ($\lambda=0$ to 3) and j and b are $SU(3)$ indices (j and b go from 1 to 3), we can say that $\langle B_a{}^i|J_\lambda(x)_b{}^j|B_c{}^k\rangle$ is an invariant tensor transforming according to $SU(3)$. Such a tensor can be written in terms of the invariant tensors $d_{abc}{}^{ijk}$ and $f_{abc}{}^{ijk}$ (which are, respectively, symmetric and antisymmetric under interchange of any two pairs of indices). Making a further separation of the current into vector and axial vector octets, we have

$$\begin{aligned}\langle B_a{}^i(p_2)|(O_\lambda{}^V(x))_b{}^j|B_c{}^k(p_1)\rangle&=u^\dagger(p_2)\gamma_0\\&\times(D_\lambda{}^Vd_{abc}{}^{ijk}+F_\lambda{}^Vj_{abc}{}^{ijk})u(p_1)e^{i(p_2-p_1)x}\\\langle B_a{}^i(p_2)|(O_\lambda{}^A(x))_b{}^j|B_c{}^k(p_1)\rangle&=u^\dagger(p_2)\\&\times\gamma_0\gamma_5(D_\lambda{}^Ad_{abc}{}^{ijk}+F_\lambda{}^Af_{abc}{}^{ijk})u(p_1)e^{i(p_2-p_1)x}.\end{aligned} \tag{2}$$

[The spinors $u(p)$ are solutions of the free Dirac equation, normalized so that $u^\dagger(p)u(p)=1$.] The tensors f and d are determined by their invariance and symmetry properties, and the traceless conditions of the baryon and current octets:

$$\begin{aligned}d_{abc}{}^{ijk}=(4/9)\delta_a{}^i\delta_b{}^j\delta_c{}^k-\tfrac{2}{3}(\delta_a{}^i\delta_c{}^j\delta_b{}^k+\delta_b{}^i\delta_a{}^j\delta_c{}^k+\delta_c{}^i\delta_b{}^j\delta_a{}^k)\\+\delta_c{}^i\delta_a{}^j\delta_b{}^k+\delta_b{}^i\delta_c{}^j\delta_a{}^k\,,\end{aligned} \tag{3}$$

$$f_{abc}{}^{ijk}=\delta_c{}^i\delta_a{}^j\delta_b{}^k-\delta_b{}^i\delta_c{}^j\delta_a{}^k.$$

From Lorentz invariance, time reversal invariance,

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[1] Nicola Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[2] W. Willis *et al.*, Phys. Rev. Letters 13, 291 (1964).
[3] N. Brene, B. Hellesen, and M. Roos, Phys. Letters 11, 344 (1964); W. Drechsler, Nuovo Cimento 38, 345 (1965).
[4] After this work was completed, a report prior to publication was received from N. Brene, L. Veje, M. Roos, and C. Cromström who were doing similar work [N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. 149, 1288 (1966)]. I wish to thank Dr. M. Kugler for calling this to my attention.
[5] For a review of weak interaction theory, see T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 381 (1965).

TABLE I. g_A as a function of D and F for all allowable decays.

Reaction	g_A
$n \to pl\nu$	$(D+F)\cos\theta$
$\Lambda^0 \to pl\nu$	$(1/\sqrt{6})(D+3F)\sin\theta$
$\Sigma^- \to nl\nu$	$(D-F)\sin\theta$
$\Sigma^- \to \Lambda^0 l\nu$	$-(\sqrt{\frac{2}{3}})D\cos\theta$
$\Sigma^+ \to \Lambda^0 l\nu$	$-(\sqrt{\frac{2}{3}})D\cos\theta$
$\Sigma^0 \to pl\nu$	$(1/\sqrt{2})(D-F)\sin\theta$
$\Xi^- \to \Lambda^0 l\nu$	$(1/\sqrt{6})(D-3F)\sin\theta$
$\Xi^0 \to \Sigma^+ l\nu$	$(D+F)\sin\theta$
$\Xi^- \to \Sigma^0 l\nu$	$(1/\sqrt{2})F\sin\theta$
$\Xi^- \to \Xi^0 l\nu$	$(D-F)\cos\theta$

and Hermiticity, we have

$$\begin{aligned} D_\lambda^V &= \gamma_\lambda D_1^V + i\sigma_{\lambda\tau}q^\tau D_2^V, \\ F_\lambda^V &= \gamma_\lambda F_1^V + i\sigma_{\lambda\tau}q^\tau F_2^V, \\ D_\lambda^A &= \gamma_\lambda D_1^A + q_\lambda D_2^A, \\ F_\lambda^A &= \gamma_\lambda F_1^A + q_\lambda F_2^A, \end{aligned} \tag{4}$$

where $q=p_2-p_1$ and $\sigma_{\lambda\tau}=\frac{1}{2}i[\gamma_\lambda, \gamma_\tau]$.

We are intesested in the decay $A \to A'+l+\nu$, where A and A' are baryons, l is a lepton, and ν is its neutrino. The previous notation its a bit cumbersome to use in doing calculations; let us write the current matrix elements in the following conventional form. The nonleptonic current can be divided into vector and axial vector parts, $J_\lambda = V_\lambda + A_\lambda$; these are related by Cabibbo's theory to their respective octets, $(V_\lambda)_{\text{str.cons.}} = (G_V/\sqrt{2})(O_\lambda{}^V)_2{}^1$, etc. The matrix elements of these currents between any two baryons A and A' can be written as

$$\begin{aligned} \langle A'(p_2)|V_\lambda(x)|A(p_1)\rangle &= (G/\sqrt{2})u^\dagger(p_2) \\ &\quad \gamma_0[\gamma_\lambda g_V(q_2)+i\sigma_{\lambda\tau}q^\tau g_M(q^2)]\times u(p_1)e^{+iq\cdot x}, \\ \langle A'(p_2)|A_\lambda(x)|A(p_1)\rangle &= (G/\sqrt{2})u^\dagger(p_2) \\ &\quad \times\gamma_0\gamma_5[\gamma_\lambda g_A(q^2)+q_\lambda g_P(q^2)]u(p_1)e^{+iq\cdot x}, \end{aligned} \tag{5}$$

where $G=G_V$ or G_S depending on whether we are dealing with the strangeness-conserving or strangeness-changing part of the current. One can easily see how to relate g_V (which is different for different baryons) to $D_1{}^V$ and $F_1{}^V$, and similarly for g_M, g_A, and g_P. In Table I, g_A is given in terms of $D_1{}^A(=D)$ and $F_1{}^A(=F)$ for all allowable decays.

The vector current can be determined explicitly if we assume that the vector current and the electromagnetic current belong to the same octet [this is essentially the conserved-vector-current (CVC) hypothesis]. Taking

$$(1/e)J_{\text{elec}} = I_z + \tfrac{1}{2}Y = \tfrac{1}{3}(2O_1{}^1 - O_2{}^2 - O_3{}^3),$$

we have

$$\langle n|1/eJ_{\text{elec}}|n\rangle = -\tfrac{2}{3}D^V$$

and

$$\langle p|1/eJ_{\text{elec}}|p\rangle = \tfrac{1}{3}D^V + F^V.$$

Hence, knowing the electromagnetic form factors of the neutron and proton,[6] one can solve for D^V and F^V. To first order in q^2,

$$\begin{aligned} D_1{}^V(q^2) &= 0, \\ F_1{}^V(q^2) &= 1.00 + 2.00(q^2/M^2), \\ D_2{}^V(q^2) &= 1.44M^{-1}(1+2.37(q^2/M^2)), \\ F_2{}^V(q^2) &= 0.42M^{-1}(1+3.02(q^2/M^2)). \end{aligned} \tag{6}$$

(M is the proton's mass.)

Calculations proceed straightforwardly from the phenomenological Hamiltonian. The momentum dependence of the form factors was taken into account by expanding to first order in q^2:

$$g(q^2) = g(0) + g'(0)q^2,$$

where

$$g'(0) = \left.\frac{dq(q^2)}{dq^2}\right|_{q^2=0}.$$

The dependence of the axial form factors on the momentum transfer q^2 is not known. High-energy neutrino experiments indicate it is not too different from that of the vector form factors and so g_A was taken to be proportional to $(1+2.00(q^2/M^2))$. The results proved quite insensitive to the coefficient of q^2/M^2.

A formula for the electronic decay rate can be calculated exactly if the electron's mass is neglected and the form factors are expanded as stated. This formula is given in the appendix. Note that the induced pseudoscalar term $g_P(q^2)$ does not appear (in the limit in which the lepton's mass is zero). The muonic decay rates were evaluated numerically. The induced pseudoscalar term was put in by using[5]

$$g_P(q^2) = \frac{M_A + M_{A'}}{q^2 - m_i{}^2} g_A(q^2), \tag{7}$$

where M_A and $M_{A'}$ are the masses of the initial and final baryons, and m_i is the mass of the π or K, depending on whether we have a strangeness-conserving or strangeness-changing decay.

TABLE II. Experimental data used.

Data used

Axial vector/vector coupling constant ratios

	Reference
$(g_A/g_V)^{np} = -1.18 \pm 0.02 = D+F$	13
$(g_A/g_V)^{\Lambda p} = -0.9 \pm 0.3 = \frac{1}{3}D+F$	14

Leptonic decays

Reaction	Branching ratio	Ref.	Total mean lifetime of primary (sec)	Ref.
$\Lambda \to pe\nu$	$(8.5\pm0.9)\times10^{-4}$	8	2.52×10^{-10}	7
$\Sigma^- \to ne\nu$	$(12.7\pm1.7)\times10^{-4}$	10	1.65×10^{-10}	9
$\Sigma^- \to \Lambda e\nu$	$(0.75\pm0.28)\times10^{-4}$	11	1.65×10^{-10}	9
$\Lambda \to p\mu\nu$	$(1.4\pm0.6)\times10^{-4}$	12	2.52×10^{-10}	7
$\Sigma^- \to n\mu\nu$	$(6.6\pm1.4)\times10^{-4}$	11	1.65×10^{-10}	9

[6] L. Hand *et al.*, Rev. Mod. Phys. **35**, 335 (1963).

TABLE III. Summary of results.

	This work	Willis solution A
θ	0.245 ± 0.010	0.272
D	-0.766 ± 0.037	-0.74
F	-0.415 ± 0.035	-0.44
Probability (χ^2)	93% (0.78)	
	This work	Experimental value
$(g_A/g_V)^{np}$	-1.18	-1.18
$(g_A/g_V)^{\Lambda p}$	-0.67	-0.9
Branching ratios:		
$\Lambda\to pe\nu$	8.5×10^{-4}	8.5×10^{-4}
$\Sigma^-\to ne\nu$	13.0×10^{-4}	12.7×10^{-4}
$\Sigma^-\to\Lambda e\nu$	0.67×10^{-4}	0.75×10^{-4}
$\Sigma^+\to\Lambda e\nu$	0.20×10^{-4}	$\sim0.2\times10^{-4}$
$\Xi^-\to\Lambda e\nu$	5.6×10^{-4}	$\leq17\times10^{-4}$
$\Xi^0\to\Sigma^+ e\nu$	2.8×10^{-4}	
$\Xi^-\to\Sigma^0 e\nu$	0.27×10^{-4}	
$\Xi^-\to\Xi^0 e\nu$	0.27×10^{-9}	
$\Lambda\to p\mu\nu$	1.4×10^{-4}	1.4×10^{-4}
$\Xi^-\to n\mu\nu$	6.2×10^{-4}	6.6×10^{-4}
$\Xi^-\to\Lambda\mu\nu$	1.6×10^{-4}	

III. FIT TO EXPERIMENTAL DATA

The preceding theory can be compared with experiment There are three parameters still undetermined: the Cabibbo angle θ and the two parameters D and F in the axial-vector current.

The experimental data used are presented in Table II.[7–14] The values of θ, D, and F which best fit these data were found by making a least-squares fit. Using the input data of Willis *et al.*, reproduces their result closely.

Some further comment is needed on the Λ^0 data. It is hard to say with certainty what the Λ^0 lifetime is; while many measurements have been made, the results are not all compatible with each other. Good measurements give mean lifetime values[7] in the range (2.3 to $2.7)\times10^{-10}$ sec with errors on the order of 0.1×10^{-10} sec.[5]; the value in the table represents some average. also there are discrepancies in the $\Lambda\to pe\nu$ branching ratio. Using only the two experiments which each have more events than all other experiments combined gives a value[8] of 8.0×10^{-4} instead of 8.5×10^{-5}. However, neither of these possible changes can affect the results within the stated errors.

The results are summarized in Table III. Most of the change from the results of Willis *et al.* is due to the new data. The value of θ quoted here is smaller: this is because the number used for the $\Lambda\to pe\nu$ and $\Sigma^-\to ne\nu$ branching ratios are smaller; since both of these are strangeness-changing decays, these branching ratios are proportional to $\sin^2\theta$. Inclusion of the q^2 dependence of the form factors proved to be a small correction; and and the weak magnetism corrections were about $\frac{1}{5}$ as big; both corrections together accounted for about 30% of the change in θ.

It has also been suggested[1] that the weak coupling constant in μ decay G_μ is related to the weak coupling constant in β decay by $G_V/G_\mu=\cos\theta$; or, stated differently that $G_\mu^2=G_V^2+G_S^2$. This constraint on $\cos\theta$ was not used in the fitting data because it is not implied by the octet current hypothesis.

Checking this second proposal against experiment is difficult because there are uncertainties in calculating the radiative corrections to β decay. The uncorrected experimental value of G_V/G_μ is 0.988.[13] The corrected value can be expressed as follows:

$$G_V/G_\mu=0.988-r. \tag{8}$$

If one assumes the existence of the intermediate boson W, one finds for a bare nucleon,[15]

$$r=\frac{\alpha}{4\pi}\left(\pi^2-\frac{25}{4}+6\ln\frac{M_W}{M}+3\ln\frac{M}{2E_m}-2.85\right)-\frac{3}{10}\left(\frac{M_\mu}{M_W}\right)^2, \tag{9}$$

where E_m=maximum energy of β particle and M_W=mass of W. (A calculation proceeding from the effective Hamiltonian, with four fields interacting at one point, yields a divergent result; explicitly, one gets the above formula, without the last term, and with M_W replaced by some cutoff energy.)

At this time, all that can honestly be said about the $G_V/G_\mu=\cos\theta$ hypothesis is that it is not inconsistent with experiment. However, one could take the value $\cos\theta=0.970$ (as found here) and use the formula for r to calculate the mass of W: $M_W=9$ GeV.

The Cabibbo angle can also be calculated from the decays of the π and K. These data were not included here since there are some objections to using them. Because of the large mass difference between the π and K, it is not a good assumption to say that the form factors are independent of momentum transfer (as is often done). The momentum dependence of these form factors is not well known, and the calculated Cabibbo angle is fairly sensitive to it.[16]

[7] For a list of experimental references see A. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965). To this list add T. Buran *et al.*, Phys. Letters **20**, 318 (1966). The number in the table is an average of all measurements with error less than or equal to 0.10×10^{-10} sec, weighted by the inverse of the stated error.

[8] C. Baglin *et al.*, Nuovo Cimento **35**, 977 (1965); V. Lind *et al.*, Phys. Rev. **135**, B1483 (1964); R. Ely *et al.*, Phys. Rev. **131**, 868 (1963); B. Aubert *et al.*, Nuovo Cimento **25**, 479 (1962).

[9] C.-Y. Chang, Nevis Report No. 145 (unpublished); W. Humphrey *et al.*, Phys. Rev. **127**, 1305 (1962).

[10] H. Courant *et al.*, Phys. Rev. **136**, B1791 (1964); D. Miller *et al.*, Phys. Letters **11**, 262 (1964); U. Nauenberg *et al.*, Phys. Rev. Letters **12**, 679 (1964); C. Murphy Phys. Rev. **134**, B188 (1964).

[11] H. Courant *et al.*, Ref. 10.

[12] B. Ronne *et al.*, Phys. Letters **11**, 357 (1964).

[13] C. S. Wu (private communication).

[14] J. Barlow *et al.*, Phys. Letters, **18**, 64 (1965).

[15] T. D. Lee, in Proceedings of the Oxford Conference on Weak Interactions and Questions of C, P, T Noninvariances, 1965 (unpublished).

[16] S. Odena and J. Sucher, Phys. Rev. Letters **15**, 927 (1965); erratum 1049.

From the decays $K\to\mu+\nu$ and $\pi\to\mu+\nu$, which are purely axial vector, we find (taking the form factor as constant)

$$\theta=0.266\pm0.006.$$

From $K\to\pi^0+e+\nu$ and $\pi\to\pi^0+e+\nu$, which involves the vector current, we find[16]

$$\theta=0.222\pm0.006.$$

(Both numbers are without radiative corrections. Radiative corrections to the first have been calculated[17]; they increase the number by 0.12%.) Thus there seems to be some indication that there are different Cabibbo angles for vector and axial vector currents. The significance of this is not yet clear. It could, in principle, be checked in baryon decay. However, better data will be required before one can reasonably fit separate angles for the vector and axial vector currents from baryon decay measurements.

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APPENDIX

The decay rate is given for the process $A\to A'+e+\nu$, where A and A' are baryons. The notation used for the coupling constants is given in Eq. (5); M_A=mass of baryon A, and m=mass of baryon A'/mass of baryon A.

$$W=(G^2/(2\pi)^3)\{|g_V|^2\times[-\tfrac{1}{2}M_A{}^5(m^3+m^4+m^5)\ln m+C_1+C_2]+\mathrm{Re}g_V{}^*g_V{}'\times[-\tfrac{1}{2}M_A{}^7(2m^3+3m^4+6m^5+3m^6+2m^7)$$
$$\times\ln m+C_3+C_4]+|g_M|^2\times[-\tfrac{1}{2}M_A{}^7(m^3+2m^4+3m^5+2m^6+m^7)\ln m+C_5]+\mathrm{Re}g_M{}^*g_M{}'$$
$$\times[-\tfrac{1}{6}M_A{}^9(6m^3+15m^4+36m^5+40m^6+36m^7+15m^8+6m^9)\ln m+C_6]+\mathrm{Re}g_V{}^*g_M$$
$$\times[-\tfrac{1}{2}M_A{}^6(2m^3+3m^4+3m^5+2m^6)\ln m+C_7]+\mathrm{Re}(g_V{}^*g_M{}'+g_M{}^*g_V{}')\times[-M_A{}^8(m^3+2m^4+4m^5$$
$$+4m^6+2m^7+m^8)\ln m+C_8]\}+(\text{term in } g_A=\text{ terms in } g_V \text{ with } m\to-m;\ \ln m\to\ln m).$$

$$C_1=(1/48)M_A{}^5(1-8m^2+8m^6-m^8),$$
$$C_2=-(1/24)M_A{}^5(m+9m^3-9m^5-m^7),$$
$$C_3=(1/80)M_A{}^7(1-15m^2-80m^4+80m^6+15m^8-m^{10}),$$
$$C_4=-(1/24)M_A{}^7(m+28m^3-28m^7-m^9),$$
$$C_5=(1/240)M_A{}^7(2-5m-30m^2-140m^3-160m^4+160m^6+140m^7+30m^8+5m^9-2m^{10}),$$
$$C_6=(1/720)M_A{}^9(5-18m-120m^2-1050m^3-1875m^4-1800m^5$$
$$+1800m^7+1875m^8+1050m^9+120m^{10}+18m^{11}-5m^{12}),$$
$$C_7=(1/48)M_A{}^6(1-3m-12m^2-44m^3-36m^4+36m^5+44m^6+12m^7+3m^8-m^9),$$
$$C_8=(1/120)M_A{}^8(1-4m-20m^2-155m^3-220m^4-80m^5+80m^6+220m^7+155m^8+20m^9+4m^{10}-m^{11}).$$

[17] T. Kinoshita, Phys. Rev. Letters 2, 477 (1959).