# Chiral Algebra, Configuration Mixing, Magnetic Moments, and Pion Photoproduction

I. S. GERSTEIN* AND B. W. LEE†
*Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania*
(Received 28 June 1966)

We develop a model for baryon states as a mixture of representations of the chiral $SU(3)\otimes SU(3)$ algebra generated by the time components of vector and axial-vector currents. Our model predicts a sum rule $G_A=1+\frac{1}{4}G^{*2}$, where $G_A$ is the axial-vector coupling constant of the nucleon and $G^*$ is the nucleon-(3,3)-resonance matrix element of the axial current. We show how to derive $G^*$ from experiment, and using this information get $G_A=1.25$. Using the electric dipole operator we also obtain that the magnetic moment of the nucleons is pure isovector, as is the photoproduction amplitude of the isospin-$\frac{1}{2}$ pion-nucleon resonances from nucleons. A value for the magnetic dipole transition from the nucleon to the (3,3) resonance is found which also agrees with experiment.

## I. INTRODUCTION

In a previous paper we presented a model for baryon states based on the chiral $U(3)\otimes U(3)$ algebra at infinite momentum.[1] The model was based on the idea that the expectation values of the commutators of the time components of vector and axial vector currents between stable octet baryons, at infinite momentum, are saturated by single-particle states and resonances, and therefore these states form a representation of the chiral $U(3)\otimes U(3)$ algebra. The known behavior of the moment operators of the electromagnetic current under the chiral algebra then allows us further to deduce the consequences of the model for the electromagnetic form factors and various electromagnetic transition moments of the baryons.

It is the purpose of this paper to give a detailed analysis of the results presented in the previous paper, together with subsequent developments on the photoproduction of pseudoscalar mesons. In preparing this article it was felt worthwhile to reiterate our basic assumptions as to, in the first place, what subalgebra of the algebra of current components we assume to be "good," i.e., to be saturated by a finite number of low-lying excitations, and secondly, what sort of states are supposed to saturate the selected current commutators. We shall discuss the scheme based on the idea of "orbital helicity excitation" and the concomitant problem of configuration mixing of the "quark helicity" and "orbital helicity." This is done in Sec. II. Nothing really new is developed in this section, but some confusion on this matter in the literature prompts us to include this material.

In Sec. III, the expectation values of the commutators are decomposed according to their $SU(3)$ transformation properties so as to provide us with a convenient means of discussing the solutions to the sum rules resulting from the chiral current commutators. Here we also discuss the connection between the Weisberger-Adler[2] sum rule and the sum rule saturated by (idealized) discrete single-particle states and resonances. This discussion relies on the Wigner-Eisenbud theory[3] of nuclear reactions. As a byproduct of this discussion, we suggest a relatively simple prescription for analytic continuation of resonant amplitudes in the pion mass, since, in the Weisberger-Adler sum rule, the pion-nucleon scattering amplitude refers to that for zero-mass (external) pions.

In comparing the predictions of our model with experiment, we felt that the effects of $SU(3)$ symmetry breaking could be very important and that without a detailed study of symmetry-breaking corrections, comparison of $SU(3)$ symmetric solutions with experiment would be of little value. Fortunately, many of the predictions of this model are true also in the chiral $U(2)\otimes U(2)$ subalgebra, as pointed out in Ref. 1, so we test our predictions for the pion-nucleon system where there exist some reliable experimental data. In Sec. IV we discuss the magnetic moments of baryons and the pion photoproduction off the proton based on our model.

## II. CHIRAL ALGEBRA AND CONFIGURATION MIXING

The commutation relations of space-integrated current components we wish to make use of are

$$[A_0^i,A_0^j]=if_{ijk}V_0^k,\quad i,j,k=0,1,\cdots,8 \qquad (1)$$

where $V_0^k$ is the space integral of the time component of the conserved vector current $\mathcal{V}_0^k(\mathbf{x},t)$ of $SU(3)$

* Research supported by the National Science Foundation. Present address: Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts.
† Supported in part by the U. S. Atomic Energy Commission and the Alfred P. Sloan Foundation. Present address: Department of Physics, State University of New York at Stony Brook, Stony Brook, New York.
[1] I. S. Gerstein and B. W. Lee, Phys. Rev. Letters 16, 1060 (1966).
[2] W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965); Phys. Rev. 143, 1302 (1965); S. Adler, Phys. Rev. Letters 14, 1051 (1965); Phys. Rev. 140, B736 (1965).
[3] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. X.

type $k$:

$$V_0{}^k=\int d^3x\, \mathcal{V}_0{}^k(\mathbf{x},t)\,,$$

while

$$A_0{}^i(t)=\int d^3x\, \mathcal{A}_0{}^i(\mathbf{x},t)$$

is the space integral of the time component of the octet axial vector current $\mathcal{A}_0{}^i(\mathbf{x},t)$ of $SU(3)$ type $i$. $A_0{}^i(t)$ is not conserved in time; for definiteness we take $A_0{}^i$ to be $A_0{}^i(t=0)$. The Lie algebra generated by $A_0{}^i$ and $V_0{}^k$ is the chiral $U(3)\otimes U(3)$ algebra. The rest of the algebra states that $V_0{}^i$ is the generator of the $U(3)$ group and that $A_0{}^i$ is a tensor operator (octet+singlet) under $SU(3)$.

We shall assume, in this section, that $SU(3)$ is an exact invariance. We consider the expectation value of the commutation relation (1) between the stable baryons (spin parity $\frac{1}{2}^+$) of $SU(3)$ type $l$ and $m$ at infinite momentum ($p_z=\infty$, $p_x=p_y=0$) and helicity $\frac{1}{2}$:

$$\begin{aligned}\langle B_{1/2}{}^l(\mathbf{p}=\infty\hat{e}_3)|[A_0{}^i,A_0{}^j]|B_{1/2}{}^m(\mathbf{p}=\infty\hat{e}_3)\rangle\\ =if_{ijk}\langle B_{1/2}{}^l(\mathbf{p}=\infty\hat{e}_3)|V_0{}^k|B_{1/2}{}^m(\mathbf{p}=\infty\hat{e}_3)\rangle.\end{aligned}\quad(2)$$

The reasons we choose to consider the expectation value of the commutator (1) of the chiral algebra at infinite momentum are the following[4]:

(a). As emphasized by Fubini and Furlan,[5] this choice of the momentum leads immediately to a sum rule which is relativistically invariant.

Thus, as shown by Adler and Weisberger, the expectation value (2) of the commutation relation

$$[A_0{}^1+iA_0{}^2,\, A_0{}^1-iA_0{}^2]=2V_0{}^3\quad(3)$$

leads to the sum rule

$$G_A{}^2+\left(\frac{mG_A}{g_r}\right)^2\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\frac{4}{3}\times\mathrm{Im}[T_{1/2}(\nu,0)-T_{3/2}(\nu,0)]=1\,,\quad(4)$$

where $G_A$ is the weak axial vector coupling constant, $m$ is the nucleon mass, $g_r{}^2/4\pi\simeq15$, and $T_I(\nu,0)$ is the forward scattering amplitude of the zero-mass pion off the proton in the isospin-$I$ channel at the incident laboratory energy $\nu$. $T_I(\nu,0)$ is normalized so that

$$\mathrm{Im}T_I(\nu,0)=\nu\sigma_I\,,$$

where $\sigma_I$ is the cross section in the isospin-$I$ channel. In deriving (4), use is made of the hypothesis of partially conserved axial-vector current (PCAC):

$$\partial_\mu A_\mu{}^i=-i\mu^2 mG_A(1/g_r)\varphi_\pi{}^i(x)\,,\quad i=1,2,3\quad(5)$$

where $\varphi_\pi{}^i$ is the pion field and $\mu$ is the pion mass.

(b). Unlike the case of static $SU(6)$, the invariant momentum transfer between the initial (or final) state and the intermediate state is zero.

This circumstance leads to a considerable simplification, since we need not concern ourselves with the dependence of the scattering amplitude on the momentum transfer between the initial (or final) state and the intermediate states. Furthermore, this momentum transfer is fixed at zero, so that the PCAC hypothesis should be a good approximation.

(c). We have reasons to believe that the sum over intermediate states is rapidly convergent.

The work of Dashen and Frautschi[6] may be used to infer that when a complete set of intermediate states is inserted on the left-hand side of Eq. (2), lower mass states will dominate the sum and the sum will converge. In Eq. (4), the Pomeranchuk limit of the cross section guarantees the convergence of the integral. In fact, the amplitude $T_{1/2}(\nu,0)-T_{3/2}(\nu,0)$ is proportional to the forward charge-exchange amplitude $p+\pi^-\to n+\pi^0$, so that it should be proportional to $\nu^{\alpha_\rho(0)}$, where $\alpha_\rho(t)$ is the $\rho$-meson Regge trajectory: $\alpha_\rho(0)\simeq0.5$. Further, our previous work[7] indicates that the contribution of at least a certain class of higher mass intermediate states (those that exist even in a free-field model) will vanish in this limit.

Thus we may hope that the left-hand side of Eq. (2) is approximately saturated by a few low-lying excitations of the baryon spectrum. This means that the contribution of a few resonances will saturate the dispersion integral on the left-hand side of (4). These states then form a representation of the chiral $U(3)\otimes U(3)$ algebra. The representation may or may not be reducible.

It was shown in previous papers[7,8] that the chiral algebra in the infinite-momentum limit is equivalent to the collinear $U(3)\otimes U(3)$ algebra, in the sense that the two algebras are isomorphic and the diagonal matrix elements of the two algebras coincide. We shall label irreducible representations of either algebra by $(n,m)_\lambda$, where $n$ and $m$ are the dimensions of the $SU(3)$ representations generated by $\frac{1}{2}(V_0{}^i+A_0{}^i)$ or $\frac{1}{2}(V_0{}^i+A_3{}^i)$ and $\frac{1}{2}(V_0{}^i-A_0{}^i)$; $\lambda$ is the eigenvalue of the operator $A_3{}^0$ which we shall refer to as the quark helicity.

In a first approximation,[9] the octet of $\frac{1}{2}^+$ baryons

[4] A similar discussion has been given also by R. F. Dashen and M. Gell-Mann, in *Proceedings of the 1966 Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1966).

[5] S. Fubini and G. Furlan, Physics **1**, 229 (1964).

[6] R. F. Dashen and S. C. Frautschi, Phys. Rev. **145**, 1287 (1966).

[7] I. S. Gerstein and B. W. Lee, Phys. Rev. **144**, 1142 (1966).

[8] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966); Ref. 4; R. Oehme, Phys. Rev. **143**, 1138 (1966); D. Amati and S. Bergia, Nuovo Cimento **45**, 15 (1966); C. Ryan, Phys. Rev. **147**, 1139 (1966).

[9] I. S. Gerstein, Phys. Rev. Letters **16**, 114 (1965).

and the $\frac{3}{2}^+$ decimet are assigned to the representation $(6,3)_{1/2}$ and this classification leads to the well-known $SU(6)$ results. Since some of these predictions are not satisfactory, we must allow the baryons to transform reducibly under the algebra. Furthermore, as we shall see later, in order to obtain nonzero magnetic moments we must allow an additional degree of freedom associated with an orbital-angular-momentum excitation. To be more precise, we must introduce a degree of freedom which we define as the orbital helicity $\Lambda_3 \equiv J_3 - \lambda$, where $J_3$ is the true helicity. In a pure quark model, we have

$$J_3 = \Lambda_3 + \tfrac{1}{2} A_3{}^0,$$
$$\Lambda_3 = -i \int d^3x\, q^\dagger(\mathbf{x}) \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) q(\mathbf{x}), \tag{6}$$
$$A_3 = \int d^3x\, q^\dagger(\mathbf{x}) \sigma_{12} q(\mathbf{x}).$$

The structure of the relevant algebra is $U(1) \otimes U(3) \otimes U(3)$, and we are lead to classify physical states as linear combination of representations $[(n,m)_\lambda, \Lambda_3]$.

We must emphasize here that we do *not* require the states which comprise a representation of the chiral $U(3) \times U(3)$ algebra to be a linear combination of a few irreducible representations of $SU(6)_W$. We allow the possibility that the states which form a representation of the chiral algebra may have projections on a very large (perhaps infinite) number of $SU(6)_W$ irreducible representations. Since we do not consider $SU(6)_W$ to be a "good" algebra in the sense of Dashen and Gell-Mann,[10] there need be no constraint between the $U(3) \otimes U(3)$ contents of, say, the helicity $\frac{1}{2}$ decuplet states and of the helicity $\frac{3}{2}$ decuplet states. On the other hand, if we assumed the helicity $\frac{1}{2}$ decuplet states, for instance, to belong to $[(6,3)_{1/2},0]$ of the $SU(6)_W$ 56-plet with $L=0$, we would have to assign the helicity $\frac{3}{2}$ decimet states to $[(10,1)_{3/2},0]$.[11]

The angular momentum operator for particles at infinite momentum, $\mathfrak{J}$, is obtained by a Lorentz transformation from $\mathbf{J}$, the spin defined in the rest frame. Noting that the components of $\mathbf{J}$ are components of a 4-dimensional antisymmetric tensor $J_{\mu\nu}$, we obtain

$$\mathfrak{J}_1 = cJ_1 - sK_2,$$
$$\mathfrak{J}_2 = cJ_2 + sK_1,$$
$$\mathfrak{J}_3 = J_3,$$

where

$$c = \lim_{p\to\infty} m^{-1}(p^2+m^2)^{1/2},$$
$$s = \lim_{p\to\infty} m^{-1} p.$$

We would like to point out that the $\mathbf{W}$ spin, whose components are $\frac{1}{2} A_3{}^0$,

$$\frac{1}{2}\int d^3x\, T_{23}{}^0(\mathbf{x}) \quad \text{and} \quad \frac{1}{2}\int d^3x\, T_{31}{}^0(\mathbf{x})$$

in a quark model where $T_{\mu\nu}(\mathbf{x})$ has the structure $q^\dagger(\mathbf{x})\beta\sigma_{\mu\nu}q(\mathbf{x})$ is in general *not a vector* transforming like $\mathfrak{J}=1$. This implies that $\mathfrak{J}-\mathbf{W}$ does not commute with $\mathbf{W}$ and therefore $\mathbf{L}_W \equiv \mathfrak{J} - \mathbf{W}$ and $\mathbf{W}$ cannot be added vectorially to form the total angular momentum at infinite momentum $\mathfrak{J}$. Dashen and Gell-Mann[4] suggested a way out of this difficulty. They suggested that it may be possible to write $\mathfrak{J} = \mathbf{\Lambda} + \frac{1}{2}\mathbf{\Sigma}$ so that $[\Lambda_i, \Sigma_j] = 0$ for all $i$ and $j=1, 2, 3$, and $\frac{1}{2}\mathbf{\Sigma}$ is unitarily equivalent to $\mathbf{W}$. Such a scheme would enable us to classify physical states as linear combinations of representations of $0(3) \times SU(6)$. This is an alternative to our scheme where we restrict ourselves to helicities $\mathfrak{J}_3$, $\Lambda_3$, and $\frac{1}{2} A_3{}^0$ and the chiral algebra. Another departure of our considerations from the configuration-mixing schemes of other authors[11] is that we do not insist that the representations we consider are necessarily those predicted on the basis of a simple three-quark model. Since we do not assume $SU(6)_W$ to be a good algebra, there hardly seems to be any justification to be so restrictive.

We shall consider, in addition to the expectation values of $A_0{}^i$, the magnetic moments of the baryons. The operator that gives the anomalous magnetic moment at infinite momentum is the electric dipole operator[12]:

$$\langle B_{1/2}(\mathbf{p}=\infty \hat{e}_3) | D_+ | B_{-1/2}(\mathbf{p}=\infty \hat{e}_3) \rangle = \sqrt{2} F_2(0) = \sqrt{2} \mu_B{}', \tag{7}$$

where $\mu_B{}'$ is the *anomalous* magnetic moment of the baryon $B$ and

$$D_\pm = D_\pm{}^3 + \tfrac{1}{3}\sqrt{3} D_\pm{}^8,$$
$$D_\pm{}^i = -i \int d^3x\, \tfrac{1}{2}\sqrt{2}(x \pm iy) \mathcal{V}_0{}^i(\mathbf{x}).$$

Under the chiral algebra $D_\pm{}^i$ transforms like a linear combination of $(8,1)_0$ and $(1,8)_0$:

$$D_\pm{}^i \sim \tfrac{1}{2}(V_0{}^i + A_3{}^i) + \tfrac{1}{2}(V_0{}^i - A_3{}^i). \tag{8}$$

A direct computation shows

$$[\mathfrak{J}_3, D_\pm{}^i] = \pm D_\pm{}^i, \tag{9}$$

[10] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 275 (1965). See also, S. Fubini, G. Segrè, and J. D. Walecka, Ann. Phys. (N. Y.) **39**, 381 (1966).

[11] Schemes of this type have been pursued by many groups. See, for example, R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. Letters **16**, 377 (1965); H. J. Lipkin, H. Rubinstein, and S. Meshkov, Phys. Rev. **148**, 1405 (1966).

[12] N. Cabibbo and L. A. Radicati (Ref. 8).

which means that $D_\pm{}^i$ has $\Lambda_3=\pm1$. Thus the expectation values of $D_\pm{}^i$ between two states of $\Lambda_3=0$ must vanish, and this is one of the prime motivations for exploring the possibility of an orbital excitation. The commutation relation

$$[A_0{}^i,D_\pm{}^j]=if_{ijk}\left[-i\int d^3x\tfrac{1}{2}\sqrt{2}(x\pm iy)\mathfrak{A}_0{}^k(\mathbf{x})\right], \quad (10)$$

when specialized to the isospin structure of the pion-nucleon system yields the analog of (4):

$$G_A\mu_V'+\frac{G_Am}{g_r}\frac{1}{e}\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\times\mathrm{Im}\left[\frac{8}{3}S_{3/2}{}^V(\nu,0)+\frac{4}{3}S_{1/2}{}^V(\nu,0)\right]=0\,, \quad (11)$$

where $e^2/4\pi=(137)^{-1}$, $\mu_V'$ is the isovector anomalous magnetic moment of the nucleon, and $S_I{}^V(\nu,0)$ is the forward (zero-mass) pion production amplitude by isovector photons in the isospin-$I$ channel. (We treat the photon as a member of an octet. This is permissible in processes which are first-order in $e$.) It is so normalized that the forward pion production cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta=0}=\frac{p_\pi}{p_\gamma}\left|\frac{1}{4\pi}\frac{m}{W}S(\nu,0)\right|^2,$$

neglecting the isopsin dependence. Here $W$ is the center-of-mass energy, $p_\pi$ and $p_\gamma$ are the pion and photon momenta in the barycentric system, respectively, and for zero-mass pions, $p_\pi=p_\gamma$. In deriving Eq. (11), use is made of the conservation of the vector current:

$$\partial\mathcal{V}_0{}^i/\partial t=\nabla\cdot\mathcal{V}^i,$$

which implies

$$\langle\alpha|D_\pm|\beta\rangle=\frac{i}{E_\beta-E_\alpha}\langle\alpha|\int d^3x\left(\frac{x\pm iy}{\sqrt{2}}\right)\nabla\cdot\mathcal{V}|\beta\rangle\,,\qquad E_\alpha\neq E_\beta$$

$$=\frac{-i}{E_\beta-E_\alpha}\langle\alpha|\frac{1}{\sqrt{2}}\int d^3x[\mathcal{V}_1(\mathbf{r})\pm i\mathcal{V}_2(\mathbf{r})]|\beta\rangle\,,$$

where $E_\alpha$ is the energy of the state $|\alpha\rangle$. Equation (11) was derived previously by Fubini, Furlan, and Rossetti[13] from a dispersion-theoretic approach and extended to finite-momentum transfer by Riazuddin and Lee.[14]

[13] S. Fubini, G. Furlan, and C. Rosetti, Nuovo Cimento **40**, 1171 (1965); 43, 161 (1966).

[14] Riazuddin and B. W. Lee, Phys. Rev. **146**, 1202 (1966).

## III. ADLER-WEISBERGER SUM RULE

We observe that the spin $\frac{3}{2}^+$ decimet, $\frac{1}{2}^-$ singlet (1405 MeV), $\frac{3}{2}^-$ and $\frac{5}{2}^+$ octets make dominant contributions to the Adler-Weisberger sum rule[2] and its strange-particle analogs.[15] We shall make the approximation that the expectation values of the commutators (2) for the $\frac{1}{2}^+$ stable baryon states are saturated by these states and the $\frac{1}{2}^+$ baryon states. When we treat these resonances as single-particle states, there result a set of nonlinear equations from (2):

$$\begin{aligned}&-G_sG_a+\tfrac{1}{4}\sqrt{5}|F(10,\tfrac{3}{2}^+)|^2-\mathrm{Re}\sum_{J^\pi=\frac{3}{2}^-,\frac{5}{2}^+}F^*(8_s,J^\pi)F(8_a,J^\pi)=0\,,\\&\tfrac{1}{2}G_s{}^2+\tfrac{1}{2}G_a{}^2+\tfrac{1}{2}\sum_{J^\pi}|F(8_s,J^\pi)|^2+\tfrac{1}{2}\sum_{J^\pi}|F(8_a,J^\pi)|^2+\tfrac{1}{8}|F(1,\tfrac{1}{2}^-)|^2=\tfrac{3}{2}\,,\\&-\tfrac{2}{5}G_s{}^2-\tfrac{2}{5}\sum_{J^\pi}|F(8_s,J^\pi)|^2+\tfrac{1}{4}|F(10,\tfrac{3}{2}^+)|^2+\tfrac{1}{8}|F(1,\tfrac{1}{2}^-)|^2=0\,,\end{aligned} \quad (12)$$

where

$$\begin{aligned}G_a&=\sqrt{3}\alpha G_A\,,\\G_s&=(5/3)^{1/2}(1-\alpha)G_A\,,\end{aligned} \quad (13)$$

$\alpha/(1-\alpha)$ being the $F/D$ ratio for the weak axial vector current coupling, and

$$\langle(N,\nu,J^\pi)_{1/2}|A_0{}^{(\lambda)}|(8,\mu,\tfrac{1}{2}^+)_{1/2}\rangle_{p\to\infty}=\sum_\xi\begin{pmatrix}8&8&N_\xi\\\mu&\lambda&\nu\end{pmatrix}F(N_\xi,J^\pi)\,. \quad (14)$$

Here $|(N,\nu,J^\pi)_h\rangle$ is the state of $\nu\equiv(I,I_Z,Y)$ belonging to an $N$-plet of $SU(3)$ and of spin parity $J^\pi$ and helicity $h$.

A possible (but not unique) solution to (12) is obtained if we assume these states to form a reducible representation $[(6,3)_{1/2},0]\oplus[(3,\bar{3})_{-1/2},1]\oplus[(8,1)_\lambda,\ \tfrac{1}{2}-\lambda]$ ($\lambda$ arbitrary) of the chiral algebra. Of various possible assignments we have looked into,[16] this choice appears to be physically significant, especially with respect to the electromagnetic moments of these states. Note that the $SU(3)$ content of (6,3), $(3,\bar{3})$ and (8,1) is exactly that of the states appearing in Eq. (12).

[15] C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters **15** 715 (1965); L. K. Pandit and J. Schecter, Phys. Letters **19,** 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, Phys. Rev. Letters **19,** 59 (1965); W. I. Weisberger, Phys. Rev. **143,** 1302 (1965).

[16] One of the possibilities is discussed by H. Harari, Phys. Rev. Letters **16,** 964 (1966); **17,** 56 (1966).

We write

$$
\begin{aligned}
|(8,\tfrac{1}{2}^+)_{1/2}\rangle &= \cos\beta\,|(6,3)_{1/2},0\rangle \\
&\quad +(\sin\beta)[\cos\alpha\,|(3,\bar{3})_{-1/2},1\rangle \\
&\qquad +\sin\alpha\,|(8,1)_\lambda,\tfrac{1}{2}-\lambda\rangle], \\
|(10,\tfrac{3}{2}^+)_{1/2}\rangle &= |(6,3)_{1/2},0\rangle, \\
|(1,\tfrac{1}{2}^-)_{1/2}\rangle &= |(3,\bar{3})_{-1/2},1\rangle, \\
|(8,\tfrac{3}{2}^-)_{1/2}\rangle &= -\sin\beta\,\sin\gamma\,|(6,3)_{1/2},0\rangle \\
&\quad +(\cos\gamma\,\sin\alpha+\cos\beta\,\sin\gamma\,\cos\alpha) \\
&\qquad \times|(8,1)_\lambda,\tfrac{1}{2}-\lambda\rangle \\
&\quad +(-\cos\gamma\,\cos\alpha+\cos\beta\,\sin\gamma\,\sin\alpha) \\
&\qquad \times|(3,\bar{3})_{-1/2},1\rangle, \\
|(8,\tfrac{5}{2}^+)_{1/2}\rangle &= -\sin\beta\,\cos\gamma\,|(6,3)_{1/2},0\rangle \\
&\quad +(-\sin\gamma\,\sin\alpha+\cos\beta\,\cos\gamma\,\cos\alpha) \\
&\qquad \times|(8,1)_\lambda,\tfrac{1}{2}-\lambda\rangle \\
&\quad +(\sin\gamma\,\cos\alpha+\cos\beta\,\cos\gamma\,\sin\alpha) \\
&\qquad \times|(3,\bar{3})_{-1/2},1\rangle.
\end{aligned}
\tag{15}
$$

We have not written down the $SU(3)$ "magnetic" quantum numbers explicitly, nor the relevant $SU(3)$ contents of $(n,m)_\lambda$ on the right-hand sides. This should not cause any ambiguity. The solution of Eq. (12) is characterized by three parameters $\alpha$, $\beta$, $\gamma$. A straightforward calculation using the Wigner-Eckart theorem gives

$$
\begin{aligned}
G_s &= (5/3)^{1/2}(\cos^2\beta+\sin^2\beta\,\sin^2\alpha), \\
G_a &= \sqrt{3}(\tfrac{2}{3}\cos^2\beta+\sin^2\beta\,\cos^2\alpha),
\end{aligned}
\tag{16a}
$$

and

$$
\begin{aligned}
F(1,\tfrac{1}{2}^-) &= \tfrac{4}{3}\sqrt{3}\,\sin\beta\,\sin\alpha, \\
F(10,\tfrac{3}{2}^+) &= (8/3)^{1/2}\cos\beta, \\
F(8_s,\tfrac{3}{2}^-) &= (5/3)^{1/2}[-\cos\beta\,\sin\beta\,\sin\gamma \\
&\quad +\sin\beta\,\sin\alpha(-\cos\gamma\,\cos\alpha \\
&\qquad +\sin\gamma\,\cos\beta\,\sin\alpha)], \\
F(8_a,\tfrac{3}{2}^-) &= \sqrt{3}[-\tfrac{2}{3}\cos\beta\,\sin\beta\,\sin\gamma \\
&\quad +\sin\beta\,\cos\alpha(\cos\gamma\,\sin\alpha \\
&\qquad +\sin\gamma\,\cos\beta\,\cos\alpha)], \\
F(8_s,\tfrac{5}{2}^-) &= (5/3)^{1/2}[-\cos\beta\,\sin\beta\,\cos\gamma \\
&\quad +\sin\beta\,\sin\alpha(\sin\gamma\,\cos\alpha \\
&\qquad +\cos\gamma\,\cos\beta\,\sin\alpha)], \\
F(8_a,\tfrac{5}{2}^-) &= \sqrt{3}[-\tfrac{2}{3}\cos\beta\,\sin\beta\,\cos\gamma \\
&\quad +\sin\beta\,\cos\alpha[-\sin\gamma\,\sin\alpha \\
&\qquad +\cos\gamma\,\cos\beta\,\cos\alpha)].
\end{aligned}
\tag{16b}
$$

From Eqs. (13) and (16) we obtain

$$
\begin{aligned}
G_A &= (5/3)\cos^2\beta+\sin^2\beta, \\
\frac{\alpha}{1-\alpha} &\equiv (F/D)_{\text{axial vector}} = \frac{\tfrac{2}{3}\cos^2\beta+\sin^2\beta\,\sin^2\alpha}{\cos^2\beta+\sin^2\beta\,\cos^2\alpha},
\end{aligned}
\tag{17}
$$

and

$$
G_A = 1+\tfrac{1}{4}|F(10,\tfrac{3}{2}^+)|^2. \tag{18}
$$

The relation (18) holds even under the chiral $U(2)\otimes U(2)$ subalgebra when the $SU(3)$ symmetry is broken and $F(10,\frac{3}{2}^+)$ is appropriately defined as

$$
F(10,\tfrac{3}{2}^+)\equiv G^* = \langle\Delta_{1/2}^{++}|A_0^1+iA_0^2|P_{1/2}\rangle_{p\to\infty},
$$

so that we can test this relation for the pion-nucleon system.

The above discussion is based on the approximation of treating resonances as single particles. We digress here to discuss how the predictions such as (16a) and (18) should be compared with experiment and to establish the connection between the Adler-Weisberger formula (4) and the single-particle representation of the current algebra such as Eq. (12). We first decompose the amplitude $T_I(\nu,0)$ into partial waves:

$$
T_I(\nu,0) = \frac{4\pi}{q}\left(\frac{W}{m}\right)\sum_{J^\pi}(J+\tfrac{1}{2})t_{J^\pi}{}^I, \quad q=\frac{W^2-m^2}{2W},
$$

where $t_{J^\pi}{}^I$ is the quantity represented by $e^{i\delta(I,J^\pi)}\times\sin\delta(I,J^\pi)$ when the external pions are on the mass shell. In some sense the single-particle representation must correspond to the Breit-Wigner resonance approximation to $t_{J^\pi}{}^I$. Thus, for example, the quantity $G^*$ must be identified with[17]

$$
\tfrac{1}{2}|F(10,\tfrac{3}{2}^+)|^2 = \tfrac{1}{2}G^{*2} = \left(\frac{mG_A}{g_r}\right)^2\frac{1}{\pi} \times\int\frac{d\nu}{\nu^2}\frac{4\pi}{q}\left(\frac{W}{m}\right)^2[t_{3/2^+}{}^{3/2}]_{\text{resonance}}, \tag{19}
$$

where $W=m^2+2m\nu$, and $[t_{3/2^+}{}^{3/2}]$ resonance stands for the Breit-Wigner formula for the $[(3,3)]$ resonance. The factor of $\frac{1}{2}$ on the left-hand side comes from the square of the isoscalar factor of deSwart:

$$
\left(\begin{array}{cc|c} 8 & 8 & 10 \\ \tfrac{1}{2},1 & 1,0 & \tfrac{3}{2},1 \end{array}\right).
$$

$$
[t_{3/2^+}{}^{3/2}(W)]_{\text{resonance}} = \frac{\tfrac{1}{2}\Gamma_{N\pi}{}^0(W)}{W-W_\Delta-\tfrac{1}{2}i\Gamma(W)}, \tag{20}
$$

where $W_\Delta\simeq1236$ MeV, $\Gamma(W)$ is the (energy-dependent) total width, and $\Gamma_{N\pi(W)}{}^0$ is the partial width of the (unphysical, zero-mass) pion-nucleon channel. For $\Gamma(W)$ we may use the expression suggested by Gell-Mann and Watson

$$
\begin{aligned}
\Gamma(W) &= \gamma(p_\pi a)^3/[1+(p_\pi a)^2], \\
p_\pi &= [W^2-(m+\mu)^2]^{1/2}[W^2-(m-\mu)^2]^{1/2}/2W.
\end{aligned}
\tag{21}
$$

$\Gamma(W)$ is the imaginary part of the self-energy of the resonance and therefore the momentum $p_\pi$ here refers

[17] See also D. Amati and S. Fubini, Ann. Rev. Nucl. Sci. **12**, 359 (1962); R. Gatto and G. Veneziano, Phys. Letters **20**, 439 (1966).

to the barycentric momentum of the (physical) pion-nucleon system which gives the on the energy-shell part of the self-energy. On the other hand, $\Gamma_{N\pi}{}^0(W)$ is the decay width of the resonance into the nucleon and (unphysical, zero-mass) pion, and therefore it appears appropriate to continue $p_\pi$ to $\mu=0$:

$$\Gamma_{N\pi}{}^0(W)=\gamma(qa)^3/[1+(qa)^2]. \tag{22}$$

Combining Eqs. (19), (20), (21), and (22), and using the values for $\gamma$ and $a$ given by Dalitz and Sutherland,[18]

$$\gamma=63.5 \text{ MeV}, \quad a=1.23/\mu,$$

[which gives the width of the (3.3) resonance $\Gamma(W_\Delta)=110$ MeV] we obtain

$$F(10,\tfrac{3}{2}^+)=G^*=1.03. \tag{23}$$

[We truncate the integral on the right-hand side of (19) at $W=3.58$ BeV. The truncation error is estimated to be at most a few percent.] Adopting the value $G^*=1.0$ for convenience, we get from (18)

$$G_A=1.25, \tag{24}$$

and from, $G^*=(8/3)^{1/2}\cos\beta$,

$$\cos\beta=(\tfrac{3}{8})^{\frac{1}{2}}. \tag{25}$$

The choice $\cos^2\alpha=\frac{3}{5}$ in Eq. (17) gives, independently of the value of $\cos\beta$,

$$(F/D)_{\text{axial vector}}=\tfrac{2}{3}. \tag{26}$$

The relation (18), $G_A=1+\frac{1}{4}(G^*)^2$ is especially significant since it is independent of the values taken by the mixing angles, $\alpha$, $\beta$, and $\gamma$.

For the second and third resonances, we deduce, just as (19),

$$|(\tfrac{3}{10}\sqrt{5})F(8_s,J^\pi)+\tfrac{1}{2}F(8a,J^\pi)|^2\equiv\tfrac{3}{4}[G_A'(8,J^\pi)]^2$$
$$=\left(\frac{mG_A}{g_r}\right)^2\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\frac{4\pi}{q}(J+\tfrac{1}{2})[t_{J^\pi}{}^{1/2}(W)]_{\text{resonance}}$$
$$J^\pi=\tfrac{3}{2}^-,\ \tfrac{5}{2}^+. \tag{27a}$$

Since these resonances are far away from the $\pi N$ threshold, we make the approximation that $\Gamma_{N\pi}$ and $\Gamma$ are constant.

$$[t_{J^\pi}{}^{1/2}(W)]=\frac{\frac{1}{2}\Gamma_{N\pi}}{W-W_0-\frac{1}{2}i\Gamma},$$
$$\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\frac{4\pi}{q}\left(\frac{W}{m}\right)(J+\tfrac{1}{2})[t_{J^\pi}{}^{1/2}(W)]_{\text{resonance}}$$
$$\cong\frac{4\pi}{q_0{}^2}(J+\tfrac{1}{2})\frac{\Gamma_{N\pi}W_0}{W_0{}^2-m^2};\quad q_0{}^2=\frac{W_0{}^2-m^2}{2W_0}.$$

For the second resonance $\frac{3}{2}^-$, we use $W_0=1518$ MeV, $\Gamma=120$ MeV, $\Gamma_{N\pi}/\Gamma=0.75$.[19] We obtain for the right-hand side of Eq. (27) 0.090; For the third resonance $\frac{5}{2}^+$, $W_0=1688$ MeV, $\Gamma=100$ MeV, $\Gamma_{N\pi}/\Gamma=0.85$.[19] The right-hand side of Eq. (27a) is 0.068. These are to be compared with the left-hand side of (27a), for which we obtain, with $\cos\beta=\frac{3}{8}^{1/2}$, $\cos\alpha=\frac{3}{5}^{1/2}$,

$$\begin{aligned}&[G_A'(8,\tfrac{3}{2}^-)]^2=\tfrac{3}{4}(\tfrac{1}{4}(5/3)^{1/2})^2\sin^2\gamma,\\&[G_A'(8,\tfrac{5}{2}^+)]^2=\tfrac{3}{4}(\tfrac{1}{4}(5/3)^{1/2})^2\cos^2\gamma.\end{aligned} \tag{27b}$$

With $\cos\gamma=(3/7)^{1/2}$, the left-hand side of (27a) is 0.045 for $\frac{3}{2}^-$ and 0.033 for $\frac{5}{2}^+$. [The value $\cos\gamma=(3/7)^{1/2}$ is obtained by fitting the relative contributions of the $J^\pi=\frac{3}{2}^-$ and $\frac{5}{2}^+$ states.]

## IV. MAGNETIC MOMENTS AND PHOTOPRODUCTION OF PIONS

As pointed out in Sec. II, the expectation value of the electric dipole operator $D_\pm$ for the baryon state is proportional to the anomalous magnetic moment of the baryon. From the commutation relation (10), we obtain

$$\langle B_{1/2}{}^l|[A_0{}^i,D_+{}^j]|B_{-1/2}{}^m\rangle_{\mathbf{p}\to\infty\mathbf{e}_3}=0, \tag{28}$$

where we have used the fact that

$$\langle B_{1/2}{}^l|\int d^3x\, x\mathcal{a}_0{}^i(\mathbf{x})|B_{-1/2}{}^m\rangle_{\mathbf{p}\to\infty\mathbf{e}_3}=0. \tag{29}$$

When a complete set of physical states is inserted between the two operators in Eq. (28), there result relativistic sum rules of the kind Eq. (11). Saturation of the commutator (10) by the same set of states as discussed in the previous section, on the other hand, gives a set of equations for the reduced matrix elements of the operator $D_+{}^j$. The matrix elements of $D_+{}^j$, however, can be determined from the transformation properties of the operator $D_+{}^j$ under the chiral algebra and under the rotation about the direction of the momentum. These are given by Eqs. (8) and (9).

We denote

$$\langle B_{1/2}{}^{(\alpha)}|D_+{}^{(\lambda)}|B_{-1/2}{}^{(\beta)}\rangle=\sqrt{2}\sum_\xi\begin{pmatrix}8&8&8_\xi\\\beta&\lambda&\alpha\end{pmatrix}\mu_\xi', \tag{30}$$

where in the $SU(3)$ limit, we have

$$\begin{aligned}&\mu_a'=\sqrt{3}(\mu_p'+\tfrac{1}{2}\mu_n),\\&\mu_s'=-(\tfrac{1}{2}\sqrt{15})\mu_n.\end{aligned} \tag{31}$$

The state $|B_{-1/2}{}^{(\beta)}\rangle$ can be generated from $|B_{1/2}{}^{(\beta)}\rangle$ by the mirror operation[20] $M\equiv Pe^{i\pi J_2}$, where $P$ is the parity operator. The operation $M$ is an automorphism of the chiral algebra, such that under $M$, $J_3\to -J_3$ $V_0{}^i\to V_0{}^i$, $A_0{}^i\to -A_0{}^i$ and $[(n,m)_\lambda,\Lambda_3]\to[(m,n)_{-\lambda},-\Lambda_3]$. Thus

[18] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

[19] A. H. Rosenfeld *et al.*, University of California Radiation Laboratory Report No. UCRL-8030, 1965 (unpublished); Rev. Mod. Phys. **37**, 633 (1965).

[20] This is also discussed by R. Gatto *et al.* (Ref. 11); H. J. Lipkin and S. Meshkov, Phys. Rev. **143**, 1269 (1966).

from (15)

$$|B_{-1/2}^{(\beta)}\rangle = M|B_{1/2}^{(\beta)}\rangle = \cos\beta|(3,6)_{-1/2},0;8\beta\rangle + \sin\beta[\cos\alpha|(\bar{3},3)_{1/2},-1;8\beta\rangle + \sin\alpha|(1,8)_{-\lambda},-\tfrac{1}{2}+\lambda;8\beta\rangle],$$

where the state $|(n,m)_\lambda,\Lambda_3;N\nu\rangle$ is the state of $\nu=(I,I_Z,Y)$ of an $SU(3)$ $N$-plet. We have therefore

$$\begin{aligned}\langle B_{1/2}^{(\alpha)}|D_+^{(\lambda)}|B_{-1/2}^{(\beta)}\rangle_{p\to\infty e_3} &= \text{const}\times(\cos\beta\sin\beta\cos\alpha)\langle\Lambda_3=1|\tfrac{1}{2}\sqrt{2}(x+iy)|\Lambda_3=0\rangle\\ &\times[\langle(6,3)_{1/2};8\alpha|\tfrac{1}{2}(V_0^{(\lambda)}+A_3^{(\lambda)})|(\bar{3},3)_{1/2};8\beta\rangle\\ &+\langle(3,\bar{3})_{-1/2};8\alpha|\tfrac{1}{2}(V_0^{(\lambda)}-A_3^{(\lambda)})|(3,6)_{-1/2};8\beta\rangle]. \quad (32)\end{aligned}$$

In Eq. (32) we have retained only the nonvanishing terms: only the products of $(3,\bar{3})$ and $(6,3)$, and of $(3,6)$ and $(\bar{3},3)$ can couple to $(8,1)$ and $(1,8)$. Furthermore, the two terms in the square bracket are equal, as one can show easily by the $M$ operation. Therefore, all the matrix elements of $D_+$ can be expressed in terms of one parameter. When we reduce the expression (32) in the form of (30) we find

$$\begin{aligned}\mu_s' &= (\sqrt{5})m\cos\beta\sin\beta\cos\alpha,\\ \mu_a' &= m\cos\beta\sin\beta\cos\alpha,\end{aligned} \quad (33)$$

where $m$ is an undetermined constant. Equations (31) and (33) imply

$$\mu_p' = -\mu_n, \quad (34)$$

i.e., the isoscalar anomalous magnetic moment $\mu_S'$ of the nucleon vanishes. The $F/D$ ratio of the anomalous magnetic moments of the stable baryons is

$$(F/D)_{\mu'} = \tfrac{1}{3}. \quad (35)$$

Equation (34) is in excellent agreement with experiment.

We decompose transition moments as

$$\langle(N,\nu,J^\pi)_{1/2}|D_+^{(\lambda)}|(8,\mu,\tfrac{1}{2}^+)_{-1/2}\rangle_{p\to\infty e_3} = \sqrt{2}\sum_\xi \begin{pmatrix}8 & 8 & N_\xi\\ \mu & \lambda & \nu\end{pmatrix} E(N_\xi,J^\pi). \quad (36)$$

A similar calculation as for the stable baryon octet yields for the $\frac{3}{2}^+$ decimet

$$E(10,\tfrac{3}{2}^+) = (\tfrac{3}{5})^{\frac{1}{2}}(1/\cos\beta)\mu_p'. \quad (37)$$

The quantity $\langle\Delta_{1/2}^+|D_+|p_{1/2}\rangle$ at infinite momentum is actually a linear combination of the $M1$ and $E2$ transition moments. Experiment indicates that the $E2$ moment is negligible. In this approximation

$$\mu^* = \langle\Delta_{1/2}^+|\mathfrak{M}_3|p_{1/2}\rangle_{p=0} = \sqrt{2}\langle\Delta_{1/2}^+|D_+|p_{-1/2}\rangle_{p=\infty e_3},$$

so that

$$\mu^* = \sqrt{2}(\sqrt{\tfrac{2}{3}})E(10,\tfrac{3}{2}^+) = (\sqrt{2}/\cos\beta)\mu_p'. \quad (38)$$

In our previous paper, the value for $\mu^*$ as obtained from (38) with $\cos\beta\simeq(\frac{3}{8})^{\frac{1}{2}}$ was compared with the phenomenological value deduced by Dalitz and Sutherland. The following method of deriving a value for $E(N,J^\pi)$ directly from experiment [analogously to the computation of $F(N,J^\pi)$ above] seems more appropriate, however. When we compare the sum rule (11) and the equation one obtains by saturating the commutator (10) by a finite number of discrete states, we see the correspondence:

$$\tfrac{1}{2}F(10,\tfrac{3}{2}^+)E(10,\tfrac{3}{2}^+) = \frac{G_A m}{g_r}\frac{1}{e}\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\,\text{Im}[S_{3/2}^V]_{\text{resonance}}, \quad (39)$$

where $[S_{3/2}^V]_{\text{resonance}}$ is the resonant $T=\frac{3}{2}$, $J^\pi=\frac{3}{2}^+$ amplitude. We decompose the amplitude $S(\nu,0)$ as

$$S(\nu,0) = \frac{4\pi}{q}\left(\frac{W}{m}\right)\sum_J\frac{(2J+1)^{1/2}}{2}\times\{[(J+\tfrac{3}{2})^{1/2}\mathcal{E}_+^J+(J-\tfrac{1}{2})^{1/2}\mathfrak{M}_+^J] + [(J-\tfrac{1}{2})^{1/2}\mathcal{E}_-^J-(J+\tfrac{3}{2})^{1/2}\mathfrak{M}_-^J]\},$$

where our amplitudes $\mathcal{E}_+^J$, $\mathfrak{M}_+^J$, $\mathcal{E}_-^J$ and $\mathfrak{M}_-^J$ are related to those of Chew, Goldberger, Low, and Nambu (CGLN)[21] by

$$\begin{aligned}\mathcal{E}_+^J &= q[(l+1)(l+2)]^{1/2}E_{l+},\\ \mathfrak{M}_+^J &= q[l(l+1)]^{1/2}M_{l+},\\ \mathcal{E}_-^J &= q[l(l+1)]^{1/2}E_{(l+1)-},\\ \mathfrak{M}_-^J &= q[(l+1)(l+2)]^{1/2}M_{(l+1)-}.\end{aligned}$$

In terms of these amplitudes, the total cross-section for photoproduction of pion is

$$\sigma_{\gamma\to\pi} = \frac{4\pi}{q^2}\tfrac{1}{4}\sum_J(2J+1)[|\mathcal{E}_+^J|^2+|\mathfrak{M}_+^J|^2+|\mathcal{E}_-^J|^2+|\mathfrak{M}_-^J|^2].$$

Thus

$$[S_{3/2}^V(\nu,0)]_{\text{resonance}} = \frac{4\pi}{q}\left(\frac{W}{m}\right)[\sqrt{3}\,\mathcal{E}_+^{3/2}+\mathfrak{M}_+^{3/2}],$$

and we use the Breit-Wigner formula for $\sqrt{3}\,\mathcal{E}_+^{3/2}+\mathfrak{M}_+^{3/2}$,

$$\sqrt{3}\,\mathcal{E}_+^{3/2}+\mathfrak{M}_+^{3/2} = \frac{\frac{1}{2}[\Gamma_{N\pi^0}(W)\Gamma_{N\gamma}(W)]^{1/2}}{W-W_\Delta-\frac{1}{2}i\Gamma(W)},$$

where $\Gamma_{N\gamma}(W)$ is the partial width of the $N\gamma$ channel. $\Gamma_{N\gamma}(W)$ is $\frac{3}{2}$ times $\Gamma_\gamma$ of Dalitz and Sutherland.[18] (We treat the photon effective in the $N^*$ production as an isotriplet.) Experimentally $\mathcal{E}_+^{3/2}\simeq0$, and we have

$$\sigma_{33}(\gamma p\to\pi^0 p) = \frac{4}{9}\frac{4\pi}{q^2}\frac{\frac{1}{4}\Gamma_{N\pi^0}(W)\Gamma_{N\gamma}(W)}{(W-W_\Delta)^2+\frac{1}{4}\Gamma^2(W)}.$$

[21] G. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

It is reasonable to assume

$$\Gamma_{N\pi^0}(W)/\Gamma_{N\gamma}(W)=\Gamma_{N\pi^0}(W_\Delta)/\Gamma_{N\gamma}(W_\Delta)\,,\ \text{for all } W\,,$$

if we adopt the usual interpretation of the formula (21) where the constant $a$ is the radius of the (3,3) resonance. (Of course the pionic radius and the photonic radius need not be equal.) Note also that the channel momenta of the zero-mass pion-nucleon channel and of the $\gamma N$ channel are equal. Therefore

$$\tfrac{1}{2}F(10,\tfrac{3}{2}^+)E(10,\tfrac{3}{2}^+)=\frac{g_r}{mG_A}\frac{1}{e}\frac{1}{2}\left[\frac{\Gamma_{N\gamma}(W_\Delta)}{\Gamma_{N\pi}(W_\Delta)}\right]^{1/2}\times\tfrac{1}{2}|F(10,\tfrac{3}{2}^+)|^2. \quad (40)$$

We combine Eqs. (19), (38), and (40), to obtain

$$\mu^*=\frac{1}{\sqrt{3}}\frac{1}{m}\left(\frac{G^*}{G_A}\right)\left(\frac{g_r}{e}\right)\left[\frac{\Gamma_{N\gamma}(W_\Delta)}{\Gamma_{N\pi^0}(W_\Delta)}\right]^{1/2}. \quad (41)$$

Thus Eq. (38) represents the value given by our model which is to be compared with the experimental value given by Eq. (41).

With $\cos\beta=\frac{3}{8}^{1/2}$, we get from Eq. (38) (with $\mu_p'\simeq 2\mu_p/3$):

$$(\mu^*)_{\text{theory}}=\tfrac{4}{3}\sqrt{3}\mu_p'\approx(8/3)^{1/2}(\tfrac{2}{3}\sqrt{2}\mu_p)=1.6(\tfrac{2}{3}\sqrt{2}\mu_p)\,, \quad (42)$$

while Eq. (41) gives

$$(\mu^*)_{\text{expt}}\approx 1.4(\tfrac{2}{3}\sqrt{2}\mu_p)\,, \quad (43)$$

with $\Gamma_{N\gamma}(W_\Delta)\simeq\frac{3}{2}\times 0.65$ MeV [the value $2\sqrt{2}\mu_p/3$ is the prediction of the static $SU(6)$ for $\mu^*$], $\Gamma_{N\pi}(W_\Delta)\simeq 110$ MeV, $G^*/G_A=\frac{4}{5}$.

For the octet $J^\pi=\frac{3}{2}^-$ and $\frac{5}{2}^+$ resonances, we obtain

$$\begin{aligned}
E(8_s,\tfrac{3}{2}^-)&=(\tfrac{1}{2}\sqrt{5})m[(\cos^2\beta-\sin^2\beta)\sin\gamma\cos\alpha-\cos\beta\cos\gamma\sin\alpha]\,,\\
E(8_a,\tfrac{3}{2}^-)&=\tfrac{1}{2}m[(\cos^2\beta-\sin^2\beta)\sin\gamma\cos\alpha-\cos\beta\cos\gamma\sin\alpha]\,,\\
E(8_s,\tfrac{5}{2}^+)&=(\tfrac{1}{2}\sqrt{5})m[(\cos^2\beta-\sin^2\beta)\cos\gamma\cos\alpha+\cos\beta\sin\gamma\sin\alpha]\,,\\
E(8_a,\tfrac{5}{2}^+)&=\tfrac{1}{2}m[(\cos^2\beta-\sin^2\beta)\cos\gamma\cos\alpha+\cos\beta\sin\gamma\sin\alpha]\,.
\end{aligned} \quad (44)$$

The constant $m$ is the same one appearing in Eq. (33), and is expressible in terms of $\mu_p'$:

$$m=\tfrac{1}{2}\sqrt{3}\,(1/\cos\beta\sin\beta\cos\alpha)\mu_p'\,.$$

Since the $F/D$ ratios for the production of these resonances are the same as that for the anomalous magnetic moments of the baryons [see Eq. (33)], the *photoproduction of the* $N^{**}(J^\pi=\frac{3}{2}^-)$ *and* $N^{***}(J^\pi=\frac{5}{2}^+)$ *resonances are caused by the isovector photon.*

To test the predictions of Eq. (44), we again make use of the correspondence:

$$[(\tfrac{3}{10}\sqrt{5})F(8_s,J^\pi)+\tfrac{1}{2}F(8_a,J^\pi)]^*\times[(\tfrac{3}{10}\sqrt{5})E(8_s,J^\pi)+\tfrac{1}{2}E(8_a,J^\pi)]=\frac{mG_A}{g_r}\frac{1}{e}\frac{1}{\pi}\int\frac{d\nu}{\nu^2}\,\text{Im}[S_{1/2}{}^V(\nu,0)]_{\text{resonance}}\,, \quad (45)$$

where

$$[S_{1/2}{}^V(\nu,0)]_{\text{resonance}}=\frac{4\pi}{q}\left(\frac{W}{m}\right)\times\begin{cases}(\mathcal{E}_{-}{}^{3/2}-\sqrt{3}\mathfrak{M}_{-}{}^{3/2})\,; & J^\pi=\frac{3}{2}^-\\ \frac{1}{2}\sqrt{3}[\sqrt{2}\,\mathcal{E}_{-}{}^{5/2}-2\mathfrak{M}_{-}{}^{5/2}]\,; & J^\pi=\frac{5}{2}^+\end{cases}$$

and we use the Breit-Wigner formula for $\mathcal{E}_{-}{}^J$ and $\mathfrak{M}_{-}{}^J$:

$$\mathcal{E}_{-}{}^{3/2}-\sqrt{3}\mathfrak{M}_{-}{}^{3/2}=\frac{\frac{1}{2}(\Gamma_{N\pi}\Gamma_{N\gamma})^{1/2}}{W-W_0-\frac{1}{2}i\Gamma}\,,\ \text{etc.}$$

We shall discuss the case of $J^\pi=\frac{3}{2}^-$ in detail and summarize the results for the $J^\pi=\frac{5}{2}^+$ case. Making use of Eqs. (27a) and (27b), we rewrite Eq. (45) in the form

$$(\tfrac{3}{10}\sqrt{5})E(8_s,\tfrac{3}{2}^-)+\tfrac{1}{2}E(8_a,\tfrac{3}{2}^-)=\frac{1}{m}\left(\frac{g_r}{e}\right)\frac{1}{2}\left(\frac{\Gamma_{N\gamma}}{\Gamma_{N\pi}}\right)^{1/2}\frac{1}{G_A}\times[(\tfrac{3}{10}\sqrt{5})F(8_s,\tfrac{3}{2}^-)+\tfrac{1}{2}F(8_a,\tfrac{3}{2}^-)]. \quad (46)$$

We shall compare the prediction of (46) with experiment in the following manner. We shall compute $\Gamma_{N\gamma}$ from (46) using our mixing scheme ($\cos\alpha=\sqrt{\frac{3}{5}}$, $\cos\beta=\sqrt{\frac{3}{8}}$, $\sin\beta=(4/7)^{\frac{1}{2}}$. The forward pion photoproduction cross section at the resonance energy in the $I=\frac{1}{2}$, $J^\pi=\frac{3}{2}^-$ channel is directly related to $\Gamma_{N\gamma}$:

$$\left[\left(\frac{d\sigma}{\partial\Omega}\right)_{\theta=0,I=\frac{1}{2}}\right]_{\text{resonance}}\simeq\frac{1}{q^2}|\mathcal{E}_{-}{}^{3/2}-\sqrt{3}\mathfrak{M}_{-}{}^{3/2}|^2=\frac{1}{q_0{}^2}\left(\frac{\Gamma_{N\gamma}}{\Gamma_{N\pi}}\right)\left(\frac{\Gamma_{N\pi}}{\Gamma}\right)^2$$

$$q_0=(W_0{}^2-m^2)/2W_0$$

$$W_0=1518\ \text{MeV}.$$

We compare the value for $(d\sigma/d\Omega)_{\theta=0}$ thus obtained with experiment. We obtain from (46), (44), and (27b),

$$\left(\frac{\Gamma_{N\gamma}}{\Gamma_{N\pi}}\right)=G_A{}^2\left(\frac{e^2}{4\pi}\frac{4\pi}{g_r{}^2}\right)(4m\mu_p')^2,$$

and this gives (with $\Gamma_{N\pi}/\Gamma\simeq 0.75$)[19]

$$[(d\sigma/d\Omega)_{\theta=0,I=\frac{1}{2}}]_{\text{resonanec}}=61\ \mu\text{b/sr} \quad (47)$$

at the resonance. Experimentally,[22]

$$(d\sigma/d\Omega)_{\theta=0}(\gamma+p\rightarrow\pi^0+p)=1.4\ \mu\text{b/sr},$$
$$(d\sigma/d\Omega)_{\theta=0}(\gamma+p\rightarrow\pi^++n)=18\ \mu\text{b/sr}\quad\text{at}\quad W\approx1520\text{ MeV}.$$

If the above two processes proceed predominantly through the $I=\frac{1}{2}$ resonance, the ratio of the forward cross section should be $\frac{1}{2}$. Therefore, we see that there is a large background in the $I=\frac{3}{2}$ state.[23] On the basis of charge independence of strong interactions, we can derive the inequality

$$\sqrt{\left[2\left(\frac{d\sigma}{d\Omega}\right)_+\right]}-\sqrt{\left(\frac{d\sigma}{d\Omega}\right)_0}\leqslant\sqrt{\left(\frac{d\sigma}{d\Omega}\right)_{I=\frac{1}{2}}}\leqslant\sqrt{\left[2\left(\frac{d\sigma}{d\Omega}\right)_+\right]}+\sqrt{\left(\frac{d\sigma}{d\Omega}\right)_0},$$

where the subscripts $+$, 0 denote the processes $\gamma+p\rightarrow n+\pi^+$ and $\gamma+p\rightarrow p+\pi^0$, respectively. This inequality gives $(d\sigma/d\Omega)_{I=\frac{1}{2}}$ between 21 to 51 $\mu$b/sr, which includes background from nonresonant angular momentum-parity channels with $I=\frac{1}{2}$.

In much the same way, we deduce from our model

$$[(d\sigma/d\Omega)_{\theta=0,I=\frac{1}{2}}]_{\text{resonance}}=6.3\ \mu\text{b/sr},\quad W=1688\text{ MeV}.$$

Experimentally we find $(d\sigma/d\Omega)_{\theta=0}(\gamma+p\rightarrow\pi^0+p)=0.2$ $\mu$b/sr,[24] and $(d\sigma/d\Omega)_{\theta=0}(\gamma+p\rightarrow\pi^++n)=(8\pm1)\ \mu$b/sr.[25] Thus from the triangular inequality, we find

$$(d\sigma/d\Omega)_{\theta=0,I=\frac{1}{2}}\simeq16\ \mu\text{b/sr}.$$

## V. CONCLUSIONS

We have presented a method for comparing matrix elements of the axial vector and electric dipole currents, taken between states of infinite momentum, with experiment and used these results to study a particular model of the baryonic states. In this section we wish to comment on these predictions.

For the Gamow-Teller matrix elements, our model yields the sum rule of Eq. (18), whose evaluation gives $G_A=1.25$. The remaining experimental information serves to fix the mixing angles. At first sight it might appear that the values of $G_A'(8,\frac{3}{2}^+)$ and $G_A'(8,\frac{5}{2}^-)$ are also independent predictions, but it is clear that the $SU(2)$ sum rule derived from Eq. (12)

$$1=G_A{}^2-\tfrac{2}{3}G^{*2}+G_A'^2(8,\tfrac{3}{2}^-)+G_A'^2(8,\tfrac{5}{2}^+)$$

fixes their absolute magnitude [in virtue of the sum rule Eq. (18)] once the angle $\gamma$ has been fixed. In fact, the difference

$$[\sum_{J^\pi}G_A'^2(8,J^\pi)]_{\text{expt}}-[\sum_{J^\pi}G_A'^2(8,J^\pi)]_{\text{theory}}=0.08$$

represents the difference between $G_A=1.25$ and the evaluation of the Weisberger-Adler sum rule directly from experiment.

Turning to photoproduction, our model has no free parameters left. The results of Eqs. (34) and (42) are quite good. We also predict that the $J^\pi=\frac{3}{2}^-$, $\frac{5}{2}^+$ octets are excited purely by isovector photons, which apparently has some experimental confirmation.[26] On the other hand, our evaluation of the magnitude of the electric dipole operator for these states was inconclusive. It is apparent that more effort is needed to fully understand these resonances and extract the relevant parameters.

It is possible that some further chiral representations should be mixed into our Eqs. (15) for the $J^\pi=\frac{3}{2}^-$, $\frac{5}{2}^+$ resonances. This can be done, in the framework of our model, without changing the representation for the stable baryons and $J^\pi=\frac{3}{2}^+$ resonances. Thus we can maintain our results for the low-lying states while gaining more freedom to fit the higher data.

It is worthwhile considering which of our predictions depend on the details of our model. In the first place, as we have remarked, we are testing sum rules for the pion-nucleon system. All our results except for those involving $F/D$ ratios (essentially the statement about the vanishing isoscalar amplitudes) hold true in the chiral $SU(2)\otimes SU(2)$ limit where our model is

$$|N_{1/2}\rangle=\cos\beta|(3,2)_{1/2},0\rangle+\sin\beta[\cos\alpha|(2,1)_{-1/2},1\rangle+\sin\alpha|(2,1)_\lambda,\tfrac{1}{2}-\lambda\rangle].\quad(48)$$

We have continued to use the dimension to stand for a chiral multiplet. Now the results for the magnetic moments are obtained by embedding (3,2) in a (6,3), the first (2,1) in a $(3,\bar{3})$ of $SU(3)\otimes SU(3)$, while the second (2,1) may be embedded in any representation which is not connected by the electric dipole operator to either (6,3) or $(3,\bar{3})$. We accomplished this by embedding it in an (8,1) and using $SU(3)$ selection rules. However, other choices are possible, for example, choosing the orbital helicity large enough.

[22] The cross section for $p+\gamma\rightarrow p+\pi^0$ is obtained from H. De Staebler *et al.*, Phys. Rev. **140**, B336 (1965). The cross section for $p+\gamma\rightarrow n+\pi^+$ is read off the graph, Fig. 17, in P. Salin, Nuovo Cimento **28**, 1294 (1963).

[23] For this reason, any phenomenological analysis which fits either of the cross sections, $p+\gamma\rightarrow p+\pi^0$ or $p+\gamma\rightarrow n+\pi^+$ by a pure $I=\frac{1}{2}$ resonance is open to question. For example, A. Bietti, Phys. Rev. **142**, B1258 (1966) quotes Beder's result [D. S. Beder, Nuovo Cimento **33**, 94 (1964)] that $\mathcal{E}_-^{3/2}=\sqrt{3}\mathfrak{M}_-^{3/2}$ for the (1,3) resonance excitation. This is based on the observation that the forward cross section for $p+\gamma\rightarrow p+\pi^0$ is very small. However, our argument indicates that the $T=\frac{3}{2}$ amplitude is not negligible, so we cannot conclude that the $T=\frac{1}{2}$ amplitudes $\mathcal{E}_-^{3/2}(T=\frac{1}{2})$ and $\mathfrak{M}_-^{3/2}(T=\frac{1}{2})$ satisfy the above relation.

[24] R. Tolman *et al.*, Phys. Rev. Letters **9**, 177 (1963).

[25] R. L. Walker, in *Proceedings of the Tenth International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tincot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1961), p. 17. Actually, the value quoted is for $\theta=5\sim10°$.

[26] F. S. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966).

## APPENDIX

In this Appendix we summarize our definitions of the matrix elements of $A_0^{(\lambda)}$ and $D_+^{(\lambda)}$ and indicate how we obtain the predictions of our model. The relevant matrix elements were defined in Eqs. (14) and (36):

$$\langle (N,\nu,J^\pi)_{1/2}|A_0^{(\lambda)}|(8,\mu,\tfrac{1}{2}^+)_{1/2}\rangle_{p\to\infty e_3} \equiv \sum_\xi \begin{pmatrix} 8 & 8 & N_\xi \\ \mu & \lambda & \nu \end{pmatrix} F(N_\xi,J^\pi), \quad \text{(A1)}$$

$$\langle (N,\nu,J^\pi)_{1/2}|D_+^{(\lambda)}|(8,\mu,\tfrac{1}{2}^+)_{-1/2}\rangle_{p\to\infty e_3} \equiv \sqrt{2}\sum_\xi \begin{pmatrix} 8 & 8 & N_\xi \\ \mu & \lambda & \nu \end{pmatrix} E(N_\xi,J^\pi). \quad \text{(A2)}$$

As special cases, we have

$$G_a = F(8_a,\tfrac{1}{2}^+), \quad \text{(A3a)}$$

$$G_s = F(8_s,\tfrac{1}{2}^+), \quad \text{(A3b)}$$

$$\mu_a' = E(8_a,\tfrac{1}{2}^+), \quad \text{(A3c)}$$

$$\mu_s' = E(8_s,\tfrac{1}{2}^+), \quad \text{(A3d)}$$

$$G^* = F(10,\tfrac{3}{2}^+), \quad \text{(A3e)}$$

$$\mu^* = \tfrac{2}{3}\sqrt{3}E(10,\tfrac{3}{2}^+), \quad \text{(A3f)}$$

and

$$G_A = \tfrac{1}{3}\sqrt{3}G_a + (\tfrac{3}{5})^{\frac{1}{2}}G_s, \quad \text{(A4a)}$$

$$\mu_V' = \mu_p' - \mu_n = \tfrac{1}{3}\sqrt{3}\mu_a' + (\tfrac{3}{5})^{\frac{1}{2}}\mu_s^-. \quad \text{(A4b)}$$

We have purposely included a factor of $\sqrt{2}$ in the definition Eq. (A2) since, for a spin $\frac{1}{2}\to\frac{1}{2}$ transition.

$$\mu = \langle J_3=+\tfrac{1}{2}|\mathfrak{M}_3|J_3=+\tfrac{1}{2}\rangle = \tfrac{1}{2}\sqrt{2}\langle J_3=+\tfrac{1}{2}|\mathfrak{M}_+|J_3=-\tfrac{1}{2}\rangle.$$

Definitions (A3e), (A3f), (A4a), and (A4b) are relevant when we restrict our attention to the chiral $U(2)\otimes U(2)$ symmetry and the pion nucleon system.

Chiral symmetry predicts relations between the $E$'s and $F$'s for different $SU(3)$ representations and fixes the $F/D$ ratio for octets since, in general, more than one $SU(3)$ multiplet is contained in a chiral multiplet. One method for deriving these relations is the familiar one of tensor algebra. We give, in the following, the tensors which represent the various representations of chiral $U(3)\otimes U(3)$ we have considered. The notation is that greek indices refer to the $SU(3)$ generated by $\frac{1}{2}(V_0^{(\lambda)}+A_0^{(\lambda)})$ while latin indices refer to that generated by $\frac{1}{2}(V_0^{(\lambda)}-A_0^{(\lambda)})$

$$(6,3):\ B^{\alpha\beta c} = D^{\alpha\beta c} + (\tfrac{1}{6}\sqrt{6})[\epsilon^{\alpha c\lambda}B^\beta{}_\lambda + \epsilon^{\beta c\lambda}B^\alpha{}_\lambda], \quad \text{(A5a)}$$

$$(3,\bar{3}):\ B^{\alpha bc} = \tfrac{1}{2}\sqrt{2}\epsilon^{bc\lambda}B^\alpha{}_\lambda + (\tfrac{1}{6}\sqrt{6})\epsilon^{\alpha bc}S, \quad \text{(A5b)}$$

$$(3,6):\ B^{\alpha bc} = D^{\alpha bc} + (\tfrac{1}{6}\sqrt{6})[\epsilon^{b\alpha\lambda}B^c{}_\lambda + \epsilon^{c\alpha\lambda}B^b{}_\lambda], \quad \text{(A5c)}$$

$$(\bar{3},3):\ B^{\alpha\beta c} = \tfrac{1}{2}\sqrt{2}\epsilon^{\alpha\beta\lambda}B^c{}_\lambda + (\tfrac{1}{6}\sqrt{6})\epsilon^{c\alpha\beta}S, \quad \text{(A5d)}$$

where $S$ is the singlet, $D^{\alpha\beta\gamma}$ is the totally symmetric decuplet, and $B^\alpha{}_\beta$ is the usual baryon matrix.

It is now easy to compute matrix elements of the axial vector current

$$A_0^{(\lambda)} = \tfrac{1}{2}(V_0^{(\lambda)}+A_0^{(\lambda)}) - \tfrac{1}{2}(V_0^{(\lambda)}-A_0^{(\lambda)}). \quad \text{(A6)}$$

It is important to recognize that Eq. (A6) is much stronger than merely prescribing the tensor transformation properties of $A_0^{(\lambda)}$. Because $A_0^{(\lambda)}$ is expressed as the difference of two generators its matrix elements between arbitrary chiral multiplets are fixed in magnitude (i.e., there is no unknown reduced matrix element for each different chiral multiplet). The relative magnitudes of the matrix elements of

$$\tfrac{1}{2}(V_0^{(\lambda)}\pm A_0^{(\lambda)})$$

is fixed by requiring that

$$V_0^{(\lambda)} = \tfrac{1}{2}(V_0^{(\lambda)}+A_0^{(\lambda)}) + \tfrac{1}{2}(V_0^{(\lambda)}-A_0^{(\lambda)}) \quad \text{(A7)}$$

generate simultaneous $SU(3)$ rotations and then the over-all magnitude of the matrix elements of the generators is fixed by the magnitude of $V_0^{(\lambda)}$. These somewhat obvious remarks are important to keep in mind for the electric dipole operator where they are no longer true. We summarize our results for the Gamow-Teller matrix element in Table I.

The electric dipole operator $D_i^{(\lambda)}$ transforms as a tensor operator under the chiral algebra, so we write

$$D_i^{(\lambda)} \sim \tfrac{1}{2}(\text{“}V_0^{(\lambda)}+A_0^{(\lambda)}\text{”}) + \tfrac{1}{2}(\text{“}V_0^{(\lambda)}-A_0^{(\lambda)}\text{”}). \quad \text{(A8)}$$

The terms on the right of Eq. (A8) are to be understood as representing the transformation properties of $D_i^{(\lambda)}$, and are tensor operators rather than the generators themselves. This has the consequence that the chiral algebra only predicts relations between members of the same chiral multiplet since there is an independent reduced matrix element for each multiplet. Furthermore, even for a given pair of chiral representations the operators in the right-hand side of Eq. (A8) have, in general, no relations between their matrix elements since the reduced matrix elements for each of them are independent quantities, that is, $D_i^{(\lambda)}$ only transforms like $V_0^{(\lambda)}$; it is not equal to $V_0^{(\lambda)}$. There is a relation,

TABLE I. Matrix elements of the axial vector current for various chiral multiplets. The normalization is such that the vector current is normalized to $G_V=1$.

| $B_{1/2}$ | (6,3) | (8,1) | $(3,\bar{3})$ | (3,6) | (1,8) | $(\bar{3},3)$ |
|---|---|---|---|---|---|---|
| $F(8_a,J^\pi)$ | $\frac{2}{3}\sqrt{3}$ | $\sqrt{3}$ | $\cdots$ | $-\frac{2}{3}\sqrt{3}$ | $-\sqrt{3}$ | $\cdots$ |
| $F(8_s,J^\pi)$ | $\sqrt{(5/3)}$ | $\cdots$ | $\sqrt{(5/3)}$ | $-\sqrt{(5/3)}$ | $\cdots$ | $-\sqrt{(5/3)}$ |
| $F(10,J^\pi)$ | $-\sqrt{(8/3)}$ | $\cdots$ | $\cdots$ | $\sqrt{(8/3)}$ | $\cdots$ | $\cdots$ |
| $F(1,J^\pi)$ | $\cdots$ | $\cdots$ | $\frac{4}{3}\sqrt{3}$ | $\cdots$ | $\cdots$ | $-\frac{4}{3}\sqrt{3}$ |

TABLE II. Matrix elements $\langle B_{1/2}|\frac{1}{2}(\text{“}V_0 \pm A_0\text{”})|B_{-1/2}\rangle$ needed to obtain the electric dipole matrix elements. Each column is independently multiplied by an unknown reduced matrix element; only the relative values within a column are relevant. We have, however, used the same normalization as in Table I.

| $B_{-1/2}$ <br> $B_{1/2}$ <br> $X$ | $(\bar{3},3)$ <br> $(6,3)$ <br> $+$ | $(3,\bar{3})$ <br> $(3,\bar{3})$ <br> $+$ | $(3,\bar{3})$ <br> $(3,\bar{3})$ <br> $-$ |
|---|---|---|---|
| $\sqrt{2}E^X(8,8;a)$ | $-\frac{1}{2}$ | $\frac{1}{2}\sqrt{3}$ | $\frac{1}{2}\sqrt{3}$ |
| $\sqrt{2}E^X(8,8;s)$ | $-\frac{1}{2}\sqrt{5}$ | $\frac{1}{2}\sqrt{(5/3)}$ | $-\frac{1}{2}\sqrt{(5/3)}$ |
| $\sqrt{2}E^X(10,8)$ | $-\sqrt{2}$ | $\cdots$ | $\cdots$ |
| $\sqrt{2}E^X(1,8)$ | $\cdots$ | $-\frac{2}{3}\sqrt{3}$ | $\frac{2}{3}\sqrt{3}$ |
| $\sqrt{2}E^X(8,1)$ | $1$ | $-\sqrt{\frac{1}{6}}$ | $\sqrt{\frac{1}{6}}$ |

however, implied by the mirror operation $M$

$$\langle (m,n)_\lambda|\tfrac{1}{2}(\text{“}V_0{}^{(\lambda)}+A_0{}^{(\lambda)}\text{”})|(m',n')_{\lambda'}\rangle = \langle (n',m')_{-\lambda'}|\tfrac{1}{2}(\text{“}V_0{}^{(\lambda)}-A_0{}^{(\lambda)}\text{”})|(n,m)_{-\lambda}\rangle. \quad \text{(A9)}$$

This relation ensures that the matrix element of the axial dipole operator between spin-$\frac{1}{2}$ baryons satisfies

$$\langle B_{1/2}{}^l|D_{5i}{}^{(\lambda)}|B_{-1/2}{}^m\rangle_{p=\infty\hat{e}_3}=0.$$

We have discussed the above point in detail since there is apparently some confusion about it in the literature. We summarize our results in Table II, where we use the definitions

$$\langle (N,\nu)_{1/2}|\tfrac{1}{2}(\text{“}V_0{}^{(\lambda)}\pm A_0{}^{(\lambda)}\text{”})|(N',\nu')_{-1/2}\rangle \equiv \sqrt{2}\sum_\xi \begin{pmatrix} N' & 8 & N_\xi \\ \nu' & \lambda & \nu \end{pmatrix} E^{\pm}(NN'\xi).$$

It is easy to obtain the $E(N_\xi,J^\pi)$ from these using Eq. (A8).

PHYSICAL REVIEW VOLUME 152, NUMBER 4 23 DECEMBER 1966

# Possible Existence of an $I=0$, $D$-Wave $\pi\pi$ Resonance in the Vicinity of the $\rho$ Meson*

BIPIN R. DESAI
*Department of Physics, University of California, Riverside, California*
(Received 21 April 1966)

A second peak in the vicinity of the $\rho^0$ peak has been seen in several recent experiments involving $\pi^-+p \to \pi^++\pi^-+n$ and $K^-+p \to \pi^++\pi^-+\Lambda$. We suggest that this peak may be due to an $I=0$, $D$-wave $\pi\pi$ resonance. On the basis of the experimental mass and angular distribution of the $2\pi$ system, the resonance is predicted to be at an energy of about 780 MeV with a width of 5 MeV. A strong $S$ wave at lower energies is also needed. Our $D$ resonance is the most logical candidate for the $2^+$ SU(3) singlet which is, as pointed out recently by Desai and Freund, needed to explain the high-energy behavior of the meson-baryon and baryon-baryon total cross sections. It is suggested that, it is this particle, not $f$, which should lie on the Pomeranchuk trajectory. It is argued that the strong attraction which the Pomeranchuk particle is known to receive and a strong centrifugal barrier typical of an $l=2$ state may well conspire to produce a narrow resonance at a low mass of the kind proposed here. A narrow-resolution $\pi^-+p \to \pi^0+\pi^0+n$ and $K^-+p \to \pi^0+\pi^0+\Lambda$ are perhaps the best experiments to isolate the $D$ wave and to determine whether it is this resonance or some other mechanism, such as $\omega \to 2\pi$, which is responsible for the second peak.

BY now a number of different independent groups have seen an excess of events in $\pi^-+p \to \pi^++\pi^-+n$ at $M_{\pi\pi}$ around 780 MeV.[1-3] A secondary peak in the vicinity of the $\rho$ peak at the same $M_{\pi\pi}$ is also seen in $K^-+p \to \pi^++\pi^-+\Lambda$.[4] In view of the fact that such an enhancement has been seen repeatedly leads us to believe that it may be a real effect and not a statistical fluctuation. Since no such excess has been seen in $\pi^++p \to \pi^++\pi^++n$ or in $\pi^-+p \to \pi^-+\pi^0+p$, the system that gives rise to the secondary peak must be in an $I=0$ state. We would like to suggest that the peak is due to a narrow $I=0$, $D$-wave $\pi\pi$ resonance.[5]

Our work is motivated by the following considerations. Recently it was pointed out that the $SU(3)$ nonet of $2^+$ mesons is not sufficient to describe the high-energy behavior of the sum $\Sigma$ of the particle and antiparticle total cross sections for meson-baryon and baryon-

* Work supported in part by Atomic Energy Commission Contract No. AEC AT(11-1)34 P107A.

[1] E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **9**, 170 (1962); W. D. Walker, E. West, A. R. Erwin, and R. H. March, in *Proceedings of the 1962 Annual International Conference on High Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 42; L. Bondar *et al.*, Nuovo Cimento **31**, 729 (1964); Saclay-Orsay-Bari-Bologna Collaboration, *ibid.* **29**, 515 (1963).

[2] V. Hagopian, W. Selove, J. Alitti, J. P. Baton, and M. Neveu-René, Phys. Rev. **145**, 1128 (1966).

[3] L. D. Jacobs, University of California Radiation Laboratory Report No. UCRL16877 (unpublished).

[4] S. M. Flatté, D. O. Huwe, J. J. Murray, J. Button-Shafer, F. T. Solmitz, M. L. Stevenson, and C. Wohl, Phys. Rev. Letters **14**, 1095 (1965). It is pointed out by these authors that incoherent $\rho$ and $\omega$ production is a more reasonable possibility because the data come from all incident momenta, from all momentum transfers, and from all decay angles.

[5] This should not be confused with $\epsilon^0$ (or $s_0$) which is an $S$-wave effect and presumably exists at a lower energy.