Weinstein's Predecay Mixing Effect*

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The aim of this paper is to compare the predictions of a model with the recent conjecture of Weinstein stating that in particular circumstances the mass and width of a resonance could be altered by the final-state interactions following the production process. This effect, which he calls predecay resonance mixing, should be observed in a reaction in which there is the possibility of producing either one of two resonances of comparable mass as, for example, ρ and ω. Our model refers explicitly to the ρ-ω case but the conclusions we draw are entirely general and can be applied to any other similar case. Our results can be formulated as follows. No finite-range final-state interaction can displace a resonance or change its width. However, the production amplitude acquires a modulating factor which can, in principle, considerably modify the production cross section. Additional peaks may appear simulating spurious resonances. These could be discriminated from true resonances by studying their energy dependence.

I. INTRODUCTION

RECENTLY, Weinstein[1] has suggested an interesting phenomenon which might, in principle, occur in any nuclear reaction in which there is the possibility of producing either one of two resonances of comparable mass. The effect, called predecay resonance mixing, can be described in the following terms: Let us for definiteness call the two resonances in question ρ and ω and suppose that we are studying the two-pion decay of the ρ meson in an experiment designed for the production of this resonance. Since the ρ meson is produced in an intense nuclear field, it is legitimate to study the influence of this field on the behavior of the decay products we are observing. More precisely; do we have to expect a dependence of the ρ width and mass or of the two-pion angular distributions on the characteristics of the nuclear field in which the production process takes place? Now it is clear that, in the case we are considering, the external field can very well induce (for example, by exchanging a pion) a ρ-ω transition, which, in view of the very close values of the masses of these particles, could happen with quite high probability. In other words, it is reasonable to expect a mixing of the two states due to the external field. Although formulated in a slightly different context (instead of the ω, he considers another hypothetical resonance), the conjecture of Weinstein is that the width and mass of the ρ meson would be altered by the mixing, and, in particular, a dependence of these quantities on the ρ energy should be observed. The latter conclusion, of course, is based on the fact that the effects due to the interaction in the final state depend on the time spent by the particle within the range of the interaction.

In this paper we make an attempt to compare the above conjecture with the predictions of a simplified model in order to reach a more detailed understanding of the effect and eventually give an estimate of its strength. Our model will be oversimplified in many respects. First of all, we shall deal with scalar particles only and we shall consider the ω stable as compared to the ρ. All our considerations will be based on the use of Schwinger variational principle for the T matrix.[2] The answer of the variational principle, however, looks very sensible, and in a limiting case is even exact. Finally, in order to avoid a proliferation of the number of channels, we shall consider the ω, the two-pion system, and its resonant state ρ as the only coupled systems. This means that the ρ is produced only by incident ω's and vice versa. However, this restriction is essentially irrelevant for the final result which can easily be generalized to a realistic case.

The conclusions based on our model can be summarized as follows. For any finite-range nuclear potential the mixing does not produce a displacement of the ρ mass nor does it alter the ρ width. However, the production amplitude acquires a modulating factor which can, in principle, modify considerably the shape of the ρ peak in a way which is energy dependent. The structure of the modulating factor is interesting because, as we shall see, it can possibly produce two additional peaks in the cross section. These peaks would represent a sort of memory of an "unsuccessful attempt" to displace the ρ and ω masses. The fact that a finite-range potential cannot displace the masses can presumably be understood with the help of the uncertainty principle which requires an infinite time to build up a state of definite energy.

In Sec. II we shall outline the nonrelativistic theory and derive the basic formula of the paper. We shall then describe a generalization of the model with relativistic kinematics, and we shall finally discuss numerical estimates to investigate the relevance of the effect in

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[1] R. Weinstein (private communication). This effect is discussed, together with certain others, in D. O. Caldwell and R. Weinstein, Nuovo Cimento 39, 991 (1965).

[2] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 320.

the real world. A discussion of the results is given in the last section.

II. NONRELATIVISTIC THEORY

A. The Variational Principle

In our model we have originally three kinds of stable particles that will be assumed to be all scalar. We have a heavy particle of infinite mass that will be called the nucleon, then a light particle of mass μ that we shall call the π meson, and an intermediate particle, the ω, of mass m_ω. The interactions among these particles that we shall consider are the following:

(1) a π-π attractive force V_{22};

(2) a nuclear potential V_{12} that induces transitions $\omega \rightleftarrows 2\pi$. The pion-pion force will be assumed to be strong enough to give a resonance but not a bound state. In these conditions the scattering matrix $T_{\alpha,\beta}$ will be 2×2 as α and β can take only the values ω or 2π and the problem is a typical two-potential scattering. We proceed first in a purely formal way. Let $\Psi_\alpha^\pm$ be the wave functions for the scattering states, satisfying

$$\begin{aligned}\Psi^+&=\Phi-\frac{1}{\tilde{H}_0-E-i\epsilon}\tilde{V}\Psi^+=\Phi-\frac{1}{A}\tilde{V}\Psi^+,\\ \Psi^-&=\Phi-\Psi^-\tilde{V}\frac{1}{\tilde{H}_0-E-i\epsilon}=\Phi-\Psi^-\tilde{V}\frac{1}{A},\end{aligned}\tag{2.1}$$

where

$$\tilde{V}=\begin{pmatrix}0 & V_{12}\\ V_{12}^\dagger & V_{22}\end{pmatrix},\tag{2.2}$$

$$\tilde{H}_0=\begin{pmatrix}H_{01} & 0\\ 0 & H_{02}\end{pmatrix}=\begin{pmatrix}p_\omega^2/2m_\omega & 0\\ 0 & (p_{\pi 1}^2+p_{\pi 2}^2)/2\mu\end{pmatrix}.\tag{2.3}$$

There is an extra label to attach to the wave functions $\Psi^{(\alpha)\pm}$ which again can take the two values ω and 2π according to whether we have in (1) a $\Phi^{(\omega)}$ or a $\Phi^{(2\pi)}$.

Having established our notation, we can write down the variational expression for $T_{2\pi,\omega}$ which is the matrix element of T relevant for our problem:

$$T_{2\pi,\omega}=\frac{(\Phi^{(2\pi)},\tilde{V}\Psi^{(\omega)+})(\Psi^{(2\pi)-},\tilde{V}\Phi^{(\omega)})}{(\Psi^{(2\pi)-},[\tilde{V}+\tilde{V}(1/A)\tilde{V}]\Psi^{(\omega)+})}.\tag{2.4}$$

This expression is stationary with respect to variations of either $\Psi^{(\omega)+}$ or $\Psi^{(2\pi)-}$. We will now show that because of the structure of the interaction operator (2), (4) depends actually only on $\Psi_\omega^{(\omega)+}$ and $\Psi_{2\pi}^{(2\pi)-}$ and is stationary with respect to variations of these quantities. Let us specialize the first of Eqs. (2.1) to the case of $\Psi^{(\omega)+}$:

$$\begin{aligned}\Psi_\omega^{(\omega)+}&=\Phi^{(\omega)}-\frac{1}{H_{01}-E-i\epsilon}V_{12}\Psi_{2\pi}^{(\omega)+},\\ &=\Phi^{(\omega)}-\frac{1}{a_1}V_{12}\Psi_{2\pi}^{(\omega)+},\\ \Psi_{2\pi}^{(\omega)+}&=-\frac{1}{H_{02}-E-i\epsilon}[V_{22}\Psi_{2\pi}^{(\omega)+}+V_{12}^\dagger\Psi_\omega^{(\omega)+}],\\ &=-\frac{1}{a_2}[V_{22}\Psi_{2\pi}^{(\omega)+}+V_{12}^\dagger\Psi_\omega^{(\omega)+}].\end{aligned}\tag{2.5}$$

The second of these equations can be solved formally for $\Psi_{2\pi}^{(\omega)+}$:

$$\Psi_{2\pi}^{(\omega)+}=-\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}^\dagger\Psi_\omega^{(\omega)+};\tag{2.6}$$

then we have

$$\Psi^{(\omega)+}=\begin{bmatrix}\Psi_\omega^{(\omega)+}\\ -\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}^\dagger\Psi_\omega^{(\omega)+}\end{bmatrix},\tag{2.7}$$

where $\Psi_\omega^{(\omega)+}$ satisfies

$$\Psi_\omega^{(\omega)+}=\Phi^{(\omega)}+\frac{1}{a_1}V_{12}\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}^\dagger\Psi_\omega^{(\omega)+}.\tag{2.8}$$

Having eliminated $\Psi_{2\pi}^{(\omega)+}$, we now verify that the three matrix elements appearing in (2.4) do not depend on $\Psi_\omega^{(2\pi)-}$. The statement is obvious for the matrix elements appearing in the numerator. They can be written more explicitly

$$\begin{aligned}(\Phi^{(2\pi)},\tilde{V}\Psi^{(\omega)+})&=\left(\Phi^{(2\pi)},a_2\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\times\frac{1}{a_2}V_{12}^\dagger\Psi_\omega^{(\omega)+}\right),\\ (\Psi^{(2\pi)-},\tilde{V}\Phi^{(\omega)})&=(\Psi_{2\pi}^{(2\pi)-},V_{12}^\dagger\Phi^{(\omega)}).\end{aligned}\tag{2.9}$$

For the matrix element appearing in the denominator of (2.4) the calculation is lengthier. The operator connecting $\Psi^{(2\pi)-}$ and $\Psi^{(\omega)+}$ can be written

$$\tilde{V}+\tilde{V}\frac{1}{A}\tilde{V}=\begin{bmatrix}V_{12}\frac{1}{a_2}V_{12}^\dagger & V_{12}+V_{12}\frac{1}{a_2}V_{22}\\ V_{12}^\dagger+V_{22}\frac{1}{a_2}V_{12}^\dagger & V_{22}+V_{12}^\dagger\frac{1}{a_1}V_{12}+V_{22}\frac{1}{a_2}V_{22}\end{bmatrix}.\tag{2.10}$$

Then

$$\left(\Psi^{(2\pi)-},\left[\tilde{V}+\tilde{V}\frac{1}{A}\tilde{V}\right]\Psi^{(\omega)+}\right)=\left(\Psi_{\omega}{}^{(2\pi)-},V_{12}\frac{1}{a_2}V_{12}{}^{\dagger}\Psi_{\omega}{}^{(\omega)+}\right)+\left(\Psi_{\omega}{}^{(2\pi)-},\left[V_{12}+V_{12}\frac{1}{a_2}V_{22}\right]\Psi_{2\pi}{}^{(\omega)+}\right)$$
$$+\left(\Psi_{2\pi}{}^{(2\pi)-},\left[V_{12}{}^{\dagger}+V_{22}\frac{1}{a_2}V_{12}{}^{\dagger}\right]\Psi_{\omega}{}^{(\omega)+}\right)+\left(\Psi_{2\pi}{}^{(2\pi)-},\left[V_{22}+V_{12}{}^{\dagger}\frac{1}{a_1}V_{12}+V_{22}\frac{1}{a_2}V_{22}\right]\Psi_{2\pi}{}^{(\omega)+}\right).$$

It is now immediately seen, using (2.6), that the two first terms of this sum cancel and that the other two can be simplified to give

$$\left(\Psi^{(2\pi)-},\left[\tilde{V}+\tilde{V}\frac{1}{A}\tilde{V}\right]\Psi^{(\omega)+}\right)=\left(\Psi_{2\pi}{}^{(2\pi)-},\left[V_{12}{}^{\dagger}-V_{12}{}^{\dagger}\frac{1}{a_2}V_{12}\right]\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}{}^{\dagger}\Psi_{\omega}{}^{(\omega)+}\right). \tag{2.11}$$

We have, finally,

$$T_{2\pi,\omega}=\frac{\left(\Phi^{(2\pi)},a_2\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}{}^{\dagger}\Psi_{\omega}{}^{(\omega)+}\right)(\Psi_{2\pi}{}^{(2\pi)-},V_{12}{}^{\dagger}\Phi^{(\omega)})}{\left(\Psi_{2\pi}{}^{(2\pi)-},\left[V_{12}{}^{\dagger}-V_{12}{}^{\dagger}\frac{1}{a_1}V_{12}\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}{}^{\dagger}\right]\Psi_{\omega}{}^{(\omega)+}\right)}. \tag{2.12}$$

The variational properties of this equation with respect to $\Psi_{\omega}{}^{(\omega)+}$ follow now from (2.8). Those with respect to $\Psi_{2\pi}{}^{(2\pi)-}$ follow from the specialization of (2.1) to the case of $\Psi^{(2\pi)-}$. The verification is left to the reader. Equation (2.12) will be the basic equation for the subsequent discussion.

B. The Plane-Wave Approximation

The approximation we shall consider consists in replacing $\Psi_{\omega}{}^{(\omega)+}$ and $\Psi_{2\pi}{}^{(2\pi)-}$ with plane waves. The only real justification for doing this is given by the fact that when the nuclear potential becomes of infinite range (in a way that will be explained in a moment), Eq. (2.12) gives the exact solution of the problem.

We indicate with p_{ω} the initial momentum of the ω, with p the momentum of the center of mass of the two-pion system, and with q the relative momentum of the two pions in the final state. We have the equations

$$E=p_{\omega}{}^2/2m_{\omega}\,,$$
$$E_{2\pi}=p^2/4\mu+q^2/\mu\,. \tag{2.13}$$

On the energy shell we have also $E=E_{2\pi}$.

With the understanding that primed variables are integrated over, we can write (2.12) more explicitly:

$$T_{2\pi,\omega}(\mathbf{p},\mathbf{q},\mathbf{p}_{\omega},E)=\frac{[a_2(p,q,E)G(\mathbf{p},\mathbf{q},\mathbf{p}',\mathbf{q}',E)V_{12}{}^{\dagger}(\mathbf{p}',\mathbf{q}',\mathbf{p}_{\omega})]V_{12}{}^{\dagger}(\mathbf{p},\mathbf{q},\mathbf{p}_{\omega})}{V_{12}{}^{\dagger}(\mathbf{p},\mathbf{q},\mathbf{p}_{\omega}')[\delta(\mathbf{p}_{\omega}'-\mathbf{p}_{\omega})-(1/a_1(p_{\omega}',E))V_{12}(\mathbf{p}_{\omega}',\mathbf{p}'',\mathbf{q}'')G(\mathbf{p}'',\mathbf{q}'',\mathbf{p}',\mathbf{q}',E)V_{12}{}^{\dagger}(\mathbf{p}',\mathbf{q}',\mathbf{p}_{\omega})]}, \tag{2.14}$$

where $G(\mathbf{p}',\mathbf{q}',\mathbf{p},\mathbf{q},E)$ is the two-pion kernel obtained from

$$\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}.$$

The case of interest to us is when the two-pion potential is strong enough to give a resonance. In such a case we can ignore the internal structure of the system and replace the two-pion kernel G with a Breit-Wigner-type expression

$$G(\mathbf{p}',\mathbf{q}',\mathbf{p},\mathbf{q},E)\propto\delta(\mathbf{q}-\mathbf{q}')\delta(\mathbf{p}-\mathbf{p}')\frac{1}{p^2-p_{\rho}{}^2}, \tag{2.15}$$

where $p_{\rho}{}^2$ has a negative imaginary part. The variable in which the effect is expected is the two-pion energy in their center-of-mass system. More precisely, we want to study the behavior of the production amplitude as a function of the c.m. energy of the two pions for fixed initial energy. This is the nonrelativistic counterpart of studying the behavior of the production amplitude as a function of the total mass of the two pions in the final state. On the energy shell, when $E=E_{2\pi}$, we have, from (2.13),

$$-p^2/4\mu+E=E_{\mathrm{c.m.}}. \tag{2.16}$$

We then see that the dependence of (2.14) on $E_{\mathrm{c.m.}}$ is given both by the explicit q dependence and the p dependence. It is sensible to assume that the p dependence is the relevant one as it is not connected with the detailed structure of the two-pion state.

As we mentioned, there is at least one case where (2.14) gives the exact answer. This is obtained by

FIG. 1. Diagrams for the numerator of Eq. (2.14).

considering the following separable potentials:

$$V_{22}(\mathbf{q}',\mathbf{q})=-f(\mathbf{q}')f(\mathbf{q}),$$

$$V_{12}^{\dagger}(\mathbf{p},\mathbf{q},\mathbf{p}_\omega)=V_{12}(\mathbf{p}_\omega,\mathbf{p},\mathbf{q})=V_{12}^{0}f(\mathbf{q})\delta(\mathbf{p}_\omega-\mathbf{p}). \quad (2.17)$$

An easy calculation gives

$$G(\mathbf{p}',\mathbf{q}',\mathbf{p},\mathbf{q},E)=\delta(\mathbf{q}-\mathbf{q}')\delta(\mathbf{p}-\mathbf{p}')\frac{1}{1-\psi(p,E)} \times\frac{1}{(q^2/\mu)+(p^2/4\mu)-E}, \quad (2.18)$$

where we have defined

$$\psi(p,E)=\int d^3q\frac{f^2(q)}{(q^2/\mu)+(p^2/4\mu)-E}. \quad (2.19)$$

The two-pion resonance is described by the propagator $[1-\psi]^{-1}$.

Inserting (2.17) and (2.18) in (2.14), we obtain

$$T_{2\pi,\omega}=\delta(\mathbf{p}-\mathbf{p}_\omega)\frac{V_{12}^{0}f}{1-\psi}\left[1-\frac{(V_{12}^{0})^2}{a_1}\frac{\psi}{1-\psi}\right]^{-1}. \quad (2.20)$$

That this is the exact expression can be verified by first solving Eq. (2.8) and then calculating

$$T_{2\pi,\omega}=\left(\Phi^{(2\pi)},a_2\left[1+\frac{1}{a_2}V_{22}\right]^{-1}\frac{1}{a_2}V_{12}^{\dagger}\Psi_\omega^{(\omega)+}\right).$$

The nuclear potential given by (2.17) corresponds to a constant interaction in configuration space. In this limiting case, the ω and the 2π states are completely mixed, and the physical states contributing to T are given by the zeros of the real part of the bracketed expression in (2.20). It can easily be seen that the ω and the 2π states tend to repel each other.

Let us consider now the more realistic case in which the nuclear potential is of finite range. In order to deal with simple formulas we simply replace V_{12} given by (2.17) with the expression

$$V_{12}^{\dagger}(\mathbf{p},\mathbf{q},\mathbf{p}_\omega)=V_{12}(\mathbf{p}_\omega,\mathbf{p},\mathbf{q})=f(\mathbf{q})U(\mathbf{p}_\omega-\mathbf{p}), \quad (2.21)$$

where U could be for example a Yukawa potential. By

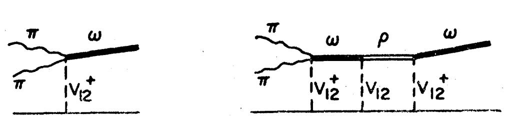

FIG. 2. Diagrams for the denominator of Eq. (2.14).

exhibiting the relevant momentum dependence we write now

$$T_{2\pi,\omega}=\frac{1}{1-\psi(p)}fU^2\left[U(\mathbf{p}-\mathbf{p}_\omega)-U(\mathbf{p}-\mathbf{p}_\omega')\frac{1}{a_1(p_\omega')} \times U(\mathbf{p}_\omega'-\mathbf{p}')\frac{\psi(p')}{1-\psi(p')}U(\mathbf{p}'-\mathbf{p}_\omega)\right]^{-1}. \quad (2.22)$$

By comparing (2.22) with (2.20), we immediately realize that in this case the factor $[1-\psi]^{-1}$ in front of (2.22) cannot be cancelled by the similar expression appearing in the brackets. In other words, as soon as the potential is of finite range, no level displacement is possible, and if the cross section was peaked because of a two-pion resonance, this peak will stay there. However, the real part of the expression in brackets could still vanish and, in general, we may expect additional peaks which represent a sort of memory of the mass displacement which is no longer exactly possible. The displacement can be recovered also if we let the two U factors on the right of (2.22) become δ functions.

In the next section we shall discuss a simple generalization of (2.22) with relativistic kinematics. The new expression will also be evaluated numerically to get an idea of the strength of the effect in realistic cases.

III. A GENERALIZATION WITH RELATIVISTIC KINEMATICS

If we examine the structure of (2.14) or (2.22), we recognize that the variational approximation to $T_{2\pi,\omega}$ is obtained in the following way. Let us call the two-pion resonant state ρ. Then the numerator is obtained by taking the product of the amplitudes corresponding to the diagrams shown in Fig. 1, while the denominator corresponds to the diagrams in Fig. 2. The generalization we have in mind consists in evaluating the amplitudes represented above as if they were Feynman diagrams and in combining them with the same rule as in the nonrelativistic case to get the T matrix. The nucleon will now be assumed to have a finite mass m_N, and we shall take the potential V_{12} to be given by the exchange of one pion. In this way four types of vertices appear: the pion-nucleon coupling, the ω-ρ-π coupling, the ρ-2π coupling, and the ω-3π coupling. To obtain a usable formula we shall make the further approximation of neglecting retardation effects in the exchange of the pions, and the dashed lines of the diagrams will be approximated by Yukawa potentials. We first briefly describe the kinematics. We call p_1 and p_2 the nucleon four-momenta in the initial and the final state, respectively. q_1 is the momentum of the incoming ω and q_2 is the total momentum of the two pions in the final state. s is the total energy squared in the center-of-mass system of the incoming particles. We have the relationships

$$\mathbf{q}_1^2=(1/4s)[s^2-2s(m_\omega^2+m_N^2)+(m_\omega^2-m_N^2)^2],$$
$$\mathbf{q}_2^2=(1/4s)[s^2-2s(M^2+m_N^2)+(M^2-m_N^2)^2], \quad (3.1)$$

where we have indicated with M the total mass of the two pions in the final state. We are interested in the dependence of the transition probability on M for fixed s. We denote by M_1, M_2, and M_3 the expressions of the three Feynman diagrams to be evaluated. We have, with an obvious meaning of the symbols,

$$M_1=g_{\pi,N}g_{\omega,3\pi}\frac{1}{\mu^2+(\mathbf{q}_2-\mathbf{q}_1)^2}, \quad (3.2)$$

$$M_2=g_{\pi,N}g_{\omega,\rho\pi}g_{\rho,2\pi}\frac{1}{\mu^2+(\mathbf{q}_2-\mathbf{q}_1)^2}\frac{1}{M^2-(m_\rho-i\Gamma/2)^2}, \quad (3.3)$$

$$M_3=g_{\pi,N}{}^3g_{\omega,\rho\pi}{}^2g_{\omega,3\pi}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{[(\mathbf{q}_1-\mathbf{k}_1)^2+\mu^2][(\mathbf{k}_2-\mathbf{k}_1)^2+\mu^2][(\mathbf{q}_2-\mathbf{k}_2)^2+\mu^2]}$$
$$\times\frac{1}{[(q_1+p_1-k_1)^2+m_N^2][(q_1+p_1-k_2)^2+m_N^2][k_1^2+(m_\rho-i\Gamma/2)^2][k_2^2+m_\omega^2]}$$
$$=\frac{g_{\pi,N}{}^3g_{\omega,\rho\pi}{}^2g_{\omega,3\pi}}{2^9\pi^6}\int d\Omega_1\int_0^\infty k_1^2dk_1\int d\Omega_2\int_{(m_\omega+m_n)^2}^\infty ds_2\frac{k_2(s_2)}{s_2^{1/2}(s_2-s-i\epsilon)(k_1^2+m_N^2)^{1/2}}$$
$$\times\frac{(k_1^2+m_N^2)^{1/2}+[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}}{[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}\{[(k_1^2+m_N^2)^{1/2}+(k_1^2+(m_\rho-i\Gamma/2)^2)^{1/2}]^2-s\}} \quad (3.4)$$
$$\times[(\mathbf{k}_1-\mathbf{q}_1)^2+\mu^2][(\mathbf{k}_2-\mathbf{k}_1)^2+\mu^2][(\mathbf{q}_2-\mathbf{k}_2)^2+\mu^2]$$

where

$$k_2^2(s_2)=(1/4s_2)[s_2^2-2s_2(m_\omega^2+m_N^2)+(m_N^2-m_\omega^2)^2]. \quad (3.5)$$

The asymmetry in the integration variables comes from our desire to write the simplest possible formula while keeping the integration variables real.

Further considerable simplification is now obtained if we consider one partial wave at a time. The amplitudes for the lth partial wave are

$$M_1^{(l)}=g_{\pi,N}g_{\omega,3\pi}\frac{1}{q_1q_2}Q_l\left(\frac{q_1^2+q_2^2+\mu^2}{2q_1q_2}\right), \quad (3.6)$$

$$M_2^{(l)}=g_{\pi,N}g_{\omega,\rho\pi}g_{\rho,2\pi}\frac{1}{q_1q_2}\frac{1}{M^2-(m_\rho-i\Gamma/2)^2}Q_l\left(\frac{q_1^2+q_2^2+\mu^2}{2q_1q_2}\right), \quad (3.7)$$

$$M_3^{(l)}=\frac{g_{\pi,N}{}^3g_{\omega,\rho\pi}{}^2g_{\omega,3\pi}}{8(2\pi)^4q_1q_2}\int_0^\infty dk_1\int_{(m_\omega+m_N)^2}^\infty ds_2\frac{(k_1^2+m_N^2)^{1/2}+[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}}{(k_1^2+m_N^2)^{1/2}[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}k_2(s_2)s_2^{1/2}(s_2-s-i\epsilon)}$$
$$\times\frac{1}{((k_1^2+m_N^2)^{1/2}+[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}-s)^2}Q_l\left(\frac{k_1^2+q_1^2+\mu^2}{2q_1k_1}\right)Q_l\left(\frac{k_1^2+k_2^2+\mu^2}{2k_1k_1}\right)Q_l\left(\frac{k_2^2+q_2^2+\mu^2}{2k_2q_2}\right). \quad (3.8)$$

From these expressions we can calculate

$$T_{2\pi,\omega}{}^{(l)}=\frac{M_1^{(l)}M_2^{(l)}}{M_1^{(l)}-M_3^{(l)}}. \quad (3.9)$$

In the following section we shall proceed to numerical estimates based on this formula.

IV. NUMERICAL CALCULATIONS

The main problem we have to solve is how to obtain a reasonable estimate of the coupling constants. Since all our particles are scalar, the values that the various couplings have in real life are not immediately apparent.

If we look at (3.9) we see that $T_{2\pi,\omega}$ actually depends on three of the four couplings we have introduced.

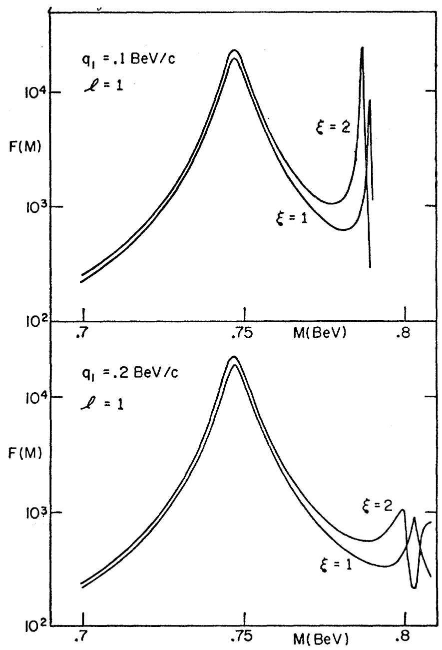

FIG. 3. Qualitative behavior of

$$F(M)=\frac{1}{|M^2-(m_\rho-i\Gamma/2)^2|^2}\frac{1}{|D^{(l)}|^2}$$

for the following choice of the masses (in BeV): $\mu=0.14$, $m_\rho=0.75$, $\Gamma=0.12$, $m_\omega=0.78$, $m_N=0.94$.

$g_{\omega,3\pi}$ cancels between the numerator and the denomivator, and this is perfectly natural because $g_{\omega,3\pi}$, $g_{\omega,\rho\pi}$, and $g_{\rho,2\pi}$ are clearly related and in principle it must be possible to express any one of them in terms of the other two. We can also disregard the $g_{\rho,2\pi}$ coupling because the effect we are interested in is given only by the denominator where this coupling is absent. We are then left with $g_{\pi,N}$ and $g_{\omega,\rho\pi}$. We shall estimate these quantities in the following way. We shall obtain an upper limit for $g_{\pi,N}$ by assuming that the S-wave amplitude for the scattering of scalar nucleons in Born approximation saturates unitarity. On the other hand, $g_{\omega,\rho\pi}$ will be estimated by assuming that the ω width is small in terms of the characteristic masses appearing in the problem.

The total cross section for scattering of scalar nucleons in the Born approximation can be written in terms of partial wave contributions,

$$\sigma_T=\frac{g_{\pi,N}^4}{4\pi}\frac{1}{16W^2}\frac{1}{q^4}\sum_\rho(2l+1)Q_l^2\left(1+\frac{\mu^2}{2q^2}\right),\quad(4.1)$$

where W is the total energy in the center-of-mass system and q is the momentum of the individual nucleons. If we now require the S wave to saturate the unitarity limit for some momentum q_0 we obtain

$$\frac{1}{q_0}\simeq\frac{g_{\pi,N}^2}{4\pi}\frac{1}{4W_0}\frac{1}{q_0^2}\frac{1}{2}\ln\left(1+\frac{4q_0^2}{\mu^2}\right).\quad(4.2)$$

For $q_0\sim\mu$ we have the estimate

$$g_{\pi,N}^2\sim32\pi m_N\mu.\quad(4.3)$$

If instead of the N-N system we had considered pion-nucleon scattering, a calculation similar to the above would have given a value for $g_{\pi,N}^2$ larger by a factor of four. The reason why (4.3) is perhaps a better estimate is seen in the following way. Our calculation can have some sense only in a case where the scattering is not dominated by a particular intermediate state but the cross section has still large enough values. For the value of q_0 we have considered, real nucleons seem to meet this situation. In the following, we shall use for $g_{\pi,N}$ the value given by (4.3), and we hope that this will correctly express the fact that the pion-nucleon interaction is strong.

We now consider $g_{\omega,\rho\pi}$. The information we have about this coupling comes from the 3π decay of the ω. One usually assumes that the ω decays first into ρ and π, this process being followed by the much more rapid decay of the ρ. We have to evaluate $\Gamma_{\omega\to3\pi}$ and $\Gamma_{\rho\to2\pi}$. A straightforward calculation gives

$$\Gamma_{\rho\to2\pi}=\frac{g_{\rho,2\pi}^2}{16\pi}\frac{(m_\rho^2-4\mu^2)^{1/2}}{m_\rho^2},\quad(4.4)$$

$$\Gamma_{\omega\to3\pi}=\frac{g_{\omega,\rho\pi}^2g_{\rho,2\pi}^2}{2^8\pi^3m_\omega^3}\int_{4\mu^2}^{(m_\omega-\mu)^2}d\kappa^2\frac{(\kappa^2-4\mu^2)^{1/2}}{\kappa|m_\rho^2-\kappa^2|^2}\times[(m_\omega^2+\kappa^2-\mu^2)^2-4\kappa^2m_N^2]^{1/2}.\quad(4.5)$$

By eliminating $g_{\rho,2\pi}$ we obtain

$$g_{\omega,\rho\pi}^2=\frac{\Gamma_{\omega\to3\pi}}{\Gamma_{\rho\to2\pi}}\frac{2^4\pi^2}{I}\frac{(m_\rho^2-4\mu^2)^{1/2}}{m_\rho^2}m_\omega^3,\quad(4.6)$$

where I represents the value of the integral. We next need an estimate of I. It is easily seen that $I<1$. A rough calculation taking $m_\omega/_\pi\sim m_\rho/\mu\sim5$ gives $I\sim\frac{1}{2}$. Finally, by considering a ratio $\Gamma_{\omega\to3\pi}/\Gamma_{\rho\to2\pi}\sim10^{-1}$, we have the order of magnitude

$$g_{\omega,\rho\pi}^2\sim3\pi^2m_\omega^2.\quad(4.7)$$

We now have all the necessary numbers. If we look at (3.9) we see that the cross section for ρ production due to the conversion of an ω in the nuclear field contains the "modulating" factor

$$\frac{1}{|D^{(l)}|^2}=\frac{|M_1^{(l)}|^2}{|M_1^{(l)}-M_3^{(l)}|^2}.\quad(4.8)$$

Using (3.6), (3.8), (4.3), and (4.7), this can be written explicitly

$$\frac{1}{|D^{(l)}|^2}=Q_l^2(q_1,q_2)\Bigg|\Bigg[Q_l(q_1,q_2)-\xi G\int_0^\infty dk_1\int_{(m_\omega+m_N)^2}^\infty ds_2\frac{1}{k_2(s_2)s_2^{1/2}(s_2-s-i\epsilon)}$$
$$\times\frac{(k_1^2+m_N^2)^{1/2}+[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}}{(k_1^2+m_N^2)^{1/2}[k_1^2+(m_\rho-i\Gamma/2)^2]^{1/2}\{[(k_1^2+m_N^2)^{1/2}+(k_1^2+(m_\rho-i\Gamma/2)^2)^{1/2}]^2-s\}}$$
$$\times Q_l(q_1,k_1)Q_l(k_1,k_2)Q_l(k_2,q_2)\Bigg]^{-1}\Bigg|^2, \qquad (4.9)$$

where

$$G=(3/4\pi)m_\omega^2 m_N\mu. \qquad (4.10)$$

ξ is a parameter which has been introduced to take into account the uncertainties in the calculation of the couplings. In Fig. 3 a plot of

$$\frac{1}{|M^2-(m_\rho-i\Gamma/2)^2|^2}\frac{1}{|D^{(l)}|^2}$$

is given for various values of the parameters involved.

V. DISCUSSION

In this section we want to add a few comments on the result we have obtained. The numerical results shown in Fig. 3 give a qualitative picture of the effect for particular choices of the various quantities involved. Their value is purely indicative, especially because of the simplicity of our model and of the uncertainties which, as we have seen, affect our estimates. With our values of $g_{\pi,N}$ and $g_{\omega,\rho\pi}$, we seem to be just on the border line of the region where the effect becomes important. Furthermore, the numerical evaluation of $D^{(l)}$ is rather complicated and requires long runs on a computer. Because of the simplified character of our model, we did not carry out an extensive numerical exploration of our formula and it is possible that more favorable conditions for the effect be discovered in a more detailed analysis. It is our hope to present, in the near future, a more realistic and detailed calculation and more reliable predictions for the experimentalist.

There are, however, a certain number of conclusions which can be established at this stage. The effect is strongly energy dependent and it decreases when the energy increases. This is perfectly natural because the time spent by the final-state particles in the nuclear field is reduced when the energy is increased.

We have also investigated in some detail the l dependence. The l dependence of $D^{(l)}$ seems to be less dramatic than the energy dependence. However, since the production amplitude will go down with increasing l (because of the extra Q_l), the effect will be due mainly to the low partial waves.

Another critical quantity besides the coupling is the ρ-ω mass difference. In our numerical examples all the masses and widths have been taken at realistic values.

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