Meson-Baryon and Baryon-Baryon Reactions in a Quark Model

H. J. LIPKIN, F. SCHECK,* AND H. STERN
The Weizmann Institute of Science, Rehovoth, Israel
(Received 7 June 1966)

An "independent-quark model" for mesons and baryons which recently was applied very successfully to *elastic* scattering at high energies, is extended to inelastic meson-baryon and baryon-baryon reactions. A number of relations between cross sections are derived, most of which are independent of $SU(3)$. The agreement with the available data is fair except for one case, but more data at higher energies are needed for a detailed test of the model.

I. INTRODUCTION

RECENTLY a very simplified "independent-quark model" for mesons and baryons has been successfully applied to elastic baryon-baryon and meson-baryon scattering,[1,2] neutral-meson production cross sections,[3] proton-antiproton annihilation at rest,[4] strong decays of the decuplet baryons and the calculation of electromagnetic and strong mass differences.[5] This quark model sometimes gives relations which are consistent with $SU(3)$ and $SU(6)_W$ symmetries and also makes predictions which are not obtainable from these symmetries, most of which agree well with experiment. In the particular case of certain relations for elastic forward-scattering amplitudes which do *not* depend upon $SU(3)$, the agreement with experiment is much better than what one is used to call agreement in the case of $SU(3)$ and $SU(6)_W$ relations.[6] As pointed out in Ref. 3, however, this remarkable success of the quark model for elastic forward scattering may perhaps be due to the particular simplicity of such processes where the four-momentum transfer is zero and where there are no corrections for mass differences and form factors. It is therefore tempting to apply the same model to inelastic meson-baryon and baryon-baryon reactions, where these effects can become appreciable, and to compare the obtained predictions with experiment. In Sec. II we construct the wave functions for the mesons and baryons and summarize our assumptions for the reaction mechanism. In Sec. III we consider a particular set of reactions which are independent of $SU(3)$ and which do not require the quark model for the baryons. In Sec. IV we derive relations which depend on $SU(3)$ and some of which are equivalent to $SU(6)_W$ predictions and in Sec. V we then discuss the necessary corrections to the cross sections (phase-space and form factors) and compare our results with experiment. Section VI finally contains a review and discussion of our assumptions and relations with symmetries.

II. PARTICLE WAVE FUNCTIONS AND ADDITIVITY ASSUMPTION FOR QUARK-QUARK SCATTERING AMPLITUDES

We assume that any 0^- or 1^- meson is a quark-antiquark pair, whereas any of the octet or decuplet baryons consists of three quarks coupled to the appropriate internal quantum numbers of the particle under consideration. For the case of the mesons the coupling of the spins and isospins of the quarks is unique. Thus the π^0 meson, e.g., is given by

$$\pi^0=\tfrac{1}{2}(\mathcal{P}\uparrow\bar{\mathcal{P}}\downarrow-\mathcal{P}\downarrow\bar{\mathcal{P}}\uparrow-\mathcal{N}\uparrow\bar{\mathcal{N}}\downarrow+\mathcal{N}\downarrow\bar{\mathcal{N}}\uparrow). \quad (1)$$

We denote the isodoublet quarks by $\mathcal{P}$ and $\mathcal{N}$ with charges $+\frac{2}{3}$ and $-\frac{1}{3}$, respectively, the strange quark with charge $-\frac{1}{3}$ by λ, whereas the physical proton, neutron, and Λ states will be denoted by p, n, and Λ. The arrow represents the spin projection on an arbitrary z direction. For the baryons one has the freedom to couple two of the quarks to intermediate spin zero or one and/or to intermediate isospin zero or one. The coupling is fixed, however, if one requires the baryon wave function to be totally symmetric in the internal degrees of freedom, i.e., classifying the baryons into the 56 multiplet of $SU(6)$. The dynamical single-quark states in a baryon (analogous to the single-particle states in a shell model) shall be symbolized by their position in the state vector. The notation $(q^\alpha q^\beta q^\gamma)$ therefore means that q^α occupies the single-quark state with energy E_1, q^β that with energy E_2, and q^γ that with energy E_3. The wave function of the physical Λ, e.g., is then given by

$$\Lambda_m=(1/\sqrt{12})\sum_\pi([\mathcal{P}\uparrow\mathcal{N}\downarrow-\mathcal{N}\uparrow\mathcal{P}\downarrow]\lambda_m), \quad (2)$$

where $\sum_\pi$ means the sum over all permutations of the three quarks and where m is the spin projection on the z axis. Similarly an N^{*++} with spin projection $m=\frac{3}{2}$, e.g., is given by

$$N^{*++}{}_{m=3/2}=\tfrac{1}{6}\sum_\pi \mathcal{P}\uparrow\mathcal{P}\uparrow\mathcal{P}\uparrow. \quad (3)$$

Clearly all other particles of the meson octets and singlets, the baryon octet and decuplet can be generated

* On leave of absence from the University of Freiburg, Germany, on a fellowship of the Volkswagenwerk Foundation.
[1] E. M. Levin and L. L. Frankfurt, JETP Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: JETP Letters 2, 65 (1965)]. V. V. Anisovich, JETP Pis'ma v Redaktsiyu 2, 554 (1965) [English transl.: JETP Letters 2, 344 (1965)].
[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1965).
[3] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).
[4] H. R. Rubinstein and H. Stern, Phys. Letters 21, 447 (1966).
[5] H. R. Rubinstein, Phys. Rev. Letters 17, 41 (1966); P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Letters 22, 208 (1966); H. R. Rubinstein, *ibid.* 22, 210 (1966).
[6] See, for example, relation (3a) of Ref. 2.

from formulas (1) to (3) by use of isospin, U-spin, and V-spin raising and lowering operators.

This assignment of mesons and baryons to the known multiplets does not mean, of course, that $SU(3)$ or $SU(6)$ is assumed to be a good symmetry. They are only used for the *classification* of the particles but the transition amplitudes will in general not be assumed to be invariant under any of these groups.[7] This point is discussed further in Sec. VI.

We again assume that all scattering amplitudes for any meson-baryon or baryon-baryon reaction are simply given by the sum of all possible quark-quark and quark-antiquark reaction amplitudes. As a particular example consider the charge-exchange amplitude $\pi^- p \to \pi^0 n$, leading from a proton with spin projection $+\frac{1}{2}$ to, e.g., a neutron with spin projection $-\frac{1}{2}$, which then is given by

(a) decomposing first the mesons,

$$\langle \pi^- p\uparrow | \pi^0 n\downarrow\rangle = (1/2\sqrt{2}) \times (\langle \mathcal{N}\uparrow p\uparrow | \mathcal{P}\uparrow n\downarrow\rangle - \langle \bar{\mathcal{P}}\downarrow p\uparrow | \bar{\mathcal{N}}\downarrow n\downarrow\rangle + \langle \mathcal{N}\downarrow p\uparrow | \mathcal{P}\downarrow n\downarrow\rangle - \langle \bar{\mathcal{P}}\uparrow p\uparrow | \bar{\mathcal{N}}\uparrow n\downarrow\rangle); \quad (4)$$

(b) then decomposing the baryons,

$$\langle \pi^- p\uparrow | \pi^0 n\downarrow\rangle = (5\sqrt{2}/12) \times (\langle \mathcal{N}\uparrow \mathcal{P}\uparrow | \mathcal{P}\uparrow \mathcal{N}\downarrow\rangle - \langle \bar{\mathcal{P}}\downarrow \mathcal{P}\uparrow \bar{\mathcal{N}}\downarrow \mathcal{N}\downarrow\rangle + \langle \mathcal{N}\downarrow \mathcal{P}\uparrow | \mathcal{P}\downarrow \mathcal{N}\downarrow\rangle - \langle \bar{\mathcal{P}}\uparrow \mathcal{P}\uparrow | \bar{\mathcal{N}}\uparrow \mathcal{N}\downarrow\rangle). \quad (5)$$

The justification for this additivity assumption of quark amplitudes is not clear at this time. Our approach to the "independent quark model" is based on the analogy with the independent-particle model of the nucleus in the early days of the nuclear shell model. At that time the success of the model in explaining experimental data seemed incompatible with the obvious fact that nucleons interact with one another with a very strong short-range interaction and cannot move independently in nuclear matter. It was only a number of years later that this point was clarified by the work of Brueckner, Bethe, Weisskopf, and others. In the same way we consider quarks as interacting independently without justification and look for predictions which can be compared with experiment. We feel that the success obtained so far by this approach justifies looking further for other predictions of the model and probing the region of validity of the model by comparison with experiment. A rigorous justification of the use of the model is a much deeper and more difficult problem and can reasonably be left for a later time.

The following example is presented as a guide to intuition in the use of the model. Consider the scattering of a high-energy electron by an α particle in which the α particle receives a momentum transfer without breaking up. The scattering amplitude for this process is given to a very good approximation by the Born approximation since the electromagnetic coupling constant is small. The scattering amplitude is found to have the following form: It is the sum of the amplitudes for scattering of the electron by the individual protons in the α particle multiplied by a form factor or structure factor which is just the Fourier transform of the proton wave functions in the α particle.

In this example the additivity approximation is valid because the process is treated in the Born approximation; i.e., by one-photon exchange. In a similar way it is easily seen that the additivity approximation using the independent-quark model is also valid for scattering of mesons and baryons if the process is treated in Born approximation; i.e., with a one-meson-exchange model. Thus in a theory where hadron scattering processes are described by meson exchange and the fundamental coupling is a quark-quark-meson vertex, the sum of all *possible one-meson exchange* diagrams gives an expression for the scattering amplitude which satisfies our additivity assumption. The effects of absorption can also be accounted for within the framework of this model to the extent that these absorption effects are representable by some kind of "optical potential." In this case the individual quark-quark scattering amplitudes considered are not those of free quark-quark scattering but represent scattering amplitudes between single-particle quark states corresponding to motion in the optical potential. This corresponds to the distorted-wave Born approximation commonly used in studying nuclear reactions. With this picture we expect the model to be valid primarily for reactions which are peripheral and have a strongly forward-peaked angular distribution.

The quark-particle and quark-quark amplitudes at the right-hand side of Eqs. (4) and (5) are unknown parameters, for which invariance under $SU(3)$ may or may not be assumed. (Isospin invariance is always assumed.) In order to obtain relations between cross sections for physical particles, however, one has to eliminate if possible these parameters from the sum of squares of amplitudes like (4) or (5). Such a meson-baryon reaction (and similarly a baryon-baryon reaction) may be represented symbolically by diagrams of the type shown in Fig. 1. Reactions involving double-

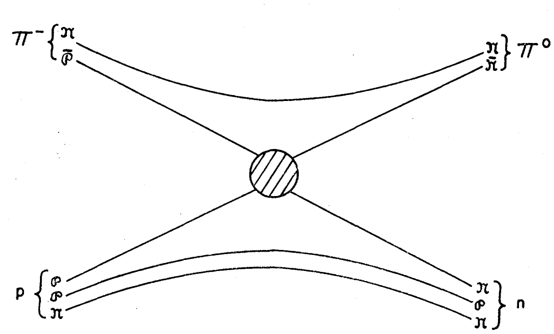

FIG. 1. Typical diagram for inelastic reactions in the quark model.

[7] In particular we will not assume invariance under W spin. It is therefore irrelevant whether one classifies according to $SU(6)_S$ or $SU(6)_W$.

charge and/or double-strangeness exchange clearly are forbidden in this model since then more than one quark state in the initial meson and the initial baryon would have to be changed.

III. $SU(3)$-INDEPENDENT RELATIONS

We first consider a set of relations which do not assume $SU(3)$ and which depend only on the quark model for the mesons. These relations involve cross sections where the target baryon and the produced octet or decuplet baryon are always the same in one particular relation and which can all be expressed in terms of quark-baryon amplitudes as in Eq. (4). These relations therefore are independent of the quark structure of the target *baryons*. Some examples are the following:

$$\bar{\sigma}(\pi^- p \to K^0\Lambda) = (9/4)\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda) - (1/12)\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Lambda)\,, \tag{6a}$$

$$\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Lambda) = 3\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda) + 3\sigma(\bar{p}p \to \bar{Y}_1{}^{*0}\Lambda)\,, \tag{6b}$$

$$\bar{\sigma}(\pi^- p \to K^{*0}\Lambda) = (9/4)\bar{\sigma}(\bar{p}p \to \bar{Y}_1{}^{*0}\Lambda)\,, \tag{6c}$$

$$\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^-\Sigma^+) = \bar{\sigma}(\pi^+ p \to K^+\Sigma^+) + \tfrac{1}{9}\bar{\sigma}(\pi^+ p \to K^{*+}\Sigma^+)\,, \tag{7a}$$

$$\bar{\sigma}(\pi^+ p \to K^{*+}\Sigma^+) = (9/8)\bar{\sigma}(\bar{p}p \to \bar{Y}_1{}^{*-}\Sigma^+)\,, \tag{7b}$$

$$\bar{\sigma}(pp \to nN^{*++}) = \bar{\sigma}(K^+ p \to K^0 N^{*++}) + (25/9)\bar{\sigma}(K^+ p \to K^{*0}N^{*++})\,, \tag{8a}$$

$$\bar{\sigma}(K^+ p \to K^{*0}N^{*++}) = (9/8)\bar{\sigma}(pp \to N^{*0}N^{*++})\,. \tag{8b}$$

The notation $\bar{\sigma}$ means cross sections averaged over all initial and summed over all final polarization states.

In relation (6) the outgoing Λ may be replaced in all cross sections by Σ^0 or $Y_1{}^{*0}$ and similarly in relation (7) the Σ^+ by $Y_1{}^{*+}$. Relations (8) hold also if the target proton is replaced by a neutron and the produced N^{*++} by a proton. The choice of the particular relations (6) to (8) among a variety of similar relations has been made in view of the available experimental data. Each of the sets of cross sections appearing in Eqs. (6), (7), and (8), respectively, are found to depend linearly upon only two independent functions of the two-body amplitudes.[8] One can therefore find three independent relations between the five cross sections appearing in Eq. (6), and two relations between each of the sets of four cross sections in Eqs. (7) and (8).

IV. RELATIONS FROM THE QUARK MODEL WHICH ARE EQUIVALENT TO $SU(6)_W$ PREDICTIONS

It is clear that none of the above relations can be obtained from any of the higher symmetries which put mesons and baryons into different multiplets. It has been shown, however, that for certain classes of meson-baryon reactions like, e.g.,

$$P+B \to P+B\,,$$

the predictions of $SU(6)_W$ hold already in the quark model under the weaker assumption of $SU(3)$ and a subgroup of the Lorentz group including reflections.[9] Examples of this are relations between scattering *amplitudes* like

$$\langle K^- p|\pi^0\Lambda\rangle = \sqrt{3}\langle K^- p|\pi^0\Sigma^0\rangle\,, \tag{9a}$$

$$\langle \pi^- p|K^0\Lambda\rangle = (\sqrt{\tfrac{3}{2}})\langle K^- p|\bar{K}^0 n\rangle\,, \tag{9b}$$

$$\langle \pi^- p|K^0\Lambda\rangle = -\sqrt{3}\langle \pi^- p|K^0\Sigma^0\rangle\,, \tag{9c}$$

etc., which in the quark model hold only, like in $SU(6)_W$, for the *forward* direction. Predictions from $SU(6)_W$ for processes of the form

$$P+B \to P+B^*$$

are irrelevant for the quark model since these are all forbidden in the forward direction.[9]

There are, however, some reactions of the type $P+B \to P+B$ for which the quark model gives stronger relations than $SU(6)_W$. These are relations for cross sections rather than forward scattering amplitudes and which therefore do not follow from $SU(6)_W$. Some examples are the following:

$$\bar{\sigma}(K^+ n \to K^0 p) = 4\bar{\sigma}(K^- p \to \pi^0\Lambda) - 2\bar{\sigma}(K^- p \to \pi^-\Sigma^+)\,, \tag{10}$$

$$\bar{\sigma}(K^- p \to \bar{K}^0 n) = 2(\bar{\sigma}(\pi^- p \to K^0\Lambda) - \bar{\sigma}(\pi^+ p \to K^+\Sigma^+))\,, \tag{11}$$

$$\bar{\sigma}(K^- p \to \pi^0\Lambda) = \tfrac{3}{4}(\bar{\sigma}(K^- p \to \pi^-\Sigma^+) + \bar{\sigma}(K^- p \to \pi^- Y_1{}^{*+}))\,, \tag{12}$$

$$\bar{\sigma}(K^- p \to \bar{K}^0 N^{*0}) = \bar{\sigma}(\pi^+ p \to K^+ Y_1{}^{*+})\,, \tag{13}$$

$$\bar{\sigma}(K^- p \to \pi^- Y_1{}^{*+}) = \tfrac{1}{3}\bar{\sigma}(K^+ p \to K^0 N^{*++})\,. \tag{14}$$

[8] This is easily understood if one notes that the matrix elements between the two meson states (or the two baryon states), in the quark model, behave under spin rotations either as a vector (V_{-1}, V_0, V_1) or as a scalar S. Since the cross sections $\bar{\sigma}$, after averaging and summing over all initial and final magnetic substates, respectively, are scalar quantities in spin space, they can only depend on the two independent scalar operators $|V_{-1}|^2+|V_0|^2+|V_{+1}|^2$ and $|S|^2$. The same is true for relations (6) and (7).

[9] H. J. Lipkin, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, California, 1966), p. 96.

In relations (9b), and (11) to (14), use has been made of $SU(3)$ invariance of the quark amplitudes, whereas (9a) and (9c) are independent of $SU(3)$. All relations (9) to (14) depend only on the quark model for the baryons and are independent of the quark constitution of the mesons.

The relations between these predictions and those of $SU(6)_W$ can be seen as follows. From the diagram of Fig. 1, it is evident that the $SU(6)_W$ coupling in the t channel is restricted to those representations which are present in quark-antiquark scattering; i.e., to the singlet and 35. For reactions involving charge or strangeness exchange the singlet is excluded and only the 35 remains.

V. PHASE-FACTOR AND FORM-FACTOR CORRECTIONS AND COMPARISON WITH EXPERIMENT

In contrast to relations for elastic forward scattering amplitudes where the momentum transfer is exactly zero, these relations can clearly not be expected to hold, in the form given here, in the presence of appreciable mass differences between the initial and final states and of nonforward contributions to the cross sections. When comparing them with experiment we must apply two corrections: The first is a trivial kinematic correction for phase space and accounts for the main part of the mass differences between the initial and the final states and between different final states in related reactions. The second correction is a "structure factor" which accounts for the *spatial* structure, in terms of the quark model, of the colliding (composite) particles and is defined in analogy to the Debye-Waller factor in x-ray scattering and the form factor in the scattering of high-energy electrons by complex nuclei (cf. the example of electron scattering by an α particle). Consider, for example, the scattering of a π^- meson by a proton in the center-of-mass system. The colliding quarks (in the example of Fig. 1 the $\mathcal{P}$ quark of the proton and the $\bar{\mathcal{P}}$ of the pion) both get an additional momentum.

$$\mathbf{\Delta}=\mathbf{p}_{\text{in}}{}^{\text{c.m.}}-\mathbf{p}_{\text{out}}{}^{\text{c.m.}}, \tag{15}$$

where $\mathbf{p}_{\text{in}}{}^{\text{c.m.}}$ and $\mathbf{p}_{\text{out}}{}^{\text{c.m.}}$ are the center-of-mass momenta of the incoming and outgoing particles. Since both quarks are bound in composite particles the scattering cross section has to be multiplied by a form factor for the pion and a form factor for the proton. For the scattering of a free quark by a physical particle the form factor is given by

$$F_S(\mathbf{\Delta})=\langle\psi_f{}^S|\exp(i\mathbf{\Delta}\cdot\mathbf{x})|\psi_i{}^S\rangle, \tag{16}$$

where $\psi_i{}^S$ and $\psi_f{}^S$ are the initial and final bound-state wave functions and $\mathbf{x}$ is the coordinate of the interacting quark in the particle S. Any πp reaction, e.g., then has to be multiplied by the factor

$$F_\pi(\mathbf{\Delta})F_p(\mathbf{\Delta}). \tag{17}$$

It is well known that for moderate values of the four-momentum transfer t and sufficiently high energy all *elastic* meson-meson and meson-baryon differential cross sections fit well to an exponential

$$e^{A_S t}=\exp(-A_S\cdot\mathbf{\Delta}^2), \quad (t=\mathbf{\Delta}^2) \tag{18}$$

with exponents A_S which are fairly energy-independent. This in fact suggests that the t dependence of these cross sections is dominated by the form factor and that the angular dependence of quark-quark elastic scattering can be neglected. Since in the quark model inelastic reactions differ from the elastic channel only through the fact that quarks exchange (internal) quantum numbers, we assume that this situation also applies to the *inelastic* channels.

The actual values of the exponent A_S are taken from experimental data for elastic scattering. Some typical values are[10]

$$\begin{aligned}(A_\pi+A_p)&\approx 8\ (\text{GeV}/c)^{-2},\\(A_K+A_p)&\approx 8\ (\text{GeV}/c)^{-2},\\(A_{\bar{p}}+A_p)&\approx 13\ (\text{GeV}/c)^{-2},\\(A_p+A_p)&\approx 10\ (\text{GeV}/c)^{-2}.\end{aligned} \tag{19}$$

Thus the form-factor correction does not introduce any additional free parameters and is uniquely defined for each reaction.

Following the prescriptions of Meshkov, Snow, and Yodh[11] the comparison of related cross sections with experiment is done at the same Q values, Q being the kinetic energy of the outgoing particles. Each experimental cross section then has to be divided by the total correction

$$F=\frac{p_{\text{out}}{}^{\text{c.m.}}}{S p_{\text{in}}{}^{\text{c.m.}}}\exp[-(A_B+A_M)\mathbf{\Delta}^2] \tag{20}$$

in order to obtain the quantities

$$\bar{\sigma}=\sigma_{\text{exp}}/F \tag{21}$$

which appear in the quark-model relations.

In Figs. 2, 3, and 4 we have plotted the correction $C=F^{-1}$ in the forward direction for various meson-baryon and baryon-baryon reactions as a function of Q. The solid lines represent the phase-space correction alone, the dashed lines the *total* correction (20) including the form factor.

The forward direction form factors have been used in all calculations, rather than averages over angular distributions. This introduced a negligible error for reactions which are strongly forward-peaked as indicated by the expression (20).

All curves have been divided by the correction for

[10] S. Focardi *et al.* Phys. Letters **19**, 441 (1965); M. L. Perl *et al.* Phys. Rev. **132**, 1252 (1963); C. Czyzewski *et al.*, Phys. Letters **15**, 188 (1965).

[11] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

the charge-exchange process $\pi^- p \to \pi^0 n$. Let δm be the mass difference between the initial and the final states of any one of these reactions. The flat curves correspond to low states of δm, the steep curves to high δm. The comparison with experiment of related cross sections with widely different δm's, therefore, is meaningful only at sufficiently high Q values, say $Q > 1$ GeV, where the corrections in all reactions have approximately the same Q dependence. This applies, for example, to our relation (11). For those relations, however, where all mass differences δm are roughly the *same* a comparison even at lower Q values, say $0.4 < Q < 1$ GeV, may still be justified. This applies, e.g., to the relations (6c), (7b), and (8b) which are discussed below.

We now proceed to the comparison of these relations with experiment. In relation (6b) the cross section for

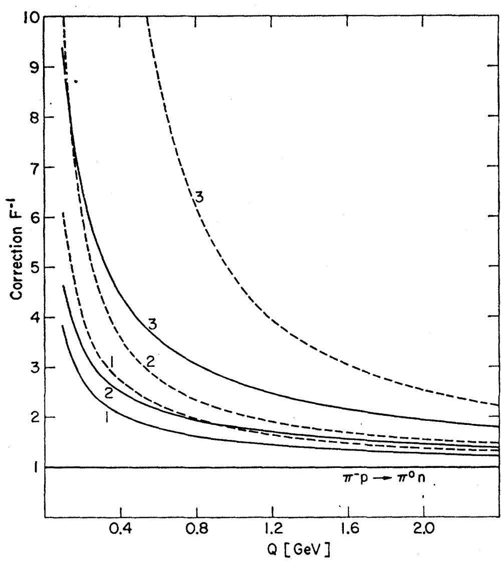

FIG. 2. Corrections to $\pi^- p$ cross sections with (dashed lines) and without (solid lines) form factor. 1: for $\pi^- p \to \pi N^*$, 2: for $\pi^- p \to K^0\Lambda$ and 3: for $\pi^- p \to K^{*0}\Sigma^0$. All curves have been divided by the correction for the reaction $\pi^- p \to \pi^0 n$.

$\bar{p}p \to \bar{Y}_1^{*0}\Lambda$ is known to be much smaller than the other two[12,13]; relation (6b) therefore can be simplified to

$$\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Lambda) \approx 3\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda). \qquad (6b')$$

Using this, relation (6a) then reduces to the simpler relation

$$\bar{\sigma}(\pi^- p \to K^0\Lambda) \approx 2\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda). \qquad (6a')$$

Experimental data for relations (6a′) and (6b′) are plotted in Fig. 5. Both relations are in rough agreement with experiment,[12–14] but clearly more data at higher

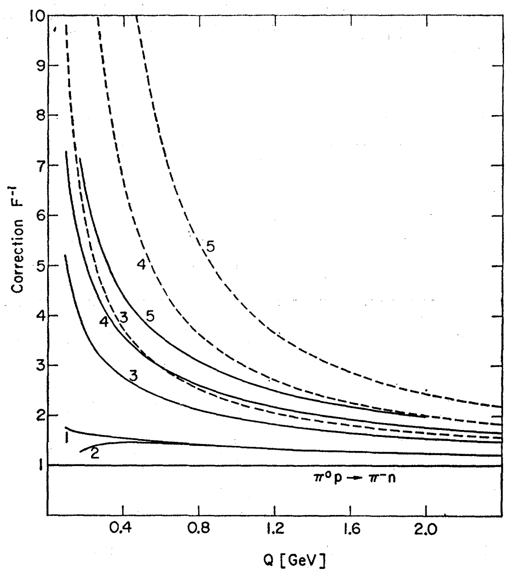

FIG. 3. Corrections to $K^- p$ cross sections with (dashed lines) and without (solid lines) form factor. 1: for $K^- p \to \bar{K}^0 n$, 2: for $K^- p \to \pi^-\Sigma^+$, 3: for $K^- p \to \bar{K}^0 N^{*0}$, 4: for $K^- p \to \rho^-\Sigma^+$ and 5: for $K^- p \to K^{*-}N^{*+}$. For 1 and 2 the solid and dashed lines coincide. All curves are divided by the correction for $\pi^- p \to \pi^0 n$.

Q values are needed. For relation (6c), as well as relations (7b) and (8b), there is little information about the $\bar{p}p$ cross sections on the right-hand sides. The only experiment known to us (for such Q values where data for the meson-baryon cross sections exist) gives cross sections which are averaged over different momenta of the incident antiproton and which therefore cannot be used for a detailed comparison.[12] Table I lists some predicted $\bar{p}p$ cross sections on the basis of relations

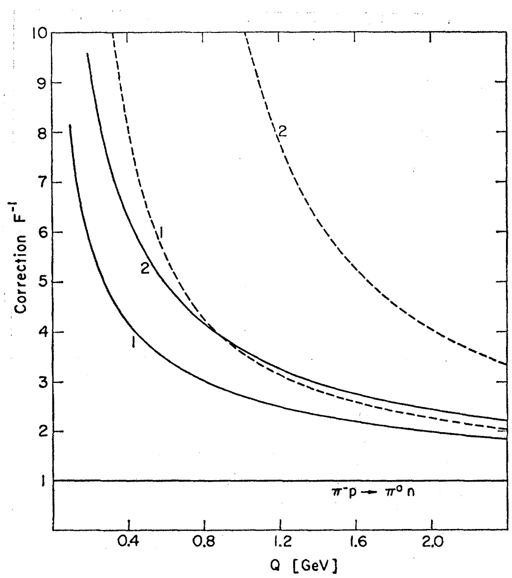

FIG. 4. Corrections to $\bar{p}p$ reactions with (dashed lines) and without (solid lines) form factor. 1: for $\bar{p}p \to \bar{\Lambda}\Lambda$ and 2: for $\bar{p}p \to \bar{Y}_1^*\Sigma$. The curves are divided by the correction for $\pi^- p \to \pi^0 n$.

[12] B. Musgrave *et al.*, Nuovo Cimento **35**, 735 (1965).
[13] C. Baltay *et al.*, Phys. Rev. **140**, B1027 (1965).
[14] D. H. Miller *et al.*, Phys. Rev. **140**, B360 (1965); G. A. Smith *et al.*, in *Proceedings of the Second Topical Conference on Resonant Particles* (Ohio University, Athens, Ohio, 1965); T. P. Wangler, A. R. Erwin, and W. D. Walker, Phys. Rev. **137**, B414 (1965).

TABLE I. Some $\bar{p}p$ cross sections predicted on the basis of relations (6c), (7b), and (8b). F_M and F_B are the phase-space and form-factor corrections for the meson-baryon and baryon-baryon reactions, respectively [cf. Eq. (20)].

Q [GeV]	$p_{\text{meson}}^{\text{lab.}}$ [GeV/c]	Reaction	Expt. cross section [μb]	Reference	F_B/F_M	Predicted cross section [μb]		$p_{\text{baryon}}^{\text{lab.}}$ [GeV/c]	Eq.
0.44	2.7	$\pi^-p \to K^{*0}\Lambda$	53±8	a	0.39	$\bar{p}p \to \bar{Y}_1^{*0}\Lambda$	9.2±1.4	3.5	(6c)
0.36	2.7	$\pi^-p \to K^{*0}\Sigma^0$	52±8	a	0.24	$\bar{p}p \to \bar{Y}_1^{*0}\Sigma^0$	5.5±0.9	3.5	(6c)
0.82	4.0	$\pi^+p \to K^{*+}\Sigma^+$	23±7	b	0.41	$\bar{p}p \to \bar{Y}_1^{*-}\Sigma^+$	8.4±2.6	5.1	(7b)
0.43	3.5	$\pi^+p \to K^{*+}Y_1^{*+}$	<20±8	c	0.15	$\bar{p}p \to \bar{Y}_1^{*-}Y_1^{*+}$	<2.7±1.4	4.4	(7b)
0.49	3.0	$K^+p \to K^{*0}N^{*++}$	1660±290	d	0.63	$\bar{p}p \to N^{*0}N^{*++}$	924±16	3.6	(8b)

a Reference 12.
b Reference 14.
c See reference in S. Meshkov *et al.*, Phys. Letters **12**, 87 (1964).
d Reference 16.

(6c), (7b), (8b) and the available data on the meson-baryon cross sections on the left-hand sides. An accurate measurement of these $\bar{p}p$ reactions at the corresponding $\bar{p}$ momenta should provide a sensitive test of the model.

Replacing the outgoing Λ by Σ^0 in relations (6a) and (6b), one easily derives from these two relations (trying to eliminate those cross sections for which no data are available) the *inequality*

$$\bar{\sigma}(\pi^-p \to K^0\Sigma^0) < \tfrac{2}{3}\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Sigma^0). \qquad (22)$$

In Fig. 6 we have plotted some experimental data for these cross sections. These data are in clear disagreement with relation (21).

In relation (7a) the cross section for the reaction $\pi^+p \to K^{*+}\Sigma^+$ with the factor $\frac{1}{9}$ is again negligible compared to the left-hand side so that the relation reduces to

$$\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^-\Sigma^+) \approx \bar{\sigma}(\pi^+p \to K^+\Sigma^+). \qquad (7a')$$

Experimental data for relation (7a′) are plotted in Fig. 7. Although the data have the correct order of magnitude, a detailed comparison is not yet possible since there are only two experimental points on the reaction $\pi^+p \to K^+\Sigma^+$ at 2.77 GeV/c [15] and 4.0 GeV/c.[16] Relation (8a) cannot yet be compared with experiment since the reported data have widely different Q values.[17,18]

We now turn to the discussion of relations (9) to (14). Relations for *forward* scattering like our relations (9) have already been discussed in the context of $SU(6)_W$ predictions and do not, in general, seem to agree with experiment. It has been pointed out, however, that owing to the large experimental uncertainties for scattering in the forward direction the actual values of the cross

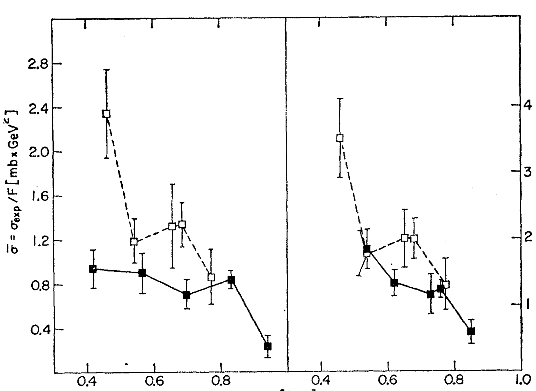

FIG. 5. (a) Experimental test of relation (6a′). ○ $=2\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda)$, ■ $=\bar{\sigma}(\pi^-p \to K^0\Lambda)$. The data for the $\bar{p}p$ cross sections are taken from Ref. 13 and from B. Musgrave *et al.*, *Proceedings of the Sienna International Conference on Elementary Particles and High Energy Physics, 1963*, edited by G. Bernardini and C. P. Puppi (Societa Italiana di Fisica, Bologna, 1963), p. 301. The cross sections for the π^-p reactions are taken from Refs. 14. (b) Experimental test of relation (6b′) ○ $=3\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda)$, ■ $=\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Lambda)$. The data for $\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Lambda)$ are taken from Ref. 12. For $\bar{\sigma}(\bar{p}p \to \bar{\Sigma}^0\Lambda)$ see caption (a).

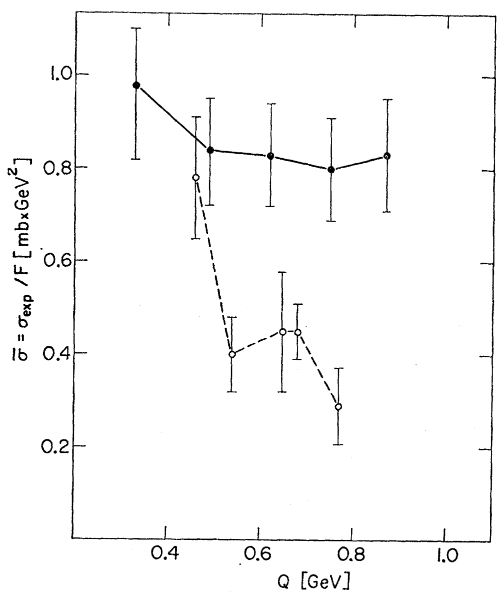

FIG. 6. Experimental test of the inequality (22). ○ $=\frac{2}{3}\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Sigma^0)$, ● $=\bar{\sigma}(\pi^-p \to K^0\Sigma^0)$. The data for $\bar{\sigma}(\pi^-p \to K^0\Sigma^0)$, are taken from Ref. 14; for $\bar{\sigma}(\bar{p}p \to \bar{\Lambda}\Sigma^0)$ see caption of Fig. 5.

15 S. S. Yamamoto *et al.*, Phys. Rev. **134**, B383 (1964).
16 Aachen-Hamburg-London-Muenchen Collaboration (to be published).
17 G. A. Smith *et al.*, Phys. Rev. **123**, 2160 (1961); M. Ferro-Luzzi *et al.*, Nuovo Cimento **36**, 1101 (1965); G. Goldhaber *et al.*, Phys. Letters **6**, 62 (1963); and G. Alexander (private communication).
18 M. Ferro-Luzzi *et al.*, Nuovo Cimento **39**, 417 (1965).

section at $\theta=0°$ may well differ appreciably from the reported values which are averaged over a finite interval of θ, in which the cross section is a rapidly varying function of the angle.[19] As long as the cross sections have not been correctly extrapolated to $\theta=0°$ a fair comparison is clearly not possible.

For the remaining relations (10) to (14) which explicitly assume invariance under $SU(3)$ there are as yet only a few data at sufficiently high energies and comparable Q values, and a meaningful test of these relations is not yet possible.

VI. REVIEW OF ASSUMPTIONS AND RELATIONS WITH SYMMETRIES

Because the foundations of this model are unclear it is advisable to keep track of the various assumptions which are used to obtain experimental predictions. In particular the minimum set of assumptions necessary to obtain any given prediction should be noted, since it is only these assumptions which are tested by comparison of the prediction with experiment. For example, some relations are obtainable between different meson-baryon scattering amplitudes which depend only on the assumption of the quark model and additivity for the meson and are independent of the structure of the baryon. Some relations are obtainable without any assumptions of relations between the individual two-body scattering amplitudes; other relations assume invariance under $SU(3)$.

In addition to explicit assumptions of the type mentioned above there are certain implicit symmetry assumptions which are inherent in the model. These involve relations between wave functions used for different particles and the assumption that the two-body quark scattering amplitude is independent of the particular state in which the quark is bound. Let us examine these assumptions in more detail in order to see to what extent they effectively imply the assumption of some higher symmetry. We first consider the hadron wave functions. If we use the Born-approximation model as a guide we see that the exact form of the wave functions for a given particle enters only in the corresponding form factor. This is an overlap integral of the form

$$\int \psi_f(\exp i\mathbf{\Delta}\cdot\mathbf{x})O\psi_i d^3x, \qquad (23)$$

where ψ_i and ψ_f are the initial- and final-state wave functions for the particle, $\mathbf{\Delta}$ is the momentum transfer, $\mathbf{x}$ is the coordinate of the particular quark making the transition, and O is the appropriate operator which changes the internal quantum number of the quark; i.e., spin, strangeness, or charge. In the present treatment the meson wave functions used are assumed to have the same radial behavior for all 36 states and the

[19] A. Dar (private communication).

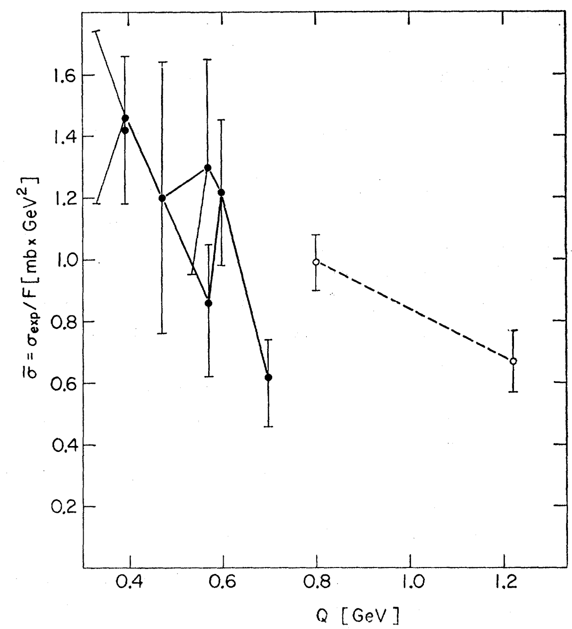

FIG. 7. Experimental data for relation (7a′). ○ $=\bar{\sigma}(\pi^+p\to K^+\Sigma^+)$ ● $=\bar{\sigma}(\bar{p}p\to\bar{\Sigma}^-\Sigma^+)$. The data for $\bar{\sigma}(\pi^+p\to K^+\Sigma^+)$ are taken from Refs. 15 and 16, the data for $\bar{\sigma}(\bar{p}p\to\bar{\Sigma}^-\Sigma^+)$ from Refs. 12, 13, and, from R. Armenteros *et al.*, in *Proceedings of the International Conference on High Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 236.

quark spin couplings are uniquely determined by the spin of the meson. The baryon wave functions are assumed to have the same radial behavior for all 56 states and spin couplings defined by $SU(6)$ wave functions. However, many predictions do not require such stringent assumptions. For example,

(1) Relations for forward elastic-scattering amplitudes are independent of the form of the wave functions since the initial and final wave functions are the same, $\mathbf{\Delta}=0$, and the form-factor integral is unity for all wave functions.

(2) Relations between elastic-scattering amplitudes at finite momentum depend upon a form factor which is essentially a Fourier transform of the wave function and is characterized by the radial size of the bound state. The experimental observation that the momentum-transfer dependence of elastic-scattering processes is independent of charge and strangeness support this assumption for mesons and nucleons. Note that no such experimental evidence as yet exists for hyperons.

(3) Relations for charge-exchange scattering processes require the same assumptions about the wave functions as elastic-scattering processes with the additional assumption of isospin which is accepted as a good symmetry for strong interactions.

(4) Strangeness-exchange processes involve overlap integrals between states differing in strangeness. If $SU(3)$ symmetry is broken and the spatial wave func-

tions of nucleons and hyperons are different, the overlap integrals for strangeness exchange processes will be reduced. Since $SU(3)$ symmetry is known to be broken in the direction of reducing strangeness-exchange processes, it is tempting to look at these overlap integrals as a possible explanation. However, in this work we have considered relations only between sets of processes which all involve strangeness exchange or all do not involve strangeness exchange, and the problem of the difference in overlap integrals does not arise.

(5) The amplitudes for vector-meson production involve overlap integrals between the vector and pseudoscalar radial wave functions.

(6) All inelastic amplitudes involve an overlap integral between the initial and final baryon states. These integrals depend on the spin coupling of the individual quark making the transition. The assumption that the baryon spin couplings are those given by $SU(6)$ is important for these processes.

The assumption that the effective two-body scattering amplitude is independent of the states in which the individual quarks are bound is a nontrivial assumption. If these amplitudes are not the free two-body scattering amplitudes but effective amplitudes "renormalized" by the binding or some average optical potential, one might expect such renormalization effects to be the same for all mesons and the same for all baryons and neglect the variation with the internal quantum numbers, charge strangeness, and spin of the states. However, the difference between a two-particle and a three-particle bound state seems sufficiently significant to cast considerable doubt on the assumption that these effects should be the same in a meson and a baryon. The successful comparison of meson-baryon and baryon-baryon total cross sections in Ref. 2, however, seem to indicate that this assumption is valid. Note, however, that the agreement obtained in relations between meson-baryon and baryon-baryon scattering is of the order of 10 to 15% and is not as good as the agreement obtained for relations involving only meson-baryon cross sections.

VII. CONCLUSIONS

We have given here a few examples of relations for inelastic reactions which follow from the quark model and the additivity assumption of quark-quark and quark-antiquark amplitudes. The first group of relations [Eqs. (6) to (8)] follow directly from the quark model without assuming any higher symmetry. The second group of relations [Eqs. (9a), (9b), and Eqs. (10) to (14)] are derived with the additional assumption of invariance under $SU(3)$. Among these, as expected from the general analysis given in Ref. 9, the relations for *forward* scattering are familiar from the $SU(6)_W$ symmetry. The other relations, however, which hold for all scattering angles are new and do not follow from $SU(6)_W$. The agreement of our relations with experiment is fair except in the case of the inequality (22), which is in disagreement with the experimental data. The discussion of the phase-space and form-factor corrections to the cross sections shows, however, that reactions with large mass differences should only be compared at rather high Q values, say $Q>1$ GeV. The existing data have Q values between 0.3 and 1 GeV and are thus not sufficient yet to achieve a fair test of the model and to distinguish clearly between the quark model and any of the higher symmetries like, e.g., $SU(6)_W$. More data at higher energies are needed.

ACKNOWLEDGMENTS

We wish to thank S. Meshkov, G. Snow, and G. Alexander for useful discussions.