New Determination of the P' Regge-Trajectory Intercept*

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A precise value of the P' Regge-trajectory intercept at $t=0$ is calculated using a modified form of the Igi sum rule and new data on pion-nucleon total cross sections.

RECENTLY, new experimental data on pion-nucleon total cross sections have appeared in the literature. This paper reports on the use of these data with the Igi sum rule[1] in an attempt to measure the possible Mandelstam-cut[2] contribution to the scattering amplitude at high energy. The high-energy cross-section data,[3] however, are not sufficiently accurate to give a precise value[4] of the P' Regge-trajectory intercept at $t=0$. (t is the crossed-channel total energy squared.) Consequently, the sum rule was used to determine this intercept. The optimum value of $\alpha_{P'}(0)$ was found to be $\alpha_{P'}(0)=0.69\pm0.01$.[5] If future high-energy data become exact enough to determine $\alpha_{P'}(0)$ independently, and if this value is found to be in conflict with our determination, we can then obtain a measure of the Regge-cut contribution to the sum rule.

The sum rule we used is essentially Igi's, but our formulas are somewhat simpler. The amplitude which describes forward ($t=0$) scattering in the s channel is[6]

$$F^+(\omega)=A^+(\omega)+\omega B^+(\omega)\,, \tag{1}$$

where ω is the pion total energy in the laboratory system, and $F^+(\omega)$ is related to the πN total cross sections (σ_T) by the optical theorem

$$\mathrm{Im}F^+(\omega)=\tfrac{1}{2}(\omega^2-\mu^2)^{1/2}[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)]\,; \tag{2}$$

$F^+(\omega)$ corresponds to pure isotopic spin$=0$ exchange in the t channel. We assume Regge behavior for the amplitude at high energy:

$$F^+(\omega)\to\sum_i -C_i\frac{e^{-i\pi\alpha_i}+1}{\sin\pi\alpha_i}\left(\frac{\omega}{\omega_0}\right)^{\alpha_i}. \tag{3}$$

Only even-signature trajectories contribute because of Bose statistics in the t channel. ω_0 is a scale factor taken to be 1 BeV in the numerical calculation, so that the C_i will correspond to those of Rarita and Phillips.[7] The arguments of C_i and α_i are $t=0$, and will be omitted. Both these functions are real in this region.

Using the normal dispersion relation for $F^+(\omega)$ and the fact that

$$\frac{1}{\pi}\int_0^\infty\frac{x^\alpha}{x-z}=-\frac{(-z)^\alpha}{\sin\pi\alpha}\,, \tag{4}$$

we can write

$$\begin{aligned}F^+(\omega)=\text{pole}+\frac{1}{\pi}\int_\mu^\infty d\omega'\left[\frac{1}{\omega'-\omega}+\frac{1}{\omega'+\omega}\right]\Bigg[\mathrm{Im}F^+(\omega')\\ -\sum_i C_i\left(\frac{\omega'}{\omega_0}\right)^{\alpha_i}\Bigg]-\sum_i C_i\Bigg\{\frac{1}{\pi}\int_0^\mu\left[\frac{1}{\omega'-\omega}+\frac{1}{\omega'+\omega}\right]\\ \times\left(\frac{\omega'}{\omega_0}\right)^{\alpha_i}d\omega'+\frac{(-\omega)^{\alpha_i}+\omega^{\alpha_i}}{(\omega_0)^{\alpha_i}\sin\pi\alpha_i}\Bigg\}\,.\end{aligned} \tag{5}$$

The second bracket in the first integral is expected to vanish in the Regge region, so we cut off the integral at x, which we arbitrarily take to be 6 BeV. The results of the calculation were found to be insensitive to variations of x down to values as low as 3 BeV. We can then write, using Eq. (2),

$$\mathrm{Re}F^+(\omega)=\text{pole}+I(\omega)-\sum_i C_iR(\alpha_i,\omega)\,, \tag{6}$$

where

$$I(\omega)=\frac{1}{\pi}P\int_\mu^x d\omega'\frac{\omega'(\omega'^2-\mu^2)^{1/2}}{\omega'^2-\omega^2}[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)]\,,$$

$$\text{pole}=\frac{g^2}{m}\frac{\omega_p^2}{\omega_p^2-\omega^2}\,,\quad \omega_p=-\frac{\mu^2}{2m}\,,\quad \frac{g^2}{4\pi}=14.6\pm0.2\,, \tag{7}$$

$$R(\alpha,\omega)=\frac{1}{\pi}P\int_0^x\frac{2\omega'}{\omega'^2-\omega^2}\left(\frac{\omega'}{\omega_0}\right)^\alpha d\omega'+\left(\frac{\omega}{\omega_0}\right)^\alpha\cot\tfrac{1}{2}\pi\alpha\,.$$

At $\omega=\mu$, threshold, $R(\alpha,\omega)$ can easily be evaluated analytically for $\alpha=1$ and $\alpha=\frac{1}{2}$. With $x\gg\mu$, $R(1,\mu)\approx(2/\pi)(x/\omega_0)$ and $R(\frac{1}{2},\mu)\approx(2/\pi)2(x/\omega_0)^{1/2}$, which suggests that we can approximate $R(\alpha,\mu)$ by the formula

$$R(\alpha,\mu)=(2/\pi)(1/\alpha)(x/\omega_0)^\alpha\,. \tag{7$'$}$$

At $\alpha=0.69$, Eq. (7′) agrees with Eq. (7) to 1 part in 4000. The use of this simple formula for $R(\alpha,\mu)$ elim-

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[3] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).
[4] The total-cross-section data alone could be fitted with a value of $\alpha_{P'}(0)$ ranging from 0.8 to 0.2.
[5] The error is determined by the usual procedure of finding the values of the varied parameter $\alpha_{P'}$ which give an upward change of one unit in the optimum χ^2. (See Table I in the text.) Perhaps a more realistic estimate would be $\alpha_{P'}(0)=0.69\pm0.05$.
[6] The formalism for πN scattering can be found in, for example, V. Singh, Phys. Rev. **129**, 1889 (1963).
[7] W. Rarita and R. Phillips, Phys. Rev. **139**, B1336 (1965).

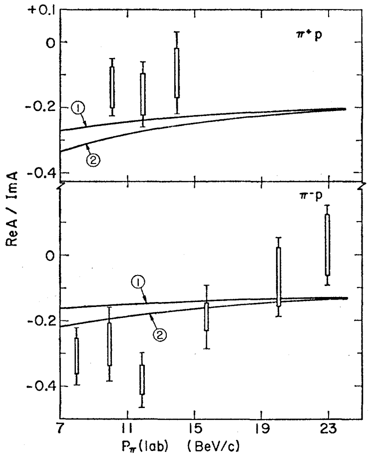

FIG. 1. The ratio of the real to the imaginary part of the forward scattering amplitude for $\pi^\pm p$ scattering. Curve 1 is the best fit to all the data obtained in the text. Curve 2 is solution 1 of Ref. 7. Data are from Ref. 11, where the error bars are explained.

inates the need for a numerical integration of Eq. (7) for each value of α chosen in the sum rule below [Eq. (8)]. For $\omega=\mu$, $\mathrm{Re}F^+(\mu)=4\pi(1+\mu/m)a_+$, where $a_+=\frac{1}{3}(a_1+2a_3)$ and a_1 and a_3 are the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering lengths. In Eq. (6) only the P and the P' trajectories are considered, denoted by subscripts 1 and 2, respectively. This gives the sum rule, assuming $\alpha_1=1$,

$$4\pi(1+\mu/m)a_+=\text{pole}+I(\mu)-C_1R(1,\mu)-C_2R(\alpha_2,\mu)\,. \quad (8)$$

The pole term with[8] $g^2/4\pi=14.6\pm0.2$ gives $-(0.152\pm0.002)$ in pion Compton wavelengths. All other numbers quoted will also be in these units. $I(\mu)$ was evaluated numerically,[9] giving 44.6 ± 0.3. Equation (8) with Eq. (7′) for the P' trajectory was put into the Rarita-Phillips program,[7] and the experimental scattering length[8] $a_+=0.0013\pm0.0030$ was taken as another datum point to be fitted. The best fit to all the data occurred for $\alpha_2=0.69$, $C_1'=5.40$, $C_2'=12.0$, $C_3'=2.43$. (The C_i' here are in BeV mb.) Subscript 3 denotes the ρ and $C_i'=C_i/\alpha_i(2\alpha_i+1)$. The variation of χ^2 with α_2 is shown in Table I. The fit corresponds to solution 1 of Ref. 7, but with the extra datum point, giving 335 πN data points with 21 adjustable parameters.

This method of determining α_2 seems much more accurate than a method in which one considers only a power-law decrease of the high-energy total cross sections. We were able to fit the total-cross-section data[3] alone with a wide range of values of α_2,[4] contrary to what has been stated in a recent paper by Sertorio and Toller.[10] It was also stated in that paper that $\mathrm{Re}F^+(\omega)$ at high energy is not given accurately by the Regge term $C_2(\omega/\omega_0)^{\alpha_2}\cot\frac{1}{2}\pi\alpha_2$. We have calculated the additional terms

TABLE I. Variation of χ^2 with α_2.

α_2	0.65	0.66	0.67	0.68	0.69	0.70	0.71
χ^2	508	503	500	499	497	499	528

$$\text{pole}+I(\omega)-\frac{C_1}{\pi}P\int_0^x d\omega'\frac{2\omega'}{\omega'^2-\omega^2}\left(\frac{\omega'}{\omega_0}\right)^{\alpha_1}-\frac{C_2}{\pi}P\int_0^x d\omega'\frac{2\omega'}{\omega'^2-\omega^2}\left(\frac{\omega'}{\omega_0}\right)^{\alpha_2}. \quad (9)$$

For $\omega=7$ BeV, we find formula (9) equals 0.03 ± 0.02 with $C_2(\omega/\omega_0)^{\alpha_2}\cot\frac{1}{2}\pi\alpha_2=15.3$, so that $\mathrm{Re}F^+(\omega)$ is indeed given by the Regge term alone in this region.

With the parameters from the $\alpha_2=0.69$ fit we checked the sum rule for the amplitude corresponding to $I=1$ exchange in the t channel, keeping only the ρ trajectory in the calculation. We obtained a scattering length of $a_-=(9.64\pm0.90)\times10^{-2}$ compared to the experimental value of[8] $a_-=(8.77\pm0.20)\times10^{-2}$.

The total cross section and the ratio of the real to the imaginary part of the elastic scattering amplitude for both π^+p and π^-p can easily be calculated in terms of Regge poles. This has been done for three sets of parameters: (a) best fit, described above; (b) solution 1 of Rarita and Phillips[7]; (c) best fit to the total cross section data from 6 to 20 BeV only,[3] with $\alpha_2=0.5$. The curves for the total cross sections are all essentially the same, and fit the data well (see Fig. 1 of Ref. 7). They continue to do so down to about $\omega=3$ BeV, where resonance formation begins. The curves for the ratio of the real to imaginary part of the amplitude for the first two sets of parameters are plotted in Fig. 1 with the experimental data.[11] The curve for parameter set (c) is essentially the same as that for parameter set (b). The curves fit the ratio equally badly, a fate of most fits to these particular data.

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