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Photonic Decay Rates and Nuclear-Coulomb-Field Coherent Production Processes

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The relationship between the $A \to B+\gamma$ photonic decay process and the "inverse" nuclear-Coulomb-field coherent production process $B+Z \to B+\gamma'+Z \to A+Z$ [$Z\equiv$high-Z nucleus, $\gamma'\equiv$exchanged virtual Coulomb photon] is studied in some detail for *arbitrary* particles (A,B). A procedure is described for the determination of the rate of $A \to B+\gamma$ from measurement of the differential cross section for A-particle coherent production by a B particle incident on a high-Z nucleus. A numerical application of the formulas derived is worked out in the case $K^-+Z \to K^{*-}+Z$, and it is estimated that a K^- energy in the range 3–15 BeV is required for a successful determination of the rate of $K^{*-} \to K^-+\gamma$.

I. INTRODUCTION

It has been suggested that the rate of photonic decay $A \to B+\gamma$, with $(A,B)=(\pi^0,\gamma)$, (η,γ), $(\Sigma^0,\Lambda)(\rho^\pm,\pi^\pm)$, or $(K^{*\pm},K^\pm)$, can be determined by measurement of the differential cross section for A-particle coherent production by a B particle incident upon a high-Z nucleus.[1–5] This general conclusion follows because the coherent production process $B+Z \to A+Z$ is dominated by the photon-exchange pole for sufficiently small momentum transfer to the nucleus and because a type of microscopic reversibility holds for electromagnetic processes [see Eq. (2.3) below]. We shall denote the photon-exchange process by $B+Z \to B+\gamma'+Z \to A+Z$, where γ' is the exchanged virtual photon associated with the Coulomb field of the nucleus Z.

An essential feature possessed by all the processes listed above is the common angular dependence of the coherent production cross sections. With the exception of the $\Lambda+Z \to \Sigma^0+Z$ process, they behave, apart from the factor associated with the photon propagator, like θ^2 for small values of the production angle $\theta(\theta\equiv\cos^{-1}(\hat{p}_B\cdot\hat{p}_A))$, and even the $\Lambda+Z \to \Sigma^0+Z$ process has this behavior provided that $(m_\Lambda/E_\Lambda)^2 \ll 1$. If the nucleus Z is regarded as a charged spin-zero particle, then, with the exception of the $\Lambda+Z \to \Sigma^0+Z$ case, it is possible to infer the θ^2 behavior without specification of the detailed mechanism of virtual-photon exchange, i.e., this behavior is a property of the conservation laws characterizing the over-all reaction. In particular, for $\gamma+Z \to \pi^0+Z$ or $\gamma+Z \to \eta+Z$, angular-momentum conservation implies that the corresponding coherent photoproduction amplitudes vanish in the forward, i.e., $\theta=0$, direction so that the θ^2 dependence is justified; for $\rho(K^*)$ mesons, which cannot be produced with zero helicity by $\pi(K)$ mesons incident on a spin-zero nucleus because of parity conservation, an auxiliary argument, very similar to that used for the case of $\gamma+Z \to \pi^0+Z$, justifies the θ^2 dependence. In contrast, for the $\Lambda+Z \to \Sigma^0+Z$ case, a general inference regarding the small θ behavior cannot be made; in fact, without some examination of the mechanism of coherent production, one would anticipate a behavior like θ^0 rather than θ^2.

We shall develop and extend the Weizäcker-Williams approximation[6] in a manner which clearly shows that it is a consequence of *electromagnetic current conservation* alone that this θ^2 behavior holds for any $B+Z \to A+Z$ nuclear-Coulomb-field coherent-production process at high incident energies and small production angles. In particular, we shall see that in this limit only *transverse* virtual photons (as viewed in the rest frame of A) are important, and that as a consequence the helicities of A and B differ by one unit; this circumstance, together with angular-momentum conservation, is in fact responsible for the θ^2 dependence.

Proceeding along similar lines, we shall also obtain a relation between the cross section for $B+\gamma \to X$ and the nuclear-Coulomb-field production cross section, $B+Z \to B+\gamma'+Z \to X+Z$, where X represents an arbitrary set of particles. If one thinks of X as representing the decay products of particle A, this relation is particularly

[1] (π^0,γ): H. Primakoff, Phys. Rev. **81**, 899 (1951); V. Glaser and R. A. Ferell, *ibid.* **121**, 886 (1961); C. Chiuperi and G. Morpurgo, Nuovo Cimento **19**, 497 (1961).

[2] (η,γ): C. M. Andersen, A. Halprin, and H. Primakoff, Phys. Rev. Letters **9**, 512 (1962); G. Belletini, C. Bemporad, P. L. Braccini, L. Foà, and M. Toller, Phys. Letters **3**, 170 (1963).

[3] (Σ^0,Λ): J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).

[4] $(\rho^\pm,\pi^\pm)$: S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964); G. Morpurgo, *ibid.* **131**, 2205 (1963); G. Morpurgo, Nuovo Cimento **31**, 569 (1964).

[5] $(K^{*\pm},K^\pm)$: M. A. B. Bég, P. C. DeCelles, and R. B. Marr, Phys. Rev. **124**, 622 (1961).

[6] K. F. Weizsäcker, Z. Physik **88**, 612 (1934); E. J. Williams, Phys. Rev. **45**, 729 (1934); V. N. Gribov, V. A. Kolkunov, L. B. Okun and U. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **41**, 1839 (1961) [English transl.: Soviet Phys.—JETP **14**, 1308 (1962)]; I. Pomeranchok and I. Shmushkevich, Nucl. Phys. **23**, 452 (1961).

useful when discussing the production of extremely short-lived particles A.

In Sec. IV, we give a brief general discussion of strong-interaction-induced $B+Z\to A+Z$ coherent production and define the relevant signal-to-noise ratio to be considered in assessing the potential success of any procedure to extract the $A\to B+\gamma$ decay rate. *A priori* estimates of the signal-to-noise ratios when A and B are, respectively, vector and pseudoscalar SU_3 octet members are given in Sec. V; particular attention is given to $(A,B)=(K^{*-},K^-)$.

II. RELATIONSHIP BETWEEN THE PHOTONIC DECAY RATE $A\to B+\gamma$ AND THE CROSS SECTION FOR $B+Z\to A+Z$ COHERENT PRODUCTION IN THE COULOMB FIELD OF A HIGH-Z NUCLEUS

We consider any two particles, A and B, having mass m_A and m_B, respectively, and spin values S_A and S_B. The $A\to B+\gamma$ and $B+\gamma\to A$ vertex functions can be written as

$$\epsilon^*(\lambda_\gamma)\cdot T(\lambda_A,\lambda_B)\,,\quad \epsilon(\lambda_\gamma)\cdot \bar{T}(\lambda_A,\lambda_B)\,;$$
$$x\cdot y\equiv x_\alpha y_\alpha=\mathbf{x}\cdot\mathbf{y}-x_0y_0\,, \tag{2.1}$$

where, up to a proportionality factor, $T(\lambda_A,\lambda_B)$ and $\bar{T}(\lambda_A,\lambda_B)$ are the $A\to B$ and $B\to A$ electromagnetic transition currents, λ_A, λ_B, and λ_γ are the helicity quantum numbers of the three particles, and $\epsilon(\lambda_\gamma)$ is a unit 4-vector describing the polarization of a photon of helicity λ_γ. In the language of an effective Lagragian, $\mathcal{L}_{\rm em}(x)=j_{\rm em}(x)\cdot A(x)$,[7]

$$\begin{aligned} T(\lambda_A,\lambda_B)&=(2E_A2E_B)^{1/2}\langle B;\text{out}|\,j_{\rm em}(0)\,|A;\text{in}\rangle\,,\\ \bar{T}(\lambda_A,\lambda_B)&=(2E_A2E_B)^{1/2}\langle A;\text{out}|\,j_{\rm em}(0)\,|B;\text{in}\rangle\,,\end{aligned} \tag{2.2}$$

whence, as a consequence of the Hermiticity of $j_{\rm em}(0)$ and of the equivalence of "in" and "out" electromagnetic single-particle states, we obtain a type of microscopic-reversibility relation

$$\bar{T}(\lambda_A,\lambda_B)=T^*(\lambda_A,\lambda_B)\,. \tag{2.3}$$

Helicity amplitudes $a(\lambda_B,\lambda_\gamma)$ are defined by

$$\epsilon^*(\lambda_\gamma)\cdot T(\lambda_A,\lambda_B)\equiv a(\lambda_B.\lambda_\gamma)d(S_A,\lambda_B,\lambda_B-\lambda_\gamma;\Psi)\,, \tag{2.4}$$

where, in the rest frame of A, Ψ is the angle between an arbitrary quantization direction along which the spin of A is measured and the direction of motion of B, and $d(J,\lambda,\lambda';\Psi)$ is an appropriate symmetric-top eigenfunction.[8] Using Eqs. (2.1) and (2.4), the photonic decay rate $\Gamma(A\to B+\gamma)$ is given in the rest frame of A by

$$\Gamma(A\to B+\tau)=\frac{1}{(2\pi)^3}\int\frac{d^3p_B}{2E_B}\frac{d^3p_\gamma}{2E_\gamma}\frac{1}{2m_A}(2\pi)^4\delta(\mathbf{p}_B+\mathbf{p}_\gamma)$$
$$\times\delta(E_B+E_\gamma-m_A)\eta_B\sum_{\lambda_B}\sum_{\lambda_\gamma=\pm1}|\epsilon^*(\lambda_\gamma)\cdot T(\lambda_A,\lambda_B)|^2$$
$$=\frac{\eta_B}{16\pi m_A}\frac{m_A^2-m_B^2}{m_A^2}\sum_{\lambda_B}\sum_{\lambda_\gamma=\pm1}|a(\lambda_B,\lambda_\gamma)|^2$$
$$\times\int\frac{d\Omega}{4\pi}|d(S_A,\lambda_A,\lambda_B-\lambda_\gamma;\Psi)|^2$$
$$=\frac{\eta_B}{16\pi}\frac{m_A^2-m_B^2}{m_A^2}\frac{1}{(2S_A+1)}$$
$$\times\sum_{\lambda_B}\sum_{\lambda_\gamma=\pm1}|a(\lambda_B,\lambda_\gamma)|^2;$$
$$\eta_B\equiv1\quad\text{if}\quad B\neq\gamma\,,$$
$$\equiv\tfrac{1}{2}\quad\text{if}\quad B=\gamma\,, \tag{2.5}$$

where $\lambda_\gamma=+1$, -1 correspond to the two states of transverse (circular) polarization of the emitted photon.

We now turn to the $B+Z\to A+Z$ coherent production process in the Coulomb field of a high-Z nucleus, $B+Z\to B+\gamma'+Z\to A+Z$. Since we shall ultimately consider the case of relatively large nucleus mass, and ignore any nucleus magnetic-moment interaction, the nucleus can be treated as a Ze-charged spin-zero particle characterized by a form factor $F(q^2)$. The $Z\to Z+\gamma$ vertex function may then be written as

$$ZeF(q^2)(p_{Z;i}+p_{Z;f})\cdot\epsilon^*(\lambda_\gamma);$$
$$p_{\gamma'}{}^2=q^2\equiv(p_{Z;i}-p_{Z;f})^2=(p_A-p_B)^2$$
$$=2E_AE_B\left\{1-\left[1-\frac{m_{=}{}^2}{E_B{}^2}\right]^{1/2}\cos\theta-\frac{1}{2}\frac{m_A{}^2+m_B{}^2}{E_AE_B}\right\};$$
$$\theta\equiv\cos^{-1}(\hat{p}_B\cdot\hat{p}_A)\,. \tag{2.6}$$

From the diagram of Fig. 1, the $Z\to Z+\gamma$ vertex function of Eq. (2.6), and the $\gamma+B\to A$ vertex function of Eq. (2.1), the total and differential cross section for $B+Z\to A+Z$ coherent production in the nuclear

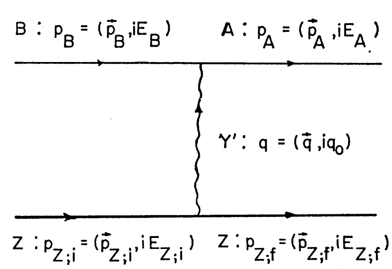

FIG. 1. Feynman diagram for $B+Z\to\gamma'+Z\to A+Z$.

[7] The factor of $(2E_A2E_B)$ in Eq. (2.2) and below is appropriate only if A and B are bosons and should be replaced by a factor of $[(E_A/m_A)(E_B/m_B)]$ if A and B are fermions. However, this replacement does not change the final result in Eq. (2.22).

[8] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959). We consistently omit the factor $\exp\{i[\lambda_A-(\lambda_B-\lambda_\gamma)]\Phi\}$ multiplying $d(S_A,\lambda_A,\lambda_B-\lambda_\gamma;\Psi)$, where Φ is an azimuthal angle.

Coulomb field are given in the laboratory frame by

$$[\sigma(B\rightarrow A)]_{\text{Coul}}=\int d\Omega_{=}\left[\frac{d\sigma(B\rightarrow A)}{d\Omega_A}\right]_{\text{Coul}}=\frac{1}{|\mathbf{p}_B|/E_B}$$
$$\times\frac{1}{(2\pi)^6}\int\frac{d^3p_{Z;f}}{2E_{Z;f}}\frac{d^3p_A}{2E_A}\frac{1}{2E_B}\frac{1}{2m_Z}(2\pi)^4$$
$$\times\delta(p_B+p_{Z;i}-p_A-p_{Z;f})Z^2e^2\frac{|F(q^2)|^2}{(q^2)^2}\frac{\eta_B'}{2(S_B+1)}$$
$$\times\sum_{\lambda_A\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2;$$
$$\eta_B'\equiv1\quad\text{if}\quad m_B=0,$$
$$\equiv\tfrac{1}{2}(2S_B+1)\quad\text{if}\quad m_B=0. \tag{2.7}$$

We now exploit the fact that, on the basis of Eqs. (2.7) and (2.5) and the "microscopic reversibility" relation in Eq. (2.3), a definite connection exists between $[\sigma(B\rightarrow A)]_{\text{Coul}}$ and $\Gamma(A\rightarrow B+\gamma)$. To deduce this connection we write

$$(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)=C^{(tr)}(\lambda_A,\lambda_B)+C^{(l,t)}(\lambda_A,\lambda_B);$$
$$C^{(tr)}(\lambda_A,\lambda_B)\equiv\sum_{\lambda_\gamma=\pm1}(p_{Z;i}+p_{Z;f})\cdot\epsilon^*(\lambda_\gamma)\times\epsilon(\lambda_\gamma)\cdot\bar{T}(\lambda_A,\lambda_B),$$
$$C^{(l,t)}(\lambda_A,\lambda_B)\equiv(p_{Z;i}+p_{Z;f})\cdot[\epsilon^*(l)\epsilon(l)+\epsilon^*(t)\epsilon(t)]\cdot\bar{T}(\lambda_B,\lambda_B), \tag{2.8}$$

where $\epsilon(\pm1)$, $\epsilon(l)$, and $\epsilon(t)$ are an orthonormal set of polarization unit 4-vectors which satisfy the completeness relation

$$\Big[\sum_{\lambda_\gamma=\pm1}\epsilon_\alpha^*(\lambda_\gamma)\epsilon_\beta(\lambda_\gamma)\Big]+[\epsilon_\alpha^*(l)\epsilon_\beta(l)+\epsilon_\alpha^*(t)\epsilon_\beta(t)]=\delta_{\alpha\beta}. \tag{2.9}$$

In order that $\epsilon(\pm1)$, $\epsilon(l)$, and $\epsilon(t)$ describe, in the rest frame of A, transverse, longitudinal, and timelike photons, respectively, in both the decay and the production processes, we take as an explicit representation

$$\epsilon'(\pm1)=\mp\tfrac{1}{2}\sqrt{2}[\epsilon'(\text{I})\pm i\epsilon'(\text{II})],$$
$$\epsilon'(\text{I})=(\hat{n}'\times\hat{q}',0),\quad\epsilon'(\text{II})=(\hat{n}',0),$$
$$\epsilon'(l)=(\hat{q}',0),\quad\epsilon'(t)=(0,i),$$
$$\hat{n}'\equiv\hat{q}'\times\hat{p}_{Z,i}',\quad\mathbf{q}'=\mathbf{p}_{Z;i}'-\mathbf{p}_{Z;f}'=\mathbf{p}_B', \tag{2.10}$$

where, in this discussion and below, primed and unprimed symbols will refer to quantities measured in the rest frames of A and of Z (laboratory frame), respectively. Then, obtaining $\epsilon(\pm1)$, $\epsilon(l)$, $\epsilon(t)$ from $\epsilon'(\pm1)$, $\epsilon'(l)$, $\epsilon'(t)$ by the Lorentz transformation connecting the rest frames of A and Z, and noting that $(p_{Z;i}+p_{Z;f})_\alpha=im_Z\delta_{4\alpha}+(p_{Z;f})_\alpha\cong2im_Z\delta_{4\alpha}$ in the case of relatively large nucleus mass, we have

$$C^{(tr)}(\lambda_A,\lambda_B)\cong-\sqrt{2}m_Z\frac{|\mathbf{p}_A|}{m_A}\sin\theta'[\epsilon(+1)-\epsilon(-1)]\cdot\bar{T}(\lambda_A,\lambda_B), \tag{2.11a}$$
$$C^{(l,t)}(\lambda_A,\lambda_B)\cong2m_Z\frac{E_A}{m_A}\left[\frac{|\mathbf{p}_A|}{E_A}\cos\theta'\epsilon(l)+\epsilon(t)\right]\cdot\bar{T}(\lambda_=,\lambda_B), \tag{2.11b}$$

where

$$\cos\theta'\equiv\frac{\hat{p}_B'\cdot(\mathbf{p}_A'-\mathbf{p}_{Z;i}')}{|\mathbf{p}_A'-\mathbf{p}_{Z;i}'|}=-\hat{p}_B'\cdot\hat{p}_{Z;i}';$$
$$\cos\theta\equiv\frac{\hat{p}_B\cdot(\mathbf{p}_A-\mathbf{p}_{Z;i})}{|\mathbf{p}_A-\mathbf{p}_{Z;i}|}=\hat{p}_B\cdot\hat{p}_A. \tag{2.12}$$

We now introduce the essential restriction of *electromagnetic current conservation*, viz.,

$$q\cdot\bar{T}(\lambda_A,\lambda_B)=0. \tag{2.13}$$

This, when applied in the rest frame of A, yields, using Eq. (2.10),

$$0=\hat{q}'\cdot\bar{\mathbf{T}}'(\lambda_A,\lambda_B)+i(q_0'/|\mathbf{q}|)\bar{T}_4'(\lambda_A,\lambda_B)$$
$$=\epsilon'(l)\cdot\bar{T}'(\lambda_A,\lambda_B)+(q_0'/|\mathbf{q}|'\epsilon'(t)\cdot\bar{T}(\lambda_A,\lambda_B)$$
$$=\epsilon(l)\cdot\bar{T}(\lambda_A,\lambda_B)+(q_0'/|\mathbf{q}|'\epsilon(t)\cdot\bar{T}(\lambda_A,\lambda_B). \tag{2.14}$$

Hence, substituting Eq. (2.14) into (2.11b),

$$C^{(l,t)}(\lambda_A,\lambda_B)\cong2m_Z\frac{E_A}{m_A}\left[1-\frac{|\mathbf{p}_A|}{E_A}\frac{q_0'}{|\mathbf{q}'|}\cos\theta'\right]\times\epsilon(t)\cdot\bar{T}(\lambda_A,\lambda_B). \tag{2.15}$$

For relatively large nucleus mass so that $E_{Z;f}\cong E_{Z;i}=m_Z$, $E_A\approx E_B$, high incident energies ($E_B\gg m_A$, m_B) and small production angles ($\theta\ll1$), the expression for q^2 in Eq. (2.5) becomes

$$q^2\cong E_B^2[\delta^2+\theta^2];\quad\delta\equiv\tfrac{1}{2}(m_A^2-m_B^2)/E_B^2, \tag{2.16}$$

and, since $(1/q^2)^2$ appears in the $B+Z\rightarrow A+Z$ coherent production cross section [Eq. (2.7)], we shall only be interested in values of $\theta\approx\delta$. Under these circumstances we also have

$$\theta'\cong2E_Bm_A\theta/(m_A^2-m_B^2)\approx m_A/E_B \tag{2.17a}$$

and

$$\frac{q_0'}{|\mathbf{q}'|}=\frac{E_{Z;i}'-E_{Z;f}'}{|\mathbf{p}_{Z;i}-\mathbf{p}_{Z;f}|}\cong\frac{|\mathbf{p}_A|}{E_A}, \tag{2.17b}$$

whence, substituting into Eqs. (2.11a) and (2.15),

$$C^{(tr)}(\lambda_A,\lambda_B)\cong -2\sqrt{2}\frac{m_Z E_B{}^2\theta}{m_A{}^2-m_B{}^2}[\epsilon(+1)-\epsilon(-1)]\cdot\bar{T}(\lambda_A,\lambda_B)$$
$$\approx -m_Z[\epsilon(+1)-\epsilon(-1)]\cdot\bar{T}(\lambda_A,\lambda_B), \quad (2.18a)$$

$$C^{(l,t)}(\lambda_A,\lambda_B)\cong 2m_Z\frac{E_B}{m_A}\left[\frac{m_A{}^2}{E_B{}^2}+\frac{1}{2}\frac{4E_B{}^2m_A{}^2\theta^2}{(m_A{}^2-m_B{}^2)^2}\right]\epsilon(t)\cdot\bar{T}(\lambda_A,\lambda_B)$$
$$\approx m_Z(m_A/E_B)\epsilon(t)\cdot\bar{T}(\lambda_A,\lambda_B). \quad (2.18b)$$

As in the decay process $A\rightarrow B+\gamma$ [see Eq. (2.4)] we write $\epsilon(\lambda_\gamma)\cdot\bar{T}(\lambda_A,\lambda_B)$ in terms of helicity amplitudes $\bar{a}(\lambda_B,\lambda_\gamma)$ as

$$\epsilon(\lambda_\gamma)\cdot\bar{T}(\lambda_A,\lambda_B)=\epsilon'(\lambda_\gamma)\cdot\bar{T}'(\lambda_A,\lambda_B)$$
$$=\bar{a}(\lambda_B,\lambda_\gamma)d(S_A,\lambda_A,\lambda_B-\lambda_\gamma;\theta'),$$
$$\theta'\equiv\cos^{-1}(-\hat{p}_B{}'\cdot\hat{p}_{Z;i}{}'); \quad (2.19)$$

note that $\epsilon'(t)$ corresponds to $\lambda_\gamma=0$, and record the following *small-angle* property of the symmetry-top eigenfunctions[9]:

$$d(J,\lambda,\lambda';\varphi)=e_{\lambda\lambda'}{}^{(J)}\varphi^{|\lambda-\lambda'|}[1+a_{\lambda,\lambda'}{}^{(J)}\varphi^2+\cdots];$$
$$e_{\lambda,\lambda}{}^{(J)}=1,\quad a_{\lambda\lambda'}{}^{(J)}\approx 1. \quad (2.20)$$

Thus, at small production angles $\theta=\cos^{-1}(\hat{p}_B\cdot\hat{p}_A)$, the dominant terms in $C^{(tr)}(\lambda_A,\lambda_B)$ and $(C^{(l,r)}(\lambda_A,\lambda_B)$ correspond to $\lambda_A=\lambda_B\pm1$ and $\lambda_A=\lambda_B$, respectively, and the quantity

$$\Sigma_{\lambda_A,\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2$$

of Eqs. (2.7) and (2.8) is given by

$$\sum_{\lambda_A,\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2$$
$$=\sum_{\lambda_A,\lambda_B}|C^{(tr)}(\lambda_A,\lambda_B)+C^{(l,t)}(\lambda_A,\lambda_B)|^2$$
$$\cong\sum_{\lambda_B}\{|C^{(tr)}(\lambda_B+1,\lambda_B)|^2+|C^{(tr)}(\lambda_B-1,\lambda_B)|^2+|C^{(l,t)}(\lambda_B,\lambda_B)|^2\}. \quad (2.21)$$

Furthermore, $\epsilon(\pm1)\cdot\bar{T}(\lambda_A,\lambda_B)$ is comparable in magnitude with $\epsilon(t)\cdot\bar{T}(\lambda_A,\lambda_B)$, since the transition currents $\bar{T}(\lambda_B\pm1,\ \lambda_B)$ and $\bar{T}(\lambda_B,\lambda_B)$ differ only in the helicity quantum number of A; with this in mind, combining Eqs. (2.21) and (2.18), we obtain, up to corrections of order $(m_A/E_B)^2$,

$$\sum_{\lambda_A,\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2$$
$$\cong\sum_{\lambda_B}[|C^{(tr)}(\lambda_B+1,\lambda_B)|^2+|C^{(tr)}(\lambda_B-1,\lambda_B|^2]$$
$$\cong\frac{8m_Z{}^2E_B{}^4\theta^2}{(m_A{}^2-m_B{}^2)^2}\sum_{\lambda_B}[|\epsilon(-1)\cdot\bar{T}(\lambda_B+1,\lambda_B)|^2+|\epsilon(+1)\cdot\bar{T}(\lambda_B-1,\lambda_B)|^2]. \quad (2.22)$$

Equation (2.22) shows that the nuclear Coulomb field is, so to speak, selective with respect to the helicity states of A that are produced in it. As an example of this "selection rule," consider the process $\Lambda+Z\rightarrow\Lambda+\gamma'+Z\rightarrow\Sigma^0+Z$; if $\lambda_\Lambda=+\frac{1}{2}$, then, according to Eq. (2.22), $\lambda_{\Sigma^0}=-\frac{1}{2}$ (or $\lambda_{\Sigma^0}=\frac{3}{2}$, which however is ruled out since $S_{\Sigma^0}=\frac{1}{2}$), i.e., the hyperon spin is "flipped" by the nuclear Coulomb field, and, therefore, in the approximation of Eq. (2.22), the nuclear-Coulomb-field coherent production amplitude does not conserve angular momentum in the forward, i.e., $\theta=0$, direction. Thus, Eq. (2.22) provides the rationale for the "mysterious" vanishing of the high-energy, forward, nuclear-Coulomb-field $\Lambda\rightarrow\Sigma^0$ coherent production amplitude discussed in the Introduction.

The expression in Eq. (2.22) for

$$\Sigma_{\lambda_A,\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2$$

involves the exchange of *transverse* virtual photons only. Thus, using Eqs. (2.19) and (2.20) we can write Eq. (2.22) as

$$\sum_{\lambda_A,\lambda_B}|(p_{Z;i}+p_{Z;f})\cdot\bar{T}(\lambda_A,\lambda_B)|^2=\frac{8m_Z{}^2E_B{}^4\theta^2}{(m_A{}^2-m_B{}^2)^2}\times\sum_{\lambda_B}\sum_{\lambda_\gamma=\pm1}|\bar{a}(\lambda_B,\lambda_\gamma)|^2, \quad (2.23)$$

and, in view of the "microscopic reversibility" relation in Eq. (2.3), and of Eqs. (2.4) and (2.19), make the crucial substitution

$$\bar{a}(\lambda_B,\lambda_\gamma=\pm1)=a^*(\lambda_B,\lambda_\gamma=\pm1). \quad (2.24)$$

Then, inserting Eqs. (2.24), (2.23), and (2.16) into Eq. (2.7), and comparing with Eq. (2.5), the high-energy, small-angle form of the differential cross section for $B+Z\rightarrow A+Z$ coherent production in the nuclear Coulomb field is given by

$$\left[\frac{d\sigma(B\rightarrow A)}{d\Omega_A}\right]_{\text{Coul}}\cong\frac{8Z^2\alpha|F(q^2)|^2\Gamma(A\rightarrow B+\gamma)/m_A}{m_A{}^2[1-m_B{}^2/m_A{}^2]^3}\times\chi_{AB}\frac{\theta^2}{[\delta^2+\theta^2]^2};$$
$$\alpha\equiv e^2/4\pi\cong 1/137,$$
$$\chi_{AB}\equiv\frac{\eta_B{}'}{\eta_B}\frac{(2S_A+1)}{(2S_B+1)};$$
$$\eta_B\equiv1,\quad B\neq\gamma,$$
$$\equiv\tfrac{1}{2},\quad B=\gamma,$$
$$\eta_B{}'\equiv1,\quad m_B\neq0,$$
$$\equiv\tfrac{1}{2}(2S_B+1),\quad m_B=0;$$
$$q^2\cong E_B{}^2[\delta^2+\theta^2],\quad \delta\equiv\tfrac{1}{2}(m_A{}^2-m_B{}^2)/E_B{}^2. \quad (2.25)$$

We should also mention that in obtaining Eq. (2.25) we have in addition neglected any deviation of the helicity

[9] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). Based on Eq. 4.1.15.

amplitudes from their $q^2=0$ values since $q^2/m_A{}^2 \approx m_A{}^2/E_B{}^2 \ll 1$ according to Eq. (2.16).

Equation (2.25) is the basic relation which connects the $B+Z\to A+Z$ coherent production in a nuclear Coulomb field with the $A\to B+\gamma$ photonic decay rate. Special cases of Eq. (2.25) correspond to

$$\pi^0\to\gamma+\gamma,\quad \eta\to\gamma+\gamma,\quad S\to\gamma+\gamma\sim\chi_{AB}=1,$$
$$f\to\gamma+\gamma \qquad \sim\chi_{AB}=5,$$
$$\rho\to\pi+\gamma,\quad K^*\to K+\gamma,\quad W\to\pi+\gamma\sim\chi_{AB}=3,$$
$$\Sigma^0\to\Lambda+\gamma \qquad \sim\chi_{AB}=1.$$

Finally, it is worth remarking that if $F(q^2)\approx F(0)$ for $q^2\lesssim 2\delta^2E_B{}^2$,[10] i.e., for $\theta^2\lesssim\delta^2$, then $[d\sigma(B\to A)/d\Omega_A]_{\rm Coul}$ has a sharp maximum for $\theta=\delta$; this sharp maximum constitutes one of the prime distinguishing features for the $B+Z\to A+Z$ nuclear-Coulomb-field coherent production process.

III. RELATIONSHIP BETWEEN THE CROSS SECTIONS FOR $B+\gamma\to X$ AND $B+Z\to X+Z$ IN THE COULOMB FIELD OF A HIGH-Z NUCLEUS ($X\equiv$MANY-PARTICLE STATE)

In this section we generalize the previous discussion by replacing the single-particle state A by an n-particle state X with $n>1$. Our goal is to obtain a relation between the $B+\gamma\to X$ total production cross section, $\sigma(B+\gamma\to X)$, and the $B+Z\to B+\gamma'+Z\to X+Z$ nuclear-Coulomb-field coherent production differential cross section, $[d\sigma(B\to X)/d\Omega]_{\rm Coul}$.

In analogy with Eq. (2.1), we write the $B+\gamma\to X$ vertex function as

$$\epsilon(\lambda_\gamma)\cdot\bar{T}([\lambda_X],\lambda_A), \tag{3.1}$$

where $[\lambda_X]$ denotes the set of n helicity numbers associated with X. The cross section $\sigma(\gamma+B\to X)$ is then given by

$$\sigma(B\to\gamma+X)=\frac{(2\pi)^4\eta_B\eta_B'}{4(2S_B+1)(s-m_B{}^2)}G(s);$$
$$G(s)\equiv\frac{1}{(2\pi)^{3n}}\int\frac{d^3p_1}{2E_1}\cdots\frac{d^3p_n}{2E_n}\delta(p_X-p_B-p_\gamma)\times\sum_{\lambda_B,[\lambda_X]}\sum_{\lambda_\gamma=\pm1}|\epsilon(\lambda_\gamma)\cdot\bar{T}([\lambda_X],\lambda_B)|^2;$$
$$p_X\equiv\sum_{j=1}^{p}p_j,\quad p_X=(\mathbf{p}_X,iE_X),\quad p_j=(\mathbf{p}_j,iE_j);$$
$$s\equiv-p_X{}^2=-(p_B+p_\gamma)_2, \tag{3.2}$$

with η_B and η_B' defined in Eq. (2.25). We now transform from the set of $3n$ integration variables, $p_1\cdots p_n$, in the above expression to the set p_X plus the remaining $3n-4$ variables internal to X,

$$d^3p_1/2E_1\cdots d^3p_n/2E_n=Jd^4p_Xdp_{\rm int}, \tag{3.3}$$

where J is the Lorentz-invariant Jacobian of the transformation and $dp_{\rm int}$ need not be made any more explicit than to say that it is a Lorentz-invariant differential. Combining Eqs. (3.2) and (3.3), we have

$$\sigma(B+\gamma\to X)=\left[\frac{4(2S_B+1)(s-m_B{}^2)}{(2\pi)^4\eta_B\eta'_B}\right]^{-1}g(s);$$
$$g(s)\equiv\frac{1}{(2\pi)^{3n}}\int dp_{\rm int}J\times\sum_{\lambda_B,[\lambda_X]}\sum_{\lambda_\gamma=\pm1}|\epsilon(\lambda_\gamma)\cdot\bar{T}([\lambda_X],\lambda_B)|^2. \tag{3.4}$$

We now turn to the $B+Z\to X+Z$ coherent production process in the Coulomb field of a high-Z nucleus: $B+Z\to B+\gamma'+Z\to X+Z$. Combining Eqs. (2.6), (2.9), and (3.1) with the analog of Fig. 1 (A replaced by X), and considering the case of relatively large nucleus mass, the nuclear-Coulomb-field induced $B\to X$ coherent cross section is given in the laboratory frame by

$$[\sigma(B\to X)]_{\rm Coul}=\frac{Z^2\alpha}{|\mathbf{p}_B|}\frac{\eta_B'}{(2S_B+1)}(2\pi)^2H,$$
$$H\equiv\frac{1}{(2\pi)^{3n}}\int\frac{d^3p_1}{2E_1}\cdots\frac{d^3p_n}{2E_n}\frac{|F(q^2)|^2}{(q^2)^2}\delta(E_B-E_X)\times\sum_{\lambda_B,[\lambda_X]}|\{[\sum_{\lambda_\gamma=\pm1}\epsilon_4{}^*(\lambda_\gamma)\epsilon(\lambda_\gamma)]+[\epsilon_4{}^*(l)\epsilon(l)+\epsilon_4{}^*(t)]\}\cdot\bar{T}([\lambda_X],\lambda_B)|^2,$$
$$p_{\gamma'}{}^2=q^2\equiv(p_{Z;i}-p_{Z;f})^2=(p_X-p_B)^2$$
$$\cong 2E_B{}^2\left\{1-[(1-s/E_B{}^2)(1-m_B{}^2/E_B{}^2)]^{1/2}\times\cos\theta-\frac{1}{2}\frac{s+m_B{}^2}{E_B{}^2}\right\},$$
$$\theta=\cos^{-1}(\hat{p}_B\cdot\hat{p}_X), \tag{3.5}$$

where the $\epsilon(\pm1)$, $\epsilon(l)$, $\epsilon(t)$ are obtained by an A-rest-frame$\to$ Z-rest-frame Lorentz transformation from the $\epsilon'(\pm1)$, $\epsilon'(l)$, $\epsilon'(t)$ defined in Eq. (2.10). Again we make the same change of variables as before [Eq. (3.3)] and obtain

$$[\sigma(B\to X)]_{\rm Coul}=\frac{Z^2\alpha\eta_B'}{|\mathbf{p}_B|(2S_B+1)}(2\pi^2)\int d^4p_X\,\delta(E_B-E_X)\times\frac{|F(q^2)|^2}{(q^2)^2}h,$$
$$h\equiv\frac{1}{(2\pi)^{3n}}\int dp_{\rm int}J\sum_{\lambda_B,[\lambda_X]}\times|\{[\sum_{\lambda_\gamma=\pm1}\epsilon_4{}^*(\lambda_\gamma)\epsilon(\lambda_\gamma)]+[\epsilon_4{}^*(l)\epsilon(l)+\epsilon_4{}^*(t)\epsilon(t)]\}\cdot\bar{T}([\lambda_X],\lambda_B)|^2. \tag{3.6}$$

[10] The goodness of this approximation depends upon the size of the nuclear radius R, i.e.,

$$q^2R^2\lesssim 2\delta^2E_B{}^2R^2=\tfrac{1}{2}[(m_A{}^2-m_B{}^2)^2/E_B{}^2]R^2$$

must be $\ll1$ for $F(q^2)\cong F(0)$.

With the exclusion of $\epsilon_4(\lambda_\gamma)$, the integrand of h is a Lorentz invariant, and since $\epsilon_4(\lambda_\gamma)$ depends only upon p_X, factors of $\epsilon_4(\lambda_\gamma)$ can be taken outside of the integral sign in h and the remaining invariant integral calculated in the rest frame of X. In this frame let us take the z axis along the 3-momentum of B. Then, for a given value of λ_B, different values of λ_γ correspond to different values of the z component of the angular momentum J_z and integration over the angles associated with dp_{int} demonstrates that there is no interference between terms with different values of λ_γ[$\epsilon(l)$ and $\epsilon(t)$ are characterized by $\lambda_\gamma=0$]. We note that in the case of the production of a single-particle state A there is no integration over any internal angles and we have to use the property of symmetric top eigenfunctions given in Eq. (2.20) in order to establish that there is no interference between terms with different λ_γ.

In analogy with the single-particle case we apply the current conservation condition [Eq. (2.13)] and consider the case of high incident energies ($E_B \gg \sqrt{s}, m_B$) and small production angles ($\theta \ll 1$). Then, bearing in mind the absence of the interference terms discussed above, we have, to order $(\sqrt{s}/E_B)^2$, $(m_B/E_B)^2$,

$$h \cong \frac{1}{(2\pi)^{3n}} \int dp_{\text{int}}\, J \sum_{\lambda_B,[\lambda_X]} \sum_{\lambda_\gamma=\pm1} |\epsilon_4(\lambda_\gamma)|^2 \times |\epsilon(\lambda_\gamma)\cdot \bar{T}([\lambda_X],\lambda_B)|^2, \quad (3.7)$$

with

$$|\epsilon_4(\pm1)|^2 \cong 2E_B{}^4\theta^2/(s-m_B{}^2)^2. \quad (3.8)$$

Equations (3.6), (3.7), and (3.8) yield

$$[\sigma(B\to X)]_{\text{Coul}} = \int d\Omega_X ds \left[\frac{d^2\sigma(B\to X)}{d\Omega_X ds}\right]_{\text{Coul}}$$
$$\cong \frac{Z^2\alpha^2}{E_B} |F(q^2)|^2 \frac{\eta_B'}{(2S_B+1)} (2\pi)^2 \int d^4p_X \times \frac{\delta(E_B-E_X)}{(q^2)^2} \frac{2E_B{}^4\theta^2}{(s-m_B{}^2)^2} g(s)$$
$$\cong \int d\Omega_X ds \left\{ \frac{Z^2\alpha^2}{E_B} \frac{\eta'_B}{(2S_B+1)} (2\pi)^2 \tfrac{1}{2}(E_B{}^2-s)^{1/2} \times \frac{|F(q^2)|^2}{(q^2)^2} \frac{2E_B{}^4\theta^2}{(s-m_B{}^2)^2} g(s) \right\}, \quad (3.9)$$

where $g(s)$ is related to $\sigma(B+\gamma\to X)$ by Eq. (3.4), and where we have made the change of variables

$$d^4p_X = dE_X d\Omega_X |\mathbf{p}_X|^2 d|\mathbf{p}^\pm| = [4(E_X{}^2-s)]^{-1/2} dE_X d\Omega_X ds (E_X{}^2-s). \quad (3.10)$$

Equations (3.4) and (3.9) give the high-energy, small-angle form of the differential cross section for $B+Z\to A+X$ coherent production in the nuclear Coulomb field as

$$\left[\frac{d^2\sigma(B\to X)}{d\Omega_X ds}\right]_{\text{Coul}} \cong \frac{Z^2\alpha|F(q^2)|^2}{\eta_B\pi^2} \times \frac{\sigma(B+\gamma\to X)}{(s-m_B{}^2)} \frac{\theta^2}{[\delta^2(s)+\theta^2]^2};$$
$$q^2 \cong E_B{}^2[\delta^2(s)+\theta^2],$$
$$\delta(s) \equiv \tfrac{1}{2}(s-m_B{}^2)/E_B{}^2,$$
$$\eta_B \equiv 1, \quad B\neq\gamma,$$
$$\equiv \tfrac{1}{2}, \quad B=\gamma, \quad (3.11)$$

which constitutes the basic result of the present section.

With reference to subsequent discussions, we note that $g(s)$ and so $\sigma(B+\gamma\to X)$ and, within our approximations, $[d^2\sigma(B\to X)/d\Omega_X ds]_{\text{Coul}}$ [see Eqs. (3.4), (3.9), (3.11)], vanish unless the "helicity" of X, $\lambda_X \equiv \lim_{|\mathbf{p}_X|\to0} \times [J_X\cdot\hat{p}_X]$ is equal to $\lambda_B\pm1$ [see Eqs. (2.7) and (2.22)]. It should also be noted that Eq. (3.11) is an appropriate generalization of Eq. (2.25); to show this, let us assume that $\sigma(B+\gamma\to X)$ is dominated by a particular resonance which we identify with the "single-particle" state A. The Breit-Wigner resonance formula for $\sigma(B+\gamma\to X)$ is then given by

$$\sigma(B+\tau\to X) = \frac{\pi\lambda\!\!\!^{-2}(2S_A+1)}{2(2S_B+1)} \times \frac{\Gamma(B+\tau\to A)\Gamma(A\to X)}{(\sqrt{s}-m_A)^2+(\Gamma_A/2)^2}, \quad (3.12)$$

where Γ_A = total decay rate of A, $\lambda\!\!\!^{-\,-1} = |\mathbf{p}_B| = |-\mathbf{p}_\gamma| = (s-m_B{}^2)/2\sqrt{s}$, $(2S_A+1)$ is replaced by 2 if $m_B=0$, and the "microscopic, reversibility" condition

$$\Gamma(B+\gamma\to A) = \Gamma(A\to B+\gamma)$$

is deduced on the basis of considerations such as those which justify Eq. (2.3). Thus, with η_B' as in Eq. (2.25),

$$\sigma(B+\tau\to X) = \frac{\pi}{2}\eta_B' \frac{(2S_A+1)}{(2S_B+1)} \times \frac{4s}{(s-m_B{}^2)^2} \frac{\Gamma(A\to B+\gamma)\Gamma(A\to X)}{(\sqrt{s}-m_A)^2+(\Gamma_A/2)^2}, \quad (3.13)$$

so that inserting Eq. (3.13) into Eq. (3.11) yields

$$\left[\frac{d^2\sigma(B\to X)}{d\Omega_X ds}\right]_{\text{Coul}} \cong 4Z^2\alpha|F(q^2)|^2 \frac{\eta_B'}{\eta_B} \frac{(2S_A+1)}{(2S_B+1)} \times \frac{s\Gamma(A\to B+\gamma)}{(s-m_B{}^2)^3} \frac{\Gamma(A\to X)}{\Gamma_X} \times \left[\frac{1}{\pi} \frac{\Gamma_A/2}{(\sqrt{s}-m_A)^2+(\Gamma_A/2)^2}\right] \frac{\theta^2}{[\delta^2(s)+\theta^2]^2}. \quad (3.14)$$

If now $\Gamma_A \ll m_A$, we may use

$$\frac{1}{\pi}\frac{\Gamma_A/2}{(\sqrt{s}-m_A)^2+(\Gamma_A/2)^2} \cong \delta(\sqrt{s}-m_A)=2m_A\delta(s-m_A{}^2), \quad (3.15)$$

and integrating Eq. (3.14) over s we obtain

$$\left[\frac{d\sigma(B\to X)}{d\Omega_X}\right]_{\text{Coul}} \equiv \int ds\left[\frac{d^2\sigma(B\to X)}{d\Omega_X ds}\right]_{\text{Coul}} \cong 8Z^2\alpha\frac{|F(q^2)|^2\Gamma(A\to B+\gamma)/m_A}{m_A{}^2(1-m_B{}^2/m_A{}^2)^3} \times\frac{\eta_B{}'}{\eta_B}\frac{(2S_A+1)}{(2S_B+1)}\frac{\theta^2}{[\delta^2+\theta^2]^2}\frac{\Gamma(A\to X)}{\Gamma_A}. \quad (3.16)$$

Equation (3.16) is in agreement with Eq. (2.25) in the limit $\Gamma(A\to X)/\Gamma_A\to 1$, so that Eq. (3.11) is indeed an appropriate generalization of this last equation.

IV. STRONG-INTERACTION–INDUCED $B+Z\to A+Z$ PRODUCTION

We write the differential cross section for the coherent production process $B+Z\to A+Z$ as

$$\frac{d\sigma(B\to A)}{d\Omega_A} \sum_{\lambda_A,\lambda_B} |M_c(E_B,\theta;\lambda_A,\lambda_B) +M_S(E_B,\theta;\lambda_A,\lambda_B)|^2, \quad (4.1)$$

where M_C and M_S are the Coulomb-field and the strong-interaction contributions to the amplitude for $B+Z\to A+Z$. $\Sigma_{\lambda_A\,\lambda_B}|M_C(E_B,\theta;\lambda_A,\lambda_B)|^2$ is the Coulomb-field cross section, $[d\sigma(B\to A)/d\Omega_A]_{\text{Coul}}$, given by Eq. (2.25). Within the context of the usual impulse approximation for a spin-zero nucleus,

$$M_S(E_B,\theta;\lambda_A,\lambda_B)=AF(q^2)\ \{(Z/A)\beta_p(E_B,\theta;\lambda_A,\lambda_B) +[(A-Z)/A]\beta_n(E_B,\theta;\lambda_A,\lambda_B)\hat{q}\}, \quad (4.2)$$

where $F(q^2)$, q^2 are defined in Eq. (2.6), and $\beta_p[\beta_n]$ is the strong-interaction-induced non-spin-flip amplitude for the process $B+p\to A+p$ $[B+n\to A+n]$. Equations (4.1), (4.2), and (2.25) yield the following expression for the signal-to-noise ratio $R(E_B)$:

$$R(E_B)\equiv\{[\sum_{\lambda_A,\lambda_B}|M_C(E_B,\theta;\lambda_A,\lambda_B)|^2] \times[\sum_{\lambda_A,\lambda_B}|M_S(E_B,\theta;\lambda_A,\lambda_B)|^2]^{-1}\}_{\theta=\delta(E_B)}$$

$$\cong 8\chi_{AB}\left(\frac{Z}{A}\right)^2\frac{(E_B/m_A)^4}{[1-m_B{}^2/m_A{}^2]^5} \times\frac{\alpha\Gamma(A\to B+\gamma)/m_A}{[\sum_{\lambda_A,\lambda_B}|(Z/A)\beta_B+[(A-Z)/A]\beta_n|^2m_B{}^2]_{\theta=\delta(E_B)}},$$

$$\delta(E_B)\equiv\tfrac{1}{2}(m_A{}^2-m_B{}^2)/E_B{}^2. \quad (4.3)$$

The numerical value of $R(E_B)$ is decisive in any evaluation of the possibility of a successful determination of $\Gamma(A\to B+\gamma)$ from a measurement of $d\sigma(B\to A)/d\Omega_A$. Unless the signal-to-noise ratio is at least of order unity, it would be difficult to extract reliable information about $\Gamma(A\to B+\gamma)$ as has recently been accomplished by Belletini *et al.*[11] in the case of $\Gamma(\pi^0\to\gamma+\gamma)$.

The above expression for the signal-to-noise ratio is somewhat smaller than a detailed treatment including the effects of "absorption" (i.e., B-Z initial-state and A-Z final-state interaction) would yield. Such a treatment has been given by Morpurgo[12] and by Engelbrecht[13] for the process $\gamma+Z\to\pi^0+Z$. These authors find that the Coulomb-field-induced amplitude is relatively unaffected by absorption, but that the magnitude of the strong-interaction-induced amplitude is reduced by a factor ≈ 2. Such an effect can be expected in general because the absorption mechanism is short range (strong interaction), and although for small momentum transfers the production takes place at relatively large distances whether it is Coulomb-field or strong-interaction-induced, the Coulomb-field effective-interaction region corresponds to much larger values of separation than the range of the absorption potential, and therefore M_C is essentially unaffected by absorption. On the other hand, the largest separations that can effectively contribute to the small-momentum-transfer strong-interaction-induced amplitude are of the same order of magnitude as the range of the absorption potential, and therefore absorption has a pronounced effect upon M_S.

V. *A PRIORI* ESTIMATES OF SIGNAL-TO-NOISE RATIO WHEN $(A,B)=(V,P)$

In this section we shall make *a priori* estimates for the signal-to-noise ratio when $(A,B)=(V,P)$, where V and P are, respectively, vector and pseudoscalar SU_3 octet members. In this case, any empirical information that can be obtained would be of great interest in testing SU_3; at present only the value of $\Gamma(\omega\to\pi^0+\gamma)\approx 1$ MeV is reliably known from experiment.

U-spin invariance predicts the following relations among the coupling constants associated with photonic decay modes of the type $V\to P+\gamma$[14]:

$$\tfrac{1}{2}a\equiv f(\rho^\pm\pi^\pm\gamma)=f(\rho^0\pi^0\gamma)=f(K^{*\pm}K^\pm\gamma) =-\tfrac{1}{2}f(K^{*0}K^0\gamma)=-\tfrac{1}{2}f(\bar{K}^{*0}\bar{K}^0\gamma); \quad (5.1)$$

in addition, the coupling constants associated with photonic decays of the φ and ω mesons are given by

$$f(\varphi\pi^0\gamma)=\tfrac{1}{2}\sqrt{3}(a\cos\theta_{\omega\varphi}-b\sin\theta_{\omega\varphi}), \quad f(\omega\pi^0\gamma)=\tfrac{1}{2}\sqrt{3}(a\sin\theta_{\omega\varphi}+b\cos\theta_{\omega\varphi}), \quad (5.2)$$

[11] G. Belletini, C. Bemporad, P. L. Braccini, and L. Foà, Nuovo Cimento **40A**, 1139 (1965).
[12] G. Morpurgo, Nuovo Cimentio **31**, 569 (1964).
[13] C. A. Engelbrecht, Phys. Rev. **133**, B988 (1964).
[14] K. Tanaka, Phys. Rev. **133**, B1509 (1964).

where b is another constant and $\theta_{\omega\varphi}$ is the ω-φ mixing angle defined by

$$|\omega\rangle=|\tilde{\varphi}\rangle\sin\theta_{\omega\varphi}+|\tilde{\omega}\rangle\cos\theta_{\omega\varphi},$$
$$|\varphi\rangle=|\tilde{\varphi}\rangle\cos\theta_{\omega\varphi}-|\tilde{\omega}\rangle\sin\theta_{\omega\varphi}, \quad (5.3)$$

with $|\varphi\rangle$, $|\omega\rangle$ and $|\tilde{\varphi}\rangle$, $|\tilde{\omega}\rangle$ the physical φ-meson, ω-meson states, and the bare unitary octet, unitary singlet states, respectively. Equations (5.3) and (5.2) show that $\sqrt{3}a/2=f(\tilde{\varphi}\pi^0\gamma)$, $\sqrt{3}b/2=f(\tilde{\omega}\pi^0\gamma)$.

Under the assumption that $\omega\to\pi+\pi+\pi$ is dominated by $\omega\to\rho+\pi$ followed by $\rho\to\pi+\pi$, one finds that $f^2(\omega\pi\rho)/4\pi\approx3$[15]; this, when combined with the fact that the decay rate for $\varphi\to\rho+\pi$ is less than 1 MeV, which implies that $f^2(\varphi\pi\rho)/4\pi\leqq0.03$, yields $f^2(\varphi\pi\rho)/f^2(\omega\pi\rho)\leqq0.01$. Furthermore, within the framework of a model in which the amplitudes for $\varphi+\pi^0\to\gamma$ and $\omega+\pi^0\to\gamma$ are dominated by a ρ^0 intermediate state (the only vector-meson state allowed by isospin conservation), $f^2(\varphi\pi^0\gamma)/f^2(\omega\pi^0\gamma)=f^2(\varphi\pi\rho)/f^2(\omega\pi\rho)$, and therefore $f^2(\varphi\pi^0\gamma)/f^2(\omega\pi^0\gamma)\leqq0.01$. Thus, we can use the condition $f(\varphi\pi^0\gamma)\cong0$ in Eq. (5.2) to eliminate the constant b and obtain

$$\tfrac{1}{2}a\cong f(\omega\pi^0\gamma)(\sin\theta_{\omega\varphi})/\sqrt{3}. \quad (5.4)$$

Combining Eqs. (5.4) and (5.1), we can now express the ρ and K^* photonic decay rates (Γ is proportional to f^2) in terms of the experimentally known rate for $\omega\to\pi^0+\gamma$ and the ω-φ mixing angle as

$$\begin{aligned}&\tfrac{1}{3}\sin^2\theta_{\omega\varphi}\Gamma(\omega\to\pi^0+\gamma)\\&\cong\Gamma(\rho^\pm\to\pi^\pm+\gamma)=\Gamma(\rho^0\to\pi^0+\gamma)\\&=\Gamma(K^{*\pm}\to K^\pm+\gamma)\\&=\tfrac{1}{4}\Gamma(K^{*0}\to K^0+\gamma)=\tfrac{1}{4}\Gamma(\bar{K}^{*0}\to\bar{K}^0+\gamma)\\&=\tfrac{1}{4}\Gamma(K_1^{*0}\to K_1^0+\gamma)=\tfrac{1}{4}\Gamma(K_2^{*0}\to K_2+\gamma).\end{aligned} \quad (5.5)$$

The preceding equations do not take into account the actual mass differences among the particles within V and within P. To do this in a very provisional way, we identify the coupling constants with those appearing in the most general Lorentz- and space-inversion-invariant expression for the decay amplitudes: $f(VP\gamma)(p_V)_\alpha\times(p_\gamma)_\beta\eta_\mu(V)\eta_\nu(\gamma)\epsilon_{\alpha\beta\mu\nu}$, where p_V, $\eta(V)$, and p_γ, $\eta(\gamma)$ are the momentum and polarization 4-vectors of the vector meson and the photon, respectively. We then obtain a kinematical correction factor proportional to $m_V^2(1-m_P^2/m_V^2)^3$ to be applied to the decay rates of Eq. (5.5); thus Eq. (5.5), applied to $\rho^\pm\to\pi^\pm+\gamma$ and corrected for kinematical factors, becomes

$$\Gamma(\rho\to\pi+\gamma)\cong\tfrac{1}{3}\sin^2\theta_{\omega\varphi}\Gamma(\omega\to\pi^0+\gamma)\times\frac{m_\rho^2[1-m_\pi^2/m_\rho^2]^3}{m_\omega^2[1-m_\pi^2/m_\omega^2]^3}. \quad (5.6)$$

TABLE I. Photonic decay rates and branching ratios with no kinematical correction (N.K.C.) and with the kinematical correction (W.K.C.).

Process	Rate (MeV) N.K.C.	Rate (MeV) W.K.C.	Branching ratio (%) N.K.C.	Branching ratio (%) W.K.C.
$\rho^\pm\to\pi^\pm+\gamma$	0.1	0.1	0.1	0.1
$\rho^0\to\pi^0+\gamma$	0.1	0.1	0.1	0.1
$K^{*\pm}\to K^\pm+\gamma$	0.1	0.05	0.2	0.1
$K_1^{*0}\to K_1^0+\gamma$	0.4	0.2	0.8	0.4
$K_2^{*0}\to K_2^0+\gamma$	0.4	0.2	0.8	0.4

In Table I we give the indicated photonic decay rates and the branching ratios relative to the corresponding total decay rates, e.g., $\Gamma(\rho^+\to\pi^++\gamma)/\Gamma_\rho$,) for $\theta_{\omega\varphi}=38°$ [15,16] and $\Gamma(\omega\to\pi^0+\gamma)=0.9$ MeV.

We now turn to an estimate of the amplitudes β_p and β_n in Eq. (4.2) for M_S. We first note that within the context of the usual impulse approximation for a spin-zero nucleus, space-rotation and space-inversion invariance require that these amplitudes be proportional to $(\mathbf{p}_V\times\boldsymbol{\eta})\cdot\mathbf{p}_V$, where $\mathbf{p}_V$ and $\boldsymbol{\eta}$ are the momentum and polarization 3-vectors of the V meson and $\mathbf{p}_P$ is the 3-momentum of the P meson. Consequently, production of helicity-zero vector-mesons is forbidden and the production cross section behaves like θ^2 for small production angles. This behavior can also be understood if one looks at the kinematics of $P+Z\to V+Z$, where Z is treated as a spin-zero particle and applies angular momentum and parity conservation. The parity of the initial state for total angular momentum J is[8] $\eta_P\eta_Z(-1)^{J-S_P-S_Z}=\eta_P\eta_Z(-1)^J$ [the η_i are intrinsic parities and the S_i are spin quantum numbers with $S_P=0$, $S_Z=0$], while the parity of the final state with the V-meson in a helicity-zero eigenstate is $\eta_V\eta_Z(-1)^{J-1}$, and since $\eta_V=\eta_P$, the helicity-zero amplitude vanishes by parity conservation. With the possibility of helicity-zero removed and considering the case $\hat{p}_V=\hat{p}_P$, one then sees that conservation of the component of angular momentum along $\hat{p}_P$ is violated by one unit so that the production amplitude must behave like $d(J,M,M\pm1;\theta)\propto\theta$ [see Eq. (2.20)].

On the basis of the above discussion we shall approximate the $P+p\to V+p$ nuclear-non-spin-flip amplitude by

$$\begin{aligned}\beta_p(E_p,\theta)&=\left[\frac{1}{2\pi}\frac{dt}{d(\cos\theta)}\right]^{1/2}g_p(s)\epsilon_{\alpha\beta\mu\nu}(p_V)_\alpha\eta_\beta(\lambda_V)\\&\quad\times(p_p)_\mu(p_p+p_{p;i})_\nu,\\ s&\equiv-(p_P+p_{p;i})^2=2E_Pm_p+m_p^2+m_P^2,\\ t&\equiv-(p_V-p_P)^2,\\ \cos\theta&\equiv\hat{p}_V\cdot\hat{p}_P.\end{aligned} \quad (5.7)$$

Furthermore, since the $P+n\to V+n$ amplitude is experimentally unknown, we assume for the purpose of an order of magnitude estimate that

$$\beta_n\cong\beta_p. \quad (5.8)$$

[15] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

[16] J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

With the approximations of Eqs. (5.7) and (5.8), the signal-to-noise ratio, Eq. (4.3), becomes

$$R(E_p)\approx 24\left(\frac{Z}{A}\right)^2 \frac{(E_p/m_V)^2}{[1-m_p{}^2/m_V{}^2]^7} \times\frac{\alpha\Gamma(V\to P+\gamma)}{m_V{}^8 m_p{}^2|g_p(s)|^2/4\pi}, \quad (5.9)$$

and to the extent that Eq. (5.7) is valid, the barycentric $P+p\to V+p$ proton-non-spin-flip, $\lambda_V\neq 0$ differential cross section is given by

$$d\sigma(P+p\to V+p:\lambda_V\neq 0,\lambda_{p;i}=\lambda_{p;f})/d\Omega_{\text{c.m.}} =[\Phi(s,m_p{}^2,m_P{}^2)\Phi(s,m_V{}^2,m_p{}^2)]^{3/2}\frac{|g_p(s)|^2}{64\pi s^2}\theta_{\text{c.m.}}{}^2,$$

$$\Phi(x,y,z)\equiv x^2+y^2+z^2-2xy-2xz-2yz. \quad (5.10)$$

In no $(A,B)=(V,P)$ case is the above cross section sufficiently well known to determine $g_p(s)$. However, for $K^-+p\to K^{*-}+p$, the entire $\lambda_V\neq 0$, differential cross section (including the nucleon-spin-flip contribution) has been determined with good precision by Friedman and Ross[17] for K^- laboratory momenta of 2.64 BeV/c. Assuming that the nuclear-spin-flip contribution does not dominate the entire $\lambda_V\neq 0$ differential cross section, we find from the data of Ref. 17 that

$$\left[\frac{d\sigma(K^-+p\to K^{*-}+p:\lambda_V\neq 0)}{d\Omega_{\text{c.m.}}}\right]_{(p_{K^-})_{\text{lab}}=2.64\,\text{BeV}/c} \cong(0.8\text{ mb})\theta_{\text{c.m.}}{}^2, \quad (5.11)$$

and therefore, comparing Eq. (5.10) and Eq. (5.11),

$$m_{K^*}{}^8 m_p{}^2|g_p\{s[(p_{K^-})_{\text{lab}}=2.64\text{ BeV}/c]\}|^2/4\pi \approx 2\times 10^{-2}. \quad (5.12)$$

Combining Table I and Eqs. (5.9) and (5.12), we thus have as an *a priori* estimate for the signal-noise ratio, when $(A,B)=(K^{*-},K^-)$ and $E_{K^-}=2.60$ BeV,

$$R(E_{K^-}=2.6\text{ BeV})\approx(2Z/A)^2\times 10^{-2}, \quad (5.13)$$
$$\delta(E_{K^-}=2.6\text{ BeV})=0.04\text{ rad}.$$

Following Ref. 2 we consider two more or less extreme possibilities for the energy dependence of $\beta_p(E_{K^-},\theta)$: (i) $\beta_p\propto\theta$, and (ii) $\beta_p\propto(E_{K^-}{}^{-2})\theta$. Situation (i) is reasonable if $K^-+p\to K^{*-}+p$ proceeds via one-nucleon intermediate states, and (ii) is realized by a vector-meson exchange model. If $\beta_p\propto\theta$, $R(E_{K^-})=1$ requires $E_{K^-}\cong 5$ BeV; and if $\beta_p\propto(E_{K^-}{}^{-2})\theta$, $R(E_{K^-})=1$ requires $E_{K^-}\cong 8$ BeV. As a consequence of the very rapid increase of $R(E_{K^-})$ with E_{K^-} it should be noted that even if Eq. (5.13) is in error by as much as a factor of 10, the estimates $E_{K^-}=5$ and 8 BeV are in error by factors of only $(10)^{1/8}$ and $(10)^{1/4}$, respectively. It would therefore seem reasonable to conclude that $R(E_{K^-})=1$ requires E_{K^-} in the range 3–15 BeV.

Estimates of $R(E_p)$ with (P,V) other than (K^-,K^{*-}) can be made in a similar way with the expected requirements on E_P again in the range of several BeV.

IV. SUMMARY AND CONCLUSIONS

We have shown, using essentially only *electromagnetic current conservation*, that in the general coherent production process $B+Z\to\gamma'+Z\to A+Z$ on a nucleus of relatively large mass (Fig. 1), at small $B\to A$ production angles ($\theta\approx\delta\ll 1$), transverse virtual photons (as viewed in the rest frame of A) are predominant, and we have inferred that the same is true when the single-particle state A is replaced by the n-particle state X ($m_A\to\sqrt{s},p_A\to p_X$). As a consequence of this effective transversality of γ', we have obtained the selection rule that the helicity of A (or X) differs from that of B by ± 1; and this selection rule, when coupled with angular-momentum conservation, implies that the nuclear-Coulomb-field coherent production amplitude vanishes like θ in the forward direction. This explains the "mysterious" vanishing in the forward direction of the nuclear-Coulomb-field $\Lambda+Z\to\Sigma^0+Z$ production amplitude: As further examples of the helicity selection rule, we would like to point out that the coherent photoproduction of the spin-two f meson in a nuclear Coulomb field, $\gamma+Z\to\gamma+\gamma'+Z\to f+Z$, gives rise to f mesons with helicity 0 and ± 2, but not ± 1; similarly, coherent production in a nuclear Coulomb field of the conjectured spin-one boson W by an incident pion $\pi+Z\to\pi+\gamma'+Z\to W+Z$ gives rise to W with helicity ± 1, but not 0. On the other hand, and as we have mentioned in the Introduction, in the nuclear-Coulomb-field $\pi\to\rho$ and $K\to K^*$ coherent production processes, $\pi+Z\to\pi+\gamma'+Z\to\rho+Z$ and $K+Z\to K+\gamma'+Z\to K^*+Z$, the spin-one ρ and K^* mesons are born with helicity ± 1 because of parity conservation, so that there the helicity selection rule does not provide an additional restriction. The predominance of transverse γ' coupled with the hermiticity of the electromagnetic current operator allows us to establish the basic proportionality between the cross section of nuclear-Coulomb-field $B+Z\to A(X)+Z$ coherent production processes and the $A\to B\neq\gamma$ decay rate ($B+\gamma\to X$ cross section) which, of course, involves only transverse photons [Eq. (2.25)].

Particular attention is given to $(A,B)=(K^{*-},K^-)$, and we estimate that a laboratory energy E_{K^-} in the range 3–15 BeV is required to produce a coherent Coulomb-field $K^-+Z\to K^{*-}+Z$ amplitude of the same order of magnitude as the strong-interaction-induced $K^-+Z\to K^{*-}+Z$ amplitude for production angles in the neighborhood of the Coulomb peak.

[17] J. H. Friedman and R. R. Ross, Phys. Rev. Letters 16, 485 (1966).