

contains a term antisymmetric in both  $a$  and  $b$ , and  $\lambda$  and  $\sigma$ , the low-energy theorem for  $f_4(\nu)$  and the corresponding sum rule (16) would have been modified. We have shown that such is not the case if one starts from the *physical S-matrix element* and applies the reduction formula to it, thereby having an explicit form (31) for the seagull term.

(ii) As is shown in the text, all exact sum rules that have been derived from the current commutation relations by making use of the method of the so-called "current algebra at  $p \rightarrow \infty$ ," can be derived from low-energy limit theorems and the assumption of unsub-

tracted dispersion relations. However, the two approaches are equivalent only if we take the point of view (a).

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## Shrinking of the $p$ - $p$ Scattering Diffraction Peak for Large Momentum Transfers

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The  $p$ - $p$  elastic-scattering cross section can be explicitly calculated in quantum field theory endowed with a fundamental length in the limit of very high energy and large momentum transfer, assuming that a simple (vector-meson) interaction is dominant. Plots of  $X \equiv \log_{10}[(d\sigma/d\Omega)_{e.m.}/(\sigma p/4\pi)^2]$  are given for various lab momenta  $P \gtrsim 10$  GeV/c and momentum transfers  $-t \gtrsim 10$  (GeV/c)<sup>2</sup>. In this range  $X$  involves only a *single, additive parameter*. A strong energy dependence, which gives the observed great shrinking, enters through the "kinematical form factors" attached to the external lines and is thus an immediate consequence of a fundamental length. The fit to the experimental points is excellent for a coupling constant of strong-interaction size if  $\lambda \approx 0.5 \times 10^{-14}$  cm.

THE curves shown in Fig. 1 are theoretical curves for  $p$ - $p$  elastic scattering at very high energies and momentum transfers obtained from quantum field theory endowed with a universal fundamental length, or high-momentum cutoff. To illustrate the typical behavior in this range of  $s$  and  $t$  caused by a fundamental length, we chose the interaction to be mediated by a single neutral vector meson for simplicity. The features arising from a fundamental length are not very sensitive to the details of a more realistic interaction. In this range the cross section and thus  $X$  depend on only a *single multiplicative unknown parameter*.<sup>1</sup> Thus in a logarithmic plot one has only the freedom of displacing all the curves rigidly up or down, without, of course, changing their shapes of relative orientation. In view of this, the fit obtained to the "shrinking" (energy dependence of the diffraction peak) seems good to us, better than any theoretical fit we have yet seen. The open circles show Serber's energy-independent theoretical curve,<sup>1</sup> obtained from an optical model, for

comparison. Although the curves have been plotted back to zero momentum transfer, the small  $-t$  [ $-t \lesssim 10$  (GeV/c)<sup>2</sup>] is not meant to be significant.

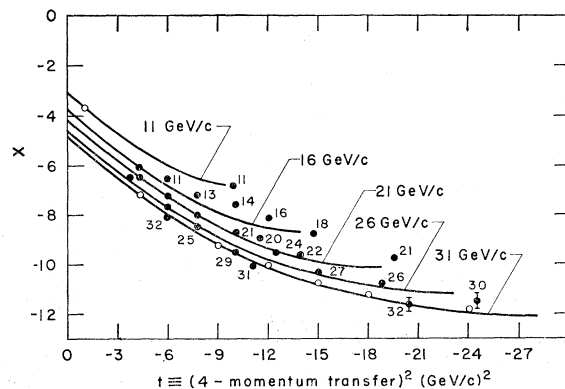


FIG. 1. The normalized  $p$ - $p$  elastic scattering cross section in the very high energy and large-momentum-transfer range, computed from Eq. (2.6) with  $X$  defined by Eq. (2.7). ● = experimental points from Ref. 12. ○ = Serber's theoretical optical model (Ref. 1). The curves correspond to  $\beta_p/\lambda^2 \approx 1.46 \times 10^4$  (GeV/c)<sup>6</sup>. The behavior for  $-t \lesssim 10$  (GeV/c)<sup>2</sup> is not significant.

<sup>1</sup> R. Serber, Rev. Mod. Phys. 36, 649 (1964).

### 1. SOME BACKGROUND

The general theory of very high energy scattering is the same as that presented in a previous paper<sup>2</sup> (herein after referred to as I); therefore we here limit ourselves to a few remarks. The basic idea is that, owing to the high-momentum cutoff  $\sim 1/\lambda$  introduced by the fundamental length  $\lambda$ , higher order graphs and radiative corrections become negligible for large enough energies and momentum transfers, so that the amplitude reduces to the sum of a certain small number of low-order Feynman graphs (the "surviving" graphs). Thus the differential cross section can be explicitly calculated in this range. Proton-proton elastic scattering has now been done at energies and momentum transfers large enough to satisfy this criterion. At the time paper I was written, sufficiently large momentum transfers had not been measured.

The cross section given here differs, however, from that given in I in two significant ways.

Recall first that in the stochastic field theory, the  $S$ -matrix elements are formed by the usual Feynman diagram rules with the sole change that an external line of momentum  $k$  gets the extra factor  $g(k)$  if incoming,  $g^*(k)$  if outgoing, and an internal line of momentum  $k$  gets the extra factor  $g(k)g^*(k) \equiv |g(k)|^2$ , where  $g(k)$  is the *kinematical form factor*. Since the theory of the general kinematical form factor, in particular that of the form factor (1.6) we have used in later work, has not been published elsewhere, we shall spend some time on it here. The reader who is only interested in the predictions of the theory for  $p$ - $p$  scattering may skip to Sec. 2.

#### The Kinematical Form Factor

All the corrections brought by stochastic field theory to the predictions of conventional field theory for scattering and production experiments—i.e., as far as  $S$ -matrix theory goes—are comprised in these kinematical form factors  $g(k)$ .  $g(k)$  is defined essentially as the Fourier transform of the frequency function over its support "with respect to the random space-time variable." That is,

$$g(k) \equiv \int_{s(x; \mathcal{L})} d\mu(\xi-x) f(\xi-x) \times \exp[ik \cdot (X(\xi; x) - x)] \quad [ \xi \in s(x; \mathcal{L}) ]. \quad (1.1)$$

The reader is referred to Ref. 3 for the notation and a complete discussion of the restrictions imposed on the frequency function  $f$ , the random space-time variable  $X$  with mean value  $x$ , and the support  $s$  with measure element  $d\mu$  for an admissible Lorentz-invariant stochastic space-time. (In particular those restrictions guarantee that the integral is independent of  $x$ .)

Given an inertial frame  $\mathcal{L}$  with 4-velocity  $n(\mathcal{L})$ ,<sup>4</sup> we have adopted in all applications the 3-dimensional planes

$$s(x; \mathcal{L}): \quad (\xi-x) \cdot n(\mathcal{L}) = 0 \quad (1.2)$$

as the supports  $s(x; \mathcal{L})$  and the usual Cartesian volume element as the measure element. This corresponds to making the random time variable  $X^4$  some function of the random spatial variables  $\mathbf{X}$ , or physically, to reducing time measurements to spatial measurements. Not only are there physical arguments for this "3-dimensional case," but other mathematical arguments single it out as the only possible physical case—in particular, there exist no stochastic space-times with 4-dimensional supports  $s(x; \mathcal{L})$ .<sup>3</sup> If now  $s(x; \mathcal{L})$  is chosen to be (1.2), then a few very general requirements, principally Lorentz invariance, fix the frequency function as a 3-dimensional spatial Gaussian.<sup>3</sup>

The form factor then becomes

$$g(k) = [(2\pi)^{1/2}\lambda]^{-3} \int_{-\infty}^{\infty} d^3\xi \exp[-(\xi-x)^2/2\lambda^2] \times \exp[ik \cdot (X(\xi; x) - x)] \quad (\xi^4 = x^4), \quad (1.3)$$

if  $x^1, \dots, x^4$  are mean coordinates referred to frame  $\mathcal{L}$ , since in these coordinates the support (1.2) becomes simply all of  $\mathcal{L}$ 's 3-space at time  $\xi^4 = x^4$ . The fundamental length  $\lambda$  is introduced as essentially the standard deviation of the frequency function.

Thus  $g(k)$  is a functional of the random variable  $X^\mu(\xi; x)$ . What function is used here depends on the physical clock adopted. In previous work, in particular in I, a simple stylized dispersionless time was used, namely,

$$\text{dispersionless time: } \mathbf{X}(\xi; x) = \xi, \quad X^4(\xi; x) = x^4 \quad (\xi^4 = x^4). \quad (1.4)$$

This leads to the (real) Gaussian form factor  $g(k) = \exp(-k^2\lambda^2/2)$ . It now seems to us that a form corresponding to measuring time in terms of spatial measurements by using light signals which have constant velocity  $c=1$  by definition ("Einstein clock") is more realistic. This can be shown<sup>5</sup> to lead to the random variable

$$\text{Einstein clock: } \mathbf{X}(\xi; x) = \xi, \quad X^4(\xi; x) = x^4 + 2\sqrt{2}\lambda/\sqrt{\pi} - |\xi-x| \quad (\xi^4 = x^4). \quad (1.5)$$

When  $c \rightarrow \infty$ , (1.5) in fact reduces to (1.4), so it is seen that the idealization previously used corresponded to infinite light velocity.

In any case, whether (1.4) or (1.5) or any other admissible random variable  $X^\mu(\xi; x)$  defined on the

<sup>2</sup> R. L. Ingraham, Nuovo Cimento 32, 323 (1964).

<sup>3</sup> R. L. Ingraham, Nuovo Cimento 34, 182 (1964).

<sup>4</sup> Geometrically,  $n(\mathcal{L})$  is the unit time-like vector pointing toward the future along the time axis of the 4-dimensional frame corresponding to the inertial frame  $\mathcal{L}$ , thus  $n(\mathcal{L}) \cdot n(\mathcal{L}) = -1$ ,  $n^{\mu'}(\mathcal{L}) > 0$  in any (primed) coordinates. Referred to  $\mathcal{L}$ 's (unprimed) coordinates,  $n^\mu(\mathcal{L}) = (0 \ 0 \ 0 \ 1)$ .

<sup>5</sup> D. T. Bailey and R. L. Ingraham (to be published).

plane (1.2) is substituted into (1.3), the form factor will be a function of  $k$  only in the form of  $k_1^2\lambda^2$  and  $k \cdot n(\mathcal{L})\lambda$ , where  $k_1^2 \equiv k^2 + [k \cdot n(\mathcal{L})]^2$ . But since  $g(k)$  enters only through the mean free fields of which the stochastic  $S$  operator is built,<sup>2</sup> for which  $k^2 = -\mu^2$  ( $\mu \equiv$  mass of the field), it can be shown that for a line of a Feynman diagram of momentum  $k$ , whether describing a real or virtual particle of mass  $\mu$  (i.e., whether external or internal),  $k \cdot n(\mathcal{L})$  in the attached form factor has the value

$$k \cdot n(\mathcal{L}) = -(k_1^2 + \mu^2)^{1/2} \quad \text{in } g(k).$$

Thus, in any case  $g(k)$  will be a function of  $k$  only in the form  $k_1^2\lambda^2$ .

When (1.5) is substituted into (1.3) and  $\mu$  is specialized to zero (which will be sufficient for our purposes in this paper), the integration yields

$$|g(k)|^2 = \exp(-4k_1^2\lambda^2) + [Z((2k_1^2\lambda^2)^{1/2})]^2, \quad (1.6)$$

for the absolute square (all we need for the cross section). Here

$$Z(x) \equiv \frac{2}{\sqrt{\pi}} e^{-x^2} \int_0^x dy e^{y^2}. \quad (1.7)$$

The first "significant way" (see above) in which the cross section given here differs from that given in I is that the form factor (1.6) is used, rather than the previous simple Gaussian  $|g(k)|^2 = e^{-k_1^2\lambda^2}$ .

Finally, we emphasize a crucial point. The 4-velocity  $n(\mathcal{L})$  to be substituted into (1.6) is that of the *measuring frame*  $\mathcal{L} \equiv$  inertial frame, in which the experiment is actually performed, i.e., the frame in which the measuring apparatus is fixed. This measuring frame for the  $p$ - $p$  scattering experiments of interest here is the so-called " $p$ - $p$  laboratory system," i.e., the frame in which one proton (the "target proton") is at rest.

### Lorentz Invariance

Although the question of the Lorentz invariance of this theory has been gone into in detail elsewhere,<sup>6</sup> we give a short qualitative discussion here to make this paper reasonably self-contained. Our aim is to convey the basic physical idea and prevent some obvious misunderstandings, rather than to present the mathematical details.

Lorentz invariance is usually understood to include the Lorentz *form invariance* of the  $S$  operator: namely, it should commute with the representation  $U(L)$  of the inhomogeneous Lorentz group  $L_4 \uparrow$  on the state vector space for all  $L \in L_4 \uparrow$ . Since our form factor and thus our  $S$  operator involve  $n(\mathcal{L})$ , the unit time-like normal associated with the measuring frame  $\mathcal{L}$ , our theory is evidently not Lorentz invariant in this sense. However, we have preferred to retain the term Lorentz invariant for this theory in spite of the semantic danger involved,

<sup>6</sup> R. L. Ingraham, *Nuovo Cimento* **27**, 303 (1963). See also Ref. 3.

because it respects the principle of relativity,<sup>7</sup> which lies at the base of special relativity. The following brief remarks will attempt to elucidate this statement.

The relativity principle requires that if two observers  $\mathcal{L}_1$  and  $\mathcal{L}_2$  (identified with their inertial frames together with synchronized clocks) in relative motion "do the same experiment," they must get identical results. For example, if the same types of particles collide with the same initial velocities relative, respectively, to the two frames, and they each measure the differential cross section, they should get the same numbers. It seems to us that it is essential to require that they "do the same experiment"—that the initial conditions are the same relative to each and that they perform the same operations, here *measuring* the particle fluxes—otherwise their physical equivalence under the Lorentz group does not guarantee this result. "Lorentz invariance" as we use it means only this statement of the physical equivalence of a class of frames, and implies nothing further.

The frame-dependent theory, as well as the current theory, is Lorentz invariant in this sense, because, essentially, only numerically the same components of the two normals  $n(\mathcal{L}_1)$  and  $n(\mathcal{L}_2)$  are involved in the matrix elements of the two  $S$  operators  $S(\mathcal{L}_1)$  and  $S(\mathcal{L}_2)$  for the two experiments. Another way of seeing this is to note that the set of all matrix elements of  $S(\mathcal{L})$  is the same as the set of all matrix elements of any other  $S(\mathcal{L}')$ , and all possible measuring frames  $n(\mathcal{L})$  enter the theory. There is no preferred frame.

This situation is essentially different from the case in which there is one preferred "rest" frame  $\mathcal{L}_0$ , say, as in ether theories, such that experiments performed in  $\mathcal{L}_0$  are distinguishable from the *same* experiment performed in another "moving" frame  $\mathcal{L}$ . This latter would constitute a real breach of Lorentz invariance.

According to the frame-dependent theory, the *different* operations of measuring  $d\sigma/d\Omega$  in the c.m. frame of the particles and measuring  $d\sigma/d\Omega$  in the lab frame of the particles and transforming it back mathematically to the c.m. frame give (slightly) different results at high energies. But this does not contradict Lorentz invariance ( $\equiv$  the relativity principle), since the two observers have performed different experiments. On the other hand, the current frame-independent theory, while also compatible with the relativity principle, goes beyond it to predict the extra property that the results of these two different experiments are the same: The cross section, referred to the c.m. frame, say, is actually independent of the frame in which it is measured.

This latter property of the usual  $S$  operator comes down to the postulate that the number of particles scattered into a given solid angle at some point per unit

<sup>7</sup> The principle of relativity as it is usually stated says that physical field equations are form-invariant under the group  $L_4 \uparrow$ . But note that this does not say that solutions of these equations must be form invariant, i.e., frame-independent, and in fact this possibility leads to the frame-dependent fields and  $S$  operator of the stochastic field theory.

time (measured by a clock at rest relative to it, say) is an absolute, regardless of how it is measured. The frame-dependent theory does not share this property; the nonabsoluteness of particle flux is presumably tied up<sup>8</sup> with the existence of a "minimal" or fundamental length, though we have no physical explanation of this predicted phenomenon. But note that there is no contradiction with quantum mechanics: Probability is conserved because each  $S(\mathcal{L})$  is unitary. What happens, therefore, is that at higher energies relative to the measuring frame the effective coupling is weakened, the particles become more mutually transparent, and more particles pass through without being scattered.

The violation of this absoluteness of particle flux in the sense explained above is certainly bizarre from the standpoint of macroscopic intuition. The only reasons that its possibility is entertained here are theoretical: If one grants that the introduction of a cutoff into  $S$  is equivalent to treating space-time as slightly stochastic ("nonlocal"), as seems likely, then the theory of stochastic space-time shows<sup>3</sup> that such a frame dependence cannot be avoided. But since elementary particles have many peculiar nonmacroscopic properties, we suggest that it be considered an experimental question.

### The Definition of the Differential Cross Section

The second and more important difference from I is that we now form the cross section for the process  $f \leftarrow i$  in exactly the conventional way<sup>9</sup> in terms of the  $S$ -operator  $S(\mathcal{L})$ , namely, in terms of the amplitude  $M_{fi}(\mathcal{L})$  defined by

$$S_{fi}(\mathcal{L}) \equiv \delta_{fi} + \delta(p_f - p_i) M_{fi}(\mathcal{L}). \quad (1.8)$$

In the preliminary definition given in I, the amplitude was defined by factoring out of this  $M_{fi}(\mathcal{L})$  all the form factors carried by the external lines [cf. Eq. I(5.1) and Ref. 28 of I]. The physical interpretation leading to this tentative definition of the cross section in stochastic field theory is now believed to be erroneous.

The practical consequence of using the amplitude  $M_{fi}(\mathcal{L})$  of (1.8) is that a strong energy dependence is introduced through the external line form factors [cf. Eq. (2.6) below], which is responsible for the great shrinking in  $p$ - $p$  scattering of the experimental order of magnitude, as is seen from Fig. 1. In I the shrinking was clearly inadequate (cf. Figs. 2 and 3 of I), even though, strictly speaking, the values of  $-t$  were too small for this theory to apply.

## 2. THE THEORETICAL $p$ - $p$ CROSS SECTION

Our intention here, as in I, is to investigate a model in which the  $p$ - $p$  scattering is mediated by an unspecified

<sup>8</sup> See A. March, *Quantum Mechanics of Particles and Wave Fields* (John Wiley & Sons, Inc., New York, 1951), especially pp. 274-277.

<sup>9</sup> See for example J. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Cambridge University Press, New York, 1955); Sec. 8-6.

neutral vector meson  $B_\mu^0$  of mass  $\kappa$  and  $B^0 p p$  coupling constant  $f_p$ . One justification is the fact that surviving pion-exchange graphs are negligible compared with the surviving vector-meson exchange graphs in the very high energy limit.<sup>10</sup> In any case, the model should illustrate the fundamental features imposed by a high-momentum cutoff, which should carry over to a more realistic interaction. It then turns out that the value of the mass  $\kappa$  is irrelevant for the high momentum transfers of interest, while the value of the fundamental length  $\lambda$  depends very insensitively on  $\beta_p \equiv f_p^2/4\pi$ .

The notation and formulas of Sec. 5 of I are carried over intact except for the new form factor (1.6) and the presence of the extra form factors from the external lines. Thus the  $p$ - $p$  differential cross section as measured in the  $p$ - $p$  laboratory system and transformed to the  $p$ - $p$  c.m. system is given by the general equation I(5.2) with  $M_{fi}(\mathcal{L})$  the amplitude defined by (1.8), or

$$(d\sigma/d\Omega)_{c.m.} = (2\beta_p^2/s) |g(p_1)g(p_2)g(p_1')g(p_2')|^2 \\ \times [|g(q)|^4 A_1/(-t+\kappa^2)^2 + |g(\bar{q})|^4 \\ \times A_2/(-u+\kappa^2)^2 + |g(q)g(\bar{q})|^2 \\ \times 2A_3/(-t+\kappa^2)(-u+\kappa^2)],$$

where

$$\beta_p \equiv f_p^2/4\pi, \quad \mathcal{L} \equiv p\text{-}p \text{ laboratory system.} \quad (2.1)$$

Here  $p_1$  and  $p_2$  ( $p_1'$  and  $p_2'$ ) are the initial (final) 4-momenta,  $w \equiv p_1 + p_2$ ,  $q \equiv p_1 - p_1'$ ,  $\bar{q} \equiv p_1 - p_2'$ , and  $s \equiv -w^2$ ,  $t \equiv -q^2$ ,  $u \equiv -\bar{q}^2$ , as usual.

The  $A_i$  are expressions in  $s$ ,  $t$ , and  $u$  which were given in I(5.6). The values of  $k_i^2$  for the various momenta involved in the form factors are easily computed to be

$$p_{11}^2 = P^2, \quad p_{21}^2 = 0, \quad p_{11'}^2 = \bar{q}_1^2, \quad p_{21'}^2 = q_1^2, \\ q_1^2 = -t(1-t/4M^2), \quad \bar{q}_1^2 = -u(1-u/4M^2), \quad (2.2) \\ \mathcal{L} = p\text{-}p \text{ laboratory system } (p_2=0),$$

where  $P \equiv$  lab 3-momentum and  $M \equiv$  proton mass.

In the very high energy and large momentum-transfer regime, the following approximations are good:

$$A_1 \approx 2M^2 P^2 + M(M+P)t + \frac{1}{4}t^2; \\ A_2 \approx M^2 P(P-2M) - 2M^2 t + \frac{1}{4}t^2; \\ A_3 \approx M^2 P(P-2M), \quad (M^2 \ll P^2, \kappa^2 \ll -t, -u); \quad (2.3) \\ s \approx 2M(M+P), \quad -t + \kappa^2 \approx -t; \\ -u + \kappa^2 \approx -u \approx 2M(P-M) + t, \\ -u(1-u/4M^2) \approx (P+t/2M)^2,$$

cf. I(5.9) and I(5.10).

### Large-Momentum Approximation of $g(k)$

For large values of the momentum  $k$ , ( $k_1^2 \lambda^2 \gg 1$ ) the exponential term in the form factor (1.6) is negligible compared to the last term, and for this term we can

<sup>10</sup> See I, Sec. 5.

use the asymptotic approximation<sup>11</sup>

$$Z(x) \sim 1/\pi^{1/2}x \quad (|x| \gg 1). \quad (2.4)$$

Thus

$$|g(k)|^2 \approx 1/2\pi k_1^2 \lambda^2 \quad (k_1^2 \lambda^2 \gg 1). \quad (2.5)$$

This approximation will be good for all the form factors occurring in (2.1) for the lab 3-momenta of interest,  $P \geq 11$  GeV/c and for momentum transfers  $-t \gtrsim 10$  (GeV/c)<sup>2</sup>, with the exception of  $|g(p_2)|^2$ , which  $\approx 1$  since  $p_{21}^2 = 0$ . Thus, the dependence on the fundamental length becomes multiplicative. In fact, the cross section in the approximations (2.3) and (2.5) reads

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{e.m.}} &\approx 2 \left(\frac{\beta_p}{\lambda^5}\right)^2 \frac{1}{2M(M+P)} \frac{1}{(2\pi)^5} \\ &\times \frac{1}{P^2(P+t/2M)^2(-t)(1-t/4M^2)} \\ &\times \left[ \frac{1}{t^4(1-t/4M^2)^2} A_1 + \frac{1}{(P+t/2M)^4 u^2} A_2 \right. \\ &\left. + \frac{1}{t^2(-u)(1-t/4M^2)(P+t/2M)^2} 2A_3 \right], \quad (2.6) \end{aligned}$$

where  $A_1$ ,  $A_2$ ,  $A_3$ , and  $-u$  are to be replaced by the approximations (2.3). Thus, as asserted before, the cross section depends only on the single multiplicative unknown parameter  $(\beta_p/\lambda^5)^2$ .

We define the quantity  $X$  by

$$X \equiv \log_{10}[(d\sigma/d\Omega)_{\text{e.m.}}/(\sigma p/4\pi)^2], \quad (2.7)$$

where  $p \equiv$  c.m. 3-momentum  $\approx [M(M+P)/2]^{1/2}$  and  $\sigma \equiv$  experimental total cross section  $\approx 40$  mb. This quantity was plotted and compared with the experimental points<sup>12</sup> shown in Fig. 1. A best fit for large

momentum transfers  $-t \gtrsim 10$  (GeV/c)<sup>2</sup> determined this single parameter as

$$\beta_p/\lambda^5 \approx 1.46 \times 10^4 \text{ (GeV/c)}^5. \quad (2.8)$$

Fortunately, this makes  $\lambda$  a very insensitive function of the strong coupling constant  $\beta_p$ . Taking  $\beta_p = 15$ , for example, we get

$$\lambda = 0.25 \text{ (GeV/c)}^{-1} = 0.49 \times 10^{-14} \text{ cm}, \quad (2.9)$$

while taking  $\beta_p$  as small as 1 reduces  $\lambda$  only by a factor of  $(15)^{-1/5} \approx 0.6$ .

For small momentum transfers  $[-t \lesssim 10 \text{ (GeV/c)}^2]$  the form factor approximation (2.5) breaks down, while for very small momentum transfers the approximation  $-t \gg \kappa^2$  breaks down as well. In this range of  $-t$  the curves shown in Fig. 1 were extrapolated back to their intercepts with the axis  $t=0$  determined from the formulas (2.1) and (1.6) with  $\kappa$  chosen as 750 MeV. However, because we expect that the large-momentum-transfer approximation leading to the simple formula (2.1) ( $\equiv$  negligible radiative corrections) breaks down in this region, we claim no significance for the very fair agreement with the experimental points in the range  $-t \lesssim 10 \text{ (GeV/c)}^2$ .

### 3. DISCUSSION

The strong energy dependence introduced by the external line form factors (the second line of Eq. (2.6)) is seen clearly from the formula (2.6); these factors are absent from the corresponding formula of I. They split apart the curves for the various energies sufficiently to give the great observed shrinking. In addition, they also serve to depress all curves so that the value of the fundamental length  $\lambda$  now found from  $p$ - $p$  scattering is only of the order of tenths of  $10^{-14}$  cm, rather than the value  $\lambda \approx 1 \times 10^{-14}$  cm found in I from  $p$ - $p$  scattering and also from low-order perturbation-theoretic calculations of electromagnetic mass shifts in isotopic multiplets.<sup>13</sup>

<sup>11</sup> R. L. Ingraham and D. T. Bailey, *Nuovo Cimento* **33**, 246 (1964), Eq. (6).

<sup>12</sup> G. Cocconi *et al.*, *Phys. Rev. Letters* **11**, 499 (1963); W. Baker *et al.*, *ibid.* **12**, 132 (1963).

<sup>13</sup> R. Genolio, doctoral thesis, New Mexico State University, 1963 (unpublished).