Renormalized ω-ϕ Mixing Angle and $J^P=1^-$ Meson Decays

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We use simple field-theoretic arguments to define a renormalized ω-ϕ mixing angle and compare predictions between the normalized and renormalized theories and experiment. The sensitivity of the predictions to the initial assumptions is briefly discussed.

IN a previous paper[1] (hereafter known as I) we generalized the field-theoretic approach of Coleman and Schnitzer[2] [for estimating the field renormalization constants and mixing parameters of the $J^P=0^-$, and $J^P=1^-$ nonet mesons due to $SU(3)$ symmetry-breaking interactions] to mesons of arbitrary spin, with particular reference to the $J^P=2^+$ meson nonet. It has been emphasized that this approach is not equivalent in general to the usual particle-mixing formalism[3] where the $SU(3)$-breaking interaction is taken to mix fields (with the same medium-strong quantum numbers) but not to renormalize them. In this paper we further discuss the renormalization and mixing of the $J^P=0^-$, 1^- meson nonets by introducing renormalized and unrenormalized mixing angles (as suggested in I). This enables us to give a simple ansatz for calculating the effects of renormalization on the predictions of $SU(3)$ with conventional particle mixing.

As in Ref. 2 and I we make the assumptions that:

(a) The $SU(3)$ symmetry breaking does not affect the vacuum expectation values of the equal-time commutators and their first time derivatives.

(b) Integrals containing the difference of the continuum parts of the perturbed and unperturbed two-point spectral functions have simple order-of-magnitude properties.

We make the further assumptions that, in the absence of $SU(3)$ breaking:

(c) The unperturbed ρ, K^* and unmixed isospin singlets ϕ_0 and ω_0 form a *degenerate* nonet of mass M of the form

$$V_9=V_8+(3)^{-1/2}\omega_0 1\,, \tag{1}$$

where V_8 is the vector-meson octet.[4]

(d) The unperturbed π, K and the unmixed isospin singlets η_0, X_0 form a *degenerate* nonet,

$$P_9=P_8+(3)^{-1/2}X_0 1\,, \tag{2}$$

where P_8 is the pseudoscalar-meson octet.[5]

From I, we see that the field renormalization constants are given by

$$Z_\rho=m_\rho^2/M^2\,,\quad Z_{K^*}=m_{K^*}^2/M^2\,, \tag{3}$$

and

$$Z_\omega^{ij}/m_\omega^2+Z_\phi^{ij}/m_\phi^2=\delta^{ij}/M^2 \tag{4}$$

for the 1^- nonet, and

$$Z_\pi=Z_K=1\,, \tag{5}$$

$$Z_\eta^{ij}+Z_X^{ij}=\delta^{ij} \tag{6}$$

for the 0^- nonet. The merit of (3) and (4) is that charge conservation in a vector-meson pole-dominance model of electric form factors is preserved under $SU(3)$ mass-breaking.

Since the matrix Z's are real, tracless, and symmetric, we can express them in terms of real angles γ and β as

$$Z_\eta=\begin{pmatrix}\cos^2\gamma & -\cos\gamma\sin\gamma\\ -\cos\gamma\sin\gamma & \sin^2\gamma\end{pmatrix}, \tag{7}$$

$$Z_X=\begin{pmatrix}\sin^2\gamma & \cos\gamma\sin\gamma\\ \cos\gamma\sin\gamma & \cos^2\gamma\end{pmatrix}, \tag{8}$$

$$Z_\phi=Z_\phi^0\begin{pmatrix}\cos^2\beta & -\cos\beta\sin\beta\\ -\cos\beta\sin\beta & \sin\beta\end{pmatrix}, \tag{9}$$

$$Z_\omega=Z_\omega^0\begin{pmatrix}\sin^2\beta & \cos\beta\sin\beta\\ \cos\beta\sin\beta & \cos^2\beta\end{pmatrix}, \tag{10}$$

where

$$Z_\phi^0=m_\phi^2/M^2\,,\quad Z_\omega^0=m_\omega^2/M^2\,. \tag{11}$$

We can interpret β, γ by considering vertices in which η, X, ω, ϕ interact. Since γ offers immediate interpretation, let us consider vertices, η, $X\rightarrow A+B$ where for simplicity A and B are unmixed particles belonging to irreducible representations of $SU(3)$. In the absence of $SU(3)$ breaking, η_0, $X_0\rightarrow A$, B vertices have coupling constants $G_{\eta_0 AB}$ and $G_{X_0 AB}$, respectively. We assume that we can neglect vertex renormalization[2] and that the coupling-constant renormalization is due entirely to field renormalization and mixing. We can thus consider the coupling-constant renormalization as taking place in two stages: (1) η_0, X_0 are mixed and renormalized; (2) A, B are renormalized.

After (1) the coupling constants $G_{\eta AB}$, G_{XAB} for

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[1] R. J. Rivers, Phys. Rev. **150**, zxx (1966).
[2] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).
[3] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).
[4] S. Okubo, Phys. Letters **5**, 165 (1963).
[5] This is shown to be equivalent to the η-X mixing in Ref. 1. See S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

physical η and X are given by

$$G_{\eta AB}{}^2=G^T Z_\eta G\,,\quad G_{XAB}{}^2=G^T Z_X G\,, \tag{12}$$

where $G^T=(G_{\eta_0 AB},G_{X_0 AB})$. From (7) and (8) we see that Eqs. (12) are satisfied by

$$G_{\eta AB}=G_{\eta_0 AB}\cos\gamma-G_{X_0 AB}\sin\gamma\,,$$

and

$$G_{XAB}=G_{\eta_0 AB}\sin\gamma+G_{X_0 AB}\cos\gamma\,. \tag{13}$$

This is the result that we would obtain by the conventional particle mixing in which we take

$$\eta=\eta_0\cos\gamma-X_0\sin\gamma\,,\qquad X=\eta_0\sin\gamma+X_0\cos\gamma\,. \tag{14}$$

Thus in this scheme 0^- particles are mixed but not renormalized by $SU(3)$ breaking.

We can treat vector mesons in an analogous way. For $\omega_0,\phi_0\rightarrow A$, B vertices with unperturbed coupling constants $G_{\omega_0 AB}$, $G_{\phi_0 AB}$, the coupling constants

$$G_{\phi AB}{}^0=G_{\phi_0 AB}\cos\beta-G_{\omega_0 AB}\sin\beta\,,$$

and

$$G_{\omega AB}{}^0=G_{\phi_0 AB}\sin\beta+G_{\omega_0 AB}\cos\beta \tag{15}$$

correspond to conventional particle mixing

$$\phi=\phi_0\cos\beta-\omega_0\sin\beta\,,\qquad \omega=\phi_0\sin\beta+\omega_0\cos\beta\,, \tag{16}$$

but no renormalization. If the coupling constants for physical renormalized ω, ϕ (in the absence of renormalization of A, B) are $G_{\omega AB}$, $G_{\phi AB}$, we see from (9), (10), (15), and (16) that

$$G_{\omega AB}=(Z_\omega{}^0)^{1/2}G_{\omega AB}{}^0\,,\qquad G_{\phi AB}=(Z_\phi{}^0)^{1/2}G_{\phi AB}{}^0\,. \tag{17}$$

Thus for vector mesons the $SU(3)$ breaking causes mixing through an angle β and renormalization. The effects of the renormalization of A and B can be calculated in like manner, even when A and B are mixed by the symmetry breaking.

Thus the renormalized mixed coupling constants G_{ABC} (where A, B, C are physical members of the 0^- and 1^- nonets) are calculated from the unperturbed coupling constants in the following way. We calculate the unrenormalized mixed coupling constants $G_{ABC}{}^0(\beta,\gamma)$ by means of the mixing (14) and (16) with mixing angles γ and β in the usual way. From (3), (5), (11), and (17), G_{ABC} is given in terms of $G_{ABC}{}^0(\beta,\gamma)$ as

$$G_{ABC}=Z_A{}^{1/2}Z_B{}^{1/2}Z_C{}^{1/2}G_{ABC}{}^0(\beta,\gamma)\,, \tag{18}$$

where $Z_V{}^{1/2}=M_V/M$ for all members of the vector nonet and $Z_P{}^{1/2}=1$ for all members of the pseudoscalar nonet.

We have yet to express β and γ in terms of the physical masses. Assuming Okubo-type octet mass breaking[6] with a quadratic mass operator[7] (differing slightly from Refs. 2 and I, where an inverse quadratic mass operator was taken), we have (from I)

$$\sin^2\gamma=[m_\eta{}^2-m_8{}^2(P)](m_\eta{}^2-m_X{}^2)^{-1}\,, \tag{19}$$

where

$$m_8{}^2(P)=\tfrac{1}{3}[4m_K{}^2-m_\pi{}^2]\,, \tag{20}$$

which is the conventional renormalized mixing angle for η-X mixing.[8] Inserting physical masses,[9] we have $\gamma=\pm10°$. The angle β is given by

$$\sin\beta=[m_\omega/m_8(V)]\sin\alpha\,,\qquad \cos\beta=[m_\phi/m_8(V)]\cos\alpha\,, \tag{21}$$

where α is the renormalized ω-ϕ mixing angle,

$$\sin^2\alpha=[m_\phi{}^2-m_8{}^2(V)](m_\phi{}^2-m_\omega{}^2)^{-1}\,, \tag{22}$$

and

$$m_8{}^2(V)=\tfrac{1}{3}[4m_{K^*}{}^2-m_\rho{}^2]\,. \tag{23}$$

We also have

$$M^2=\tfrac{1}{3}[m_\rho{}^2+2m_{K^*}{}^2]\,. \tag{24}$$

Thus for the vector mesons, in addition to renormalizing the fields, the $SU(3)$ breaking renormalizes the mixing angle. Inserting physical masses we have $\alpha=39.9°$, $\beta=32.7°$.[10] The maximal mixing angle, given by the Okubo ansatz that $\mathrm{Tr}V_9$ never appears in the mass operator,[11] is $\theta=\sin^{-1}(1/\sqrt{3})=35.3°$. We note that the renormalized angle β is closer to θ than the renormalized mixing angle α.

Using the Okubo nonet ansatz for vector-meson interaction Lagrangians that terms proportional to $\mathrm{Tr}V_9$ do not appear, and a similar ansatz for the pseudoscalar nonet, we compare the predictions of the renormalized and unrenormalized theories [with coupling constants $G_{ABC}{}^0(\alpha,\gamma)$] with experiment for particular cases, especially when ω, ϕ, η are in the initial or final states.

$1^-\rightarrow0^-+0^-$ *decays*. The experimental partial widths[12] are $\Gamma_{\rho\to\pi\pi}=106\pm5$ MeV, $\Gamma_{K^*\to K\pi}=50\pm2$ MeV, and $\Gamma_{\phi\to K\bar{K}}=2.5\pm0.8$ MeV. Using the above value of β and (19) we predict[13]

$$\Gamma_{\rho\to\pi\pi}:\Gamma_{K^*\to K\pi}:\Gamma_{\phi\to K\bar{K}}=106:46.5:2.8\,, \tag{25}$$

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[6] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[7] For the effect of general mass operators on the unrenormalized theory, see A. J. MacFarlane and R. H. Socolow, Phys. Rev. **144**, 1194 (1966).

[8] Obtained in Lagrangian formalism by diagonalizing the η-X sector of the mass-squared operator

$$aP_{8b}{}^aP_{8a}{}^b+bP_{8a}{}^3P_{83}{}^a+cX_0X_0+dP_{83}{}^3X_0$$

through a mixing of the form (15).

[9] Unless otherwise stated, masses and partial widths are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).

[10] α and β have been taken positive to reproduce the known small $\phi\rho\pi$ coupling.

[11] Obtained by taking a mass-squared operator of the form $aV_{9b}{}^aV_{9a}{}^b+bV_{9a}{}^3V_{93}{}^a$.

[12] J. Yellin, Phys. Rev. **147**, 1080 (1966).

[13] G. Fraser (private communication).

in excellent agreement with experiment. Unrenormalized $SU(3)$ gives the poorer prediction

$$\Gamma_{\rho\to\pi\pi}:\Gamma_{K^*\to m\pi}:\Gamma_{\phi\to K\bar{K}}=106:31.7:2.5. \quad (26)$$

We stress that the satisfactory prediction of $\Gamma_{\rho\to\pi\pi}/\Gamma_{K^*\to K\pi}$ in (25) is independent of β and hence of the mass operator, and is dependent only on (3).

3π decays of ω and ϕ. We use the recent calculation by Yellin[12] on the basis of the Gell-Mann–Sharp–Wagner model[14] for 3π decays via $\rho\pi$ intermediate states that gives $(\Gamma_{\phi\to3\pi}/\Gamma_{\omega\to3\pi})$ in terms of $(G_{\phi\rho\pi}/G_{\omega\rho\pi})^2$. From the nonet ansatz and (18) we have

$$\frac{\Gamma_{\phi\to3\pi}}{\Gamma_{\omega\to3\pi}}=(34\pm2.5)\tan^2(\theta-\beta)(m_\phi^2/m_\omega^2). \quad (27)$$

Taking $\Gamma_{\omega\to3\pi}=10.8\pm1.6$ MeV we find

$$\Gamma_{\phi\to3\pi}/\Gamma_\phi^{\text{tot}}=0.39\pm0.16, \quad (28)$$

in comparison with the unrenormalized predicted ratio of 0.73 ± 0.30 [obtained by replacing $\tan^2(\theta-\beta)\times(m_\phi^2/m_\omega^2)$ by $\tan^2(\theta-\alpha)$ in (27)]. The experimental data[15,16] favor the prediction of the renormalized theory.

Electromagnetic decays of ω and ϕ. The only electromagnetic decays for which there are reliable data are $\omega\to\pi+\gamma$ and $\phi\to\eta+\gamma$. If we assume that the photon couples via ρ, ω, ϕ poles in the usual U-spin scalar combination, the mass differences are taken care of by the coupling-constant renormalization (18) and by the $V\gamma$ coupling-constant renormalization

$$G_{V\gamma}=(m_V/M)G_{V\gamma}^0, \quad (29)$$

where $G_{V\gamma}^0$ are the mixed but unrenormalized coupling constants.

Using simple p^3 phase space, we find

$$\begin{aligned}\Gamma_{\phi\to\eta\gamma}/\Gamma_{\omega\to\pi\gamma}&=0.52 \quad \text{for} \quad \gamma=+10°,\\ &=0.31 \quad \text{for} \quad \gamma=-10°,\end{aligned} \quad (30)$$

in comparison with 0.33 for $\gamma=+10°$ and 0.20 for $\gamma=-10°$ in the unrenormalized theory.[6,17] The experimental data[12,16] are not yet sufficiently good to distinguish between the predictions and are consistent with all of them. Very roughly, $\Gamma_{\phi\to\eta\gamma}/\Gamma_{\omega\to\pi\gamma}\approx0.4$ with large errors.[18]

Leptonic decays of ρ, ω, ϕ. Using an intermediate photon model in which the unrenormalized mixed coupling constants are given by

$$G_{Ve^+e^-}{}^0=G_{V\gamma}{}^0e/m_V{}^2, \quad (31)$$

the renormalized coupling constants (from (18)] become

$$G_{Ve^+e^-}=G_{V\gamma}{}^0e/M_Vm. \quad (32)$$

This gives

$$\frac{\Gamma_{\omega\to e^+e^-}}{\Gamma_{\rho\to e^+e^-}}\cong\frac{\sin^2\beta}{3}\frac{m_\rho}{m_\omega}=0.10 \quad (33)$$

and

$$\frac{\Gamma_{\phi\to e^+e^-}}{\Gamma_{\rho\to e^+e^-}}\cong\frac{\cos^2\beta}{3}\frac{m_\rho}{m_\phi}=0.18, \quad (34)$$

to be compared with 0.14 and 0.26, respectively, in the unrenormalized theory.[19] The experimental data of $\Gamma_{\omega\to e^+e^-}=0.5$ to 8 keV[20] and $\Gamma_{\rho\to\mu^+\mu^-}=(4.0_{-0.8}^{+1.9})$ keV [21] are barely consistent with (33) (μ decay differs from e decay by about 1% using the same model) or with the prediction of the unrenormalized theory. It is to be hoped that better data will become available soon.

ϕ suppression in K^-p interactions. It has been suggested[22] that the suppression of ϕ production (relative to ω) in K^-p interactions can be well accounted for. If we assume the *generalized* Okubo ansatz of Ref. 19 for meson-baryon coupling in which the meson indices are only allowed to couple to the indices of the *three-index* baryon octet representation and $\mathrm{Tr}V_9$ terms are not allowed, we find that[23]

$$G_{\phi pp}{}^0/G_{\omega pp}{}^0=\tan(\theta-\beta) \quad (35)$$

for *both* electric and magnetic couplings, irrespective of the d/f ratio. This gives the ratio of ϕ to ω production roughly as

$$R=(m_\phi^2/m_\omega^2)\tan^2(\theta-\beta)\cong\tfrac{1}{3}\%, \quad (36)$$

about half that of the unrenormalized theory using the same ansatz.

Although the renormalized theory is at least as successful as the unrenormalized theory and probably more so, we ought not to treat the theoretical predictions that depend on β in the combination $\theta-\beta$ too seriously for the reason that our major assumption has been an exactly degenerate vector nonet in the absence of $SU(3)$ breaking. If we think in terms of a quark model in which the $SU(3)$ breaking arises from the quark

[14] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[15] J. S. Lindsey and G. A. Smith, Bull. Am. Phys. Soc. **10**, 502 (1966) give $\Gamma_{\phi\to3\pi}/\Gamma_\phi^{\text{tot}}=0.18\pm0.08$. K. London *et al.*, Phys. Rev. **143**, 1034 (1966) give $\Gamma_{\phi\to\rho\pi}/\Gamma_{\phi\to K\bar{K}}=0.30\pm0.15$ whence $\Gamma_{\phi\to3\pi}/\Gamma_\phi^{\text{tot}}=0.25\pm0.12$.

[16] J. Badier *et al.*, Phys. Letters **17**, 337 (1965).

[17] See also R. H. Dalitz and S. Sutherland, Nuovo Cimento **37**, 1777 (1965); 38, 1945(E) (1965).

[18] The ratio (30) depends on γ approximately as $\Gamma_{\phi\to\eta\gamma}/\Gamma_{\omega\to\pi\gamma}\propto\cos^2(\theta-\gamma)$. The effects of using different mass operators for the 0^- nonet can be obtained from Ref. 7. Since γ is very sensitive to the mass operator used, the ratio (30) is also. For example, using a linear mass operator to give $\gamma=\pm23.4$, we have

$$\Gamma_{\phi\to\eta\gamma}/\Gamma_{\omega\to\pi\gamma}=0.61(\gamma=23.4°) \quad \text{and} \quad 0.16(\gamma=-23.4°).$$

[19] H. Sugawara and F. Von Hippel, Phys. Rev. **145**, 1331 (1966). See also R. F. Dashen and D. H. Sharp, *ibid.* **133**, B1585 (1964).

[20] D. M. Binnie *et al.*, Phys. Letters **18**, 348 (1965).

[21] J. K. de Pagter *et al.*, Phys. Rev. Letters **16**, 35 (1966).

[22] H. M. Fried and J. G. Taylor, Phys. Rev. Letters **15**, 709 (1965).

[23] This corresponds to an interaction Lagrangian proportional to

$$f\bar{B}_b{}^a[V_9,B]_a{}^b+(1-f)\bar{B}_b{}^a\{V_9,B\}_a{}^b+(2f-1)(\mathrm{Tr}V_9)\bar{B}_b{}^aB_a{}^b.$$

mass differences, this degeneracy implies $SU(3)$-independent quark forces in the idealized limit of equal-mass quarks. Suppose that in this idealized limit the quark forces were not completely $SU(3)$-independent so that the singlet had a mass M_0 slightly different from the octet mass M with $M/M_0=I$. From Ref. 1 we see that Eqs. (15) become

$$\begin{aligned} G_{\phi AB}{}^0 &= G_{\phi_0 AB}\cos\beta - IG_{\omega_0 AB}\sin\beta\,,\\ G_{\omega AB} &= G_{\phi_0 AB}\sin\beta + IG_{\omega_0 AB}\cos\beta\,. \end{aligned} \tag{37}$$

For the sake of argument, let us suppose that $G_{\omega_0 AB}$ and $G_{\phi_0 AB}$ are still related by the nonet ansatz. Then, for example,

$$G_{\phi\rho\pi}{}^0/G_{\omega\rho\pi}{}^0=\tan(\theta-\beta')\,, \tag{38}$$

where the effective mixing angle β' is given by

$$\beta'\approx\beta+(\sqrt{2}/3)(I-1)\,. \tag{39}$$

We see that we only need $I\approx 1.02$ to halve the $\phi\to 3\pi$ partial width and $I\approx 1.08$ would completely suppress $\phi\to 3\pi$ in the model considered. The predictions of the renormalized theory that are least sensitive to slight deviations from nonet degeneracy will be decays involving real or virtual photons and thus unfortunately are susceptible to the merits of the models used to describe these decays.

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High-Energy Behavior of the Scattering Amplitude in the Unphysical Region $0<t<4m^2$*

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It is proved within the framework of axiomatic field theory that the scattering amplitude must have a Regge behavior in the unphysical region $0<t<4m^2$ in the sense that the logarithmic derivative of the absorptive part is bounded by $C\ln s$ for very large s.

It can be shown in axiomatic field theory that the absorptive part $A(s,t)$ of the elastic scattering amplitude is positive and monotonically increasing in the interval $0\leq t<4m^2$ and that the rate of increase is restricted by the upper bound[1,2]

$$A(s,t)<Cs^{1+(t/4m^2)^{1/2}+\epsilon}\,,\quad \epsilon>0\,. \tag{1}$$

This suggests strongly that $A(s,t)$ has a Regge-type behavior in this interval. However, since the inequality (1) does not tell much about the actual t dependence of $A(s,t)$, it is necessary to examine the behavior of $A(s,t)$ more closely in order to settle this question. The purpose of this paper is to show that $A(s,t)$ has in fact a Regge behavior in the interval $0<t<4m^2$ in the sense that t dependence stronger than that of the Regge type is ruled out. At present, it is not known whether or not this result can be extended to the physical region $t\leq 0$.

We start from the result of axiomatic field theory[3] that the absorptive part $A(s,t)$ for a fixed physical value of s is analytic in an ellipse in the t plane with foci $t=0$, $-4k^2$ and semimajor axis $4m^2+2k^2$. Thus $A(s,t)$ can be expanded in this ellipse into partial waves:

$$A(s,t)=(\sqrt{s}/2k)\sum_{l=0}^{\infty}(2l+1)a_l(s)P_l(1+t/2k^2)\,, \tag{2}$$

where $a_l(s)$ is the absorptive part of the lth partial-wave amplitude satisfying the unitarity restriction

$$0\leq a_l(s)\leq 1\,,\quad l=0,1,2,\cdots\,, \tag{3}$$

as well as the analyticity requirement[4]

$$a_l(s)\leq Cs^2\exp[-2l(4m^2/s)^{1/2}]\,. \tag{4}$$

As is well known, for $s>4m^2$, $A(s,t)$ and all derivatives of $A(s,t)$ with respect to t are positive in the inter-

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[1] For simplicity, we treat only the elastic scattering of spinless particles with equal mass m. As usual, s and t are the square of the total energy and the momentum transfer in the center-of-mass system. We also use the center-of-mass momentum k, related to s by $s=4(m^2+k^2)$.

[2] The inequality (1) was first derived by K. Bardakci [Phys. Rev. **127**, 1832 (1962)] starting from the Mandelstam representation. A more general derivation was given by A. Martin (Ref. 4). We have put $N=2$ in their formulas, taking account of the recent result of Martin (Ref. 3).

[3] A. Martin, Nuovo Cimento **42A**, 930 (1966); **44A**, 1219 (1966).

[4] A. Martin, in *Strong Interactions and High Energy Physics* (Oliver and Boyd, London, 1964), p. 105.