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Is the Nucleon a πN Bound State?

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The P_{11} partial wave of the πN system is characterized by a large absorption and a resonance (possibly inelastic) at 575-MeV laboratory pion energy (Roper resonance). In view of these features of the P_{11} channel, the πN system may be rendered sufficiently weak not to bind the nucleon. Using the experimental mass of the nucleon and the recently obtained extensive energy-dependent, complex phase-shift results on πN scattering data, we examine here, through a nonperturbative S-matrix approach, the existence of a nucleon bound state in the πN channel in the absence of forces arising from the inelastic states. The formalism is independent of any model for the contribution of the left-hand cut. We discuss two different possibilities concerning the nature of the Roper resonance: (a) It is an elastic resonance primarily due to the forces in the πN channel; (b) it is an inelastic resonance mainly due to the forces from the inelastic states. In either case we find it unlikely that the πN system by itself can sustain the nucleon bound state.

I. INTRODUCTION

RECENTLY, Roper and Wright,[1] Auvil, Donnachie, Lea, and Lovelace,[2] and others[3] have reported extensive energy-dependent phase-shift analyses of the pion-nucleon scattering data, for incident-pion laboratory kinetic energies E_π up to 700 MeV. Many prominent features emerge as a result of this analysis. Of particular interest is the existence of the Roper resonance in the P_{11} partial wave at $E_\pi \sim 575$ MeV accompanied by a very small value (~ 0.265) for the absorption parameter η at this energy. Because of the rapidly decreasing value of η above $E_\pi = 170$ MeV as well as the possibility that the Roper resonance may itself be an inelastic resonance, arising mainly from the presence of inelastic channels, we investigate here the reasonable possibility that the nucleon is an inelastic bound state in the πN system. If this were so, it would imply that the simple-minded model of the nucleon as a pion-nucleon bound state is quantitatively not a believable model. Thus the nucleon would emerge as a more complicated object involving other states (possibly three-body states) besides the πN for its adequate description. Clearly the new status of the nucleon would have obvious implications for other calculations involving the structure of the nucleon—e.g., nucleon bootstraps.

II. FORMULATION OF THE PROBLEM

It is well known that kinematic singularities introduced by the nucleon spin in the πN partial-wave amplitudes can be avoided by working in the total energy variable W.[4] We consider the amplitude $T_{2J}(W)$ defined by (we suppress the isospin index)[5]

$$T_{2J}(W) = [\eta_{2J}(W)e^{2i\delta_{2J}(W)} - 1]/2i\rho_{2J}(W), \quad (2.1)$$

where the kinematical factor $\rho_{2J}(W)$ is given by

$$\rho_{2J}(W) = (E-M)(q/W)^{2J}, \quad (2.2)$$

with M the nucleon mass, q the (center-of-mass) momentum, and E the nucleon energy $[=(W^2+M^2-1)/2W]$. δ_{2J} is the real part of the phase shift, and η_{2J} $(=e^{-2\delta_{2J}^I})$ the inelasticity parameter $(1 \geqslant \eta_{2J} \geqslant 0)$. δ_{2J} and η_{2J} satisfy identical symmetries in the W plane:

$$\begin{aligned}\eta_{2J}(W) &= \eta_{2J}(l=J+\tfrac{1}{2}, W), \quad W > W_E \\ &= \eta_{2J}(l=J-\tfrac{1}{2}, W), \quad W < -W_E, \quad (2.3)\end{aligned}$$

where $W_E = M+1$ is the elastic threshold. We also define the elastic-scattering amplitude $\tilde{T}_{2J}(W)$ which describes the scattering of the πN system in the absence of inelastic effects:

$$\tilde{T}_{2J}(W) = [\exp 2i\tilde{\delta}_{2J}(W) - 1]/2i\rho_{2J}(W). \quad (2.4)$$

The $I=\frac{1}{2}$, $J=\frac{1}{2}$ amplitude has a pole at the nucleon mass with a residue $R = -\frac{3}{2}g_{\pi NN}^2/4\pi$, where $g_{\pi NN}^2/4\pi$ is the renormalized πN coupling constant. We explicitly display this pole and write[6]

$$T_{11}(W) = \frac{N_{11}(W)}{D_{11}(W)} = t_{11}(W) + \frac{R}{W-M}, \quad (2.5)$$

where $t_{11}(W)$ is the remainder function. The elastic πN scattering amplitude $\tilde{T}_{11}(W)$ (obtained by switching off the inelastic effects) would have the pole shifted to

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[1] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964); L. D. Roper and R. M. Wright, Phys. Rev. **138**, B921 (1965).

[2] P. Auvil, A. Donnachie, A. Lea, and C. Lovelace, Phys. Letters **12**, 76 (1964).

[3] L. Roper, R. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

[4] W. Frazer and J. Fulco, Phys. Rev. **119**, 1420 (1960); S. Frautschi and J. Walecka, *ibid.* **120**, 1486 (1960).

[5] We shall put $\hbar = c = m_\pi = 1$.

[6] N/D is written in the sense of G. Frye and R. L. Warnock, Phys. Rev. **130**, 478 (1963).

a new value $\tilde{M}$ with a modified residue $\tilde{R}$ so that

$$\tilde{T}_{11}(W)=\frac{\tilde{N}_{11}(W)}{\tilde{D}_{11}(W)}=\tilde{t}_{11}(W)+\frac{\tilde{R}}{W-\tilde{M}}. \quad (2.6)$$

Now the modification $T_{11}(W)-\tilde{T}_{11}(W)$ can be dispersed along the inelastic unitarity cut to obtain[7]

$$\tilde{T}_{11}(W)=T_{11}(W)-\frac{Z(W;\tilde{M})}{\tilde{D}_{11}(W)D_{11}(W)}, \quad (2.7a)$$

where[8]

$$Z(W;\tilde{M})=\frac{1}{\pi}\int_{W_I}^{\infty}\frac{|\tilde{D}_{11}(W)D_{11}(W)|\,(1-\eta_{11}(W')\cos[\delta_{11}(W')-\tilde{\delta}_{11}(W')]dW'}{(W'-W)2\rho_1(W')}. \quad (2.7b)$$

Here W_I is the threshold for the inelastic unitarity cut.

Thus, assuming that the nucleon can be sustained as a bound state with mass $\tilde{M}$ in the elastic pion-nucleon amplitude, we have

$$\tilde{D}_{11}(W;\tilde{M})=(W-\tilde{M})\times\exp\left[-\frac{(W-\tilde{M})}{\pi}\int_{W_E}^{\infty}\frac{\tilde{\delta}_{11}(W')dW'}{(W'-W)(W'-\tilde{M})}\right], \quad (2.8)$$

with $\tilde{\delta}_{11}(\infty)=-\pi$. Now we are led into a self-consistency problem in the following sense. We use (2.8) to compute $Z(W;\tilde{M})$ from (2.7b) and then compute $\tilde{D}_{11}{}^L(M;M)$ from (2.7a) by taking the limit $W\rightarrow M$.

$$\tilde{D}_{11}{}^L(M;\tilde{M})=Z(M;\tilde{M})/RD_{11}'(M). \quad (2.9)$$

If $\tilde{M}$ is truly the new position of the bound state, it must satisfy the self-consistency condition

$$\tilde{D}_{11}(M;\tilde{M})=\tilde{D}_{11}{}^L(M;\tilde{M}). \quad (2.10)$$

We define, for convenience, in searching for $\tilde{M}$ on a computer:

$$\sin\xi=\frac{|\tilde{D}_{11}(M;\tilde{M})-\tilde{D}_{11}{}^L(M;\tilde{M})|}{[2(\tilde{D}_{11}(M;\tilde{M})^2+\tilde{D}_{11}{}^L(M;\tilde{M})^2)]^{1/2}}. \quad (2.11)$$

It is clear that zero or a low minimum in $\sin\xi$ indicates consistency or near consistency. It should also be emphasized that there is no contribution from the left-hand cut to (2.7a) and hence our results will be independent of any model for the actual forces arising from the exchanges in the crossed channels. In the next section we give the details of the calculation and discussion.

III. CALCULATION AND DISCUSSION

The results of the extensive phase-shift analyses determine the behavior of δ_{11} and η_{11} up to $E_\pi\sim700$ MeV.[1-3] δ_{11} starts off with a small negative value, then changes sign and goes through $\frac{1}{2}\pi$ at $E_\pi\sim575$ MeV to produce the Roper resonance. Now, assuming that the Roper resonance is produced mainly by forces in the πN channel (we shall call this a "non-CDD-type" situation[9]) we can write, using the normalization $\delta_{11}(\infty)=-\pi$,

$$D_{11}(W)=(W-M)\exp\left[-\frac{W-M}{\pi}\times\int_{W_E}^{\infty}\frac{\delta_{11}(W')dW'}{(W'-W)(W'-M)}\right]. \quad (3.1)$$

To construct D_{11} from (3.1), existing experimental phase-shift analyses are supplemented by various asymptotic forms which join smoothly to the experimental values. In order to test the sensitivity of our results to the high-energy behavior of the phase shifts we use the following set of significantly different asymptotic forms:

$$\delta_{11}(W)=-\pi+A/W, \quad (3.2)$$

$$\delta_{11}(W)=-\pi+1/\ln(W/A), \quad (3.3)$$

$$\delta_{11}(W)=-\pi+Ae^{-W}. \quad (3.4)$$

Since in the non-CDD situation the major force producing the Roper resonance comes from the πN channel itself, the resonance will be present even in the elastic amplitude T_{11}, perhaps at a different energy. Shaw and Wong[10] have given the following fit to $\delta_{11}(W)$:

$$\delta_{11}(W)=-\frac{8\pi q^3(W-8.5)(W-W_0)}{(W-2)^5},$$

where W_0 is adjusted to yield the Roper resonance at the experimental value. We use the same expression for $\tilde{\delta}_{11}(W)$, but adjust W_0 to yield the resonance at different values ($\tilde{W}_{\text{Rop}}$) in order to produce widely different energy dependences for $\tilde{\delta}$. On the other hand, if the Roper resonance is a CDD-type resonance[9] and is produced mainly by inelastic channels, as is strongly

[7] P. Nath and Y. N. Srivastava (to be published).

[8] We have checked numerically that the $-W$ contribution to the dispersion integral is negligible in the present case.

[9] M. Bander, P. Coulter, and G. Shaw, Phys. Rev. Letters **14**, 270 (1965); J. Hartle and C. Jones, Phys. Rev. **140**, B90 (1965); Phys. Rev. Letters **14**, 534 (1965). (CDD≡Castillejo-Dalitz-Dyson.)

[10] G. L. Shaw and D. Y. Wong, Phys. Rev. **147**, 1028 (1966).

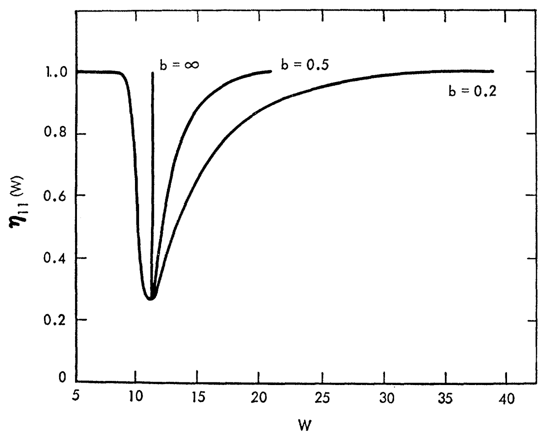

FIG. 1. The inelasticity parameter η_{11} for various values of the parameter b. Because of the exponential form for η beyond the experimental limit, η rapidly approaches 1 for finite values of b.

suggested by the small value of η_{11} at the resonant energy, then a pair of CDD zeros at $W=W_R\pm iW_I$ appears on the physical sheet in the S-matrix element S_{11} and we use[10]

$$D_{11}(W)=(W-M)\left(\frac{(M-W_R)^2+W_I^2}{(W-W_R)^2+W_I^2}\right)^{1/2}$$

$$\times\exp\left[-\frac{W-M}{\pi}\int_{W_E}^{\infty}\frac{\delta_{11}(W')dW'}{(W'-W)(W'-M)}\right], \quad (3.5)$$

with $\delta_{11}(\infty)=0$. Again we use the Shaw-Wong[10] fit to $\delta_{11}(W)$:

$$\delta_{11}(W)=-\frac{W_p q^3(W-8.5)}{(W-2.0)^5}, \quad (3.6)$$

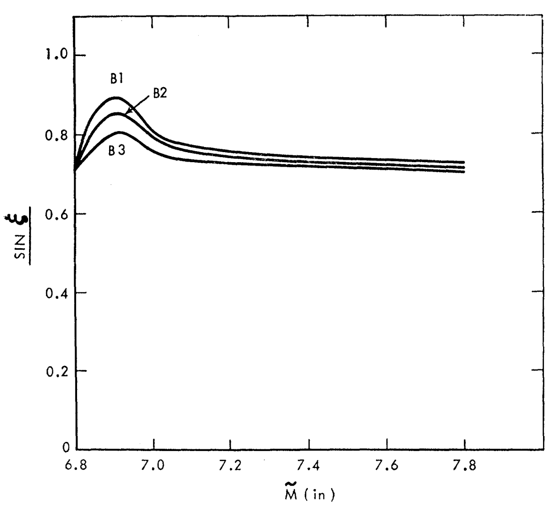

FIG. 2. The Roper resonance is assumed to be an inelastic resonance in this case. $\eta\equiv1$ beyond the experimental limit, corresponding to $b=\infty$. The various parametrizations for $\bar{\delta}_{11}$ used in the numerical analysis are labeled B_1, B_2, and B_3 corresponding to (3.7), (3.8), and (3.9), respectively. As indicated by the curves, no bound-state solution is found.

were W_p is adjusted so that $\delta_{11}=\frac{1}{2}\pi$ at the Roper resonance ($W=10.7$) and $W_R=16$ and $W_I=2$. We also consider a cutoff form of δ_{11}, where $\delta_{11}(W)$ is set equal to zero after $W\approx15$.[5]

Since in the CDD situation, the Roper resonance is not present in the elastic πN scattering, $\bar{\delta}_{11}$ is expected to be significantly different from δ_{11}. Assuming that the nucleon is still present in the elastic scattering as a bound state, we have, using Levinson's theorem,[11] $\bar{\delta}_{11}(0)-\bar{\delta}_{11}(\infty)=\pi$. Now we use the normalization $\bar{\delta}_{11}(\infty)=-\pi$ consistent with (2.8) and three significantly different parametrizations of $\bar{\delta}_{11}(W)$:

$$\bar{\delta}_{11}(W)=-\pi+\frac{\pi(M+1)}{W}, \quad (3.7)$$

$$\bar{\delta}_{11}(W)=-\pi+\frac{\pi}{\{1+\ln[W/(M+1)]\}}, \quad (3.8)$$

$$\bar{\delta}_{11}(W)=-\pi+\pi e^{-(W-M-1)}. \quad (3.9)$$

We have also considered a possible case where $\bar{\delta}_{11}(W)$, instead of decreasing uniformly from 0 to $-\pi$, attains small positive values and then asymptotically tends to $-\pi$. Again the vulnerability of our results to the different parametrizations will indicate how general our conclusions are.

The single most important parameter in the present calculation is η_{11}. The absorption parameter η_{11} reaches a minimum at $E_\pi\sim616$ MeV and rises slowly up to $E_\pi\sim700$ MeV. Beyond this value there is no restriction on η_{11} (except that it is constrained to lie between 0 and 1). Even though the contribution to η from any given inelastic channel goes to zero asymptotically, an infinite number of channels open up, which could presumably shift $\eta(\infty)$ from 1. On the other hand, it has

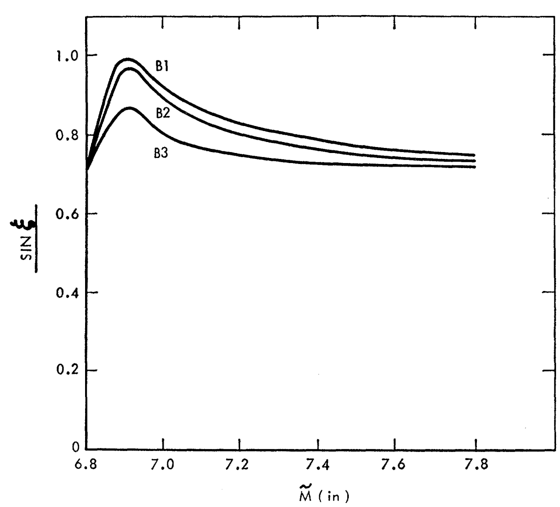

FIG. 3. Same as Fig. 2 except that a sharp cutoff on δ_{11} was used, so that $\delta_{11}(W)\equiv0$ for $W\geqslant15$.

[11] See, e.g., R. Warnock, Phys. Rev. **131**, 1320 (1963).

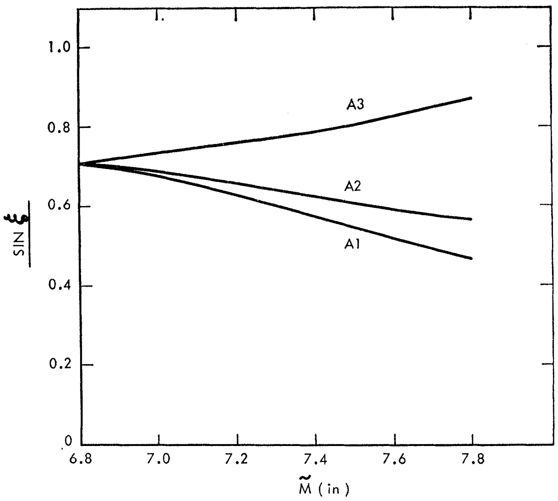

FIG. 4. The Roper resonance is assumed to be an elastic resonance in this case. $W_{\mathrm{Rop}}=10.9$ and $b=0.2$. The curves have been labeled A_1, A_2, A_3, corresponding to (3.2), (3.3), and (3.4), respectively. As is clear from the figure, no bound-state solution exists.

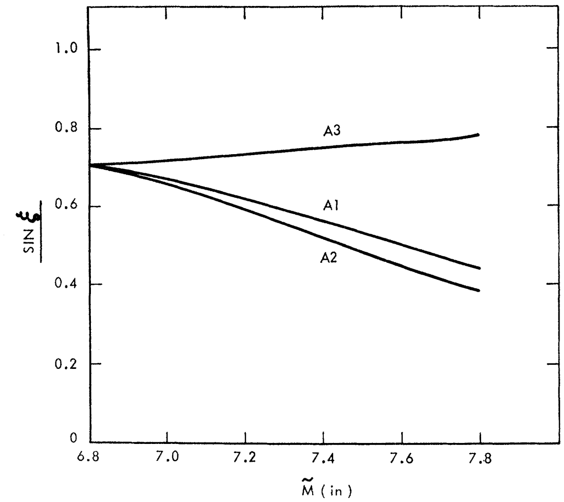

FIG. 5. Sams as Fig. 4 except that $W_{\mathrm{Rop}}=14$.

recently been conjectured,[12] on the basis of certain specific models, that even though an infinite number of channels may open up as energy goes to infinity, $\eta(\infty)$ does in fact equal 1. In the present calculation we shall use as conservative an estimate of η_{11} beyond experimental energies as necessary to remove any doubt as to its reasonableness. First, we shall in fact set $\eta_{11}(\infty)=1$. We still have a number of choices left for the high-energy behavior of $(1-\eta_{11})$.[13] Again we take the conservative estimate that asymptotically $1-\eta_{11}$ behaves exponentially. We write

$$\eta_{11}=1-(1-\eta_0)e^{-b(W-W_0)}, \tag{3.10}$$

where η_0 is the value at the last experimentally known energy W_0. Equation (3.10) is a very conservative estimate for η. In fact some recent calculations[14] have used far more liberal parametrizations of η. In Fig. 1 we have plotted $\eta_{11}(W)$ for various values of the parameter b.

Some typical results are given in Figs. 2–7 where various parametrizations have been indicated. First consider the CDD-type situation, which seems to be the more reasonable one because of the small value of η near the resonance. In this case, as an extreme example, we set $\eta\equiv1$ (i.e., $b=\infty$) after the last experimental point. Figures 2 and 3 show the results for the parametrizations (3.7), (3.8), and (3.9) for $\tilde{\delta}_{11}(W)$. The curves of $\sin\xi$ versus $\tilde{M}$(in) do not give any zeros or minima, and hence no solution exists between 6.8 and 7.8.[5] These curves will probably slope downwards in the physical region and a resonant solution may be found.[15] In the case when $\tilde{\delta}_{11}(W)$ attains positive values, the curves were found to slope downwards in the bound-state region, but we did not get a solution for any reasonable value of b. Only for large values of b ($\eta\approx1$ beyond the experimental limit) did we find bound-state solutions for certain parametrizations of $\tilde{\delta}_{11}(W)$. It can be readily seen that any parametrization of η with $\eta(\infty)<1$ will indeed increase the magnitude of the integrals asymptotically on the right-hand side of Eqs. 2.7(a) and 2.7(b) and hence will yield no solution.

The results for the non-CDD case are shown in Figs. 4–7. Figures 4 and 5 are given for $b=0.2$ (when $\eta\rightarrow1$ near $W\approx35$) and Fig. 6 for $b=0.5$ ($\eta\rightarrow1$ near $W\approx18$). Here again we find that although some of the curves slope downwards, they still yield so solution. Again we present the extreme case of $\eta\equiv1$ beyond the

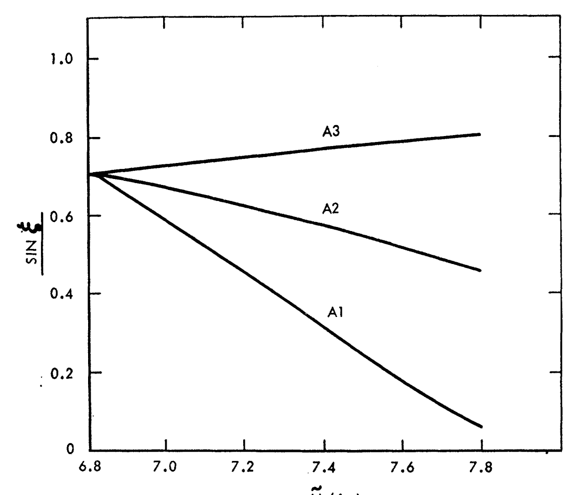

FIG. 6. Same as Fig. 4 except $b=0.5$.

[12] R. E. Kreps and P. Nath, Phys. Rev. **148**, 1436 (1966).

[13] Various possible asymptotic behaviors for $1-\eta_{11}$ are: $O(1/\ln W)$, $O(W^{-|a|})$, $O(e^{-|b|W})$, etc.

[14] P. Coulter, A. Scotti, and G. Shaw, Phys. Rev. **136**, B1379 (1964).

[15] This was verified explicitly in some cases where we looked for solutions for $\tilde{M}$ in the region above the threshold.

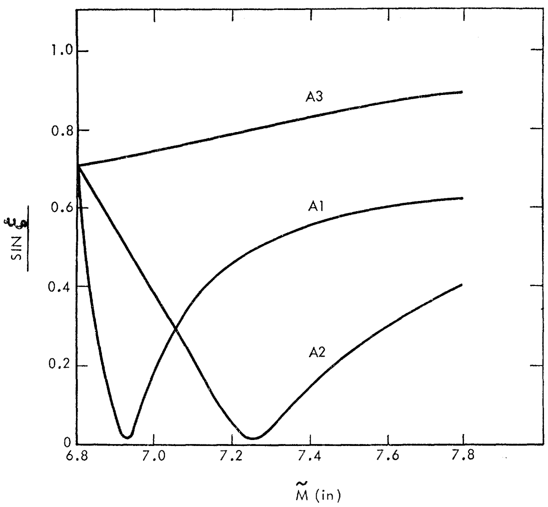

FIG. 7. Same as Fig. 4 except that $\eta \equiv 1$ beyond the experimental limit. In this case we find that bound-state solutions exist for asymptotic forms (3.2)[A_2] and (3.3)[A_2] of δ_{11} but not for (3.4)[A_3].

experimental limit, in Fig. 7. Here we find that bound-state solutions are present. Thus in both the CDD and non-CDD cases we can force a bound state for larger values of b and certain parametrizations of the phase shifts, although this is not true for all the parametrizations. As expected, because of the additional attractive forces in the non-CDD situation, the value of b needed to force a bound state is much larger in the CDD case than in the non-CDD case. However, the particular situations in which bound-state solutions were obtained seem to be very artificial. It seems unlikely that the experimental value of η will tend to 1 so fast. For any realistic parametrization of η we do not obtain any bound-state solution. Thus we conclude that given the presently known experimental phase shifts and inelasticity parameters, the πN system by itself is unlikely to be able to sustain a nucleon bound state, and higher inelastic channels must play a considerable role in binding the nucleon. It is interesting to note that the $SU(6)$ static baryon bootstrap model also suggests that a substantial part of the binding of the nucleon comes from the channels other than the πN channel.[16]

Finally we make some remark regarding the relevance of the present conclusions to the results obtained by Coulter and Shaw.[17] In that work it was not possible to produce a nucleon bound state with the correct residue even in the presence of inelasticity. When they forced the nucleon bound state to appear with the correct residue by including it in the direct-channel contribution, they obtained reasonably good agreement for the phase shifts. From their results one can see that their unperturbed ($\eta = 1$) and perturbed ($\eta \neq 1$) phase shifts differ considerably. Their formalism did not ask for the location of the unperturbed pole. This is indeed what was considered in the present work. It seems natural to us that when the predicted phase shifts change considerably by the inclusion of inelasticity, the location of the bound state should also change appreciably.

ACKNOWLEDGMENTS

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[16] See, e.g., R. H. Capps, Phys. Rev. Letters **14**, 31 (1965).
[17] P. W. Coulter and G. L. Shaw, Phys. Rev. **141**, 1419 (1966).