(III) If quarks satisfy $SU(3)$ symmetry, Eqs. (5) and (6) can be rewritten in a much simpler form:

$$\sigma(\Lambda p)=2\sigma(pn)-\sigma(pp)\,,\qquad \text{(7a)}$$
$$\sigma(\Xi^- p)=4\sigma(pn)-3\sigma(pp)\,,$$

$$\sigma(\Sigma^+ p)=\sigma(pn)\,,\qquad \text{(7b)}$$
$$\sigma(\Sigma^- p)=\sigma(\Xi^0 p)=3\sigma(pn)-2\sigma(pp)$$

Discussion. Equations (2) and (5) follow from the assumption of additivity alone. They should be tested experimentally. Equations (2) are the easiest to test. With the present knowledge on the experimental data up to the highest energy available, they differ from the predictions by approximately 10–20%. The errors on the experimental data are still large. To clear up this point, more data with better accuracy are needed on the nucleon-nucleon and nucleon-antinucleon total cross sections. It is also of interest to have some data at higher energy on hyperon-proton scattering total cross sections. Then Eqs. (5) would serve as an independent check on the assumption of additivity.

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Capture of a $\bar{K}$ Meson from the $2P$ Orbit in ^{4}He Followed by π^- and Λ^0 Production

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The reaction $\bar{K}+{}^4\mathrm{He}\rightarrow\pi^-+\Lambda^0+{}^3\mathrm{He}$ is studied under the assumption of capture from the $2P$ Bohr orbit. The outgoing pion and the recoiling ^{3}He momentum distributions are calculated with the inclusion of $Y_1{}^*$-resonance and Σ-Λ conversion amplitudes. The Σ-Λ conversion process is of particular importance. The main conclusion is that the pion spectrum tends to favor the assumption that the stopping $\bar{K}$ are captured from a P-state atomic orbit. The conclusion is consistent with the results of the $\bar{K}$-capture x-ray experiment of Burleson *et al.*

IN a recent article by Said and the author[1] (referred to as I), the reaction $\bar{K}+{}^4\mathrm{He}\rightarrow\pi^-+\Lambda^0={}^3\mathrm{He}$ has been extensively studied from the point of view of the recoil-nucleus and the pion momentum distributions. Three processes have been considered: a direct nonresonant production of a π^- and a Λ in the absorption of the $\bar{K}$ meson by a neutron; the formation of the $Y_1{}^*$ resonance in the intermediate state; and the reaction with a Σ production in the first stage and its subsequent conversion into a Λ in a successive collision with another nucleon. The initial-state kaon Bohr orbits considered were nS and mP; however, only simplified calculations were performed for the P-wave case with no elastic Λ-^{3}He distortion and the Σ-Λ conversion included. All the detailed computations referred to nS orbits, presumed, at that time, to be the most important ones for the direct nuclear $\bar{K}$ capture.

From the computed momentum distribution $R_3(p_3)$ of the recoiling ^{3}He, it was concluded that the formation of the $Y_1{}^*$ resonance should be an important process (cf. also Letessier and Vinh Mau[2]); from $R_3(p_3)$ and from the momentum distribution $R_\pi(p_\pi)$ of the outgoing pion, rather strong evidence was obtained for the great importance of the Σ-Λ conversion process, particularly at large p_3 and at small p_π of the two respective distributions. The elastic Λ-^{3}He final-state interaction was found to be of little importance. Semiquantitative fits to the observed spectra $R_3(p_3)$ and $R_\pi(p_\pi)$ were obtained.

Contrary to many previous speculations and expectations (cf., e.g., some of the references quoted in I), it appears from the most recent experimental results of Burleson *et al.*[3] that about 80% of all the kaons undergo the direct $\bar{K}$ capture from the $2P$ Bohr orbit. It is the aim of the present note to supplement I with the corresponding results for this case.

Our method, mathematical approximations, and notations are all those of I.

For the non-Σ–Λ-conversion amplitude, we use the approximate expression [cf. Eq. (7) of I]:

$$M_{2P}\cong N_{2P}\{\mathfrak{F}_{(3,n,\Lambda)}([M_3/(M_3+m_\Lambda)]\mathbf{P}_f)i\mathbf{e}\cdot\nabla_{\mathbf{q}_0}\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{\mathbf{p}_K=0}+i\tfrac{3}{4}[\mathbf{e}\cdot\nabla_\omega\mathfrak{F}_{(3,n,\Lambda)}(\boldsymbol{\omega})]_{\omega=[M_3/(M_3+m_\Lambda)]\mathbf{P}_f}\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{\mathbf{p}_K=0}\}\,,\qquad(1)$$

where N_{2P} is the normalization of the kaon $2P$ Bohr orbit wave function, $\mathbf{e}$ is the polarization vector of the

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[2] J. Letessier and R. Vinh Mau, Nucl. Phys. (to be published). For a study of the $Y_1{}^*$ effects in the nonpionic $\bar{K}$ capture in deuterium, see G. Fowler and P. Poulopoulos, Nucl. Phys. 77, 689 (1966).

[3] G. R. Burleson, D. Cohen, R. C. Lamb, D. N. Michael, R. A. Schluter, and T. O. White, Jr., Phys. Rev. Letters 15, 70 (1965). The nuclear properties of the $\bar{K}$-mesonic ^{4}He atom have been studied recently by F. von Hippel and J. H. Douglas [Phys. Rev. 146, 1042 (1966)], and by J. Uretsky [Phys. Rev. 147, 906 (1966)].

$2P$ orbit, and $\mathbf{p}_K$ is the Fourier momentum of the kaon; $\mathbf{P}_f$ is the momentum of the final hypernuclear $\Lambda+{}^3\mathrm{He}$ system: $\mathbf{P}_f=\mathbf{p}_\Lambda+\mathbf{p}_3$; in the case of the capture at rest we find $\mathbf{P}_f=-\mathbf{p}_{\pi^-}$ the outgoing pion momentum. The basic $\langle t\rangle$-matrix element (vertex function) of the elementary process $\bar{K}+N\rightarrow\pi^-+\Lambda^0$ in our impulse approximation depends on the initial and final relative momenta of the particles involved, $\mathbf{q}_0$ and $\mathbf{q}_1$, respectively; in view of the momentum conservation they reduce to

$$\mathbf{q}_0=\mathbf{p}_K+\frac{m_K}{M+m_K}\mathbf{p}_3 \quad\text{and}\quad \mathbf{q}_1=\mathbf{p}_\pi+\frac{m_\pi}{m_\Lambda+m_\pi}\mathbf{p}_3.$$

Finally, the common nuclear-hypernuclear form factor is

$$\mathfrak{F}_{(3,n,\Lambda)}(\boldsymbol{\omega})\equiv\int d\mathbf{u}d\mathbf{v}\delta(\mathbf{u}-\mathbf{v})\times\exp(i\boldsymbol{\omega}\cdot\mathbf{u})\int d\xi\,\Psi_f^*(\mathbf{v},\xi)\Psi_4(\mathbf{u},\xi). \quad (2)$$

where $\Psi_4(\mathbf{u},\xi)=\phi_{n-3}(\mathbf{u})\phi_3(\xi)$ is the ${}^4\mathrm{He}$ wave function, $\mathbf{u}=\mathbf{r}_N-\mathbf{r}_3$ is the distance vector from the ${}^3\mathrm{He}$ to the nucleon N, and ξ are the intrinsic coordinates of ${}^3\mathrm{He}$. In our computations we have used the Hulthén form of ϕ_{n-3} as specified in I.

It has been demonstrated in I that the elastic final hypernuclear Λ-${}^3\mathrm{He}$ distortion can affect the most important distribution, $R_3(p_3)$, only a little. Recently, Said[4] has confirmed this point explicitly even for the Y_1^*-resonant amplitude avoiding the "zero-range" approximation of I. Consequently, in order to simplify the rather complex kinematics and analysis, we neglect this distortion, and represent the Λ-${}^3\mathrm{He}$ relative motion by a plane wave; then we have: $\Psi_f(\mathbf{v},\xi)=(\exp i\mathbf{q}_{\Lambda3}\cdot\mathbf{v})\times\phi_3(\xi)$; here $\mathbf{v}=\mathbf{r}_\Lambda-\mathbf{r}_3$ is the distance vector from the ${}^3\mathrm{He}$ to the Λ.

For the case of $1S$ capture the matrix element corresponding to M_{2P} of Eq. (1) is

$$M_{1S}\cong N_{1S}\mathfrak{F}_{(3,n,\Lambda)}([M_3/(M_3+m_\Lambda)]\mathbf{P}_f)\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{\mathbf{p}_K=0}. \quad (3)$$

The Σ-Λ–conversion amplitude in the $2P$ case, $\Delta M_{\mathrm{conv}}{}^{2P}$, can be approximately related to that for the $1S$ capture of Eq. (21) of I:

$$\Delta M_{\mathrm{conv}}{}^{2P}=-\frac{A_c{}^{(2P)}}{A_c{}^{(1S)}}\frac{N_{2P}}{N_{1S}}\frac{3}{4}\frac{\mathbf{e}\cdot\boldsymbol{\omega}}{\omega}i\frac{\partial}{\partial\omega}\Delta M_{\mathrm{conv}}{}^{1S}(\omega,q_{\Sigma3}), \quad (4)$$

where $\boldsymbol{\omega}=-[M_3/(M_3+m_\Sigma)]\mathbf{p}_\pi$; $A_c{}^{(i)}$ are the effective (constant) i-orbit Σ-production t-matrix elements; N_{1S} is the normalization constant of the $1S$-orbit wave function; $\mathbf{q}_{\Sigma3}$ is the momentum of the Σ intermediate state relative to the nucleus ${}^3I$. The relation of Eq. (4) permits us to avoid writing a rather lengthy explicit expression for $\Delta M_{\mathrm{conv}}{}^{2P}$. Equation (4) is obtained in analogy to the relation which follows from Eqs. (1) and (3).

The reaction rate is proportional to the trace with respect to σ_Λ of $M_{\mathrm{tot}}{}^\dagger M_{\mathrm{tot}}$, where $M_{\mathrm{tot}}\equiv M+\Delta M_{\mathrm{conv}}$. With an arbitrary normalization we can express it as

$$R_{3\pi}=d\mathbf{p}_3d\mathbf{p}_\pi\int d\mathbf{p}_\Lambda\delta(\Sigma\,\mathbf{p}_i)\delta(\Sigma\,E_i)\tfrac{1}{2}\,\mathrm{Tr}_{(\sigma_\Lambda)}\times(M_{\mathrm{tot}}{}^\dagger M_{\mathrm{tot}}). \quad (5)$$

The respective distributions $R_3(p_3)$ and $R_\pi(p_\pi)$ are then computed in the standard manner, as in I, by integrating over the respective remaining variables.[5] In the $2P$-capture case, averaging is performed over the polarizations of the $2P$ orbit, $\mathbf{e}$.

Explicit expressions for the pure direct-nonresonant and Y_1^*-resonant parts of $R_3(p_3)$ are given in Eqs. (23)–(24) of I; the corresponding interference term ($\propto A_d a_r^*$) disappears in our approximation of no quasielastic Λ-${}^3\mathrm{He}$ distortion. Here A_d stands for the (constant) "direct" part of the $\langle t\rangle$ element, while the Y_1^* coupling constant a_r is defined, as in I, by

$$\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle_{\mathrm{res}}=a_r\frac{\mathbf{q}_0\cdot\mathbf{q}_1+\frac{1}{2}i\boldsymbol{\sigma}_\Lambda\cdot(\mathbf{q}_0\times\mathbf{q}_1)}{s^{1/2}-M_1^*+i\frac{1}{2}\Gamma_1}, \quad (6)$$

where $s^{1/2}$ is the total energy in the $\Lambda+\pi$ c.m. system; M_1^* and Γ_1 are the Y_1^* parameters.[6]

The adjustable parameters of our model are the ratios of the 'effective" coupling constants, $A_d{}^{(2P)}/a_r$ and $A_d{}^{(2P)}/A_c{}^{(2P)}$; here $A_d{}^{(2P)}$ and $A_c{}^{(2P)}$ should be roughly of the same order of magnitude, as in the case of the $1S$ capture.

There remain still rather considerable uncertainties in the parameters mentioned (which are therefore actually adjustable) and in the other parameters involved, and, on the other hand, the presently available data are too scarce to make very detailed analysis warranted.

However, in spite of these uncertainties, it turns out that the main features of our results vary relatively little with any reasonable variations of our parameters. It is for just this reason that we think it interesting to present typical distributions $R_3(p_3)$ and $R_\pi(p_\pi)$ for the $2P$ capture as compared with those of I.

The most important restriction on our parameters is a reasonable order of magnitude of the ratio of the total initial Σ production amplitude squared to the total Λ nonconversion amplitude squared. It turns out that this condition is satisfied in our case for those numerical

[4] P. Said (private communication).

[5] Through a misprint, a volume factor "$p_3{}^2dp_3$" has been omitted in Eq. (20) of I.

[6] Here Γ_1 is a constant; other well-known Breit-Wigner–type forms of $\langle t\rangle_{\mathrm{res}}$ or possible refinements of it introducing a q_1 dependence of the "effective" Γ_1 would most probably lead to no essential changes in our final results.

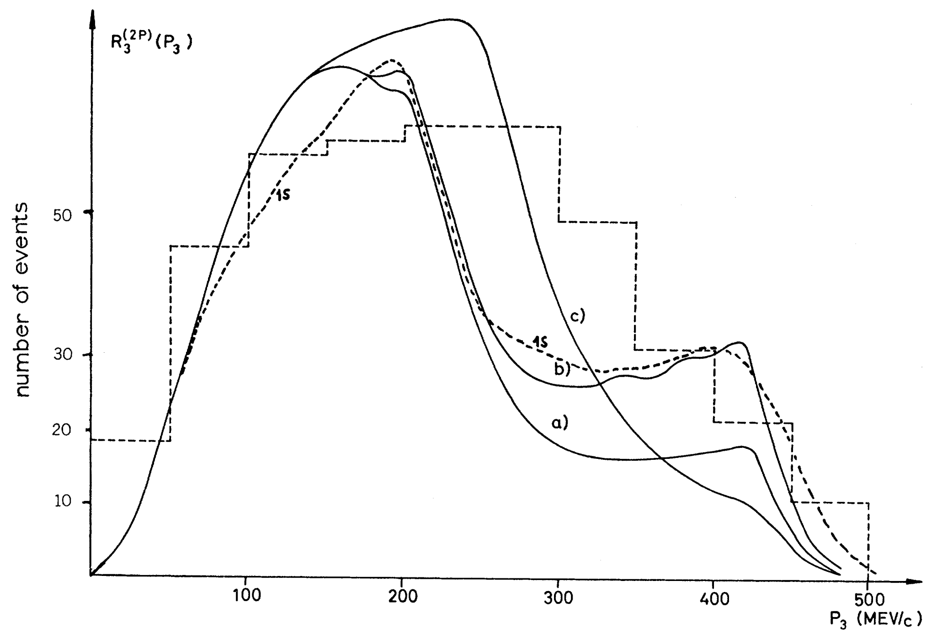

FIG. 1. The ^{3}He-recoil momentum distribution $R_3(p_3)$ for the reaction $\bar{K}+{}^4\text{He}\rightarrow\pi^-+\Lambda^0+{}^3\text{He}$ when the $\bar{K}$ capture is from the $2P$ Bohr orbit with $A_d^{(2P)}/a_r=0.2\times10^3$ and: (a) $A_c^{(2P)}/A_d^{(2P)}=15$; (b) $A_c^{(2P)}/A_d^{(2P)}=20$; (c) $A_c^{(2P)}$ is replaced by the appropriate Y_0^* t-matrix element in analogy to the curve 2(b) of Fig. 6 of I. The experimental histogram is from Ref. 4. The other Σ parameters defined in I are: $a_0^{(\Sigma)}=-2.0$ F, $\eta_0=1.0$ F, and $\lambda'=1.35$ F. For comparison, the curve 1 of Fig. 6 of I corresponding to $1S$ capture is reproduced; it is labeled $1S$. All the theoretical curves are arbitrarily normalized.

values of the parameters that seem to give optimum fits to the data.[7]

In Fig. 1 we present three numerical examples of the recoil ^{3}He momentum distribution $R_3(p_3)$ computed for the $2P$ capture of $\bar{K}$, taking into account the combined effect of the "direct" ($\propto A_d$), the Y_1^* resonant ($\propto a_r$), and the Σ-Λ–conversion ($\propto A_c$) amplitudes; this includes also all the (typically small) interference terms. The form of $R_3(p_3)$ is generally quite insensitive to A_d; in particular, the ratio A_d/A_c can be increased by a factor up to 5–10 by increasing A_d without distorting the form of $R_3(p_3)$ in any significant way, and remaining still within a "reasonable" range of parameters. This can already be easily understood from the results of Fig. 4 of I. The width parameter of the Y_1^* resonance is fixed at $\Gamma_1=53$ MeV. The Σ parameters $a_0^{(\Sigma)}$, η_0, and λ' are exactly those of I.[8]

The maximum of the Σ-Λ–conversion contribution falls at about $p_3=420$ MeV/c; the general shape and characteristics of our $R_3(p_3)$ are quite similar to those of the curve 1 of Fig. 6 of I, i.e., giving only a rather qualitative agreement with the data.[7]

A better, semiquantitative, agreement with the data[7] can be obtained when, as for the $1S$ capture in I, the Y_0^* resonance is included in the Σ-production element; cf. the corresponding curve in Fig. 1.

The outgoing pion momentum distribution $R_\pi(p_\pi)$ is presented in Fig. 2 for parameter values corresponding exactly to those of the curve (a) of Fig. 1. Here again

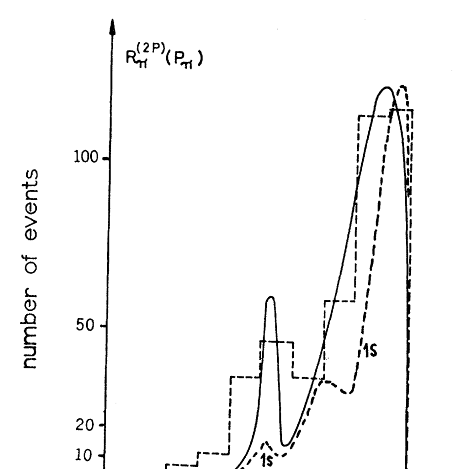

FIG. 2. The π^- momentum distribution $R_\pi(p_\pi)$ for the reaction $\bar{K}+{}^4\text{He}\rightarrow\pi^-+\Lambda^0+{}^3\text{He}$ when the kaon is captured from the $2P$ orbit; the parameters are $A_d^{(2P)}/a_r=0.2\times10^3$, and $A_c^{(2P)}/A_d^{(2P)}=15$. The experimental histogram is from Ref. 4. The other Σ parameters defined in I are $a_0^{(\Sigma)}=-2.0$ F, $\eta_0=1.0$ F, and $\lambda'=1.35$ F. For the sake of comparison, the curve "b" of Fig. 7 of I corresponding to $1S$ capture is reproduced; it is labeled $1S$. All the theoretical curves are arbitrarily normalized.

[7] Helium Bubble Chamber Collaboration Group, Nuovo Cimento **20**, 724 (1961); J. Auman *et al.*, in *Proceedings of the 1962 Annual International Conference on High Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 330.

[8] The complex Σ-I^3 scattering amplitude $(a_0^{(\Sigma)},\eta_0)$ is defined by the Σ-^3I phase shift as $\tan\delta_0^{(\Sigma)}=-q_{\Sigma 3}(a_0^{(\Sigma)}-i\eta_0)$; the parameter λ' is defined in Eq. (17) of I by the boundary-condition correction to the Σ-^3I effective radial-wave function (modified spherical outgoing wave: $[\exp(iq_{\Sigma 3}r)-\exp(-\lambda' r)]/r$).

the constant A_d could be varied considerably without any significant modification of $R_\pi(p_\pi)$. However, on the other hand, a change in A_c by a factor of $\frac{4}{3}$ would already produce too large a first peak of $R_\pi(p_\pi)$, due exclusively to the Σ-Λ–conversion amplitude.

By comparing our result of Fig. 2 with the curve (b) of Fig. 7 of I, we observe that, indeed, the assumption of $2P$ capture gives a better semiquantitative fit to the data[7] than does the assumption of the $1S$ capture. The now very pronounced first peak corresponds to the Λ-Σ threshold which occurs at $p_\pi \cong 164$ MeV/c (cf. the pygmy peak of Fig. 7 of I).

In summing up the discussion of both Figs. 1 and 2, we may say that while the rough agreement with the data[4] is about the same for the $2P$ capture as for the $1S$ capture of I, the $R_\pi(p_\pi)$ pion data appear to definitely favor the former case.[9]

[9] Actually, the smallness of the first peak of the curve $1S$ of Fig. 2 is a little exaggerated by the interference terms calculated in I with an inappropriate choice of one of the relative phases.

Obviously, one should regard our results with caution because of the crudeness of the treatment of the Σ-Λ–conversion amplitude by the two-component distorted-wave approximation as in I.

Our numerical results correspond to a simple but completely arbitrary choice of the phase of the conversion amplitude relative to the rest. In view of the inevitable smallness of the corresponding interference terms, only a very small improvement of our $R_3(p_3)$ could be achieved by any required shift of this phase, while $R_\pi(p_\pi)$ would suffer practically no change.

Many more data, particularly on angular distributions and on the absolute values of the reaction rates, should be avilable before a more detailed analysis becomes warranted.

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Uncoupled-Phase Method with Many Perturbing Channels

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The uncoupled-phase method is a nonperturbative formalism which describes the influence of any given (nth) channel on the dynamics of an n-channel scattering reaction. The method relates scattering amplitudes for the uncoupled reaction, obtained by switching off interactions to the nth channel while the interactions among the rest remain unchanged, to the scattering amplitudes describing the full reaction. We extend the uncoupled-phase method further, under exactly the same assumptions used to derive the previous uncoupled phase relations. We remove the restriction that there is only one perturbing channel and allow for the possibility of an arbitrary number of perturbing channels. The more general set of uncoupled-phase relations reduces to the previous uncoupled-phase relations when the number of perturbing channels is equal to 1. Some elementary applications of these relations is made, and their possible application in elementary particle reactions is indicated.

I. INTRODUCTION

ONE of the crucial facts about high-energy scattering is the large number of channels that become available for scattering, and a correct description of any one of them involves all the channels that are significantly coupled at the relevant energy. In many important physical situations it may be sufficient to consider only the coupled two-body channels (some of which may be closed in the energy region of interest). For example, in the meson-baryon scattering, $(\pi\Lambda,\pi\Sigma,\bar{K}N,\eta\Sigma,K\Xi)$ may be coupled significantly near the energy region of the Y_1^* resonance, and $(\pi\Xi,\bar{K}\Lambda,\bar{K}\Sigma,\eta\Xi)$ near the region of the $\Xi_{1/2}^*$ resonance. However, even though such systems can be handled by matrix N/D dispersion relations, the calculations are generally involved and in practice most calculations ignore all except the nearest channel. On the other hand, some channels may not be significantly important and could be safely neglected, but one needs some semiquantitative criterion in ignoring them.

The uncoupled-phase method (UPM) developed by Ross and Shaw[1,2] relates the actual amplitudes describing n coupled channels to the "uncoupled" amplitudes which describe the scattering, when the couplings to the nth channel are switched off, the other interactions remaining unchanged. Although the UPM was originally developed in the framework of a potential model,[1–4] it has subsequently been extended to relativistic N/D matrix calculations.[5] The usual weak-coupling approxi-

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[4] Interactions with hard cores were also investigated.
[5] P. Nath and G. Shaw, Phys. Rev. **137**, B711 (1965).