Total Cross Sections in a Quark Model*

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Meson and baryon total scattering cross sections are obtained using a quark model. It is emphasized that from the assumption of additivity alone [independent of $SU(2)$ or $SU(3)$ symmetry assumptions], two sum rules exist. Experimentally, they can be tested easily.

IT has been shown recently[1,2] that some very successful predictions concerning meson and baryon total cross sections can be easily derived from a simple quark model using the assumption that they are the sum of their individual quark-quark and quark-antiquark cross sections. In these predictions, it is observed that some of the sum rules are not in good agreement with the experimental results.[3] In this paper we would like to make a further investigation of all the known meson and baryon total scattering cross sections using the quark model. We will keep the additivity assumption for the total cross sections and introduce the internal symmetry for the quarks in steps so that we will be able to examine which of the sum rules is in better agreement with the experimental results.

We start with the assumption that all scatterings of a quark on a quark or a quark on an antiquark are different. There are altogether twelve different quark-quark and quark-antiquark cross sections. We now assume that mesons are made of a quark and an antiquark and baryons are made of three quarks. With the assumption of additivity, all the ten experimentally measured meson and baryon total cross sections can be expressed in terms of ten quark-quark and quark-antiquark cross sections. They are given in Eqs. (1).

$$\begin{aligned}
\sigma(\pi^+p)&=2\sigma(1,1)+\sigma(1,2)+2\sigma(1,\bar{2})+\sigma(2,\bar{2}),\\
\sigma(\pi^-p)&=2\sigma(1,2)+\sigma(2,2)+2\sigma(1,\bar{1})+\sigma(1,\bar{2}),\\
\sigma(pp)&=4\sigma(1,1)+4\sigma(1,2)+\sigma(2,2),\\
\sigma(pn)&=2\sigma(1,1)+5\sigma(1,2)+2\sigma(2,2),\\
\sigma(\bar{p}p)&=4\sigma(1,\bar{1})+4\sigma(1,\bar{2})+\sigma(2,\bar{2}),\\
\sigma(\bar{p}n)&=2\sigma(1,\bar{1})+5\sigma(1,\bar{2})+2\sigma(2,\bar{2}),\\
\sigma(K^+p)&=2\sigma(1,1)+\sigma(1,2)+2\sigma(1,\bar{3})+\sigma(2,\bar{3}),\\
\sigma(K^-p)&=2\sigma(1,3)+\sigma(2,3)+2\sigma(1,\bar{1})+\sigma(1,\bar{2}),\\
\sigma(K^+n)&=\sigma(1,1)+2\sigma(1,2)+\sigma(1,\bar{3})+2\sigma(2,\bar{3}),\\
\sigma(K^-n)&=\sigma(1,3)+2\sigma(2,3)+\sigma(1,\bar{1})+2\sigma(1,\bar{2}),
\end{aligned} \tag{1}$$

where we have used notation such that, for example, $\sigma(1,\bar{2})$ is the total cross section of a quark q_1 on an antiquark $\bar{q}_2$.

(I) Without any further assumption, it is interesting to note that there are two sum rules contained in Eqs. (1), namely,

$$\sigma(\pi^+p)+\sigma(\pi^-p)=\tfrac{1}{3}[\sigma(pp)+\sigma(pn)+\sigma(p\bar{p})+\sigma(\bar{p}n)], \tag{2a}$$

$$\sigma(\pi^+p)-\sigma(\pi^-p)=[\sigma(pp)-\sigma(pn)]-[\sigma(\bar{p}p)-\sigma(\bar{p}n)]. \tag{2b}$$

These two sum rules are obtained from the assumption of additivity alone. They are independent of $SU(2)$ or $SU(3)$ symmetry assumptions. Hence, an experimental test of these sum rules is important because it tests the assumption of additivity—if mesons and baryons are indeed made of quarks, can their internal structures be neglected at high-energy scattering?

Because the total $\bar{p}n$ cross section is not well determined experimentally, we combine Eqs. (2a) and (2b) to give

$$\sigma(\pi^+p)+2\sigma(\pi^-p)=\sigma(pn)+\sigma(\bar{p}p). \tag{2c}$$

A comparison of this relation and the relations (2a) and (2b) with experiment[4] is shown in Table I. The points for all reactions are taken at the same laboratory momentum, and the agreement is fair. Hopefully, the agreement will get better at higher energy.[5]

(II) If we assume that quarks satisfy $SU(2)$ symmetry, we obtain the following relations for the quark-quark and quark-antiquark cross sections:

$$\sigma(1,1)=\sigma(2,2),\qquad \sigma(1,3)=\sigma(2,3),$$
$$\sigma(1,\bar{1})=\sigma(2,\bar{2}),\qquad \sigma(1,\bar{3})=\sigma(2,\bar{3}).$$

Incorporating these conditions into Eqs. (1), we obtain two more sum rules [in addition to those of Eqs. (2)]:

$$\sigma(K^+p)-\sigma(K^+n)=\sigma(pp)-\sigma(pn), \tag{3a}$$

$$\sigma(K^-p)-\sigma(K^-n)=\sigma(\bar{p}p)-\sigma(\bar{p}n). \tag{3b}$$

(III) If quarks satisfy $SU(3)$ symmetry, their cross sections will have the following relations:

$$\sigma(1,1)=\sigma(2,2)=\sigma(3,3),\qquad \sigma(1,2)=\sigma(2,3)=\sigma(3,1),$$
$$\sigma(1,\bar{1})=\sigma(2,\bar{2})=\sigma(3,\bar{3}),\qquad \sigma(1,\bar{2})=\sigma(2,\bar{3})=\sigma(3,\bar{1}).$$

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[1] E. M. Levin and L. L. Frankfurt, JETP Pis'ma v Redaktsivu **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); J. J. J. Kokkedee and L. Van Hove, CERN Report 66/248/5-TH. 642, 1966 (unpublished).

[3] For example, in Ref. 1, their sum rules (4) to (11) can be rewritten in a much simpler form: $\sigma(\pi^+p)=\sigma(\pi^-p)=\sigma(K^+p)=\sigma(K^-p)=\sigma(K^+n)=\sigma(K^-n)=\frac{1}{3}[\sigma(pp)+\sigma(\bar{p}p)]$, $\sigma(pp)=\sigma(pn)$ and $\sigma(\bar{p}p)=\sigma(\bar{p}n)$, which are obviously not in good agreement with the experimental results.

[4] The experimental data are taken from W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965). For more references to the experimental data see V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965).

[5] At 20 BeV/c, the ratio between the right-hand side and the left-hand side of Eq. (2c) is about 1.13 ± 0.09 and is still decreasing with the energy.

TABLE I. Experimental comparisons on relations (2c), (2a), and (2b).

Momentum (BeV/c)	Experimental comparison on relation (2c) $\sigma(\pi^+p)+2\sigma(\pi^-p)$ (mb)	$\sigma(pn)+\sigma(\bar{p}p)$ (mb)	Experimental comparison on relation (2a) $\sigma(\pi^+p)+\sigma(\pi^-)$ (mb)	$\frac{1}{3}[\sigma(pp)+\sigma(pn)+\sigma(\bar{p}p)+\sigma(\bar{p}n)]$	Experimental comparison on relation (2b) $\sigma(\pi^+p)+\sigma(pn)+\sigma(\bar{p}p)$	$\sigma(\pi^-p)+\sigma(pp)+\sigma(\bar{p}n)$
6	83.2±0.5	101.9±2.0	54.7±0.4	67.3±1.5	128.1±2.0	128.6±4.1
8	80.1±0.5	98.2±1.9	52.6±0.4	65.2±1.5	123.3±1.9	124.8±4.0
10	77.8±0.5	...	51.3±0.4	...	...	...
12	76.0±0.5	92.1±1.9	50.1±0.4	61.8±1.4	116.3±1.9	119.1±3.8
14	74.7±0.5	90.9±1.9	49.3±0.4	61.1±1.4	114.8±1.9	117.9±3.8
16	73.6±0.5	89.4±1.9	48.5±0.4	60.3±1.4	112.8±1.9	116.5±3.8
18	73.5±0.5	89.5±4.0	48.5±0.4	57.5±3.3	113.0±4.0	108.1±9.0
20	73.0±0.5	...	48.2±0.4	...	...	...

TABLE II. Experimental comparison on relations (3a) and (3b).

Momentum (BeV/c)	Experimental comparison on relations (3a) $\sigma(K^+p)+\sigma(pn)$ (mb)	$\sigma(K^+n)+\sigma(pp)$ (mb)	Experimental comparison on relations (3b) $\sigma(K^-p)+\sigma(\bar{p}n)$ (mb)	$\sigma(K^-n)+\sigma(\bar{p}p)$ (mb)
6	59.6±1.7	58.1±0.7	83.5±4.0	81.2±1.2
8	59.1±1.7	57.6±0.7	80.9±3.9	76.1±0.9
10	58.8±1.7	57.4±0.7	...	...
12	57.7±1.7	57.0±0.7	75.4±3.7	71.9±0.9
14	57.6±1.7	56.6±0.7	74.9±3.7	70.8±1.0
16	57.2±1.7	56.1±0.7	74.0±3.7	69.5±1.0
18	56.3±1.7	56.3±0.7	65.4±9.0	70.6±3.8
20	56.2±1.7	56.1±0.7	...	...

TABLE III. Experimental comparison on relations (4a) and (4b).

Momentum (BeV/c)	Experimental comparison on relations (4a) $\sigma(\pi^+p)+\sigma(K^-n)$ (mb)	$\sigma(\pi^-p)+\sigma(K^+n)$ (mb)	Experimental comparison on relations (4b) $\sigma(K^+p)+\sigma(K^-p)$ (mb)	$\frac{1}{2}[\sigma(\pi^+p)+\sigma(\pi^-p)+\sigma(K^+n)+\sigma(K^-n)]$
6	48.1±0.4	46.0±0.5	41.0±0.3	47.1±0.3
8	44.8±0.4	45.1±0.5	40.9±0.2	45.0±0.3
10	45.4±0.4	44.0±0.5	39.8±0.2	44.7±0.3
12	44.4±0.4	43.5±0.5	38.9±0.2	44.0±0.3
14	44.0±0.4	42.9±0.5	38.9±0.2	43.5±0.3
16	43.7±0.6	42.5±0.5	38.3±0.4	43.1±0.4
18	43.8±1.1	42.6±0.5	38.1±0.8	43.2±0.6
20	...	42.5±0.5	39.9±4.6	...

Besides Eqs. (2) and (3), we obtain two more relations

$$\sigma(\pi^+p)-\sigma(\pi^-p)=\sigma(k^+n)-\sigma(k^-n)\,, \qquad (4a)$$

$$\sigma(k^+p)+\sigma(k^-p)=\tfrac{1}{2}[\sigma(\pi^+p)+\sigma(\pi^-p)+\sigma(k^+n)+\sigma(k^-n)]\,. \qquad (4b)$$

Equation (4a) and Eqs. (2b), (3a), and (3b) combined are just the Johnson-Treiman relations.[6] Both experimental comparisons on relations (3) and (4) are shown in Tables II and III, respectively. They are in a reasonable agreement with the experimental results.[4]

Similarly, we can make a calculation on the five measurable hyperon-proton scattering total cross sections.

(I) With the assumption of additivity alone, we obtain the following four sum rules:

$$\sigma(\Lambda p)=\sigma(pn)+\sigma(K^-p)-\sigma(\pi^-p)\,, \qquad (5a)$$

$$\sigma(\Xi^0p)=\sigma(pn)+2\sigma(K^-p)-2\sigma(\pi^-p)\,, \qquad (5b)$$

$$\sigma(\Xi^-p)=2\sigma(pn)-\sigma(pp)+2\sigma(K^+p)-2\sigma(\pi^-p)\,, \qquad (5c)$$

$$2\sigma(\Sigma^+p)+\sigma(\Sigma^-p)=2\sigma(pn)+\sigma(pp)+3\sigma(K^-p)-3\sigma(\pi^-p)\,. \qquad (5d)$$

(II) Together with the assumption that quarks satisfy $SU(2)$ symmetry, instead of one sum rule (5d), we obtain two sum rules:

$$\sigma(\Sigma^+p)=\sigma(pp)+\sigma(K^-p)-\sigma(\pi^-p) \qquad (6a)$$

$$\sigma(\Sigma^-p)=2\sigma(pn)-\sigma(pp)+\sigma(K^-p)-\sigma(\pi^-p)\,. \qquad (6b)$$

Equations (5a)–(5c) remain the same.

[6] K. Johnson and S. B. Trieman, Phys. Rev. Letters **14**, 189 (1965); R. Good and Nguyen-Huu Xuong, *ibid.* **14**, 191 (1965).

(III) If quarks satisfy $SU(3)$ symmetry, Eqs. (5) and (6) can be rewritten in a much simpler form:

$$\sigma(\Lambda p)=2\sigma(pn)-\sigma(pp),\quad (7a)$$
$$\sigma(\Xi^- p)=4\sigma(pn)-3\sigma(pp),$$

$$\sigma(\Sigma^+ p)=\sigma(pn),\quad (7b)$$
$$\sigma(\Sigma^- p)=\sigma(\Xi^0 p)=3\sigma(pn)-2\sigma(pp)$$

Discussion. Equations (2) and (5) follow from the assumption of additivity alone. They should be tested experimentally. Equations (2) are the easiest to test. With the present knowledge on the experimental data up to the highest energy available, they differ from the predictions by approximately 10–20%. The errors on the experimental data are still large. To clear up this point, more data with better accuracy are needed on the nucleon-nucleon and nucleon-antinucleon total cross sections. It is also of interest to have some data at higher energy on hyperon-proton scattering total cross sections. Then Eqs. (5) would serve as an independent check on the assumption of additivity.

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Capture of a $\bar{K}$ Meson from the $2P$ Orbit in ^{4}He Followed by π^- and Λ^0 Production

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The reaction $\bar{K}+{}^4\mathrm{He}\rightarrow\pi^-+\Lambda^0+{}^3\mathrm{He}$ is studied under the assumption of capture from the $2P$ Bohr orbit. The outgoing pion and the recoiling ^{3}He momentum distributions are calculated with the inclusion of Y_1^*-resonance and Σ-Λ conversion amplitudes. The Σ-Λ conversion process is of particular importance. The main conclusion is that the pion spectrum tends to favor the assumption that the stopping $\bar{K}$ are captured from a P-state atomic orbit. The conclusion is consistent with the results of the $\bar{K}$-capture x-ray experiment of Burleson *et al.*

IN a recent article by Said and the author[1] (referred to as I), the reaction $\bar{K}+{}^4\mathrm{He}\rightarrow\pi^-+\Lambda^0={}^3\mathrm{He}$ has been extensively studied from the point of view of the recoil-nucleus and the pion momentum distributions. Three processes have been considered: a direct nonresonant production of a π^- and a Λ in the absorption of the $\bar{K}$ meson by a neutron; the formation of the Y_1^* resonance in the intermediate state; and the reaction with a Σ production in the first stage and its subsequent conversion into a Λ in a successive collision with another nucleon. The initial-state kaon Bohr orbits considered were nS and mP; however, only simplified calculations were performed for the P-wave case with no elastic Λ-^{3}He distortion and the Σ-Λ conversion included. All the detailed computations referred to nS orbits, presumed, at that time, to be the most important ones for the direct nuclear $\bar{K}$ capture.

From the computed momentum distribution $R_3(p_3)$ of the recoiling ^{3}He, it was concluded that the formation of the Y_1^* resonance should be an important process (cf. also Letessier and Vinh Mau[2]); from $R_3(p_3)$ and from the momentum distribution $R_\pi(p_\pi)$ of the outgoing pion, rather strong evidence was obtained for the great importance of the Σ-Λ conversion process, particularly at large p_3 and at small p_π of the two respective distributions. The elastic Λ-^{3}He final-state interaction was found to be of little importance. Semiquantitative fits to the observed spectra $R_3(p_3)$ and $R_\pi(p_\pi)$ were obtained.

Contrary to many previous speculations and expectations (cf., e.g., some of the references quoted in I), it appears from the most recent experimental results of Burleson *et al.*[3] that about 80% of all the kaons undergo the direct $\bar{K}$ capture from the $2P$ Bohr orbit. It is the aim of the present note to supplement I with the corresponding results for this case.

Our method, mathematical approximations, and notations are all those of I.

For the non-Σ–Λ-conversion amplitude, we use the approximate expression [cf. Eq. (7) of I]:

$$M_{2P}\cong N_{2P}\{\mathfrak{F}_{(3,n,\Lambda)}([M_3/(M_3+m_\Lambda)]\mathbf{P}_f)i\mathbf{e}\cdot\nabla_{\mathbf{q}_0}\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{\mathbf{p}_K=0}+i\tfrac{3}{4}[\mathbf{e}\cdot\nabla_\omega\mathfrak{F}_{(3,n,\Lambda)}(\boldsymbol{\omega})]_{\omega=[M_3/(M_3+m_\Lambda)]\mathbf{P}_f}\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{\mathbf{p}_K=0}\},\quad (1)$$

where N_{2P} is the normalization of the kaon $2P$ Bohr orbit wave function, $\mathbf{e}$ is the polarization vector of the

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[3] G. R. Burleson, D. Cohen, R. C. Lamb, D. N. Michael, R. A. Schluter, and T. O. White, Jr., Phys. Rev. Letters **15**, 70 (1965). The nuclear properties of the $\bar{K}$-mesonic ^{4}He atom have been studied recently by F. von Hippel and J. H. Douglas [Phys. Rev. **146**, 1042 (1966)], and by J. Uretsky [Phys. Rev. **147**, 906 (1966)].