able because of its classical nature and an uncritical application of the correspondence principle. If the renormalized momentum is taken as the quantum counterpart of the classical momentum, all these difficulties are resolved.

V. SUMMARY

It has been shown that care needs to be exercised in discussing the classical limit of quantum electrodynamics. For vacuum phenomena, the photon density must be large and the frequency must be low, though it is not well known how the frequency must decrease as the density increases, for Maxwell theory to be valid. In the presence of electrons, forward scattering of photons introduces a new class of renormalizations, account of which must be taken in interpreting the results of classical calculations because no counterparts of these renormalizations exist in the classical theory. When these renormalizations are included, a number of apparent discrepancies between classical and quantum calculations are removed.

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Study of $\pi^- + p \rightarrow \Sigma^0 + K^0$ at 1170 MeV/c*

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We have measured the differential and total cross sections and polarization of the Σ^0 from the reaction $\pi^- + p \rightarrow \Sigma^0 + K^0$ with 1170-MeV/c pions incident on the Alvarez 72-in. hydrogen bubble chamber. Using 524 single-Λ events (where the only visible decay is that of the Λ from the decay $\Sigma^0 \rightarrow \Lambda + \gamma$), 138 single-$K^0$ events (where only the K^0 decay is visible), and 256 double-vee events (where both Λ and K^0 decays are visible), we find the coefficients in the Legendre expansion of the differential cross section $d\sigma/d\Omega = A_0P_0 + A_1P_1 + A_2P_2$ to be $A_0 = 19.68 \pm 0.60$ μb/sr, $A_1 = -0.04 \pm 1.20$ μb/sr, and $A_2 = 14.54 \pm 1.60$ μb/sr, corresponding to a total cross section $\sigma = 247 \pm 10$ μb. No polynominals higher than P_2 are needed. Using both single-Λ and double-vee events, we find the coefficients in the polarization expansion $P_\Sigma d\sigma/d\Omega = \frac{1}{2}\sin\theta_\Sigma (B_1 + B_2\cos\theta_\Sigma)$ to be $B_1 = -9.98 \pm 8.29$ μb/sr and $B_2 = -35.45 \pm 21.88$ μb/sr. In both angular-distribution and polarization studies a single-Λ event is statistically equivalent to about one half of a double-vee event.

I. INTRODUCTION

We have measured the angular distribution and polarization of the Σ^0 in the process

$$\pi^- + p \rightarrow \Sigma^0 + K^0, \tag{1a}$$

$$\Sigma^0 \rightarrow \Lambda + \gamma, \tag{1b}$$

using 1170-MeV/c π^- incident on the Alvarez 72-in. hydrogen bubble chamber. We use 256 double-vee events where both the K^0 and the Λ decay visibly via the charged modes

$$K^0 \rightarrow \pi^+ + \pi^- \tag{2}$$

and

$$\Lambda \rightarrow p + \pi^-. \tag{3}$$

We also use 524 single-Λ events where the decay (3) is observed, but (2) is not, and 138 single-K^0 events where (2) is observed and (3) is not. All three types of events are used to find the angular distribution, and both double vees and single Λ's are used to find the Σ^0 polarization. We extract from the data a maximum amount of information on the Σ^0 polarization. Our method can be applied in other reactions involving polarized Σ^0's.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

In a later paper we shall present our results for $\pi^- + p \rightarrow \Sigma^- + K^+$ at the same momentum. We defer until then a comparison of the experimental results for $\pi^- + p \rightarrow \Sigma^0 + K^0$, $\pi^- + p \rightarrow \Sigma^- + K^+$, and $\pi^+ + p \rightarrow \Sigma^+ + K^+$ with the predictions of charge independence.[1]

II. SELECTION OF EVENTS

Events corresponding to Σ^0 production must be distinguished from the topologically similar events resulting from Λ production via the reaction

$$\pi^- + p \rightarrow \Lambda + K^0. \tag{4}$$

Whenever there is a visible K^0 decay both Σ^0 and Λ production are kinematically overdetermined. We then use the fitting program PACKAGE and select events on the basis of χ^2. For these events (single K's and double vees) there is no ambiguity between Σ^0 and Λ production.

[1] For earlier experimental results on $\Sigma + K$ production at pion momenta near 1 BeV/c, see F. S. Crawford, Jr., R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959), which includes earlier references; J. A. Anderson *et al.*, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1962), p. 270; R. Kraemer *et al.*, *ibid.*, p. 273; J. R. Albright *et al.*, *ibid.*, p. 276; F. S. Crawford, F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962); Y. S. Kim, G. R. Burleson, P. I. P. Kalmus, A. Roberts, and T. A. Romanowski, *ibid.* **143**, 1028 (1966).

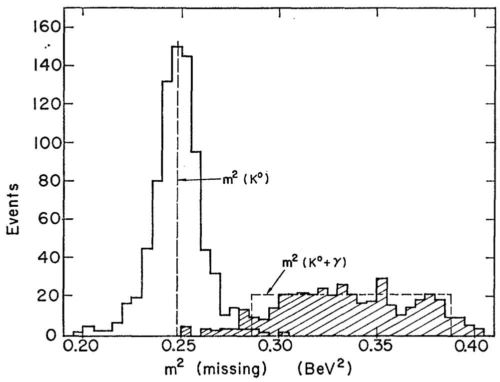

FIG. 1. Square of the missing mass recoiling against the observed Λ, for double vees treated as if they were single Λ's. Unshaded events peaking at $m^2=m^2(K^0)$ are obtained from 812 double-vee events from $\Lambda-K^0$ production. Shaded events are 426 double-vee events from Σ^0-K^0 production. The rectangular distribution labeled $m^2(K^0+\gamma)$ is the theoretical distribution in m^2 for 426 Σ^0-K^0 events at 1170 MeV/c, ignoring effects of measurement errors.

The single-Λ events are treated separately. The production vertex can be fitted with one constraint (1C) for the Λ production; it cannot be fitted for Σ^0 production because of the lack of knowledge of the momentum of the gamma ray from the Σ^0 decay, reaction (1b). We proceed as follows.

(A) We fit all single-Λ events to the production reaction (4). If $\chi^2(1C)$ for reaction (4) is less than 8.6, the event is accepted as a Λ production.[2] Some good Σ^0-production events are lost by this procedure. To estimate that loss, we study our fitted double-vee Σ^0-production events, which we *know* are Σ^0 and not Λ production. Fitting these double vees as if they were single Λ's, we find that $(4.3\pm1.6)\%$ fit reaction (4) with $\chi^2(1C)<8.6$. These would be lost if they were single Λ's from Σ^0 production.

(B) The remaining single Λ's with $\chi^2(1C)>8.6$, are mostly but not entirely due to Σ^0 production; they are slightly contaminated by Λ-production events. We estimate this contamination by studying our fitted double-vee Λ-production events, which we *know* are due to Λ production. We fit them as if they were single Λ's. We find that $(1\pm1)\%$ of them have $\chi^2(1C)>8.6$, and would therefore be included among our Σ^0 candidates if they were single Λ's. Our subsequent procedure [paragraphs (C) and (D)] reduces this contamination to $(\frac{1}{2}\pm\frac{1}{2})\%$. Since we have 1500 single Λ's with $\chi^2(1C)<8.6$, we expect approximately 8 ± 8 contaminating Λ-production events in our sample.

(C) We calculate the invariant missing mass m recoiling against the observed Λ. For Λ production m^2 should, in the absence of measuring errors, correspond to the m^2 of a K^0, $m^2(K^0)=0.248$ BeV^2. For Σ^0 production, m is the invariant mass of the system $K^0+\gamma$, where the gamma ray is from $\Sigma^0\rightarrow\Lambda+\gamma$. At 1170 MeV/$c$ incident-pion momentum $m^2(K+\gamma)$ can vary between the lower and upper limits 0.286 and 0.388 BeV^2. Between these limits the distribution $dN/d(m^2)$ is flat. (This can be shown to follow from the facts that the decay $\Sigma^0\rightarrow\Lambda+\gamma$ is spherically symmetric and that the pion beam is monoenergetic. The spherical symmetry follows from parity conservation, which is assumed to hold because the Σ^0 decay is electromagnetic.) We illustrate these two distributions in m^2 by treating all of our double vees (both Λ and Σ^0 production) as if they were single Λ's. The results are plotted in Fig. 1.

(D) For all single Λ's with $\chi^2(1C)>8.6$ we calculate m^2 and plot the result in Fig. 2. The square distribution is that expected for Σ^0 production in the absence of measurement errors. The smooth curve is obtained by folding the square distribution with the resolution function. For the resolution function we use the experimental distribution in m^2 for fitted double-vee Λ-production events, treated as if they were single Λ's; i.e., the distribution centered on $m^2(K^0)$ in Fig. 1. Of the single Λ's with $\chi^2(1C)>8.6$ we accept as Σ^0-production events those with m^2 greater than $m^2(K^0)$. We believe that the ten shaded events in Fig. 2, which have $m^2<m^2(K^0)$, are due to Λ production with $\chi^2(1C)>8.6$. Correspondingly there should be approximately 10 ± 3 contaminating Λ's in the accepted region $m^2>m^2(K^0)$. [This substantiates the estimate, made in paragraph (B), of 8 ± 8 contaminating events.] We do not expect and do not find any single-Λ events that have $\chi^2(1C)>8.6$ and have m^2 very close to $m^2(K^0)$. For example, single-Λ events with $m^2=m^2(K^0)$ must have $\chi^2(1C)=0$. Furthermore, most (70%) of the Λ-production events have a calculated standard deviation in m^2 of less than 0.01 BeV^2, according to our study of double vees. Thus we expect a hole in the distribution of Fig. 2, with width of order 0.02 BeV^2 and centered at $m^2(K^0)$, corresponding to the removal of events with $\chi^2(1C)<8.6$. Finally, we may notice in Fig. 2 a slight depletion in the m^2 distribution near the lower end of the

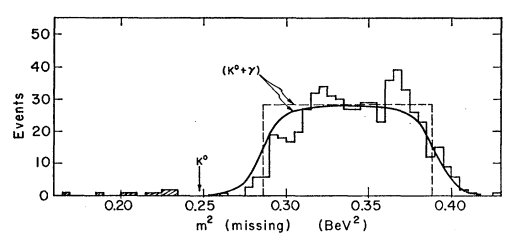

FIG. 2. Square of the missing mass recoiling against the observed Λ, for single-Λ events that do not fit $\Lambda-K^0$ production. The rectangular curve labeled $(K^0+\gamma)$ is the expected distribution for Σ^0-K^0 production, neglecting measurement errors; the smooth curve has the resolution in m^2 folded in. The resolution function is taken to be the m^2 distribution for $\Lambda-K^0$ production of double-vees treated as single Λ's, i.e., the unshaded distribution of Fig. 1.

[2] Experience has shown that for a correct hypothesis the χ^2 distribution for PACKAGE fits to 72-in. chamber events agree with the theoretical χ^2 distributions for the appropriate constraint class, provided the theoretical values of χ^2 are multiplied by 2. Thus our 1C cutoff at $\chi^2=8.6$ corresponds to a theoretical χ^2 of about 4.3.

expected distribution for Σ^0-production events. We believe this depletion corresponds to the 4.3% loss of good Σ^0's that have $\chi^2(1C)<8.6$ discussed in paragraph (A).

The above selection procedures for Σ^0-production events yield 426 double vees, 582 single Λ's, and 187 single K's. These numbers are reduced by fiducial-volume criteria to 256 double vees, 524 single Λ's, and 138 single K's. (The largest fiducial loss results from our demand that a single Λ travel at least 0.8 cm before it decays; for a double vee or single K we demand that the K^0 travel at least 0.8 cm before it decays.) In calculating physically interesting quantities each event (i) carries a weighting factor $b_i\geqslant 1$ that includes all fiducial corrections as well as correction for the attenuation of the pion beam in the chamber. We choose the decay fiducial volume larger than the production fiducial volume so as to avoid fluctuations from accidentally large values of b_i. The weighted or "true" number of events N is given by

$$\begin{aligned} N&\equiv\sum b_i\pm[\sum b_i^2]^{1/2}\\ &=N_{\text{obs}}\langle b\rangle\pm[N_{\text{obs}}\langle b^2\rangle]^{1/2}, \end{aligned} \tag{5}$$

where the sums extend over the observed events $i=1$ to N_{obs}. The brackets $\langle\ \rangle$ mean an average over the data. [We use weighted counts as in Eq. (5) not only for the entire sample, i.e., to obtain absolute cross sections and branching ratios, but also for subsamples used to find angular distributions and polarizations.] For the entire sample we find for double vees, single K's, and single Λ's, respectively, the values $\langle b\rangle=1.76$, 1.74, and 1.29, and $\langle b^2\rangle^{1/2}=1.85$, 1.84, and 1.30.

The weighted number of single-Λ events is further corrected by subtracting a fraction 10/582 of contaminating Λ-production events [see paragraph (D)] and by adding a fraction 0.043 of good Σ^0's lost because they had $\chi^2(1C)<8.6$ for the Λ-production fit [see paragraph (A)]. After correcting for scanning efficiencies of 0.972 for double vees and 0.929 for single vees, we obtain the corrected numbers of Σ^0-production events:

$$\text{Double vees:}\quad D=501.2\pm30.7\,, \tag{6a}$$

$$\text{Single } K\text{'s:}\quad K=261.4\pm23.4\,, \tag{6b}$$

$$\text{Single } \Lambda\text{'s:}\quad \Lambda=747.9\pm32.7\,. \tag{6c}$$

III. BRANCHING RATIOS AND CROSS SECTIONS

The three results D, K, and Λ are used to find the more interesting quantities N, B_K, and B_Λ, where N is the "true" number of Σ^0-production events (1), including all visible and invisible decays, and where B_K and B_Λ are the branching ratios

$$B_K\equiv\Gamma(K_1{}^0\rightarrow\pi^+\pi^-)/\Gamma(\text{all } K^0)$$

and

$$B_\Lambda\equiv\Gamma(\Lambda\rightarrow p\pi^-)/\Gamma(\text{all }\Lambda)\,.$$

The (corrected) numbers of events are related to these quantities by

$$D=NB_KB_\Lambda\,,$$

$$K=NB_K(1-B_\Lambda)\,,$$

and

$$\Lambda=NB_\Lambda(1-B_K)\,,$$

which can be solved for

$$B_K=D/(D+\Lambda)\,, \tag{7a}$$

$$B_\Lambda=D/(D+K)\,, \tag{7b}$$

and

$$N=(D+K)(D+\Lambda)/D\,. \tag{7c}$$

Inserting results (6) into Eqs. (7) yields

$$B_K=0.401\pm0.018\,, \tag{8}$$

$$B_\Lambda=0.657\pm0.025\,, \tag{9}$$

$$N=1901\pm78\,. \tag{10}$$

Our branching ratios (8) and (9) are in fair agreement with current world averages $B(K_1)\equiv2B_K=0.685\pm0.010$, and $B_\Lambda=0.663\pm0.010$.[3]

From our result (10) and our pion track length we obtain the total cross section at 1170 MeV/c:

$$\sigma(\pi^-p\rightarrow\Sigma^0K^0)=247\pm10\ \mu\text{b}\,. \tag{11}$$

IV. ANALYSIS OF ANGULAR DISTRIBUTION

A. Double Vees and Single K's

We write the differential counting rate dN_Σ for Σ^0's produced at μ in $d\mu$ in the Legendre polynomial expansion

$$dN_\Sigma=N(\mu)d\mu=\tfrac{1}{2}[A_0P_0+A_1P_1(\mu)+A_2P_2(\mu)]d\mu\,, \tag{12}$$

where $\mu\equiv\hat{\pi}\cdot\hat{\Sigma}\equiv\cos\theta_\Sigma$ is the cosine of the angle between the incident pion direction $\hat{\pi}$ and the produced Σ direction $\hat{\Sigma}$ in the c.m. system. For double vees and single K's, μ is known for each event. We find the coefficients in Eq. (12) by the method of least squares using weighted counts as given by Eq. (5). Redefining the coefficients in Eq. (12), we write the absolute differential cross section [normalized to our integrated-cross-section result (11)] in the form

$$d\sigma/d\Omega=A_0P_0+A_1P_1(\mu)+A_2P_2(\mu)\,. \tag{13}$$

We find from the double vees and single K's

$$A_0=19.68\pm1.03\ \mu\text{b/sr}\,, \tag{14a}$$

$$A_1=3.06\pm2.01\ \mu\text{b/sr}\,, \tag{14b}$$

and

$$A_2=13.97\pm2.57\ \mu\text{b/sr}\,, \tag{14c}$$

with χ^2 probability 64%. No polynomials higher than $P_2(\mu)$ are needed.

[3] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

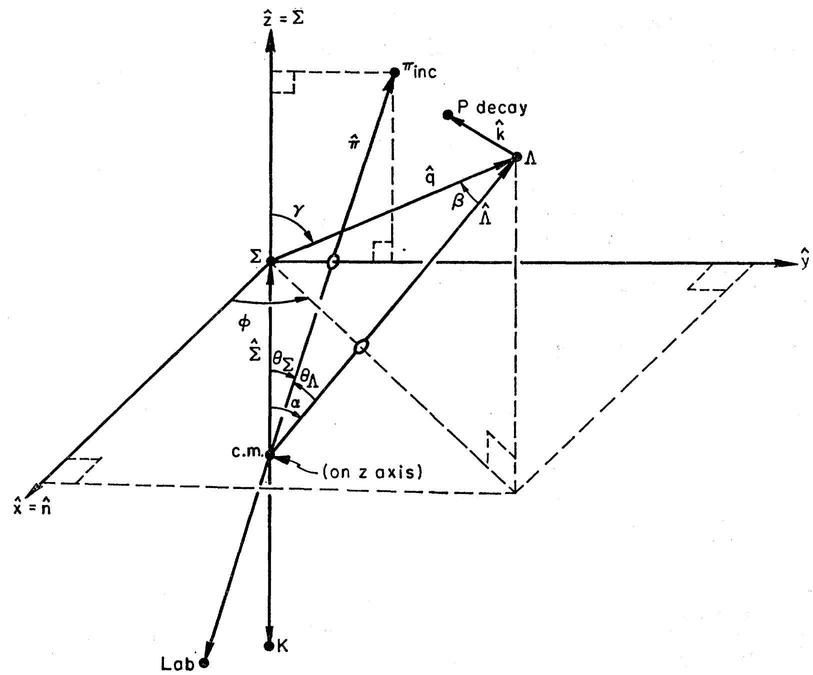

FIG. 3. Schematic diagram in velocity space for the production reaction $\pi^-+p\rightarrow\Sigma^0+K^0$, followed by the decays $\Sigma^0\rightarrow\Lambda+\gamma$ and $\Lambda\rightarrow p+\pi^-$. The dots labeled lab, c.m., π_{inc}, Σ, K, Λ and p_{decay} correspond to the rest frame of the target proton, the c.m. frame for the production reaction, and the rest frames of the incident pion, the produced Σ^0 and K^0, the Λ from Σ^0 decay, and the proton from Λ decay. The unit vectors labeled $\hat{\pi}$, $\hat{\Sigma}$, and $\hat{\Lambda}$ correspond to the direction of the velocity of the incident pion, the Σ^0, and the Λ, all with respect to the c.m. frame. The unit vector $\hat{q}$ is along the velocity of the Λ in the Σ^0 frame. Each unit vector is uniquely defined only in the Lorentz frames which it "connects." (For example, $\hat{q}$ is along the Λ velocity in the Σ^0 frame, or along the negative of the Σ^0 velocity in the Λ frame, but is not uniquely defined in the c.m., lab, or π_{inc} frames.) The angles α, β, and γ are defined by $\cos\alpha=\hat{\Sigma}\cdot\hat{\Lambda}$ (in the c.m. frame), $\cos\beta=\hat{\Lambda}\cdot\hat{q}$($\Lambda$ frame), and $\cos\gamma=\hat{q}\cdot\hat{\Sigma}$($\Sigma^0$ frame). Since we are dealing with relativistic particles, the Euclidean relation $\gamma=\alpha+\beta$ does not hold; therefore we call the diagram "schematic." The angles θ_Σ and θ_Λ are defined by $\cos\theta_\Sigma\equiv\mu=\hat{\pi}\cdot\hat{\Sigma}$ (c.m. frame), and $\cos\theta_\Lambda\equiv\mu_\Lambda=\hat{\pi}\cdot\hat{\Lambda}$(c.m. frame). The right-handed Cartesian coordinate system $\hat{x}$, $\hat{y}$, $\hat{z}$ can be defined in the Σ^0 frame by $\hat{z}=\hat{\Sigma}$, $\hat{x}=\hat{n}$, and $\hat{y}=\hat{z}\times\hat{x}$. Here $\hat{n}$ is a unit vector normal to the production plane and defined in the c.m.,lab, and Σ^0 frames. In the c.m. system it is given by $\hat{n}=(\hat{\pi}\times\hat{\Sigma})/|\hat{\pi}\times\hat{\Sigma}|$. Spherical coordinates for the unit vector $\hat{q}$ are defined in the Σ^0 frame. The polar axis is $\hat{z}$, the polar and azimuthal angles are γ and ϕ. Further relationships between the angles are given in the Appendix.

B. Single Λ's

For the single Λ's it is convenient to use the method of moments described below to find the coefficients A_L corresponding to Eq. (12). By multiplying Eq. (12) by $P_L(\mu)$ and by integrating over μ from -1 to $+1$, we project out the coefficient A_L:

$$A_L=(2L+1)\int P_L(\mu)dN_\Sigma. \tag{15}$$

In the method of moments we replace the integrals in Eq. (15) by sums over weighted counts. We thus define "experimental" values of A_L by

$$A_0=\sum b_i=N\,, \tag{16a}$$

$$A_1=3\sum b_iP_i(\mu_i)\,, \tag{16b}$$

and

$$A_2=5\sum b_iP_2(\mu_i)\,, \tag{16c}$$

with standard deviations and correlations given by generalizing the expression for the error in Eq. (5); for example, by inspection of Eq. (16), the off-diagonal error $\delta A_1\delta A_2$ is given by

$$\delta A_1\delta A_2=15\sum b_i^2P_1(\mu_i)P_2(\mu_i)\,, \tag{17}$$

with similar expressions for the other elements of the error matrix.

For single-Λ events we cannot determine $\mu\equiv\cos\theta_\Sigma$. Therefore we cannot apply Eqs. (16) directly. However, we can determine $\cos\theta_\Lambda\equiv\mu_\Lambda\equiv\hat{\pi}\cdot\hat{\Lambda}$, where θ_Λ is the angle between the incident-pion direction $\hat{\pi}$ and the observed lambda direction $\hat{\Lambda}$ in the c.m. system of the incident pion and target proton (see Fig. 3). Now, θ_Λ and θ_Σ are merely the polar angles of the incident-pion direction $\hat{\pi}$ in two different spherical-polar-coordinate systems in the c.m. frame; as such, they are related by the addition theorem for Legendre polynomials, namely,

$$P_L(\mu_\Lambda)=P_L(\cos\alpha)P_L(\mu)+2\sum_{m=1}^{L}\frac{(L-m)!}{(L+m)!}P_L{}^m(\mu)\times P_L{}^m(\cos\alpha)\cos[m(\tfrac{1}{2}\pi-\phi)]\,, \tag{18}$$

where α is defined in the c.m. frame by $\cos\alpha \equiv \hat{\Lambda}\cdot\hat{z}$, and ϕ is the azimuth of the unit vector $\hat{\Lambda}$ in the coordinate system of Fig. 3 in both the Σ^0 and c.m. frames. Note that α is a function only of γ, the polar angle of the Λ direction in the Σ^0 frame, for a given Σ^0 speed in the c.m. system. (The dependence is given explicitly in the Appendix.) The factor $(\frac{1}{2}\pi-\phi)$ in the azimuth argument of Eq. (18) is the azimuth of the pion, $\frac{1}{2}\pi$, minus the azimuth of the lambda, ϕ, in that spherical-polar-coordinate system in the c.m. frame defined by the conditions that the polar axis be $\hat{z}=\hat{\Sigma}$ and that the vector $\hat{\pi}$ be in the y-z plane with positive y component. Definitions of angles and coordinate systems are discussed in the Appendix.

Now let us define coefficients A_L', in terms of which the lambda angular distribution can be expressed in a form analogous to Eq. (12). By analogy with Eq. (15) we define

$$A_L' \equiv (2L+1)\int P_L(\mu_\Lambda)dN_\Lambda, \tag{19}$$

where the differential distribution dN_Λ of Λ's arising from Σ^0's produced at μ in $d\mu$ and decaying into solid angle $d\Omega_\Lambda$ is given by

$$dN_\Lambda = N(\mu)d\mu\frac{d\Omega_\Lambda}{4\pi} = N(\mu)d\mu\frac{d\phi}{2\pi}\frac{d(\cos\gamma)}{2}. \tag{20}$$

The distribution $N(\mu)d\mu$ is given by Eq. (12), and the factor $d\Omega_\Lambda/4\pi$ expresses the spherical symmetry of the decay $\Sigma^0 \to \Lambda+\gamma$. Using the results from single K's and double vees, we assume that no polynomials higher than $P_2(\mu)$ are needed. Then upon inserting Eqs. (20) and (18) into Eq. (19) and integrating, we note that all terms in the sum over m have a factor $\cos[m(\frac{1}{2}\pi-\phi)]$ which integrates to zero in the ϕ integration. Thus the only remaining term is

$$A_L' = (2L+1)\int P_L(\cos\alpha)P_L(\mu)dN_\Lambda. \tag{21}$$

Now inserting Eqs. (20) and (12) into Eq. (21) and using the orthogonality of the Legendre polynomials, we obtain

$$A_L' = A_L I_L, \tag{22}$$

where

$$I_L = \int P_L(\cos\alpha)\frac{d(\cos\gamma)}{2}. \tag{23}$$

Note that the I_L are purely kinematic integrals depending only on constants of nature and the incident beam momentum. Kinematic and geometric relations used in the calculation of the I_L are given in the Appendix.

Then by defining "experimental" values of the A_L' [in strict analogy to Eqs. (16)] by

$$A_0' = \sum b_i P_0(\mu_{\Lambda i}) = N, \tag{24a}$$

$$A_1' = 3\sum b_i P_1(\mu_{\Lambda i}), \tag{24b}$$

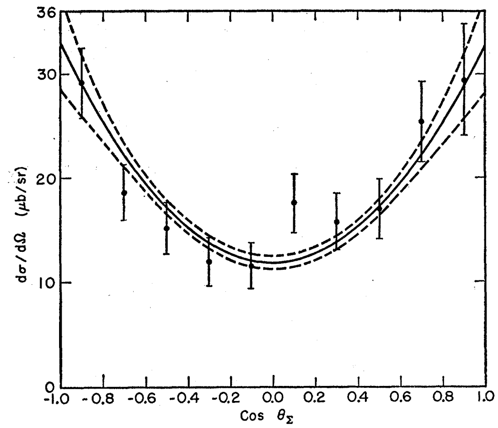

FIG. 4. Angular distribution for Σ^0 production. The data points shown are obtained from the single K's and double vees only. The smooth curve and its error band are obtained from the best-fit parameters, using all of the data including single Λ's.

and

$$A_2' = 5\sum b_i P_2(\mu_{\Lambda i}), \tag{24c}$$

we can calculate experimental values of the desired quantities A_L from the calculated quantities I_L and the measured A_L', using Eq. (22).

For an incident-pion momentum of 1170 MeV/c, we find $I_0=1$, $I_1=0.961$, and $I_2=0.884$. Note that in the limit of zero gamma-ray momentum, or of infinite Σ^0 momentum in the c.m. system, the direction of the Λ in the c.m. system would be that of the Σ^0, whence $\mu_\Lambda \equiv \mu$. In that case we would have $\alpha\equiv 0$ and then $I_L\equiv 1$ for all L, so that $A_L' \equiv A_L$, as expected.

We now redefine coefficients A_L so that they correspond to the absolute differential cross section (13), normalized to the total cross section. The single Λ's alone then give

$$A_0 = 19.68 \pm 0.75\ \mu\text{b/sr}, \tag{25a}$$

$$A_1 = -2.14 \pm 1.54\ \mu\text{b/sr}, \tag{25b}$$

and

$$A_2 = 15.66 \pm 2.07\ \mu\text{b/sr}. \tag{25c}$$

Within the statistical errors, the results (25) obtained for single Λ's agree with the results obtained in Eq. (14) for double vees and single K's. We combine (14) and (25) by least-squares analysis to obtain our final result for the entire sample for the differential cross section, Eq. (13):

$$A_0 = 19.68 \pm 0.59\ \mu\text{b/sr}, \tag{26a}$$

$$A_1 = -0.04 \pm 1.19\ \mu\text{b/sr}, \tag{26b}$$

and

$$A_2 = 14.39 \pm 1.58\ \mu\text{b/sr}, \tag{26c}$$

with off-diagonal errors given by

$$\delta A_0 \delta A_1 = +0.056(\mu\text{b/sr})^2, \tag{27a}$$

$$\delta A_0 \delta A_2 = +0.298(\mu\text{b/sr})^2, \tag{27b}$$

and

$$\delta A_1 \delta A_2 = +0.017(\mu\text{b/sr})^2. \tag{27c}$$

The angular-distribution data from single K's and double vees and the smooth curve corresponding to the complete sample are shown in Fig. 4.

V. ANALYSIS OF Σ^0 POLARIZATION

We consider a sample of Σ^0's with polarization expectation value $\mathbf{P}_\Sigma$ given by

$$\mathbf{P}_\Sigma = P_\Sigma \hat{n}, \tag{28}$$

where P_Σ must lie between -1 and $+1$, and $\hat{n}$ is a unit vector. Because parity is conserved in the strong production process $\pi^- + p \to \Sigma^0 + K^0$, $\hat{n}$ must be perpendicular to the production plane. We can thus take

$$\hat{n} = (\hat{\pi}\times\hat{\Sigma})/|\hat{\pi}\times\hat{\Sigma}| \tag{29}$$

as shown in Fig. 3.

We now derive an expression for $\mathbf{P}_\Lambda$, the polarization expectation value of the daughter Λ from the decay $\Sigma^0 \to \Lambda + \gamma$. The component of $\mathbf{P}_\Lambda$ along the line of flight of the gamma ray must be opposite (and equal in magnitude) to that component of $\mathbf{P}_\Sigma$. This is because in any eigenstate of the system the gamma ray must have an angular-momentum eigenvalue of ± 1 along its line of flight. Thus, for this component of $\mathbf{J}(\gamma) = \mathbf{J}(\Sigma^0) - \mathbf{J}(\Lambda)$ the eigenvalue combinations $\pm 1 = (\pm\frac{1}{2}) - (\mp\frac{1}{2})$ are allowed, whereas the combinations $(\pm\frac{1}{2}) - (\pm\frac{1}{2}) = 0$ are forbidden. Since for each allowed eigenstate the components of $\mathbf{J}(\Lambda)$ and $\mathbf{J}(\Sigma^0)$ along the gamma-ray line of flight are opposite, this also holds for the expectation values $\mathbf{P}_\Lambda$ and $\mathbf{P}_\Sigma$. As for the components of $\mathbf{P}_\Lambda$ perpendicular to the gamma-ray line of flight, they are proportional to the expectation values of the gamma ray's electric and magnetic fields. These components can be shown to average to zero in our experiment. Finally we can write[4]

$$\mathbf{P}_\Lambda = -(\mathbf{P}_\Sigma \cdot \hat{q})\hat{q}, \tag{30}$$

where $\hat{q}$ is a unit vector along (or opposite to) the direction of the gamma ray in the Σ^0 frame. We choose $\hat{q}$ opposite to the gamma-ray direction, along the direction of the Λ, as shown in Fig. 3. Combining Eqs. (28) and (30) we have

$$\mathbf{P}_\Lambda = -P_\Sigma(\hat{n}\cdot\hat{q})\hat{q}, \tag{31}$$

where P_Σ is a function of $\cos\theta_\Sigma \equiv \mu$. The relevant Lorentz frames are more thoroughly discussed in the Appendix.

Consider any collection of N lambdas, with polarization expectation value $\mathbf{P}_\Lambda$, decaying via $\Lambda \to p + \pi^-$. Let unit vector $\hat{k}$ be the direction of the decay proton in the Λ frame, and let $d\Omega_k$ be the corresponding element of solid angle. Then the angular distribution of decays is given by[5]

$$dN = (N/4\pi)[1 + \alpha_\Lambda \mathbf{P}_\Lambda \cdot \hat{k}]d\Omega_k, \tag{32}$$

where

$$\alpha_\Lambda = +0.66 \tag{33}$$

is the decay-asymmetry parameter.[6] Inserting Eq. (31) into Eq. (32) we obtain

$$dN = (N/4\pi)[1 - \alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})]d\Omega_k. \tag{34}$$

In our applications of Eq. (34) we use spherical polar coordinates to describe $\hat{k}$. The polar axis is along an as-yet-unspecified direction $\hat{a}$. We integrate Eq. (34) over the azimuth of $\hat{k}$ about $\hat{a}$. (We want to choose $\hat{a}$ so that this integration does not discard useful decay-asymmetry information.) Then Eq. (34) becomes

$$dN = \tfrac{1}{2}N[1 - \alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{a})(\hat{a}\cdot\hat{k})]d(\hat{a}\cdot\hat{k}). \tag{35}$$

From Eq. (35) it is apparent that the choice $\hat{a}=\hat{q}$ preserves all of the decay asymmetry. Any other choice reduces useful information by the factor $(\hat{a}\cdot\hat{q})$. For the double vees, $\hat{q}$ is known, and we naturally choose $\hat{a}=\hat{q}$. We also use the single-Λ events, where $\hat{q}$ and $\hat{n}$ are not known; in that case we cannot choose $\hat{a}=\hat{q}$. It then turns out that the choice $\hat{a}=\hat{n}_\Lambda$, with $\hat{n}_\Lambda \equiv (\hat{\pi}\times\hat{\Lambda})/|\hat{\pi}\times\hat{\Lambda}|$, yields the maximum decay asymmetry information. (Single Λ's are discussed in Sec. B below.)

A. Double-Vee Events

We choose $\hat{a}=\hat{q}$. Then Eq. (35) becomes

$$dN = \tfrac{1}{2}N[1 - \alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})]d(\hat{q}\cdot\hat{k}). \tag{36}$$

We now specify the sample N in more detail. We fix the Σ^0-production angle θ_Σ. Then $P_\Sigma(\mu)$ is fixed. Because of the spherical symmetry of the Σ^0 decay, events will be distributed uniformly in $\hat{n}\cdot\hat{q}$ between -1 and $+1$. (We verify this, within statistics, in Figs. 1 and 2.) Thus Eq. (36) may be written

$$d^2N = \tfrac{1}{4}N[1 - \alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})]d(\hat{q}\cdot\hat{k})d(\hat{n}\cdot\hat{q}), \tag{37}$$

where P_Σ is a function of μ, and where N is the number of Σ^0 productions at μ, within some interval in μ.

We now use the method of moments to find $P_\Sigma(\mu)$. We multiply Eq. (37) by the projector $(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})$ and integrate over $\hat{n}\cdot\hat{q}$ and $\hat{q}\cdot\hat{k}$. Each integral of the first term is zero, and each integral contributes a factor $\frac{2}{3}$ to the second term. Thus we have

$$\int\int (\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})d^2N = -N\alpha_\Lambda P_\Sigma/9. \tag{38}$$

In the method of moments we replace the double integral on the left side with a sum over weighted counts. Thus we have

$$-N\alpha_\Lambda P_\Sigma = 9\sum b_i(\hat{n}\cdot\hat{q})_i(\hat{q}\cdot\hat{k})_i. \tag{39}$$

The standard deviation is given by

$$\delta(N\alpha_\Lambda P_\Sigma) = 9[\sum b_i^2(\hat{n}\cdot\hat{q})_i^2(\hat{q}\cdot\hat{k})_i^2]^{1/2}. \tag{40}$$

[4] R. Gatto, Phy. Rev. **109**, 610 (1958); N. Byers and H. Burkhardt, *ibid.* **121**, 281 (1961); G. Snow and J. Sucher, Nuovo Cimento **18**, 195 (1960); R. H. Dalitz, *Strong Interaction Physics and Strange Particles* (Oxford University Press, London, 1962), Chap. XII.

[5] T. D. Lee, J. Steinberger, G. Feinberg, P. K. Kabir, and C. N. Yang, Phys. Rev. **106**, 1367 (1957).

[6] Equation (33) is obtained by averaging the results of J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963) with other results compiled in Ref. 3. The positive sign for α_Λ corresponds to the conventions we use in Eq. (32). Physically, it corresponds to the fact that the decay protons prefer positive helicity.

We use Eqs. (39) and (40), renormalized to cross section instead of counts, to calculate the experimental points in the histogram shown in Fig. 5.[7]

From the results of our study of the angular distribution, we believe that we need consider only S- and P-wave production of Σ^0's, since we need only coefficients through A_2. We wish to find the appropriate parameters for the Σ^0 polarization times differential cross section. We again use the method of moments. In Eq. (37) we specify N in more detail as the number of Σ^0 productions with a given value of $\cos\theta_\Sigma \equiv \mu$ in $d\mu$. Thus we replace N in Eq. (37) by $N(\mu)d\mu$, where $N(\mu)d\mu$ is the same as the Legendre-polynomial expansion Eq. (12), with coefficients A_L given by Eqs. (26). Similarly we replace NP_Σ in Eq. (37) by an expansion for $N(\mu)P_\Sigma(\mu)d\mu$, with coefficients B_1 and B_2, given by[5]

$$N(\mu)P_\Sigma(\mu)d\mu = \tfrac{1}{2}\sin\theta_\Sigma(B_1+B_2\mu)d\mu. \qquad (41)$$

Thus Eq. (37) becomes

$$\frac{d^3N}{d\mu d(\hat{n}\cdot\hat{q})d(\hat{q}\cdot\hat{k})} = \tfrac{1}{4}N(\mu) - \tfrac{1}{8}\alpha_\Lambda \sin\theta_\Sigma(B_1+B_2\mu)(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k}). \qquad (42)$$

Our object is to determine B_1 and B_2. First we multiply Eq. (42) by the projector $(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})$ and integrate over $\hat{n}\cdot\hat{q}$ and $\hat{q}\cdot\hat{k}$ from -1 to $+1$. As in our derivation of Eq. (39), each integral of the $N(\mu)$ term is zero, and each integral contributes a factor $\frac{2}{3}$ to the α_Λ term. The right side of Eq. (42) becomes $-(1/18)\alpha_\Lambda \sin\theta_\Sigma(B_1+B_2\mu)$. To obtain the B_1 term, we multiply by $\sin\theta_\Sigma$ and integrate over $\cos\theta_\Sigma \equiv \mu$ from -1 to $+1$.[8] The term $B_2\mu$ vanishes in this integration, and the integral of $\sin^2\theta_\Sigma$ gives a factor $\frac{4}{3}$. After these three integrations the right side of Eq. (42) becomes $-(2/27)\alpha_\Lambda B_1$. In the method of moments we interpret the left side of this triple integration as a sum over weighted counts. Thus

$$-\alpha_\Lambda B_1 = (27/2)\sum b_i(\sin\theta_\Sigma)_i(\hat{n}\cdot\hat{q})_i(\hat{q}\cdot\hat{k})_i, \qquad (43)$$

with a standard deviation analogous to Eq. (40).

Similarly, to determine B_2 we multiply Eq. (42) by the projector $\mu\sin\theta_\Sigma(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})$ and integrate over μ, $\hat{n}\cdot\hat{q}$, and $\hat{q}\cdot\hat{k}$. The $N(\mu)$ term again integrates to zero, the μ integral gives a factor 4/15, and the $\hat{n}\cdot\hat{q}$ and $\hat{q}\cdot\hat{k}$ integrals each give $\frac{2}{3}$. Thus

$$-\alpha_\Lambda B_2 = (135/2)\sum b_i\mu_i(\sin\theta_\Sigma)_i(\hat{n}\cdot\hat{q})_i(\hat{q}\cdot\hat{k})_i. \qquad (44)$$

[7] If the weight factors b_i were all equal or were independent of production and decay angles, we could simplify Eq. (40). Suppose all b_i were equal to unity. Then Eq. (40) would become $\delta(N\alpha_\Lambda P_\Sigma) = 9N^{1/2}\langle(\hat{n}\cdot\hat{q})^2(\hat{q}\cdot\hat{k})^2\rangle^{1/2}$, where $N=\Sigma b_i$, and $\langle\ \rangle$ denotes an average over the distribution, Eq. (37). Then we would have $\langle(\hat{n}\cdot\hat{q})^2(\hat{q}\cdot\hat{k})^2\rangle = \langle(\hat{n}\cdot\hat{q})^2\rangle\langle(\hat{q}\cdot\hat{k})^2\rangle = (\frac{1}{3})(\frac{1}{3})$. Dividing by N, we would have $-\alpha_\Lambda P_\Sigma = (9/N)\Sigma(\hat{n}\cdot\hat{q})_i(\hat{q}\cdot\hat{k})_i \pm (9/N)^{1/2}$. We may note that if instead of $\hat{a}=\hat{q}$ we had chosen $\hat{a}=\hat{n}$ in Eq. (35), we would have, instead of Eq. (37), $d^2N = \frac{1}{4}N[1-\alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{q})^2(\hat{n}\cdot\hat{k})]d(\hat{n}\cdot\hat{k})d(\hat{n}\cdot\hat{q})$. After integration over $\hat{n}\cdot\hat{q}$ this becomes $dN = \frac{1}{2}N[1-\frac{1}{3}\alpha_\Lambda P_\Sigma(\hat{n}\cdot\hat{k})] \times d(\hat{n}\cdot\hat{k})$. The method of moments would then give $-\alpha_\Lambda P_\Sigma = (9/N)\Sigma(\hat{n}\cdot\hat{k})_i \pm (27/N)^{1/2}$. We see that the choice $\hat{a}=\hat{n}$ is equivalent to discarding $\frac{2}{3}$ of the counts.

[8] The weighting factor $\sin\theta_\Sigma$ is not necessary. It serves to reduce the expected error by an amount that depends on the angular distribution. In our experiment it reduces the calculated error by about 10%.

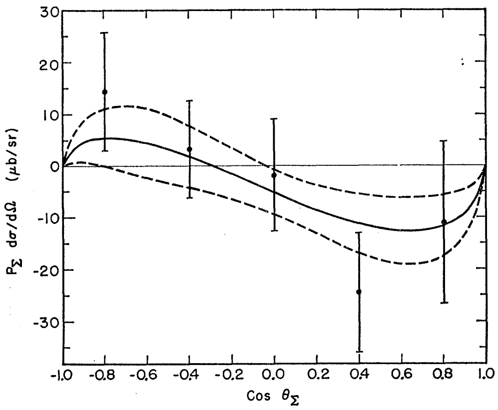

FIG. 5. Polarization times differential cross section for Σ^0 production. The data points shown are obtained using Eqs. (39) and (40) with double vees alone. The smooth curve with the error band is obtained from the best-fit parameters, using all of the data including single Λ's.

Finally we redefine B_1 and B_2 in terms of polarization times absolute differential cross section (instead of times counts), and also set $\alpha_\Lambda = +0.66$. Then B_1 and B_2 are defined by

$$P_\Sigma(\mu)\frac{d\sigma}{d\Omega} = \tfrac{1}{2}\sin\theta_\Sigma(B_1+B_2\mu). \qquad (45)$$

Our results for the double vees are

$$B_1 = -12.77 \pm 12.42\ \mu\text{b/sr} \qquad (46a)$$

and

$$B_2 = -67.21 \pm 30.41\ \mu\text{b/sr}. \qquad (46b)$$

with a correlation error

$$\delta B_1 \delta B_2 = +76.28(\mu\text{b/sr})^2. \qquad (46c)$$

B. Single-Λ Events

For a single-Λ event, $\hat{q}$ is unknown. Therefore, $\hat{q}$ cannot be chosen as the polar axis $\hat{a}$ along which to measure the Λ-decay asymmetry. The Σ^0 direction $\hat{\Sigma}$ is also unknown. Therefore, we cannot choose $\hat{a}$ to be $\hat{n} \equiv (\hat{\pi}\times\hat{\Sigma})/|\hat{\pi}\times\hat{\Sigma}|$. The only observable directions (aside from $\hat{k}$, the direction of the decay proton) are $\hat{\pi}$, $\hat{\Lambda}$, and $\hat{n}_\Lambda$, where we define

$$\hat{n}_\Lambda \equiv (\hat{\pi}\times\hat{\Lambda})/|\hat{\pi}\times\hat{\Lambda}|. \qquad (47)$$

The asymmetry term in the distribution function Eq. (35) is proportional to $(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{a})(\hat{a}\cdot\hat{k})$. Since $\hat{n}$ and $\hat{q}$ are unknown, it might seem that the single Λ's are useless. However, from Fig. 3 we see that

$$\hat{n}\cdot\hat{q} = \sin\gamma\cos\phi, \qquad (48)$$

where $\cos\gamma \equiv \hat{q}\cdot\hat{\Sigma}$ and $\cos\phi \equiv (\hat{q}\cdot\hat{x})\sin\gamma$. In the Appendix we show that, although ϕ is indeed completely unknown, γ is known for each event. If we can choose $\hat{a}$ so that $\hat{q}\cdot\hat{a}$ contains the factor $\cos\phi$, then $(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{a})$ contains $\cos^2\phi$. In dealing with the complete sample we can then replace $\cos^2\phi$ by its average value of $\frac{1}{2}$, which follows from the spherical symmetry of the Σ^0 decay. The

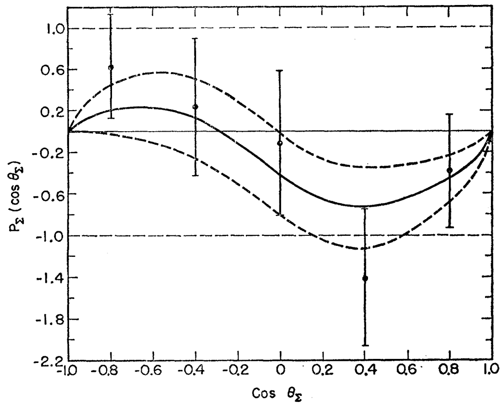

FIG. 6. Polarization of Σ^0's. The smooth curve and error band are obtained from the best-fit parameters, i.e., by dividing the values from the smooth curve of Fig. 5 by those from Fig. 4. The data points shown are obtained from the double vees alone, using Eqs. (39) and (40) for each histogram interval, and dividing by the number of weighted events in the interval.

choices $\hat{a}=\hat{\pi}$, or $\hat{a}=\hat{\Lambda}$, or any linear combination of them, do not introduce a factor $\cos\phi$, and therefore the decay asymmetry along these directions averages to zero in the integration over the unobserved angle ϕ. The choice $\hat{a}=\hat{n}_\Lambda$ does yield the desired factor $\cos\phi$ and is the only possible successful choice. In the Appendix we show

$$\hat{q}\cdot\hat{n}_\Lambda=\sin\theta_\Sigma \sin\beta \cos\phi/\sin\theta_\Lambda, \tag{49}$$

where β is defined by $\cos\beta\equiv\hat{\Lambda}\cdot\hat{q}$ in the Λ frame. Note that β depends on γ alone for a given beam momentum. The dependence is discussed in the Appendix. Inserting Eq. (48), Eq. (49), and $\hat{a}=\hat{n}_\Lambda$ into Eq. (35) we obtain

$$dN/d(\hat{n}_\Lambda\cdot\hat{k})=\tfrac{1}{2}N[1-\alpha_\Lambda P_\Sigma(\sin\theta_\Sigma/\sin\theta_\Lambda) \times(\sin\gamma \sin\beta)\cos^2\phi(\hat{n}_\Lambda\cdot\hat{k})]. \tag{50}$$

We now specify N in more detail. Because of the spherical symmetry of the Σ^0 decay, the probability for a decay with given ϕ in $d\phi$ and $\cos\gamma$ in $d(\cos\gamma)$ is $1/(4\pi)d\phi d(\cos\gamma)$. Introducing this distribution into Eq. (50), we integrate ϕ from 0 to 2π and replace $\cos^2\phi$ by its average value of $\frac{1}{2}$. We introduce $N(\mu)$ and $P_\Sigma(\mu)$ as previously defined in Eqs. (12) and (41). Then Eq. (50) becomes

$$\frac{d^3N}{d(\mu)d(\cos\gamma)d(\hat{n}_\Lambda\cdot\hat{k})}=\tfrac{1}{4}N(\mu)-\tfrac{1}{16}\alpha_\Lambda(\sin^2\theta_\Sigma/\sin\theta_\Lambda) \times(B_1+B_2\mu)\sin\gamma\sin\beta(\hat{n}_\Lambda\cdot\hat{k}). \tag{51}$$

Our object is to determine B_1 and B_2. To determine B_1 we use a projector g_1, defined in terms of quantities measurable for each event:

$$g_1\equiv\sin\gamma \sin\beta \sin\theta_\Lambda(\hat{n}_\Lambda\cdot\hat{k}). \tag{52}$$

We multiply Eq. (51) by g_1 and integrate over μ, $\cos\gamma$, and $\hat{n}_\Lambda\cdot\hat{k}$, all from -1 to $+1$. In the $\hat{n}_\Lambda\cdot\hat{k}$ integration, the $N(\mu)$ term vanishes and the asymmetry term acquires a factor $\frac{2}{3}$. The B_2 term vanishes in the μ integration, and the B_1 term acquires a factor $\frac{4}{3}$. The $\cos\gamma$ integration gives a factor

$$I_1\equiv\frac{1}{2}\int_{-1}^{1}\sin^2\gamma\sin^2\beta d(\cos\gamma)=0.4986 \tag{53}$$

at 1170 MeV/c.[9] Thus the right side of Eq. (51) integrates to $-(1/9)I_1\alpha_\Lambda B_1$. In the method of moments we interpret the integral of the left side as a sum over weighted counts. Thus we have

$$-\alpha_\Lambda B_1=(9/I_1)\sum b_i(g_1)_i. \tag{54}$$

In order to determine B_2 we multiply Eq. (51) by the projector

$$g_2\equiv\cos\theta_\Lambda g_1 \tag{55}$$

and integrate over μ, $\hat{n}_\Lambda\cdot\hat{k}$, and $\cos\gamma$. In the Appendix we show that

$$\cos\theta_\Lambda=\mu\cos\alpha+\sin\alpha\sin\theta_\Lambda\sin\phi. \tag{56}$$

In the integration over ϕ the $\sin\phi$ term in Eq. (56) integrates to zero. The remaining term, $\mu\cos\alpha$, contains the known factor $\cos\alpha$ and the factor μ, which projects out the $B_2\mu$ term in the μ integration. After the triple integration the right side of Eq. (51) thus becomes $-(I_2/45)\alpha_\Lambda B_2$, where

$$I_2\equiv\frac{1}{2}\int_{-1}^{1}\cos\alpha\sin^2\gamma\sin^2\beta d(\cos\gamma)=0.4735 \tag{57}$$

at 1170 MeV/c.[10] Thus we obtain

$$-\alpha_\Lambda B_2=(45/I_2)\sum b_i(g_2)_i. \tag{58}$$

Finally we redefine B_1 and B_2 as in Eq. (45), and set $\alpha_\Lambda=+0.66$. Then our results from the single Λ's are

$$B_1=-10.75\pm11.34\ \mu\text{b/sr} \tag{59a}$$

and

$$B_2=-0.42\pm32.20,\ \mu\text{b/sr}, \tag{59b}$$

with correlation error

$$\delta B_1\delta B_2=-27.16(\mu\text{b/sr})^2. \tag{59c}$$

The results (59) obtained from single Λ's agree within their quoted errors with the results (46) from double vees. We combine (46) and (59) by least-squares analysis to obtain

$$B_1=-9.98\pm8.29\ \mu\text{b/sr} \tag{60a}$$

and

$$B_2=-35.45\pm21.88\ \mu\text{b/sr}, \tag{60b}$$

with correlation error

$$\delta B_1\delta B_2=+11.57(\mu\text{b/sr})^2. \tag{60c}$$

In Fig. 5 we show our polarization results. The smooth curves correspond to Eq. (45), with coefficients given by

[9] We may note that in the limit of zero gamma-ray energy, or of infinite Σ^0 c.m. momentum, we have $\beta=\gamma$ (see Fig. 3). In that limit the integral (53) is $(8/15)=0.5333$.

[10] In the limit of infinite Σ^0 momentum we have $\alpha=0$, and then $I_2=I_1=8/15$.

(60). The data points shown are taken from the double vees alone and are obtained in each histogram interval by using Eqs. (39) and (40). The smooth curve with error band shown in Fig. 6 is obtained by dividing the curve for $P_\Sigma(\mu)d\sigma/d\Omega$ by that for $d\sigma/d\Omega$. The data points are obtained as in Fig. 5, dividing by the number of weighted events in each histogram interval.

APPENDIX: GEOMETRICAL QUANTITIES

Figure 3 is necessarily schematic. We emphasize that angles whose cosines are defined as the dot product of two unit vectors are each defined in only a single Lorentz frame. The unit vectors $\hat{\pi}$, $\hat{\Lambda}$, $\hat{\Sigma}$, and $\hat{q}$ are each defined as the vector momentum of the particle in the appropriate Lorentz frame divided by the magnitude of that vector momentum. A given unit vector can be defined in any frames connected by Lorentz transformations along its direction. For example, the unit vector $\hat{\Sigma}$ is defined in the c.m. Σ^0, and K^0 frames. Thus $\hat{\Sigma}$ is along the Σ momentum, $\mathbf{p}_\Sigma$(c.m.) or $\mathbf{p}_\Sigma(K^0)$, or opposite to the K momentum, along $-\mathbf{p}_K$(c.m.) or $-\mathbf{p}_K(\Sigma^0)$, where the relevant Lorentz frame is indicated in parentheses.

The angle γ is accordingly defined in the Σ^0 frame by $\cos\gamma \equiv \hat{\Sigma}\cdot\hat{q}$. The angle α is defined in the c.m. frame by $\cos\alpha \equiv \hat{\Sigma}\cdot\hat{\Lambda}$. The angle β is defined in the Λ frame by $\cos\beta \equiv \hat{\Lambda}\cdot\hat{q}$.

Directions $\hat{x}$ and $\hat{y}$ are defined in both the c.m. and Σ^0 frames, and the azimuth ϕ (Fig. 3) has the same value in both frames. This is seen as follows. We define $\hat{z}$ (c.m.)$\equiv\hat{\Sigma}$(c.m.). We define $\hat{y}$ (c.m.) to be along the component of incident pion c.m. momentum that is perpendicular to $\hat{z}$. We then define $\hat{x}$(c.m.)$\equiv\hat{y}\times\hat{z}$. When we transform from the c.m. to the Σ^0 frame, the incident-pion momentum component perpendicular to $\hat{\Sigma}$ is invariant and is used to *define* the $\hat{y}$ direction in the Σ^0 frame. Thus $\hat{y}$ is the same physical direction in each frame, namely, the direction of the incident-pion vector momentum component perpendicular to $\hat{\Sigma}$. The $\hat{x}$ direction is then given in the Σ^0 frame by $\hat{y}\times\hat{z}$. The azimuth ϕ gives the angle between $\hat{x}$ and the component of Λ momentum in the c.m. that is perpendicular to $\hat{\Sigma}$. This component is also Lorentz-invariant under transformation from the c.m. to the Σ^0 frame, so that ϕ is also invariant.

We now discuss the geometrical quantities used in the determination of the angular distribution by means of single Λ's. By Lorentz transformation of the Λ four-momentum from the Σ^0 frame to the c.m. frame, we have

$$p_\Lambda(\text{c.m.})\cos\alpha = \gamma_0 p_\Lambda(\Sigma)\cos\gamma + \eta_0 E_\Lambda(\Sigma), \quad \text{(A1)}$$

and

$$E_\Lambda(\text{c.m.}) = \gamma_0 E_\Lambda(\Sigma) + \eta_0 p_\Lambda(\Sigma)\cos\gamma, \quad \text{(A2)}$$

with $\gamma_0=(1-\beta_0^2)^{-1/2}$ and $\eta_0=\gamma_0\beta_0$, where $\beta_0=0.19116$ is the velocity of the Σ^0 in the c.m. system for incident pion momentum of 1170 MeV/c; $E_\Lambda(\Sigma)$ and $p_\Lambda(\Sigma)$ are the constants of nature 1117.9 and 75.1 MeV, corresponding to $v_\Lambda(\Sigma)=p_\Lambda(\Sigma)/E_\Lambda(\Sigma)=0.0672$. Also

$$p_\Lambda(\text{c.m.}) = [E_\Lambda(\text{c.m.})^2 - m_\Lambda^2]^{1/2}, \quad \text{(A3)}$$

where m_Λ is the mass of the Λ.

After insertion of Eqs. (A2) and (A3) into Eq. (A1), $\cos\alpha$ is written explicitly as a function only of $\cos\gamma$, constants of nature, and quantities depending only on beam momentum. Then this expression is used in evaluating the integrals I_L, Eq. (23). Integrals I_1 and I_2 were evaluated on an IBM-7094 computer. The results are $I_0=1$ (by inspection), $I_1=0.961$, and $I_2=0.884$.

Next we discuss geometrical quantities needed for the polarization determinations. In Eq. (28), $\mathbf{P}_\Sigma$ is defined in the Σ^0 frame, hence $\hat{n}$ means $\hat{n}(\Sigma)$. In Eq. (31), $\hat{q}$ occurs twice, once as defined in the Σ^0 frame, once in the Λ frame. (The Λ polarization $\mathbf{P}_\Lambda$ is defined in the Λ frame.) Thus Eq. (31) means

$$\mathbf{P}_\Lambda(\Lambda) = -P_\Sigma[\hat{n}(\Sigma)\cdot\hat{q}(\Sigma)]\hat{q}(\Lambda). \quad \text{(A4)}$$

In Eq. (34) the product $(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k})$ means

$$(\hat{n}\cdot\hat{q})(\hat{q}\cdot\hat{k}) = [\hat{n}(\Sigma)\cdot\hat{q}(\Sigma)][\hat{q}(\Lambda)\cdot\hat{k}(\Lambda)]. \quad \text{(A5)}$$

The first factor in Eq. (A5) is given by

$$\hat{n}(\Sigma)\cdot\hat{q}(\Sigma) = \sin\gamma\cos\phi, \quad \text{(A6)}$$

where γ and ϕ are defined above. Note that both factors in Eq. (A5) are defined according to the discussion in the first paragraph of this Appendix.

For single-Λ events $\hat{\Lambda}$ is known for each event, but $\hat{\Sigma}$ is not, so that ϕ is unknown. However the polar angle γ *is* known. It depends only on p_Λ (c.m.), which is measurable for each event, and β_0, which is given above. This is seen by combining the longitudinal component of the Lorentz transformation above, Eq. (A1), with the transverse component, namely,

$$p_\Lambda(\text{c.m.})\sin\alpha = p_\Lambda(\Sigma)\sin\gamma, \quad \text{(A7)}$$

and eliminating the angle α. Then $\cos\gamma$ is given in terms of quantities known or measurable for each event. Thus $\cos\alpha$ also is measurable for each event.

Next we derive Eq. (49), which means (in accordance with the discussion above)

$$\hat{q}(\Lambda)\cdot\hat{n}_\Lambda(\Lambda) = \sin\theta_\Sigma\sin\beta\cos\phi/\sin\theta_\Lambda. \quad \text{(A8)}$$

The vector $\hat{q}$ is defined in the Λ frame as the negative of the direction of the Σ:

$$\hat{q}(\Lambda) = -\mathbf{p}_\Sigma(\Lambda)/p_\Sigma(\Lambda). \quad \text{(A9)}$$

Thus we have

$$\hat{q}(\Lambda)\cdot\hat{n}_\Lambda(\Lambda) = -\mathbf{p}_\Sigma(\Lambda)\cdot\hat{n}_\Lambda(\Lambda)/p_\Sigma(\Lambda). \quad \text{(A10)}$$

We now transform from the Λ frame to the c.m. frame. The direction $\hat{n}_\Lambda$ is perpendicular to the relative velocity of the Λ and c.m. frames. Therefore from Lorentz invariance of the transverse momentum components we have

$$-\mathbf{p}_\Sigma(\Lambda)\cdot\hat{n}_\Lambda(\Lambda) = -\mathbf{p}_\Sigma(\text{c.m.})\cdot\hat{n}_\Lambda(\text{c.m.}). \quad \text{(A11)}$$

But we have

$$\hat{n}_\Lambda(\text{c.m.}) \equiv (\hat{\pi}\times\hat{\Lambda})/|\hat{\pi}\times\hat{\Lambda}| = (\hat{\pi}\times\hat{\Lambda})/\sin\theta_\Lambda \quad \text{(A12)}$$

and

$$\mathbf{p}_\Sigma(\text{c.m.}) = p_\Sigma(\text{c.m.})\hat{z}. \quad \text{(A13)}$$

We also have (see Fig. 3)

$$\hat{\pi} = \hat{z}\cos\theta_\Sigma + \hat{y}\sin\theta_\Sigma, \quad \text{(A14)}$$

and

$$\hat{\Lambda} = \hat{z}\cos\alpha + \sin\alpha(\hat{x}\cos\phi + \hat{y}\sin\phi), \quad \text{(A15)}$$

so that

$$\hat{z}\cdot(\hat{\pi}\times\hat{\Lambda}) = -\sin\theta_\Sigma \sin\alpha\cos\phi. \quad \text{(A16)}$$

Combining Eqs. (A10) through (A16) we obtain

$$\hat{q}\cdot\hat{n}_\Lambda = [p_\Sigma(\text{c.m.})\sin\alpha/p_\Sigma(\Lambda)]\cos\phi\sin\theta_\Sigma/\sin\theta_\Lambda. \quad \text{(A17)}$$

The expression in square brackets involves quantities known or measurable for each event. It is equal to $\sin\beta$, as follows from the Lorentz invariance of the transverse momentum components of $\mathbf{p}_\Sigma$ in the transformation from the c.m. to Λ frame:

$$p_\Sigma(\text{c.m.})\sin\alpha = p_\Sigma(\Lambda)\sin\beta. \quad \text{(A18)}$$

Combining Eqs. (A17) and (A18) we obtain Eq. (A8), i.e., Eq. (49). Also we see from Eq. (A18) that the angle β also depends (through α) only on the angle γ, constants of nature, and the beam momentum, and is consequently measurable for each event. (Note that the Euclidean relation that seems implied in Fig. 3, namely $\beta=\gamma-\alpha$, is not valid due to its non-Lorentz-invariant nature.)

Lastly, we derive Eq. (56). By definition $\cos\theta_\Lambda = \hat{\pi}(\text{c.m.})\cdot\hat{\Lambda}(\text{c.m.})$. Then Eqs. (A14) and (A15) give

$$\hat{\pi}\cdot\hat{\Lambda} = \cos\alpha\cos\theta_\Sigma + \sin\alpha\sin\theta_\Sigma\sin\phi, \quad \text{(A19)}$$

which is Eq. (56).

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Reactions $K^-p \to$ Hyperon+Meson at 3.5 GeV/c

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A study has been made of some of the quasi-two-body final states (in which one of the particles is a hyperon) produced by 3.5-GeV/c K^- mesons on protons. The analysis has been performed with 310 000 photographs taken in the 81-cm Saclay hydrogen bubble chamber. The cross sections for most of the reactions are lower than have been observed at lower incident momenta. Many of the reactions are characterized by a forward peaking of the production angular distribution of the final-state meson, but in a few cases a significant backward peak has been observed. Decay distributions of unstable particles have been investigated to obtain more information about the production processes. The $Y_1^{*+}(1385)$ decay is consistent with the $Y^*\pi^-$ final state being produced by K^* exchange, but in the case of the production of vector mesons, it is difficult to draw any conclusion concerning the spin of the exchanged particle. An enhancement was observed at 1645 MeV in the $\Sigma^\pm\pi^\mp$ system. It is difficult to interpret this in terms of the decay of the neutral $Y_1^*(1660)$.

1. INTRODUCTION

DESPITE intensive theoretical and experimental studies, the subject of the production mechanism of elementary-particle reactions at medium and high energies is still far from being completely understood. It is clear that experimental data are required for a large number of different reactions over a wide range of incident energies, and with high statistical accuracy. In this article we describe some of the features of the interactions of 3.5-GeV/c K^- mesons with protons.

Some results of this experiment have already been published in two articles; one[1] on the discovery of the $K^*(1400)$ and the other[2] on a determination of the parity of the $Y^*(1660)$. In this paper we present results on two-body channels involving a strange baryon. An

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