Quantum Electrodynamics and the Correspondence Principle*

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(Received 1 August 1966)

The application of the Bohr correspondence principle to quantum electrodynamics is considered, taking into account the existence of the scattering of light by light and of intensity-dependent radiative corrections. When the effect of the latter is allowed for, the discrepancy between completely quantum-mechanical and completely classical calculations of Thomson scattering is removed.

I. INTRODUCTION

THE Bohr correspondence principle states that in some appropriate limit the predictions of quantum theory approach those of classical theory. The principle was formulated in connection with nonrelativistic particle mechanics, where it is made necessary by the large realm in which classical mechanics is valid. Classical electromagnetism also has a large realm of validity, so that quantum electrodynamics must contain electrodynymics as a special case. It is our purpose here to discuss the extent to which systems involving high photon densities can be described classically.

II. MAXWELL FIELD WITHOUT SOURCES

Maxwell's equations for the vacuum are linear in the field strengths. This has a consequence that according to classical theory, light does not scatter light *in vacuo*. In material media, if the polarizability is not strictly field-independent, the equations become nonlinear and light can scatter from light, in a manner of speaking. Said more precisely, in the presence of a nonlinear medium, light can be scattered from inhomogeneities produced by another light beam. *In vacuo*, however, a superposition of light beams of arbitrary strength is a solution of Maxwell's equations.

The vacuum situation is quite different in quantum electrodynamics because of the possibility of the creation of real or virtual pairs. No such mechanism exists in Maxwell theory. Virtual pair creation makes the vacuum a polarizable medium, and starting with the fourth order of perturbation theory causes a field-dependent polarizability to appear. There is, thus, the possibility of the scattering of light by light, or by any electromagnetic field. Quantum electrodynamics is inherently a nonlinear theory even in the absence of sources, while classical electrodynamics is nonlinear only in the presence of sources which are themselves influenced by the field, or in material media, which amounts to the same thing from the microscopic point of view.

In nonrelativistic particle quantum mechanics the correspondence principle is usually stated in terms of the limit $\hbar \to 0$. Mathematically this is convenient, but it is physically an artifice, because $\hbar$ has a specific value. A more accurate statement is that in the limit of large quantum numbers, the predictions of quantum mechanics are not distinguishable from those of classical mechanics. As applied to the electromagnetic field, this is a limit as the number of photons or the expectation value of the number of photons present in the system becomes large. The classical limit is, therefore, a high-intensity limit unless a restriction is at the same time placed on the frequencies of photons present.

The center-of-momentum differential scattering cross section for two photons, averaged over polarizations, is[1]

$$\bar{\sigma}(\theta,\omega)=\frac{1}{4\pi^2}\frac{139}{8100}\left(\frac{e^2}{\hbar c}\right)^4\left(\frac{\hbar}{mc}\right)^2\left(\frac{\hbar\omega}{mc^2}\right)^6(3+\cos^2\theta)\,,\quad (1)$$

when $\hbar\omega \ll mc^2$. Given a beam of intensity $I=\rho c\hbar\omega$, where ρ is the photon density, a second beam of the same frequency traveling in the opposite direction for a distance L will be attenuated by scattering from the first beam by the fractional amount

$$\rho\bar{\sigma}_{\mathrm{tot}}L.$$

If this attenuation is to be negligible, so that a classical linear theory is a good approximation in a region of dimension L, we must have

$$\frac{\rho L}{\lambda\!\!\!^{-6}}\left(\frac{e^2}{mc^2}\right)^4\left(\frac{\hbar}{mc}\right)^4 \ll 1\,. \quad (2)$$

In a system of fixed dimensions operating at a fixed wavelength, the high-intensity limit is not the classical limit in the usual sense of Maxwell theory.

There exists a classical theory due to Born and Infeld[2] which is nonlinear and contains, therefore, a vacuum polarizability which is field-dependent. The theory contains a parameter with the dimensions of a field strength, E_0, which they interpreted as the field strength at the "surface of the electron," namely, $e/4\pi r_0^2$. The energy density of the field may be expanded in powers of E_0^{-2}, the leading terms of the expansion

* Research supported in part by U. S. Army Research Office (Durham).

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[1] R. Karplus and M. Neuman, Phys. Rev. 83, 776 (1951).

[2] M. Born and L. Infeld, Proc. Roy. Soc. (London) A144, 425 (1934).

being[3]

$$U=\tfrac{1}{2}(B^2+D^2)-[(1.236)^4/8E_0^2]\times[(B^2-D^2)^2+4(\mathbf{B}\cdot\mathbf{D})^2]. \quad (3)$$

This theory has the property that a plane wave in vacuum, of any spectral composition and any polarization, propagates as in Maxwell theory, but the superposition of two waves with different directions of propagation is not a solution of the Born-Infeld equations.

It has been shown by Euler[3] that when $\hbar\omega\ll mc^2$, when m is the electron mass, the scattering of light by light in quantum theory coincides in lowest nonvanishing order with what would be expected from a classical theory similar to that of Born and Infeld, but with U given by

$$U=\tfrac{1}{2}(B^2+D^2)-[(1.236)^4/8E_0^2]\times[1.7(B^2-D^2)^2+2.9\times4(\mathbf{B}\cdot\mathbf{D})^2]. \quad (4)$$

Thus the Euler-Born-Infeld theory[4] is a classical approximation of quantum electrodynamics which is more accurate than the Maxwell theory in the case of very high intensity fields at a fixed frequency. If the frequency is not held fixed but is decreased as the photon density increases in such a way that (2) is satisfied, then the nonlinear terms in this classical theory remain small. Even this does not establish the validity of Maxwell theory as a limit of quantum electrodynamics because only the lowest order contribution to the light-light scattering has been included. There are higher order amplitudes, such as that shown in Fig. 1, which must be added to the fourth-order diagram and which will bring in interference terms proportional to the square of the photon density. The frequency dependence of such terms is not known, but it is very plausible that the extra factor of ρ is not multiplied by a power of ω as high as the sixth, as in (1). If this is the case, the inequality (2) no longer suffices to eliminate the effect of light-light scattering, and it is not clear that even a low-frequency limit will lead to Maxwell theory at arbitrary intensity. While probably of negligible magnitude in any possible laser or maser, these terms are present in principle and render meaningless the "exactness" of any calculation containing powers of the photon density higher than the first, and based on Maxwell theory.

III. THOMSON AND COMPTON SCATTERING

The scattering of light by free electrons provides a problem which can be discussed in detail both classically and quantum mechanically, and thus can illustrate some new features associated with the correspondence principle.

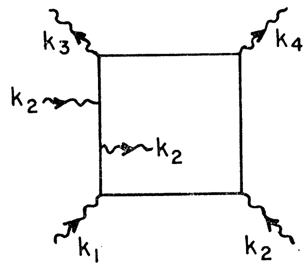

FIG. 1. A contribution to $k_1+k_2\rightarrow k_3+k_4$ which is proportional to $(N/V)^{3/2}$, where N refers to photons k_2.

When the frequency of the light to be scattered is low, the classical calculation proceeds by finding the motion of the electron in the field of the light wave without taking the electron's radiating into account. The moving electron constitutes a classical current whose radiation is then calculated. If the intensity of the light is not too high, the equation of motion of the electron is Lorentz's equation, and for low intensity the force on the electron due to the magnetic field may be neglected. In this latter case the result is the Thomson formula. Phrased as a differential cross section it is, averaged over polarizations,[5]

$$d\sigma_T=r_0^2\tfrac{1}{2}(1+\cos^2\theta)d\Omega. \quad (5)$$

When the effect of the magnetic field is included, there is generation of a second harmonic. This high-intensity effect is small to the extent that

$$eE/mc\omega\ll1. \quad (6)$$

In principle, a measurement of e/m for the electron by a mass spectrometer using static fields and of the Thomson cross section can serve to determine the charge and the mass of the electron by purely classical means.

The calculation of the Compton scattering section as given by the Klein-Nishina formula[5] proceeds in an analogous way by the use of perturbation theory. The scattering of a photon by a free electron first occurs in second-order perturbation theory. Higher order processes also contribute to the matrix element for the scattering, these processes being of two kinds. The first involves the processes called radiative corrections. Virtual photons are emitted and absorbed. If the usual box normalization procedure is used, each virtual photon emission or absorption brings a factor $V^{-1/2}$ to the amplitude, so that there is a factor V^{-1} associated with each virtual photon line. This is cancelled by a factor V in the density of photon states which enters when all possible virtual photons are summed over, and the radiative corrections are independent of the quantization volume.

A second type of higher order process involves the forward scattering of a photon from the incident beam. This process involves no change in the states of the electron or of the beam. The emission and absorption

[3] H. Euler, Ann. Physik 26, 398 (1936).

[4] A. Sommerfeld, *Electrodynamics* (Academic Press Inc., New York, 1952), p. 305ff.

[5] J. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), p. 246.

matrix elements are proportional either to $(N/V)^{1/2}$ or $[(N+1)/V]^{1/2}$, where N is the expectation value of the number of photons in the beam, depending on whether the emission or absorption takes place first. The photons concerned are real, and no integration over a density of states occurs, so no factor V appears in the numerator. For fixed N, these contributions thus vanish as $V\rightarrow\infty$. The usual radiative corrections provide a complete account of higher order processes when, but only when, the density of photons, N/V, is vanishingly small.

When

$$\hbar\omega\ll mc^2 \tag{7}$$

it is known[5–7] that the usual or vacuum radiative corrections lead only to the renormalization of the electron mass and charge. To compare the Klein-Nishina formula with experiment, the formula must be written in terms of the renormalized mass and the renormalized charge. At higher frequencies there are other, finite corrections to the Klein-Nishina formula.

If the photon density is not vanishingly small, the radiative corrections coming from the forward scattering of incident photons cannot be neglected. The relevant parameter for measuring the contribution of these processes is[8]

$$\nu^2=(N/V)\, r_0(\hbar/mc)\lambda\!\!\!^{-}, \tag{8}$$

the number of photons in a box of dimensions given by the classical electron radius, the Compton wavelength, and the wavelength of the incident light. This forward scattering of real photons also affects the vacuum radiative corrections. Thus in addition to diagrams like those of Figs. 2(a) and (b), which represent vacuum and beam-induced self energies, there are mixed diagrams like that of Fig. 3, which is a self-energy diagram of order $\nu^2\alpha^2$. This is a vacuum radiative correction to a beam-induced correction, whose effect is to renormalize the charge and mass entering the expression for the beam-induced renormalization. All mixed diagrams can be interpreted in this way.

The beam-induced radiative corrections to any electrodynamic process stand on the same footing as the vacuum radiative corrections, and neither possesses a classical analog. In making comparison with classically derived results, it must be borne in mind that the classical results are to be expressed in terms of renormalized quantities. The appearance of new, intensity-dependent renormalizations requires the classical formulas to be expressed in terms of intensity-dependent quantities. Any intensity-dependent effect derived from classical theory must, therefore, be carefully checked to see that the renormalization has been correctly carried out.

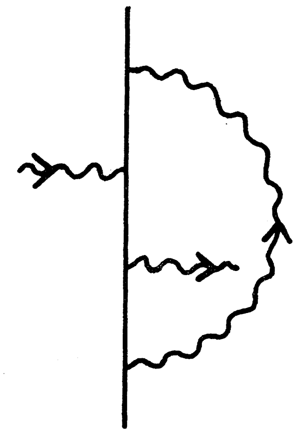

FIG. 3. Vacuum correction to beam-induced correction.

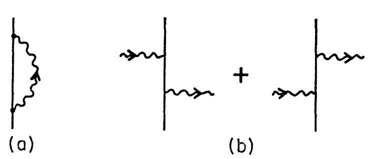

FIG. 2. Lowest order vacuum and beam-induced self-energies.

IV. BEAM-INDUCED SELF-ENERGY

The electron Green's function in quantum electrodynamics is defined in the Heisenberg picture by[9]

$$-\tfrac{1}{2}S_F'(x_1,x_2)=\langle 0|T\psi(x_1)\bar{\psi}(x_2)|0\rangle. \tag{9}$$

The presence of the physical vacuum state $|0\rangle$ means that the only radiative processes considered are those involving virtual photons. This is entirely appropriate when, in fact, almost no photons are present. When an intense beam of photons is present, the Green's function of interest is $S_B'(x_1,x_2)$, given by

$$-\tfrac{1}{2}S_B'(x_1,x_2)=\langle B|T\psi(x_1)\bar{\psi}(x_2)|B\rangle, \tag{10}$$

where the state $|B\rangle$ includes the physical photons of the beam and no physical electrons. For $|B\rangle$ to have any simple properties whatever, it must contain photons all having a common momentum direction. Otherwise the total momentum of a set of photons could be time-like rather than null, and real scattering or pair creation could take place.

The simplest way to see what is involved in $S_B'(x_1,x_2)$ is to use perturbation theory and expand it in powers of α and $\nu^2\alpha$. If $\nu^2\ll 1$, it is reasonable to keep only first-order terms in $\nu^2\alpha$ while keeping all orders of α. The Feynman diagrams involved are then the ones shown in Fig. 2(b), together with all vacuum radiative corrections to these processes. If the beam consists of low-energy photons so that $\hbar\omega\ll mc^2$, the theorem of Refs. 6 and 7 tells us that the only effect of the vacuum radiative corrections to this forward Compton-scattering amplitude is to renormalize the mass and the charge, a fact noted earlier. It is then sufficient to consider the diagrams of Fig. 2 as they stand without vacuum radiative corrections as long as the results are

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[6] F. Low, Phys. Rev. **96**, 1428 (1954).

[7] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

[8] P. Stehle, J. Op. Soc. Am. **53**, 1003 (1963).

[9] S. S. Schweber, *Relativistic Quantum Field Theory* (Row, Peterson and Company, Evanston, Illinois, 1961), p. 658.

expressed as containing the renormalized mass and charge. This is also correct for terms of higher order in $\nu^2\alpha$.

To the order considered we write, in momentum space,[10]

$$S_B^{(2)}(p)=S_F'(p)+S_F'(p)\Sigma(k,p)S_F'(p)\,, \qquad (11)$$

so that

$$\Sigma(k,p)=(-e^2/2k_0V)i(2\pi)^4\langle B|\gamma_\mu a_\mu^*\frac{1}{\gamma\cdot(p+k)-m}\gamma_\nu a_\nu +\gamma_\nu a_\nu\frac{1}{\gamma\cdot(p-k)-m}\gamma_\mu a_\mu^*|B\rangle\,, \qquad (12)$$

where the a_μ^* are photon creation operators, etc. The a's and a^*'s commute with everything but themselves and can be arranged so that a_ν stands to the right of a_μ^*. If $|B\rangle$ is a state with a definite number N of photons, then the matrix element of $a_\mu^*a_\nu$ is just $N\delta_{\mu\nu}$. If $|B\rangle$ is a coherent state, the matrix element is the mean photon number. In any case, it is the expectation value of the photon number. When this is large, we may neglect unity in comparison with it and can treat a_μ^*, a_ν as though they were classical, commuting quantities. This leads to

$$\begin{aligned}\Sigma(k,p)=&-e^2\langle N\rangle i(2\pi)^4/2k_0V\{[(p^2-m^2)(-\gamma\cdot p+m)\\&+(2p\cdot k)\gamma\cdot k](\gamma\cdot e\gamma\cdot e^*+\gamma\cdot e^*\gamma\cdot e)\\&+[(p^2-m^2)\gamma\cdot k+2p\cdot k(-\gamma\cdot p+m)]\\&\times(\gamma\cdot e\gamma\cdot e^*-\gamma\cdot e^*\gamma\cdot e)+(p^2-m^2+2p\cdot k)2e\cdot p\gamma\cdot e^*\\&+(p^2-m^2-2p\cdot k)2e^*\cdot p\gamma\cdot e\}\\&\times[(p^2-m^2)^2-(2p\cdot k)^2]^{-1}\,, \qquad (13)\end{aligned}$$

with e the polarization vector of the beam photons. The case of greatest interest is that in which p is a free-electron momentum. When this is so, both p^2-m^2 and $\gamma\cdot p-m$ can be set equal to zero and the above result reduces to

$$\Sigma(k,p)=-i(2\pi)^4\gamma\cdot\delta\,, \qquad (14)$$

with

$$\delta=(e^2\langle N\rangle/2Vk_0)\,k/p\cdot k\,. \qquad (15)$$

The propagator can now be written, correct to first order in ν^2, as

$$\begin{aligned}S_B^{(2)}(p)&=\frac{i}{(2\pi)^4}\left[\frac{1}{\gamma\cdot p-m}+\frac{1}{\gamma\cdot p-m}\gamma\cdot\delta\frac{1}{\gamma\cdot p-m}\right]\\&=[i/(2\pi)^4][\gamma\cdot(p-\delta)-m]^{-1}\,. \qquad (16)\end{aligned}$$

This last form includes all iterations of diagrams in Fig. 2, but no diagrams in which the electron line ever carries momentum $p\pm 2k$. The effect of such diagrams has been studied by Ehlotzky,[11] who finds that the form of (16) is correct, δ being expressible as a power series in ν^2. An analogous situation exists in the scalar theory of Fried and Eberly,[12] the first one which included all orders.

The propagator of (16), considered as a function of p, has a pole at

$$p^2=m^2+\Delta m^2\,, \qquad (17)$$

$$\begin{aligned}\Delta m^2&=e^2\langle N\rangle/k_0V\,,\\&=\nu^2m^2.\end{aligned} \qquad (18)$$

This can be regarded as a mass renormalization due to the presence of the beam. In the exact Green's functions of Eberly and Reiss[13] and of Ehlotzky,[11] there are also poles at other values of p^2, the function being meromorphic. The residues of these other poles decrease rapidly as the intensity of the beam gets low, so that they do not appear in the second-order calculation made here. The entire effect of the beam cannot, however, be described by a mass renormalization because the beam has a characteristic direction. Its effect can be described by regarding δ as a momentum renormalization, so that $\tilde{p}=p-\delta$ is the "renormalized momentum" of the electron in the beam. As a function of $\tilde{p}$, $S_B^{(2)}(\tilde{p})$ has a pole at the original place, $\tilde{p}^2=m^2$, and there is no mass shift.

The device of renormalizing the momentum accounts completely for the effect of the beam on the propagator, but does not account for beam effects on other processes.[14] If a vertex other than one involving forward scattering occurs, there are analogs of vertex corrections. Also, it must be recalled that the delta function describing momentum conservation at each vertex has as arguments the unrenormalized momenta.

In this situation one now may ask what the classical description of the system should be. We generalize the rule used for low intensities, which states that classical formulas are to be expressed in terms of renormalized mass and renormalized charge, by including momentum among the quantities to be renormalized. This means that wherever a momentum p appears in a classical formula, it is to be replaced by $p-\delta$, the mass remaining unaffected. This rule has the very interesting effect of canceling precisely an effect which occurs in classical theory,[15,16] the acquisition of a drift velocity by an electron initially at rest on being enveloped by a light pulse. This drift velocity corresponds to a momentum δ of the electron in the direction of the beam. This drift velocity or incremental momentum, which exists classically before renormalization, has been cited as the cause of a frequency shift of Compton-scattered light.[12] This shift is very difficult to understand on a quantum-mechanical basis,[1,17] but has been considered as inevit-

[10] The notation is that of Ref. 9.

[11] F. Ehlotzky, Acta Phys. Austriaca 23, 95 (1966).

[12] Z. Fried and J. H. Eberly, Phys. Rev. **136**, B871 (1964).

[13] J. H. Eberly and H. Reiss, Phys. Rev. **145**, 1035 (1966).

[14] G. P. DeBaryshe, Ph.D. thesis, University of Pittsburgh (unpublished).

[15] L. S. Brown and T. W. B. Kibble, Phys. Rev. **133**, A705 (1964).

[16] Vachaspati, Phys. Rev. **128**, 664 (1962); **130**, 2598 (1963).

[17] T. W. B. Kibble, Phys. Rev. **138**, B740 (1965).

able because of its classical nature and an uncritical application of the correspondence principle. If the renormalized momentum is taken as the quantum counterpart of the classical momentum, all these difficulties are resolved.

V. SUMMARY

It has been shown that care needs to be exercised in discussing the classical limit of quantum electrodynamics. For vacuum phenomena, the photon density must be large and the frequency must be low, though it is not well known how the frequency must decrease as the density increases, for Maxwell theory to be valid. In the presence of electrons, forward scattering of photons introduces a new class of renormalizations, account of which must be taken in interpreting the results of classical calculations because no counterparts of these renormalizations exist in the classical theory. When these renormalizations are included, a number of apparent discrepancies between classical and quantum calculations are removed.

PHYSICAL REVIEW VOLUME 152, NUMBER 4 23 DECEMBER 1966

Study of $\pi^-+p\rightarrow\Sigma^0+K^0$ at 1170 MeV/c*

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We have measured the differential and total cross sections and polarization of the Σ^0 from the reaction $\pi^-+p\rightarrow\Sigma^0+K^0$ with 1170-MeV/c pions incident on the Alvarez 72-in. hydrogen bubble chamber. Using 524 single-Λ events (where the only visible decay is that of the Λ from the decay $\Sigma^0\rightarrow\Lambda+\gamma$), 138 single-$K^0$ events (where only the K^0 decay is visible), and 256 double-vee events (where both Λ and K^0 decays are visible), we find the coefficients in the Legendre expansion of the differential cross section $d\sigma/d\Omega=A_0P_0+A_1P_1+A_2P_2$ to be $A_0=19.68\pm0.60$ μb/sr, $A_1=-0.04\pm1.20$ μb/sr, and $A_2=14.54\pm1.60$ μb/sr, corresponding to a total cross section $\sigma=247\pm10$ μb. No polynominals higher than P_2 are needed. Using both single-Λ and double-vee events, we find the coefficients in the polarization expansion $P_\Sigma d\sigma/d\Omega=\frac{1}{2}\sin\theta_\Sigma(B_1+B_2\cos\theta_\Sigma)$ to be $B_1=-9.98\pm8.29$ μb/sr and $B_2=-35.45\pm21.88$ μb/sr. In both angular-distribution and polarization studies a single-Λ event is statistically equivalent to about one half of a double-vee event.

I. INTRODUCTION

We have measured the angular distribution and polarization of the Σ^0 in the process

$$\pi^-+p\rightarrow\Sigma^0+K^0\,, \tag{1a}$$

$$\Sigma^0\rightarrow\Lambda+\gamma\,, \tag{1b}$$

using 1170-MeV/c π^- incident on the Alvarez 72-in. hydrogen bubble chamber. We use 256 double-vee events where both the K^0 and the Λ decay visibly via the charged modes

$$K^0\rightarrow\pi^++\pi^- \tag{2}$$

and

$$\Lambda\rightarrow p+\pi^-. \tag{3}$$

We also use 524 single-Λ events where the decay (3) is observed, but (2) is not, and 138 single-K^0 events where (2) is observed and (3) is not. All three types of events are used to find the angular distribution, and both double vees and single Λ's are used to find the Σ^0 polarization. We extract from the data a maximum amount of information on the Σ^0 polarization. Our method can be applied in other reactions involving polarized Σ^0's.

In a later paper we shall present our results for $\pi^-+p\rightarrow\Sigma^-+K^+$ at the same momentum. We defer until then a comparison of the experimental results for $\pi^-+p\rightarrow\Sigma^0+K^0$, $\pi^-+p\rightarrow\Sigma^-+K^+$, and $\pi^++p\rightarrow\Sigma^++K^+$ with the predictions of charge independence.[1]

II. SELECTION OF EVENTS

Events corresponding to Σ^0 production must be distinguished from the topologically similar events resulting from Λ production via the reaction

$$\pi^-+p\rightarrow\Lambda+K^0. \tag{4}$$

Whenever there is a visible K^0 decay both Σ^0 and Λ production are kinematically overdetermined. We then use the fitting program PACKAGE and select events on the basis of χ^2. For these events (single K's and double vees) there is no ambiguity between Σ^0 and Λ production.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] For earlier experimental results on $\Sigma+K$ production at pion momenta near 1 BeV/c, see F. S. Crawford, Jr., R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters 3, 394 (1959), which includes earlier references; J. A. Anderson *et al.*, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1962), p. 270; R. Kraemer *et al.*, *ibid.*, p. 273; J. R. Albright *et al.*, *ibid.*, p. 276; F. S. Crawford, F. Grard, and G. A. Smith, Phys. Rev. 128, 368 (1962); Y. S. Kim, G. R. Burleson, P. I. P. Kalmus, A. Roberts, and T. A. Romanowski, *ibid.* 143, 1028 (1966).