Dynamic Penetration Effects in the Internal Conversion of Electric Dipole Transitions in Lu^{175}†

G. T. EMERY
Brookhaven National Laboratory, Upton, New York, and State University of New York at Stony Brook, Stony Brook, New York
AND
M. L. PERLMAN
Brookhaven National Laboratory, Upton, New York
(Received 23 May 1966)

Internal-conversion coefficients are reported for the 396.3-, 282.5-, and 144.8-keV electric dipole transitions in Lu^{175}; these occur between the 396.3-keV $\frac{9}{2}-$ state of the [514↑] band and the $\frac{7}{2}+$, $\frac{9}{2}+$, and $11/2+$ members of the ground-state [404↓] band. Values for K and L_1 shells were determined directly, and for the L_2 and L_3 shells the coefficients were obtained from the L_1 and K values and the $L_1:L_2:L_3$ ratio measurements of Herrlander and Ewan. The dynamic penetration terms, $\mathbf{j}\cdot\nabla$ and $\mathbf{j}\cdot\mathbf{r}$, and the $M2$ admixtures were determined. For the 282.5-keV transition, additional information from Thun *et al.* on the electron-gamma angular correlation was used in the analysis. The amounts of $M2$ admixture were found to be small, the $\mathbf{j}\cdot\nabla$ nuclear matrix elements are consistent with zero, and the $\mathbf{j}\cdot\mathbf{r}$ matrix elements are in good agreement with rotational-model predictions.

I. INTRODUCTION

THE existence of penetration effects[1] in internal conversion of certain retarded low-multipolarity nuclear electromagnetic transitions is well established, and the theory of internal conversion has been extended to include these effects.[2,3] Analysis of these effects can provide information about certain new nuclear electromagnetic matrix elements, in addition to those ordinarily involved in gamma-ray emission. It may be especially useful to study these penetration matrix elements in the case of electric dipole transitions between low-lying states, for little is understood about the gamma-ray matrix elements of these transitions except that they are expected to be, and are, very small compared to "single-particle" estimates. Some experimental data are available on internal conversion of such electric dipole transitions in deformed nuclei. Explicit methods are available for extracting the new nuclear $E1$ matrix elements involved,[4,5] and with the recent publication of individual conversion-electron atomic matrix elements all the information required for meaningful analysis of experimental data is at hand.[6]

For the heavy-element region Asaro, Stephens, Hollander, and Perlman[7] have reviewed the evidence for $E1$ penetration effects. Transition rates, L-subshell ratios, and, with less precision, absolute conversion coefficients are known for about a dozen transitions. For six of these transitions Kramer and Nilsson[5] have compared the experimental results with nuclear-model calculations. In none of these cases, however, is there available a complete set of data for two transitions between the same initial and final rotational bands with which it would be possible to test branching ratios (transformation properties) of the nuclear matrix elements allowed by penetration.

In the deformed rare-earth region, however, Herrlander and Ewan[8] have reported L-subshell ratios for a number of retarded $E1$ transitions; and in one case, that of Lu^{175}, there is a clear conversion anomaly in two transitions, those of 282.5 and 396.3 keV, which connect the same excited state with two members of the ground-state rotational band. A level scheme[9] for Lu^{175} showing states populated in the decay of Yb^{175} is given in Fig. 1.

In this work measurements have been made of absolute K and L_1 conversion coefficients of the 396.3-, 282.5-, and 144.8-keV transitions. The subshell ratios of Herrlander and Ewan have been combined with these measurements to provide absolute L_2 and L_3 coefficients for the 396.3- and 282.5-keV transitions. It is of some importance to have these absolute numbers, because they provide limits on the amounts of $M2$ admixture. These measurements are discussed, and the results given, in Sec. II.

The extraction of the penetration matrix elements is discussed in Sec. III. In Sec. IV the matrix-element

† Work supported in major part by the U. S. Atomic Energy Commission.

[1] E. L. Church and J. Weneser, Phys. Rev. **104**, 1382 (1956).
[2] T. A. Green and M. E. Rose, Phys. Rev. **110**, 105 (1958).
[3] E. L. Church and J. Weneser, Ann. Rev. Nucl. Sci. **10**, 193 (1960).
[4] E. L. Church and J. Weneser, Nucl. Phys. **28**, 602 (1961). Some of the information needed is given in E. L. Church, Brookhaven National Laboratory Report No. BNL 50002, 1966 (unpublished).
[5] G. Kramer and S. G. Nilsson, Nucl. Phys. **35**, 273 (1962).
[6] I. M. Band, M. A. Listengarten, and L. A. Sliv, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 1673; in *Internal Conversion Processes*, edited by J. H. Hamilton (Academic Press Inc., New York, 1966), p. 589.
[7] F. Asaro, F. S. Stephens, J. M. Hollander, and I. Perlman, Phys. Rev. **117**, 492 (1960).
[8] C. J. Herrlander and G. T. Ewan, in *Role of Atomic Electrons in Nuclear Transformations, Proceedings of the 1963 Warsaw Conference* (Nuclear Energy Information Center, Warsaw, 1965), p. 148.
[9] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.), NRC 59-2-89.

values and branching ratios are compared with the predictions of nuclear models.

Measurements similar to those of this work have been made by Hager and Seltzer, and a brief report was presented at the 1965 Nashville Conference.[10] Their results, which were analyzed by a method somewhat different from that used here, are compared with the present results in Sec. IV.

II. METHODS AND RESULTS

The internal-conversion coefficients were determined from measurements of relative conversion-line and relative gamma-ray intensities from sources of Yb^{175} and of Au^{198}; the K-conversion coefficient of the well-known 411.8-keV $E2$ transition in Au^{198} decay was used to normalize the electron and gamma-ray data so that absolute values could be obtained. As a test of this procedure the 212.2-keV transition in Te^{121m} was checked.[11]

The sources of Yb^{175} used in these measurements were produced by irradiation of ytterbium oxide in the Brookhaven graphite reactor. This oxide,[12] enriched to 98.4% in Yb^{174}, had been deposited by vacuum sublimation onto thin aluminum supports, which were later cut into strips, approximately 1 mm by 15 mm. A source thickness of about 30 $\mu g/cm^2$ was required to get sufficient intensity in some of the weaker conversion lines. Gamma-ray and electron measurements were made on each of several sources. The Au^{198} source was prepared in the same way as was the Yb^{175}; the Te^{121m} had been produced by deuteron irradiation of natural antimony and the activity had been electropolated onto a gold foil support.[11]

Internal-conversion electron lines were measured with an iron, 50-cm, $\pi\sqrt{2}$ double-focusing spectrometer.[11,13] For most of the measurements the solid angle was set at 0.15% of 4π, and typical line widths observed were 0.11% (full width at half-maximum). Intensities were derived from the areas under the conversion line plots. The lines had "tails" on their low-momentum sides; this was especially true for transitions of low energy. These tails are the effects of energy loss in the source material and of back-scattering from the aluminum support. Backscattered electrons should make a negligible contribution to the intensity at momenta near that of the undegraded peak. At momenta just below that of the peak the rate was observed to drop approximately exponentially, and it is this part of the line which was taken to be produced by energy loss. The slope of the exponential changed with electron energy in the way

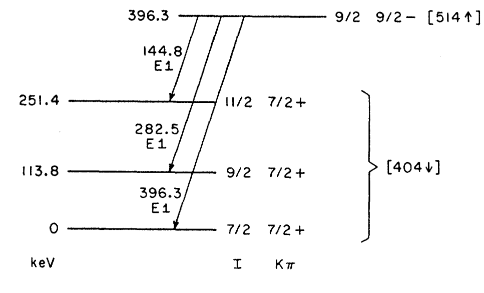

FIG. 1. Partial level scheme of Lu^{175}, showing the transitions studied in this work.

expected. The contribution of this exponential tail, which was included in the line area, was about 20% for $K114$ and $K145$ and less than 10% for the higher energy transitions. Although the L-subshell lines were incompletely resolved, the fact that the L_1 line in each case is the most intense and that the L ratios have been measured[8] made the determination of the L_1 intensities unambiguous. The uncertainties given to the intensity values include estimates of the errors involved in the analysis. Some results of checks on the methods used in this work are presented below.

There are shown in Table I relative intensities of the electron lines observed in these measurements together with the L-subshell ratios taken from the literature.

Measurements of gamma rays were made with lithium-drifted germanium detectors and multichannel analyzers. Most of this work was done with a detector[14] having a sensitive volume of 0.6 cm^3; for the rest a larger detector,[15] 3.5 cm^3, was used. Resolution of these detectors at 396 keV was 4.9 and 4.6 keV (FWHM), respectively. For the determination of the energy dependence of the photopeak efficiencies of the detectors, use was made of sources whose disintegration rates had been calibrated[16] and of sources such as Hf^{180m}, in which transitions with well-established relative abundances occur. It was found that the photopeak

TABLE I. Relative intensities of internal conversion lines of transitions in Lu^{175} following decay of Yb^{175}.

Transition energy (keV)	$I(K)^a$	$I(L_1)^a$	$L_1:L_2:L_3{}^b$
113.8	12.4±0.9		3.24±0.03:1.32±0.02:1
144.8	0.124±0.016	0.018±0.003	
282.5	0.274±0.013	0.045±0.003	12.6±1:2.6±0.4:1
396.3	1	0.152±0.008	50±5:5.7±0.6:1

a K and L_1 line intensities measured relative to K 396.3.
b L-subshell ratios for the 282.5- and 396.3-keV transitions are from Herrlander and Ewan (Ref. 8). Those for the 113.8-keV transition are from Novakov and Hollander (Ref. 20).

[10] R. Hager and E. Seltzer, in *Internal Conversion Processes*, edited by J. H. Hamilton (Academic Press Inc., New York, 1966), p. 315.

[11] Y. Y. Chu, O. C. Kistner, A. C. Li, S. Monaro, and M. L. Perlman, Phys. Rev. **133**, B1361 (1964).

[12] This material was supplied by the Y-12 Plant, Union Carbide and Carbon Corporation, through the Isotopes Division, U. S. Atomic Energy Commission, Oak Ridge, Tennessee.

[13] P. Erman, I. Bergström, Y. Y. Chu, and G. T. Emery, Nucl. Phys. **62**, 401 (1965).

[14] Purchased from R. C. A. of Canada, Ltd., Montreal.

[15] We wish to thank C. Chasman, R. A. Ristinen, and A. W. Sunyar for making this detector available to us.

[16] Standard sources supplied by International Atomic Energy Authority, Vienna.

TABLE II. Relative gamma-ray intensities of transitions in Lu^{175} following decay of Yb^{175}.

Transition energy (keV)	Previous work DeW[a]	M[b]	MBS[c]	HBMD[d]	GKM[e]	This work
113.8	25		30	31		30.2±2.0
137.6			<5	2.2		1.5±0.3
144.8			<10	5.9	13	4.6±0.5
282.5	50	58	43	62	61	47.1±2.4
396.3	100	100	100	100	100	100

[a] H. de Waard, Phil. Mag. 46, 445 (1955). NaI.
[b] N. Marty, Compt. Rend. 240, 963 (1955). NaI.
[c] J. P. Mize, M. E. Bunker, and J. W. Starner, Phys. Rev. 100, 1390 (1955). NaI.
[d] E. N. Hatch, F. Boehm, P. Marmier, and J. W. M. DuMond, Phys. Rev. 104, 745 (1956). Bent crystal.
[e] A. V. Gnedich, L. N. Kryukova, and V. V. Murav'eva, Zh. Eksperim. i Teoret. Fiz. 38, 726 (1960) [English transl.: Soviet Phys.—JETP 11, 524 (1960)]. Photoconversion.

shape was well approximated by a Gaussian except for a small part on the low-energy side and a very small part, below 1% of maximum amplitude, on the high-energy side. Each photopeak was therefore fitted to a Gaussian, and the channel number corresponding to a characteristic point (40% of maximum amplitude) on the low side of each peak was found. The area taken for the peak was the sum of all counts above this point. Before the Gaussian fit was made backgrounds were subtracted. It may be noted that considerable care must be taken with this background subtraction, especially in cases in which backscatter edges and Compton peaks from higher energy gamma rays fall in the vicinity of a photopeak. This is especially pertinent in the case of the 144.8-keV gamma ray. The results of these gamma-ray intensity measurements are shown in Table II.

The K-conversion coefficient of the 396.3-keV transition was determined by direct comparison of electron and gamma-ray intensities with those of the 411.8-keV transition of Au^{198} decay. Only a small correction for the energy dependent difference in gamma-ray detection efficiency was required. For the K-conversion coefficient of the much studied 411.8-keV transition the recent value of Bergkvist and Hultberg, $(3.02\pm0.04)\times10^{-2}$, was adopted.[17] Other conversion coefficients were derived from the K 396.3-keV result and the relative intensity data in Tables I and II. These results are given in Table III. Theoretical values from the tabulation of Sliv and Band[18] are given for comparison.[19] For the 396.3-keV transition the K-, L_1-, and L_2-shell coefficients are about 5 times larger than the Sliv and Band values; the L_3 coefficient agrees with that of Sliv and Band. In the case of the 282.5-keV transition, there is again no anomaly in the L_3 conversion; L_2 is larger than the Sliv and Band value by a factor of nearly 3, and L_1 and K are anomalous by smaller factors. There is no firm evidence for anomalous conversion of the 144.8-keV transition. As for the mixed $M1$-$E2$ 113.8-keV transition, the measured K coefficient agrees with that calculated from the Sliv and Band values with a mixing fraction defined by the L subshell ratios.[20]

The conversion-coefficient results of Table III for the 396.3- and 282.5-keV transitions agree very well with the experimental results of Hager and Seltzer.[10] In the case of the 144.8-keV transition the two sets of results differ by amounts somewhat larger than the combined uncertainties. It may be that this is due to difficulty in the measurement of the intensity of the 144.8-keV gamma ray.

III. THE PENETRATION MATRIX ELEMENTS

The conversion coefficient results have been analyzed in a manner similar to that of Church and Weneser.[4] The internal conversion coefficient in the κ_i shell for an

TABLE III. Internal conversion coefficients of transitions in Lu^{175}. Italicized figures are derived from the tabulated values of Sliv and Band, Ref. 18.

Transition energy (keV)	$\alpha(K)$	$\alpha(L_1)$	$\alpha(L_2)$	$\alpha(L_3)$
113.8	1.73±0.21(0)[a] *1.86(0)*[b]			
144.8[c]	1.14±0.19(−1) *1.11(−1)*	1.65±0.34(−2) *1.23(−2)*		
282.5[c]	2.45±0.24(−2) *2.02(−2)*	4.03±0.44(−3) *2.42(−3)*	8.3±1.6(−4) *3.24(−4)*	3.20±0.43(−4) *3.16(−4)*
396.3[c]	4.22±0.30(−2) *8.93(−3)*	6.4 ±0.6(−3) *1.10(−3)*	6.2±0.7(−4) *1.19(−4)*	1.09±0.12(−4) *1.12(−4)*

[a] The quantity 1.73±0.21(0) signifies $(1.73\pm0.21)\times10^0$.
[b] From the conversion coefficients of Sliv and Band and an $E2$ fraction of 19.5±2.5% (see Novakov and Hollander, Ref. 20), one derives $\alpha(K)=1.86\pm0.04$ for the 113.8-keV transition.
[c] The experimental values given by Hager and Seltzer in Ref. 10 are: for the 144.8-keV transition, $\alpha(K)=0.84\pm0.06(-1)$, $L_1/L_2=3.2\pm0.6$, $\alpha(L_1)=0.87\pm0.08(-2)$; for the 282.5-keV transition, $\alpha(K)=2.2\pm0.1(-2)$, $L_1/L_3=10.5\pm0.6$, $L_2/L_3=2.44\pm0.16$; for the 396.3-keV transition, $\alpha(K)=3.7\pm0.2(-2)$, $L_1/L_3=53\pm3$, $L_2/L_3=5.8\pm0.3$.

[17] K. E. Bergkvist and S. Hultberg, Arkiv Fysik 27, 321 (1965).
[18] L. A. Sliv and I. M. Band, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 1639.
[19] The values of Rose (see Ref. 21, below) are essentially the same.
[20] T. Novakov and J. M. Hollander, Nucl. Phys. 60, 593 (1964).

TABLE IV. Penetration matrix element weighting factors η and ζ, derived from Table III of Church and Weneser (Ref. 4, p. 617). Only the lowest order terms are given here. The electron wave-function weighting factors $f_\kappa(0)$ and $g_\kappa(0)$ are defined in Ref. 4; in each fg product the left member refers to the bound initial state and the right member to the continuum final state.

κ_i	κ_f	η	ζ
-1	$+1$	$(1/10)(2-5k)g_{-1}(0)f_{+1}(0)$	$g_{-1}(0)f_{+1}(0)$
	-2	$(3/10)g_{-1}(0)g_{-2}(0)$	0
$+1$	-1	$(1/10)(2+5k)f_{+1}(0)g_{-1}(0)$	$-f_{+1}(0)g_{-1}(0)$
	$+2$	$(3/10)f_{+1}(0)f_{+2}(0)$	0
-2	-1	$(3/10)g_{-2}(0)g_{-1}(0)$	0
	$+2$	0	0
	-3	0	0

electric dipole transition may be written

$$\alpha(\kappa_i)=\sum_{\kappa_f}|\mathfrak{M}(\kappa_i,\kappa_f)|^2, \quad (1)$$

in which

$$\mathfrak{M}(\kappa_i,\kappa_f)=M(\kappa_i,\kappa_f)+i[C(\kappa_i,\kappa_f)/2]^{1/2}\eta(\kappa_i,\kappa_f)k^{-3/2}X +i[C(\kappa_i,\kappa_f)/2]^{1/2}\zeta(\kappa_i,\kappa_f)k^{-3/2}Y. \quad (2)$$

In the definition of $\mathfrak{M}(\kappa_i,\kappa_f)$, $M(\kappa_i,\kappa_f)$ is the normal conversion matrix element for final electron state κ_f, as tabulated by Band, Listengarten, and Sliv[6]; the C coefficients are those given by Rose[21]; k is the transition energy in units of the electron rest energy; η and ζ are described in Table IV; and X and Y are proportional to ratios of penetration and gamma-ray matrix elements

$$X=(\pi\alpha/3)^{1/2}\mathcal{R}^2\frac{J(E1,i\rightarrow f)}{f_{\text{c.m.}}G(E1,i\rightarrow f)} \quad (3)$$

$$Y=(3/2)(\pi\alpha/3)^{1/2}\frac{(e\hbar/2mc)H(E1,i\rightarrow f)}{ef_{\text{c.m.}}G(E1,i\rightarrow f)}. \quad (4)$$

In (3) and (4) α is the fine structure constant; $\mathcal{R}$ is the nuclear oscillator radius parameter, $\mathcal{R}=(\hbar/m\omega_0)^{1/2}$; e is the electronic charge and $(e\hbar/2mc)$ the nuclear magneton; and $f_{\text{c.m.}}$ is the dipole effective-charge factor, N/A for protons, and $-Z/A$ for neutrons. $G(E1)$ is defined as the effective Nilsson matrix element for $E1$ gamma-ray emission; it is related to the gamma-ray transition probability, Λ_γ, between initial state with angular momentum I' and projection K' and final state I, K by[22]

$$\Lambda_\gamma(E1,I'K'\rightarrow IK) =\frac{4\alpha}{3\hbar}\left(\frac{\mathcal{R}}{\hbar c}\right)^2E_\gamma^3f_{\text{c.m.}}^2[C(I'1I;K',K-K')]^2 \times[G(E1,I'K'\rightarrow IK)]^2 =3.83\times10^{14}f_{\text{c.m.}}^2A^{1/3}E_\gamma^3[C(I'1I;K',K-K')]^2 \times[G(E1,I'K'\rightarrow IK]^2, \quad (5)$$

in which $C(I'1I;K',K-K')$ is a Clebsch-Gordan coefficient; in the last equality $\hbar\omega_0$ has been given its conventional value 41 $A^{-1/3}$ MeV and E_γ is measured in MeV. One may rewrite Eqs. (1–5) to give the transition probability for electric dipole conversion-electron emission from the κ_i shell:

$$\Lambda(\kappa_i,E1,I'K'\rightarrow IK) =\tfrac{4}{3}(\mathcal{R}^2/\hbar)(E_\gamma/\hbar c)^3[C(I'1I;K',K-K')]^2 \times\sum_{\kappa_f}|ef_{\text{c.m.}}G(E1,I'K'\rightarrow IK)M(\kappa_i,\kappa_f) +i[\pi\alpha C(\kappa_i,\kappa_f)/6k^3]^{1/2}\eta(\kappa_i,\kappa_f)\mathcal{R}^2eJ(E1,I'K'\rightarrow IK) +i(3/2)[\pi\alpha C(\kappa_i,\kappa_f)/6k^3]^{1/2}\zeta(\kappa_i,\kappa_f)(e\hbar/2mc) \times H(E1,I'K'\rightarrow IK)|^2. \quad (6)$$

The transition amplitudes $G(E1)$, $H(E1)$, and $J(E1)$ have been phenomenologically defined by Eqs. (5) and (6). $H(E1)$ and $J(E1)$ are proportional to the $\mathbf{j}\cdot\mathbf{r}$ and the $\mathbf{j}\cdot\nabla$ matrix elements, respectively, of Church and Weneser, who have pointed out that, in general, $H(E1)$ has a larger dimensional value than does $J(E1)$. For the transitions in Lu^{175}, $[514\uparrow]\rightarrow[404\downarrow]$, the matrix element $H(E1)$ is that of a transition allowed in the asymptotic limit, while the leading terms in $J(E1)$ and $G(E1)$ are zero in that limit. One may thus expect Y to be larger than X.

There may be admixtures of magnetic quadrupole radiation in these rather retarded $E1$ transitions. No higher multipoles are expected. The conversion coefficients may then be written

$$\alpha(\kappa_i)=(1-F)\alpha_{E1}(\kappa_i)+F\alpha_{M2}(\kappa_i), \quad (7)$$

in which $F=\delta^2/(1+\delta^2)$, in terms of the usual mixing amplitude δ.

To obtain numerical values of η and ζ the quantities $f(0)$ and $g(0)$ were taken from a tabulation of Church[4]; and the resulting numerical equations for the conversion coefficients in terms of F, X, and Y are given in the Appendix. In these equations the coefficients multiplying Y are a few times larger than those multiplying X. There are several systematic trends in the behavior of the weighting coefficients with subshell and energy which should also be noted. (1) For the K and L_1 shells the coefficient of Y and the normal matrix element have opposite signs; for the L_2 shell they have the same sign. If the anomalies are not so large that the normal matrix element can be neglected, the interference term will be constructive in one case and destructive in the other. (2) $\text{Im}M(-1,+1)$ has a sign change which, for $Z\approx71$, comes at an energy near 500 keV; this reduces the interference term for the K and L_1 shells of the 396.3-keV transition. (3) The weighting coefficients do not rise as fast with decreasing energy as the normal amplitudes; thus the penetration anomalies may be less pronounced at lower energies. This effect may be ampli-

[21] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958), p. xvii.

[22] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955); [see Nilsson's Eqs. (29) and (35)].

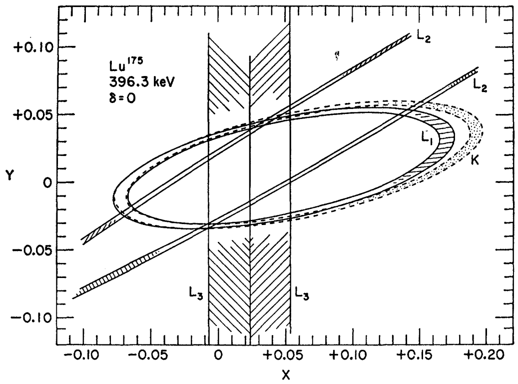

FIG. 2. Regions in the X-Y plane allowed by the internal conversion-coefficient results for the 396.3-keV transition in Lu^{175}; for this plot the $M2$ contribution has been taken to be zero. X and Y are proportional to the $\mathbf{j}\cdot\nabla$ and $\mathbf{j}\cdot\mathbf{r}$ penetration matrix elements, respectively.

fied in cases like Lu^{175}, in which the lower energy gamma-ray transitions of the triplet are less retarded.

Analysis of 396.3-keV Transition

We proceed to see if the set of equations (A1) is consistent with the experimental data for the 396.3-keV transition. Figure 2 shows, for the limiting case $F=0$, the regions allowed in the X-Y plane for the experimental conversion coefficients given in Table III. There is an allowed region near $X=-0.005$, $Y=-0.032$, and another near $X=+0.035$, $Y=+0.045$.

We have made a least-squares adjustment of F, X, and Y in the first region, and find the values

$$F=\ \ 0.0115\pm0.0127$$
$$X=-0.0015\pm0.0052$$
$$Y=-0.0313\pm0.0017.$$

$\chi^2=0.12$. The correlation matrix was

$$10^{-4}\times\begin{pmatrix}1.614 & 0.483 & 0.090\\ 0.483 & 0.273 & 0.066\\ 0.090 & 0.066 & 0.028\end{pmatrix}.$$

The experimental quantities used for the fitting were

$$\alpha(K)=(4.22\pm0.30)\times10^{-2},$$
$$\alpha(L_1)/\alpha(K)=0.152\pm0.008,$$
$$\alpha(L_1)/\alpha(L_3)=50\pm5,$$
$$\alpha(L_2)/\alpha(L_3)=5.7\pm0.6,$$

where the first two are from this work and the last two are from Herrlander and Ewan.[8]

A least-squares fit in the second region leads to negative (and nonphysical) values for F. If F is held fixed at zero, the results are

$$X=+0.0355\pm0.0039,$$
$$Y=+0.0432\pm0.0018,$$
$$\chi^2=10.79,$$
$$A=10^{-4}\begin{bmatrix}0.153 & 0.064\\ 0.064 & 0.034\end{bmatrix}.$$

The fit is not as good in this region as in the first, and such a large value of X is not expected.

Analysis of the 282.5-keV Transition

In addition to the quantities used in the analysis of the 396.3-keV transition there are available in the case of 282.5-keV transition the angular correlation results of Thun, Grabowski, El-Nesr, and Bruce.[23] These investigators measured the γ-γ, γ-K, and K-γ correlations for the 282.5–113.8-keV cascade and have given a value for the particle parameter, $b_K(282.5\text{ keV})=+0.06\pm0.12$. If this transition showed no penetration effects or $M2$ admixture the expected b_K value would be -1.52. This is the only case in which the particle parameter for an $E1$ transition has been shown to have penetration effects.[24]

The effective particle parameter for a mixed $E1-M2$ transition in which the $E1$ part may contain penetration terms may be written

$$b=\left[\frac{1+\delta^2}{1+\delta^2(\alpha_{M2}/\alpha_{E1})}\right]\left[\frac{F_2(11I'I)b_{E1}+2\delta(\alpha_{M2}/\alpha_{E1})^{1/2}F_2(12I'I)b_{E1,M2}+\delta^2(\alpha_{M2}/\alpha_{E1})F_2(22I'I)b_{M2}}{F_2(11I'I)+2\delta F_2(12I'I)+\delta^2F_2(22I'I)}\right].\qquad(8)$$

Here δ is the mixing ratio, the $F_2(L_1L_2I'I)$ are the coefficients tabulated by Ferentz and Rosenzweig,[25] and b_{E1}, $b_{E1,M2}$, and b_{M2} are defined by Biedenharn and Rose and by Ivash.[26] Appropriate values for b_{M2} have been tabulated,[27] but, when penetration effects are allowed, b_{E1} and $b_{E1,M2}$, as well as α_{E1}, must be con-

[23] J. E. Thun, Z. Grabowski, M. S. El-Nesr, and G. Bruce, Nucl. Phys. **29**, 1 (1962).

[24] P. Hornshøj and B. I. Deutch, Nucl. Phys. **67**, 342 (1965); in *Internal Conversion Processes*, edited by J. H. Hamilton (Academic Press Inc., New York, 1966), p. 459.

[25] M. Ferentz and N. Rosenzweig, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 1687.

[26] L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953); E. V. Ivash, Nuovo Cimento **9**, 136 (1958). The sign of $b_{E1,M2}$ has been corrected by E. L. Church, A. Schwarzschild, and J. Weneser, Phys. Rev. **133**, B35 (1964).

[27] I. M. Band, M. A. Listengarten, and L. A. Sliv, (edited by J. E. Thun) in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 1683.

sidered as functions of X and Y. For example

$$b_{E1}=1-\frac{|T_{E1}+2|^2}{|T_{E1}|^2+2},$$

in which

$$T_{E1}=\sqrt{2}e^{i(\delta_{+1}-\delta_{-2})}\mathfrak{M}_{+1}/\mathfrak{M}_{-2}.$$

The $\mathfrak{M}_\kappa$ are those functions of X and Y defined earlier, and the δ_κ are the continuum wave-function phases tabulated by Band, Listengarten, and Sliv.[6] A numerical equation for b_K (282.5 keV) in terms of δ, X, and Y is given in the Appendix.

As in the case of the 396.3-keV transition, there is shown in Fig. 3 for the 282.5-keV transition the regions in the X-Y plane allowed by the five experimentally determined quantities if δ is taken to be 0. The effect of taking δ to be ± 0.1 is shown by the partial plot of Fig. 4. There is but one allowed region—that near $X=0$, $Y=+0.013$—and a least-squares adjustment has been used in an attempt to ascertain the most probable values for X, Y, and δ. It turns out, as shown in Table V, that there is a region of values over which χ^2 exhibits a trough-like minimum. Unfortunately, there seems to be no model-independent method for making a selection from these values. There are, however, two model-dependent criteria which aid in the selection. First, it is to be expected that X should be much smaller than Y; its dimensional value[4] is about 1/30 that of Y and, since the Nilsson matrix element $J(E1)$ is not asymptotically allowed while $H(E1)$ is, one may expect that $|X/Y| < 1/100$. Thus, one of the sets of values in Table V has been obtained with X taken to be zero. Second, for transitions of the type here considered, δ and Y should be related by a fixed constant of proportionality. For the 282.5-keV transition the ratio expected is $\delta/Y=2.32$, if g_s(eff., $E1$) is assumed equal to g_s(eff., $M2$). The last set of values in Table V is the result of the least-squares adjustment with this constraint. It may be noted that these values are consistent with those derived under the assumption $X=0$, and they have been chosen for further analysis in the next section.

Analysis of the 144.8-keV Transition

The conversion lines of this transition are weaker than those of the two discussed above, and the information available is therefore limited. From the data presented in Table III it can be seen that there is no firm evidence for the existence of penetration effects. Indeed, because this transition is less retarded than the other two and because the effective weighting of the penetration terms is smaller, one would expect the conversion to be close to normal.

TABLE V. Values of the penetration matrix elements X and Y, and the mixing parameter δ for the 282.5-keV transition in Lu^{175}. Results have been obtained from the experimentally determined quantities by a least-squares procedure with the values in parentheses held fixed.

X	Y	δ	χ^2
-0.0045 ± 0.0032	$+0.0131\pm0.0011$	(0)	7.1
(0)	$+0.0150\pm0.0009$	$+0.044\pm0.018$	5.1
$+0.0060\pm0.0048$	$+0.0167\pm0.0015$	$(+0.05)$	3.7
$+0.0261\pm0.0072$	$+0.0216\pm0.0019$	$(+0.1)$	6.3
$+0.0019\pm0.0048$	$+0.0154\pm0.0016$	$(2.315Y=0.0357)$	5.1

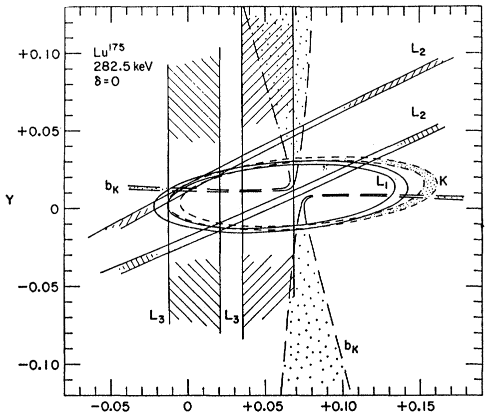

FIG. 3. Regions in the X-Y plane allowed by the internal conversion coefficients and the particle-parameter result (b_K) for the 282.5-keV transition; for this plot the $M2$ contribution has been taken to be zero (see also Fig. 4).

IV. TRANSITION MATRIX ELEMENTS AND THE ROTATIONAL MODEL

The values adopted for X, Y, and δ for the 396.3- and 282.5-keV transitions and the limits on Y for the 144.8-keV transition are given in Table VI. The lifetime of the 396.3-keV state has been measured[28] by Vartapetian and by Hauser and his colleagues. From the lifetime, from the δ values, and from decay branchings of Tables I and II (with corrections for conversion in the M and higher shells) the $G(E1)$ values of Eq. (5) have been calculated. $H(E1)$ and limits on $J(E1)$ were derived from Eqs. (3) and (4). $G(M2)$ was calculated from the appropriate Nilsson equation analogous to Eq. (5). In the last two columns ratios of the quantities $G(E1)$ and $H(E1)$ relative to those for the 282.5-keV transition are given.

[28] H. Vartapetian, Compt. Rend. **244**, 65 (1957); U. Hauser, K. Runge, and G. Knissel, Nucl. Phys. **27**, 632 (1961); U. Hauser, E. N. Hatch, K. Runge, G. Knissel, and W. Schneider, in *Electromagnetic Lifetimes and Properties of Nuclear States*, edited by P. H. Stelson (National Academy of Sciences—National Research Council, Washington, D. C., 1962), Publ. No. 974, p. 230. The two measurements agree, within their uncertainties, and we adopt the mean value, $T_{1/2}=(3.25\pm0.3)$ nsec.

Note added in proof. Two additional measurements of this lifetime have come to our attention: $T_{1/2}=(3.31\pm0.10)$ nsec, R. E. McAdams, G. W. Eakins, and E. N. Hatch, Nucl. Phys. **82**, 449 (1966); and $T_{1/2}=(3.25\pm0.10)$ nsec, E. E. Berlovich, Yu. K. Gusev, V. V. Il'in, and M. K. Nikitin, Zh. Eksperim. i Teor. Fiz. **43**, 1625 (1962) [English transl.: Soviet Phys.—JETP **16**, 1144 (1963)].

TABLE VI. Adopted values of the quantities X, Y, and δ and of the gamma-ray and penetration matrix elements for the 396.3-, 282.5-, and 144.8-keV transitions in Lu^{175}.

E_γ (keV)	X[a]	Y	δ	$\lvert G(E1)\rvert$	$\lvert H(E1)\rvert$	$\lvert J(E1)\rvert$	$\lvert G(M2)\rvert$	$G(E1)/G(E1)_{282.5}$ [b]	$H(E1)/H(E1)_{282.5}$ [b]
396.3[c]	$(-0.15\pm0.52)\times10^{-2}$	$(-3.13\pm0.17)\times10^{-2}$	$\pm(1.08_{-1.08}^{+0.49})\times10^{-1}$	$(0.19\pm0.01)\times10^{-2}$	0.98 ± 0.07	0.5 ± 1.8	$6.4_{-6.4}^{+2.9}$	-0.42 ± 0.01	0.85 ± 0.10
282.5[d]	$(+0.19\pm0.48)\times10^{-2}$	$(+1.54\pm0.16)\times10^{-2}$	$(+0.36\pm0.04)\times10^{-1}$	$(0.45\pm0.02)\times10^{-2}$	1.15 ± 0.13	1.5 ± 3.9	4.0 ± 0.5	(1)	(1)
144.8[e]	(0)	$(+0.8\ \pm1.4)\times10^{-2}$	$(+1.7\ \pm2.9)\times10^{-1}$	$(1.22\pm0.09)\times10^{-2}$	$1.6\ \pm2.8$	···	5.6 ± 9.7	$+2.71\pm0.14$	$1.4\ \pm2.5$

[a] In the calculation of the uncertainties given for X, Y, H, etc., small contributions due to the uncertainties in the normal conversion matrix elements M and in the weighting factors η and ξ have been neglected. These uncertainties are thought to be only a few percent and their inclusion would not change the quoted uncertainties appreciably.
[b] Although the signs of the matrix elements cannot be derived from experiment, relative signs can be established, as discussed in the text; the relative signs give the signs of the matrix element ratios.
[c] For this transition, values for X, Y, and δ were obtained from a least-squares fit of the expressions in the Appendix to the experimental data. If one makes the model-dependent assumption $\delta=1.82Y$, the values of X and Y are essentially unchanged, and the value of δ becomes $(-0.57\pm0.03)\times10^{-1}$.
[d] In the case of the 282.5-keV transition, the least-squares fitting procedure, even with the particle-parameter result included, led to the ambiguity described in the text, and the model-dependent constraint, $\delta=2.32Y$, was therefore used. The alternative model-dependent constraint, $X=0$, leads to $Y=(+1.50\pm0.12)\times10^{-2}$, $\delta=(+0.44\pm0.23)\times10^{-1}$.
[e] For the 144.8-keV transition, the two model-dependent constraints, $X=0$ and $\delta=2.10Y$, were used.

According to the zero-order branching-ratio rules of Alaga, Alder, Bohr, and Mottelson[29] the leading terms in the quantities $H(E1, I'K'\rightarrow IK)$, $G(E1, I'K'\rightarrow IK)$, etc. should be independent of I' and I for all transitions occurring between the same two bands. In the case of the [514↑]→[404↓] transitions in Lu^{175}, the matrix elements $G(E1)$ and $J(E1)$ are not allowed,[30] and higher order corrections may thus be important in their branching ratios. For $H(E1)$ and $G(M2)$, which are allowed, the zero-order rules should apply. It may be seen in Table VI that the $H(E1)$ values are essentially constant within their uncertainties and that the $G(M2)$ limits are consistent with a constant value. These results for $H(E1)$, which are reasonably consistent with the $\lambda(\mathbf{j}\cdot\mathbf{r})$ ratios of Hager and Seltzer,[10] thus confirm the zero-order branching rules, and the expected transformation properties of the leading penetration term.

From the Nilsson model[22] and the results of Church and Weneser[4] the lowest order value for $H(E1)$ is

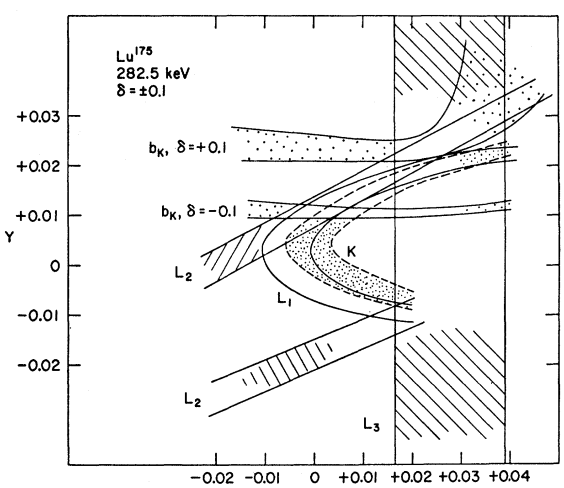

FIG. 4. Plot similar to Fig. 3 but for $\delta^2=0.01$. The conversion coefficients are independent of the sign of δ, but the particle parameter b_K is not.

[29] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 9 (1955).

[30] The selection rules for gamma-ray transitions were given by G. Alaga, Nucl. Phys. **4**, 625 (1957); for the penetration matrix elements they were given by Church and Weneser, Ref. 4.

given by

$$H(E1)=-ig_s\langle\chi_f|(\mathbf{s}\times\mathbf{r}/\mathcal{R})_{\Delta K}|\chi_i\rangle, \quad (9)$$

in which g_s is the nucleon spin g factor, $\mathbf{s}$ is the spin operator, and the vector product is defined in the spherical basis. For the [514↑]→[404↓] transitions $H(E1)$ reduces to $\frac{1}{2}g_s$. Higher order terms[4] in the expression for $H(E1)$ would decrease the value to $\simeq0.9g_s/2$; furthermore, the pairing correction factor, $UU'+VV'$, is estimated to be about 0.9. If one takes the weighted average experimental value $H(E1)=1.0$, one may thus calculate a value $g_s(\text{eff.})=0.4_5g_s(\text{free})$. There is no reason to expect that the g_s(eff.) values for transitions of various multipole orders should be the same,[31] but the value determined here is remarkably close to values derived from the analysis of $M1$ matrix elements.[32,33]

As may be seen in Table VI, the limits which can be set on the values $J(E1)$ are comparatively wide; the values are consistent with zero and are expected to be small because the matrix elements are forbidden by the Alaga selection rules.[30]

As may be expected for these forbidden transitions the values for $G(E1)$ are small compared with their dimensional value of unity, and the zero-order branching rule is not obeyed. One may write for $G(E1)$ the expression[34] good to first order

$$G(E1, I'K'\rightarrow IK) =A\{1+z[I(I+1)-I'(I'+1)]\}, \quad (10)$$

[31] See, for example, A. de Shalit, in *Electromagnetic Lifetimes and Properties of Nuclear States*, edited by P. H. Stelson (National Academy of Sciences—National Research Council, Washington, D. C., 1962), Publ. No. 974, p. 15.

[32] J. D. Rogers, Ann. Rev. Nucl. Sci. **15**, 241 (1965).

[33] The $G(M2)$ values are sufficiently uncertain that no statement can be made about $g_s(\text{eff.}, M2)$. Fortunately, relaxation of the constraint, $g_s(\text{eff.}, M2)=g_s(\text{eff.}, E1)$, used to select the best value of $Y_{282.5}$ affects the results for Y and $g_s(\text{eff.}, E1)$ by not more than $\approx15\%$.

[34] The form given is similar to that given for other cases by A. Bohr and B. R. Mottelson, At. Energ. (USSR) **14**, 41 (1963) [English transl.: Soviet J. At. Energy **14**, 36 (1963)]. Equivalent expressions are given by V. M. Mikhailov, Izv. Akad. Nauk SSSR Ser. Fiz. **28**, 308 (1964) [English transl.: Bull. Acad. Sci. USSR Phys. Ser. **28**, 225 (1964)], and by Yu. T. Grin' and I. M. Pavlichenkow, Phys. Letters **9**, 249 (1964); Zh. Eksperim. i Teor. Fiz. **47**, 1847 (1964) [English transl.: Soviet Phys.—JETP **20**, 1244 (1965)]. For application to the case of Hf^{177} see M. N. Vergnes and J. O. Rasmussen, Nucl. Phys. **62**, 233 (1965).

in which z is a Nielsen-like parameter.[35] With this expression and the $G(E1)$ values of Table VI one finds $z=+0.157\pm0.001$. The fact that a value of z could thus be obtained implies that the first-order branching rule, Eq. (10), is obeyed. The value of z requires that the signs of the $G(E1)$ ratios be those given in the ninth column of Table VI. These signs for the $G(E1)$ ratios together with the signs for Y determine the signs for the $H(E1)$ ratios, which are given in the last column and are all positive. This last result is just what is expected from an argument completely independent of Eq. (10) and of the relationship Eq. (4) connecting $G(E1)$, $H(E1)$, and Y. The simple fact that the $|H(E1)|$ values are consistent with a constant value, that is that they obey the zero-order branching rule, would lead naturally to the conclusion that the $H(E1)$ values must have the same sign.

Just as magnetic multipole radiation can be emitted by a moving charge, the nuclear operator being $g_l\mathbf{L}\cdot[\nabla(r^LY_{LM})]^*$, so can electric multipole radiation be emitted by a moving magnetic moment.[36] The appropriate nuclear operator is $-ig_s(\mathbf{s}\times\mathbf{r})\cdot[\nabla(r^LY_{LM})]^*$, which for $L=1$ becomes the same operator whose matrix element is $H(E1)$.[37] Since we have evaluated $H(E1)$ for the transitions in Lu^{175}, one may now determine how much this spin-current term contributes to the total gamma-ray-emission matrix element $G(E1)$. One finds that it contributes -10% to $|G(E1)|_{396.3}$, $+3\%$ to $|G(E1)|_{282.5}$, and about $+0.5\%$ to $|G(E1)|_{144.8}$. This seems to be the first evidence for spin-current contributions to nuclear transitions of electric multipolarity. That there is a transition-energy factor in the spin-current contribution does not affect the agreement of the $G(E1)$ values with the zero-plus-first-order branching rule: because of the approximate $I(I+1)$ level spacings within each band, the transition energies can be written to fairly high accuracy as

$$E(I'\rightarrow I)=282.5\text{ keV}-(\hbar^2/2\mathfrak{J})[I(I+1)-I'(I'+1)].$$

Thus the effects of spin-current contributions cannot in general be phenomenologically distinguished from the effects of Coriolis mixing and other possible contributions to the first-order term in Eq. (10). In this case, however, one knows the magnitude of the spin-current matrix element, and it contributes about 15% to the first-order term.

Note added in proof. Recently, J. E. Thun has remeasured the particle parameter b_k (282.5 keV) [Institute of Physics, University of Uppsala, Report No. UUIP 476, 1966 (unpublished)]. With this new result, $b_k=0.28\pm0.06$, the X and Y penetration–matrix-element values given in Table V are changed only slightly, except for the case $\delta=2.315Y$, for which Y becomes $+0.0183\pm0.0011$ and X becomes 0.0104 ± 0.0040.

ACKNOWLEDGMENTS

We thank E. L. Church and J. Weneser for many illuminating discussions and Dr. Church for making available to us his tabulations of the quantities $f(0)$ and $g(0)$.

We thank R. Hager and E. Seltzer for informing us of their results and C. J. Herrlander, J. E. Thun, and B. I. Deutch for discussions.

APPENDIX

The numerical equations for the various conversion coefficients are:

$$\begin{aligned}
\alpha(K396.3)&=0.233F+(1-F)\{0.03\times10^{-2}+|-0.0087-1.01X+5.38Y|^2+|-0.0924+1.55X|^2\},\\
\alpha(L_1396.3)&=0.0378F+(1-F)\{0.02\times10^{-3}+|-0.00294-0.415X+2.21Y|^2+|-0.0322+0.659X|^2\},\\
\alpha(L_2396.3)&=4.62\times10^{-3}F+(1-F)\{0.03\times10^{-4}+|-0.00275+0.525X-0.894Y|^2+|-0.0104+0.083X|^2\},\\
\alpha(L_3396.3)&=1.73\times10^{-3}F+(1-F)\{0.99\times10^{-4}+|-0.00363+0.155X|^2\}.
\end{aligned}\tag{A1}$$

$$\begin{aligned}
\alpha(K282.5)&=0.674F+(1-F)\{0.08\times10^{-2}+|-0.0287-0.550X+7.20Y|^2+|-0.1365+1.85X|^2\},\\
\alpha(L_1282.5)&=0.118F+(1-F)\{0.04\times10^{-3}+|-0.0094-0.231X+3.02Y|^2+|-0.0486+0.811X|^2\},\\
\alpha(L_2282.5)&=1.51\times10^{-2}F+(1-F)\{0.04\times10^{-4}+|-0.00467+0.647X-1.357Y|^2+|-0.0170+0.091X|^2\},\\
\alpha(L_3282.5)&=8.43\times10^{-3}F+(1-F)\{2.74\times10^{-4}+|-0.00650+0.234X|^2\}.
\end{aligned}\tag{A2}$$

---

[35] O. B. Nielsen, in *Proceedings of the Rutherford Jubilee Conference, Manchester, 1961* (Academic Press Inc., New York, 1964), p. 317; P. Gregers Hansen, O. B. Nielsen, and R. K. Sheline, Nucl. Phys. **12**, 413 (1959).
[36] See, for example, S. A. Moszkowski, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), pp. 863–886, especially Eqs. (50), p. 874.
[37] In Eq. (4) of Ref. 4 a factor $(2L+1)^{-1}$ has been omitted from the term involving the $\mathbf{j}\cdot\mathbf{r}$ matrix element.

$$\begin{aligned}
\alpha(K144.8)&=6.52F+(1-F)\{0.02\times10^{-1}+|-0.1265+0.748X+13.05Y|^2+|-0.303+2.82X|^2\},\\
\alpha(L_1 144.8)&=1.34F+(1-F)\{0.01\times10^{-2}+|-0.0398+0.346X+5.93Y|^2+|-0.102+1.37X|^2\},\\
\alpha(L_2 144.8)&=0.170F+(1-F)\{0.01\times10^{-3}+|-0.0126+1.11X-3.26Y|^2+|-0.0495+0.114X|^2\},\\
\alpha(L_3 144.8)&=0.204F+(1-F)\{2.52\times10^{-3}+|-0.0187+0.564X|^2\}.
\end{aligned} \tag{A3}$$

$$b(K282.5)=\left\{\frac{(1+\delta^2)}{(1+1.369\delta-0.625\delta^2)(1+0.674\delta^2/\alpha_{E1})}\right\} \times\left\{1-\frac{|T_{E1}+2|^2}{|T_{E1}|^2+2}+0.7655\frac{\mathrm{Re}[e^{i3.345}(1-T_{E1}^{-1})]\delta}{(1+2|T_{E1}|^{-2})^{1/2}\alpha_{E1}^{1/2}}-0.550\delta^2/\alpha_{E1}\right\}. \tag{A4}$$

PHYSICAL REVIEW VOLUME 151, NUMBER 3 18 NOVEMBER 1966

Gamma-Ray Yields from Nuclear Reactions and Level Densities of Deformed Nuclei*

J. O. RASMUSSEN AND T. T. SUGIHARA†
Department of Chemistry and Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 8 July 1966)

Theoretical calculations of spin distributions of compound nuclei and subsequent neutron-evaporation products are made for 27-MeV alpha particles on ^{178}Hf, 52-MeV alpha particles on ^{180}Hf, and 56-MeV ^{11}B ions on ^{165}Ho. The first two cases were chosen to match experimental work of Lark and Morinaga. We use our calculations as a basis for interpretation of their relative yield of ^{180m}W, the $J=8^-$ isomer, and the $J=4^+$ state of the ground rotational band. We find these yields consistent with the simple assumption that after the last neutron is evaporated to form the excited ^{180}W nucleus all products with spin $J\lesssim10$ feed into the 8^- isomer and those with spin $J\gtrsim10$ contribute to the prompt radiation from the first $J=4^+$ level. Possible relationships between the course of the gamma-ray cascade and the nature of the Nilsson orbitals nearest the Fermi surface are discussed. Finally, the importance of measuring gamma-ray angular distributions in addition to integrated yields is stressed, and formulas of possible value for the analysis of such angular distributions are collected and presented.

I. INTRODUCTION

THERE have been a number of studies of nuclear level densities based on the yields of isomeric pairs in compound nuclear reactions. The first analysis of such experiments was by Vandenbosch and Huizenga[1] and was intended to apply to systems involving relatively low angular momentum and excitation energy. Recently Dudey and Sugihara[2] have extended the statistical-model formalism to cover much broader ranges of angular momentum and excitation energies such as one would encounter in alpha-particle-induced reactions at several tens of MeV.

The recently published work of Lark and Morinaga[3] may offer a more stringent test of this type of calculation. Following (α,2n) and (α,4n) reactions on even-even deformed nuclei, they were frequently able to resolve several prompt gamma rays of the ground rotational bands of the product nuclei and in a few cases delayed gamma rays from isomers. Stephens, Lark, and Diamond[4] have pursued similar studies using heavy-ion beams, usually with odd-Z projectiles on odd-even targets. Hansen *et al.*[5] have investigated gamma transitions in the ground band following (p,2n) reactions on odd-even rare-earth targets.

II. ISOMER-YIELD THEORY APPLIED TO ^{180}W

Let us examine the results of Lark and Morinaga[3] for intensities of gamma-ray cascades in ^{180}W. They give prompt gamma-ray yields, from which can be deduced independent yields, of 2^+, 4^+, 6^+, 8^+, 10^+, and 12^+ levels of the ground rotational band. This nucleus also has a

* This work was done under the auspices of the U. S. Atomic Energy Commission.
† Department of Chemistry, Clark University, Worcester, Massachusetts.
[1] R. Vandenbosch and J. R. Huizenga, Phys. Rev. **120**, 1313 (1960).
[2] N. D. Dudey and T. T. Sugihara, Phys. Rev. **139**, B896 (1965).
[3] N. L. Lark and H. Morinaga, Nucl. Phys. **63**, 466 (1965).
[4] F. S. Stephens, N. L. Lark, and R. M. Diamond, Phys. Rev. Letters, **12**, 225 (1964); Nucl. Phys. **63**, 82 (1965).
[5] G. B. Hansen, B. Elbek, K. A. Hagemann, and W. F. Hornyak, Nucl. Phys. **47**, 529 (1963).