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# Total Cross Sections for Kilovolt Neutrons of Even-Odd Nuclei in the Region of the 3s Strength-Function Resonances*

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The 3s strength-function resonance has its maximum value in nuclei for which the major 28-neutron shell is near closure or just beyond. In view of this fact, it seemed desirable to investigate this mass region more extensively, and to do so we have studied the even-even compound nuclei which lie in the region of the 3s strength-function resonance. We find that the resonances in $^{53}$Cr and, to a certain extent, in $^{57}$Fe show an uncommon tendency to cluster. The result is that for $^{53}$Cr the strength function below 10 keV has a relatively large value of about $12\times10^{-4}$, whereas in the energy interval 3–60 keV the value is about $5\times10^{-4}$, in accord with the optical-model prediction. This situation suggests that the relatively large peak values of about $10\times10^{-4}$ which certain experiments show for the 3s strength-function resonance may result from a tendency for several nuclides in the vicinity of $^{53}$Cr to have rather large "local" strengths in a manner similar to that of $^{53}$Cr. This means that the maximum value of the 3s strength function is actually about $4.5\times10^{-4}$. A quasiparticle calculation is made to try to understand the spacings observed. Excepting $^{53}$Cr, the calculations showed good agreement with experiment, in that the observed levels are predominantly of the four-quasiparticle type, since two-quasiparticle types are relatively so few in number that they are not likely to be seen.

## I. INTRODUCTION

SINCE Feshbach, Porter, and Weisskopf first showed that the observed s-wave strength function could be accounted for by means of an optical model,[1] much work has been done to extend the measurements and refine the calculations. Thus, early data located the 3s and 4s resonances at about 55 and 160 mass units, respectively. However, it became evident that the 4s resonance was split into two components at $A=140$ and $A=180$, while the 3s resonance acquired an asymmetrical shape as more data accumulated. Such behavior was accounted for in terms of perturbations of the simple optical model by nonspherical rotational and spherical vibrational modes of excitation, respectively.[2–5]

Recently new interest in the subject of strength functions has been aroused by predictions that the giant resonances themselves may have structure.[6,7] Roughly speaking, the argument is that, in first order, the bombarding particle interacts with an average central potential, in second order, it interacts with one other nucleon and the rest of the nucleus, and so on. In those nuclides whose resonant energy states are highly complex, this hierarchy of configurations is, in principle, capable of manifestation. Thus, the giant resonance in the strength function is the manifestation of the single-particle state and the next stage, the two-particle, one-hole state (if for simplicity an even target is considered) is expected to give structure to the giant resonance, and so on. The problem of predicting the location and character of these second-order contributions to the strength function has for its solution less foundation for choice of parameters than has the optical model. Thus, for example, the location of the optical-model states, i.e., the single-particle states is, in general, known, while the location of compound states is unpredictable.

Among lighter nuclides, and especially for those near closed shells, e.g., 28 neutrons or protons, resonant states of simple composition ought to be observable. The mass region in the neighborhood of the 3s strength-function resonance is also the mass region of the closing of the 28-neutron shell. It seems of interest to look closely at these nuclides for possible evidence of elementary-configuration resonances, in order, if possible, to learn something about them. If one examines the strength function data on $^{51}$V, $^{55}$Mn, and $^{59}$Co, one will find that these nuclides at the peak of the 3s resonance exhibit values which seem to have relatively large or small values, depending upon whether the measurements have been taken below, say, 15 keV, or extended to 70 keV, or higher.[8–11] While for any given nuclide, the apparent fluctuation nearest dissociation energy could

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be statistical, the systematic tendency exhibited by nuclides near 28 neutrons suggests something of deeper significance.

There is an important class of nuclei which has not so far been systematically studied in the mass region $40<A<60$, viz; the even-even compound nuclides, consisting of $^{44}$Ca, $^{48}$Ti, $^{50}$Ti, $^{54}$Cr, $^{58}$Fe, and $^{62}$Ni. These nuclides have $-4$, $-2$, 0, $+2$, $+4$, and $+6$ neutrons, respectively, relative to the closed 28-neutron shell. They have not been studied extensively because of the low relative abundance of the corresponding even-odd target nuclei. These nuclides are the object of the present investigation.

## II. DESCRIPTION OF EXPERIMENT

The pulsed Van de Graaff accelerator ORNL, by utilizing the Li$(p,n)$ reaction and a relatively thick target, can be employed to measure transmissions of neutrons by time-of-flight in the energy range 2–60 keV with an energy precision of better than 2%. This order of resolution is adequate to obtain considerable information on nuclides whose level spacings exceed, say, 2 keV, such as in light nuclides and in nuclides near closed shells. The instrument is adapted for studies of separated isotopes, because the source of neutrons is small, and hence, it is possible to use very small samples. In the measurements described here, the diameter of the sample was usually about 0.5 in. With this size of sample, the resolution figures ranged downward from about 8 nsec/m for a mean energy of 5 keV to about 2 nsec/m, for a mean energy of 45 keV. All samples were in the form of pressed metal or metal-oxide powder. Specifically, $^{47}$Ti, $^{53}$Cr, the thin $^{57}$Fe, and $^{61}$Ni were in the form of pressed metal powder; the others, viz: $^{43}$Ca, $^{49}$Ti, and the thick $^{57}$Fe were oxides. Included in the series of measurements were the odd-even nuclei $^{63}$Cu and $^{65}$Cu, which were also in the form of pressed metal powders.

The data consist of the accumulation of flight-time intervals, which are stored in the memory of a 2048-or 4096-channel analyzer and read out on punched paper tape. These data are transferred to magnetic tape for input into a computer which calculates energies, transmissions, and cross sections, and in addition prepares a secondary magnetic tape for the input into a Calcomp plotter which can plot any or all of the calculated results. The data-taking time for 2000 counts per 2-nsec time interval is roughly 8 h at a given bombarding energy, or, correspondingly, for a given range of neutron energies. The instrument as a whole (pulsed Van de Graaff and time analyzer) can be run unattended for extended periods of time, provided no changes in bombarding energy are required. The samples cycle automatically between sample-in and sample-out positions, and it has become the practice to leave the instrument in an unattended running state about one-third of the running time. It is generally possible to obtain Calcomp plots of $\sigma_T$-versus $E_n$ within 4 to 6 h after a measurement has been completed.

The results are first collected as an assemblage of cross-section plots composed of data taken at different flight paths. These are sifted to obtain the corresponding energy interval which each flight path represents with best energy resolution. From these "best-resolution" fragments a composite total-cross-section plot is constructed. Smooth joining of the individual fragments is substantiation of the fact that the background subtractions have been consistent. When a satisfactory composite plot of the total cross sections has been obtained, the corresponding transmission plots are also obtained and from these the width parameters are obtained by area analysis.[12]

## III. RESULTS AND DISCUSSION

Figure 1 exhibits the total cross sections of the nuclides under investigation. Table I lists the corresponding resonance parameters, and Table II summarizes the significant statistical parameters, viz. strength function and average level spacing.

If one considers the various theories for the strength function,[13–18] one finds that the parameters are chosen, excepting Ref. 5, for data interpreted as having a value at maximum of between 10 and $15\times10^{-4}$. $^{54}$Cr is a compound nucleus whose mass places it at the position of maximum strength function and, indeed, it has been quoted as $14\times10^{-4}$.[19] It will be seen, however, in Fig. 1, that the levels of $^{54}$Cr show a tendency to cluster, or at least to overlap. The result is a group of levels near dissociation energy that has a local strength function of $12\times10^4$. However, on the energy interval 3–40 keV the value is $5\times10^{-4}$. Such fluctuations in strength function are allowed by Porter-Thomas and Wigner distributions on width and spacings. However, the systematic tendency (but in lesser degree), for the nuclides $^{51}$V, $^{55}$Mn, and $^{59}$Co to overlap, if not cluster, near dissociation energy has given support for generally accepting a peak $3s$ strength function value in excess of $10\times10^{-4}$. The results of the present experiment, and especially $^{53}$Cr, indicate that the choice of the peak value of about $5\times10^{-4}$ is more consistent with the idea of the optical model then values in excess of $10\times10^{-4}$, because fluctuations are averaged as much as possible. We are led to propose that the tendency toward overlapping of levels, at least near threshold, for nuclides near the peak of the $3s$ resonance is a real effect,

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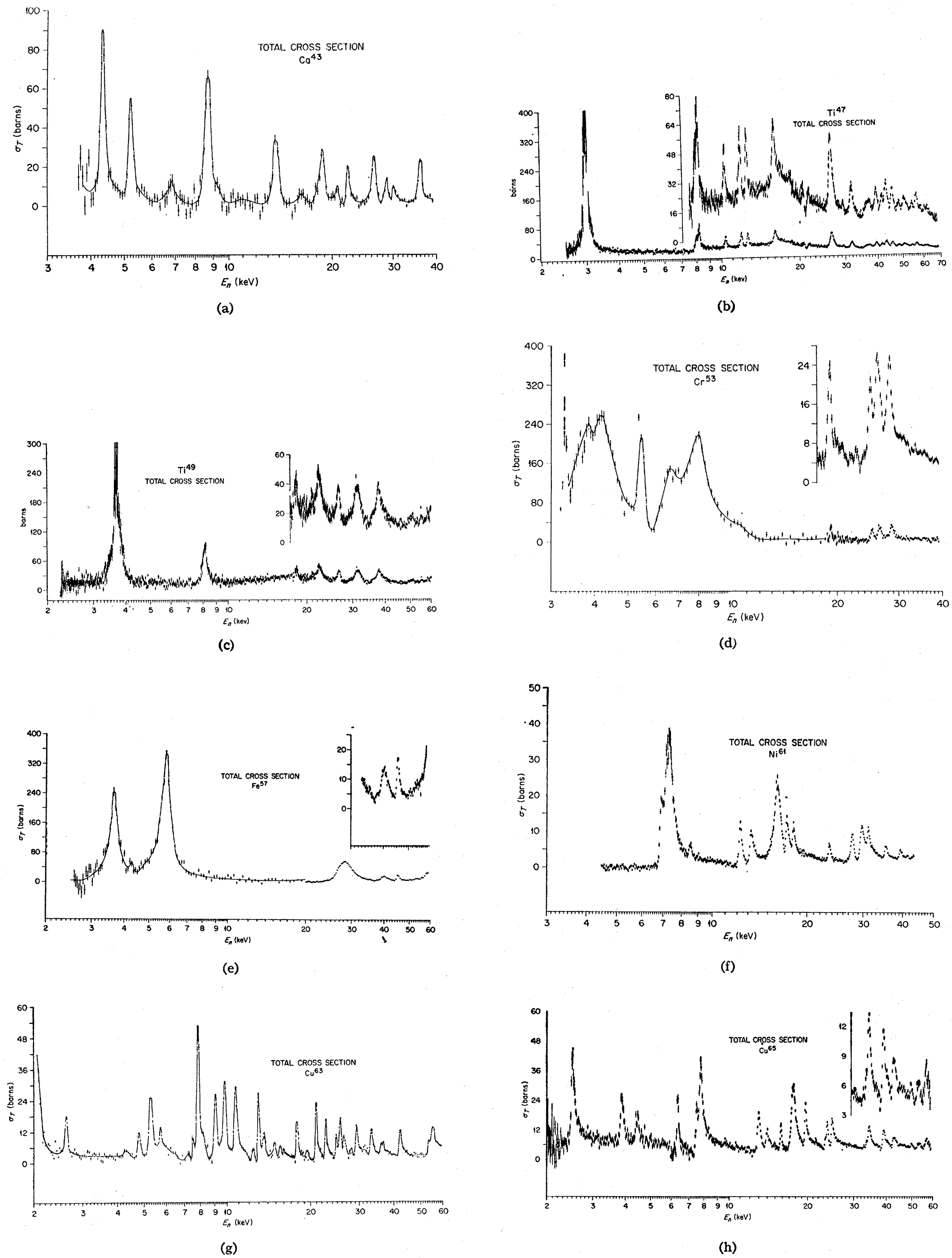


FIG. 1. Total cross sections observed in $^{43}$Ca, $^{47}$Ti, $^{49}$Ti, $^{53}$Cr, $^{57}$Fe, $^{61}$Ni, $^{63}$Cu, and $^{65}$Cu. The energy-dependent background in $^{47}$Ti and $^{49}$Ti arises from approximately 12% contamination with $^{48}$Ti.

TABLE I. Resonance parameters for the nuclides $^{43}$Ca, $^{47}$Ti, $^{49}$Ti, $^{53}$Cr, $^{57}$Fe, $^{61}$Ni, $^{63}$Cu, and $^{65}$Cu. The errors in the resonant energies can be taken as typically less than 2% and the error in the width as typically 30%.

| | $E_0$ (keV) | $\Gamma_n$ (eV) | $\Gamma_n^0$ (eV) | $J$ |
|---|---|---|---|---|
| $^{43}$Ca | 4.36 | 49 | 0.74 | |
| | 5.23 | 37 | 0.51 | |
| | 8.70 | 157 | 1.68 | |
| | 13.80 | 147 | 1.25 | |
| | 18.9 | 162 | 1.18 | |
| | 21.0 | 54 | 0.37 | |
| | 22.5 | 109 | 0.73 | |
| | 26.9 | 204 | 1.24 | |
| | 29.2 | 89 | 0.52 | |
| | 30.6 | 60 | 0.34 | |
| | 36.5 | 299 | 1.57 | |
| $^{47}$Ti | 2.98 | 117 | 2.15 | |
| | 8.21 | 98 | 1.08 | |
| | 10.4 | 60 | 0.58 | |
| | 12.0 | 79 | 0.72 | |
| | 12.7 | 83 | 0.74 | |
| | 16.2 | 184 | 1.45 | |
| | 20.4 | 708 | 4.35 | |
| | 29.1 | 79 | 0.47 | |
| | 31.5 | 504 | 2.84 | |
| | 36.1 | 202 | 1.06 | |
| | 37.0 | 200 | 1.04 | |
| | 39.2 | 369 | 1.86 | |
| | 41.4 | 268 | 1.32 | |
| | 42.9 | 555 | 2.68 | |
| | 45.3 | 484 | 2.27 | |
| | 48.0 | 221 | 1.01 | |
| | 50.3 | 370 | 1.65 | |
| | 51.9 | 94 | 0.41 | |
| | 54.0 | 223 | 0.96 | |
| | 36.2 | 528 | 2.23 | |
| $^{49}$Ti | 3.72 | 153 | 2.51 | |
| | 8.18 | 168 | 1.86 | |
| | 18.1 | 154 | 1.14 | |
| | 20.8 | 122 | 0.85 | |
| | 22.1 | 400 | 2.69 | |
| | 26.4 | 409 | 2.52 | |
| | 29.4 | 160 | 0.93 | |
| | 31.1 | 925 | 5.24 | |
| | 35.6 | 142 | 0.76 | |
| | 37.7 | 1580 | 8.14 | |
| $^{53}$Cr | 3.6 | 157 | 2.62 | |
| | 4.2 | 445 | 6.86 | |
| | 5.4 | 212 | 2.88 | |
| | 6.6 | 357 | 4.39 | |
| | 8.0 | 1073 | 10.7 | |
| | 10.5 | 224 | 2.20 | |
| | 19.3 | 132 | 0.95 | |
| | 25.3 | 237 | 1.49 | |
| | 26.4 | 350 | 2.15 | |
| | 28.8 | 555 | 3.27 | |
| $^{57}$Fe | 3.87 | 177 | 2.85 | 0 |
| | 6.10 | 396 | 5.07 | 1 |
| | 28.7 | 3018 | 17.8 | 1 |
| | 40.5 | 1258 | 4.95 | |
| | 45.5 | 404 | 1.60 | |
| $^{61}$Ni | 6.97 | 23 | 0.28 | |
| | 7.37 | 238 | 2.79 | |
| | 12.4 | 67.7 | 0.61 | |
| | 13.3 | 75.9 | 0.66 | |
| $^{61}$Ni | 13.7 | 13.0 | 0.11 | |
| | 16.3 | 411 | 3.21 | |
| | 17.5 | 174 | 1.32 | |
| | 18.3 | 181 | 1.34 | |
| | 23.8 | 100 | 0.64 | |
| | 28.2 | 236 | 1.40 | |
| | 30.2 | 423 | 2.43 | |
| | 31.6 | 392 | 2.20 | |
| | 32.7 | 120 | 0.66 | |
| | 33.8 | 123 | 0.67 | |
| | 36.0 | 294 | 1.55 | |
| | 40.0 | 243 | 1.21 | |
| | 42.2 | 133 | 0.65 | |
| | 44.0 | 169 | 0.80 | |
| | 48.4 | 83 | 0.38 | |
| $^{63}$Cu | 2.63 | 6.9 | 0.13 | |
| | 4.80 | 8.8 | 0.13 | |
| | 5.31 | 28.2 | 0.39 | |
| | 5.79 | 13.4 | 0.18 | |
| | 7.64 | 8.5 | 0.10 | |
| | 7.98 | 69.3 | 0.77 | |
| | 9.21 | 425 | 0.44 | |
| | 9.95 | 60.8 | 0.61 | |
| | 10.9 | 67.6 | 0.65 | |
| | 12.4 | 13.5 | 0.12 | |
| | 13.0 | 67.2 | 0.59 | |
| | 13.7 | 49.3 | 0.42 | |
| | 14.9 | 34.7 | 0.28 | |
| | 15.6 | 22.1 | 0.18 | |
| | 16.1 | 13.8 | 0.10 | |
| | 17.8 | 195.8 | 1.47 | |
| | 19.4 | | | |
| | 20.9 | 302.5 | 2.09 | |
| | 22.7 | 143 | 0.95 | |
| | 24.8 | 75.5 | 0.48 | |
| | 25.6 | 207 | 1.29 | |
| | 26.5 | 121 | 0.74 | |
| | 28.2 | 51.5 | 0.31 | |
| | 29.3 | 242 | 1.41 | |
| | 31.2 | 96 | 0.54 | |
| | 33.2 | 325 | 1.78 | |
| | 36.4 | 308 | 1.61 | |
| | 42.2 | 546 | 2.66 | |
| $^{65}$Cu | 2.5 | 23.2 | 0.46 | |
| | 3.87 | 19.7 | 0.32 | |
| | 4.45 | 15.4 | 0.23 | |
| | 6.45 | 30.0 | 0.37 | |
| | 7.65 | 29.8 | 0.34 | |
| | 7.92 | 155 | 1.74 | |
| | 9.85 | | | |
| | 13.2 | 72.4 | 0.63 | |
| | 14.2 | 51.6 | 0.43 | |
| | 15.2 | 9.6 | 0.08 | |
| | 15.9 | 43.9 | 0.35 | |
| | 17.8 | 296 | 2.22 | |
| | 19.8 | 194 | 1.37 | |
| | 21.8 | 27.4 | 0.18 | |
| | 24.1 | 84 | 0.54 | |
| | 25.0 | 256 | 1.62 | |
| | 34.9 | 324 | 1.73 | |
| | 39.7 | 249 | 1.25 | |
| | 43.5 | 224 | 1.07 | |
| | 50.8 | 70 | 0.31 | |
| | 54.2 | 132 | 0.57 | |
| | 58 | 320 | 1.33 | |

probably related in some way to the simultaneous proximity to the closing of the 28-neutron shell, and that the proper values to take for the strength function are those representative of the largest energy intervals, in order to average the fluctuations. If we collect the $s$-wave strength-function data for the odd-odd and

TABLE II. Summary of quantities deduced from Table I and quantities used for making comparisons with theory. Columns 2–9 give, respectively, the target ground-state spin and parity, the possible *s*-wave resonance spins and parity, the energy interval of measurement, the *s*-wave strength function, the observed mean level spacing, the theoretical mean level spacing, the binding energy, and the theoretical lower limit for six-quasiparticle excitation. See the text for columns 7 and 9.

| Isotope | Ground-state spin and parity | Resonance spin and parity | $\Delta E$ (keV) | $S_n^0$ ($10^{-4}$) | $\bar{D}_{obs}$ (keV) | $\bar{D}_{th}$ (keV) | Binding energy (MeV) | 6-quasiparticle threshold energy (MeV) |
|---|---|---|---|---|---|---|---|---|
| $^{43}$Ca | $\frac{7}{2}^-$ | $3^-,4^-$ | 0−36.5 | 1.4 ±0.7 | 3.3±0.6 | 12.2 | 11.14 | |
| $^{47}$Ti | $\frac{3}{2}^-$ | $2^-,3^-$ | 2−31 | 2.39±1.4 | | | 11.62 | 12.02 |
| | | | 31−57 | 4.12±2 | | | | |
| | | | 0−57 | 2.57±1 | 2.6±0.4 | 4.6 | | |
| $^{49}$Ti | $\frac{7}{2}^-$ | $3^-,4^-$ | 0−60 | 2.66±1.3 | 5.6±1 | 5.3 | 10.94 | 11.68 |
| $^{53}$Cr | $\frac{3}{2}^-$ | $1^-,2^-$ | 3−15 | 12.4 ±8 | | | 9.72 | 12.02 |
| | | | 15−40 | 1.6 ±1.3 | | | | |
| | | | 3−40 | 5.1 ±2.6 | 3.0±0.7 | 25 | | |
| $^{57}$Fe | $\frac{3}{2}^-$ | $1^-,2^-$ | 2−55 | 3.7 ±2.6 | 10±3 | 11.2 | 10.05 | 12.58 |
| $^{61}$Ni | $\frac{3}{2}^-$ | $1^-,2^-$ | 2−50 | 2.5 ±0.9 | 2.4±0.6 | 3.3 | 10.59 | |
| $^{63}$Cu | | | 2−60 | 2.7 ±0.8 | 1.2±0.2 | | 7.19 | |
| $^{65}$Cu | | | 2−60 | 1.5 ±0.5 | 1.7±0.3 | | 7.06 | |

even-even compound nuclides, for both classes of which about 30% accuracy in strength function on a 70-keV energy interval is possible, based upon Porter-Thomas and Wigner statistics, we obtain the picture of the 3*s* resonance shown in Fig. 2. It will be seen that the entire strength function is in rather good agreement with the optical-model calculations of Buck and Perey. We cannot, at this time, say whether it was to be expected, but we would like to note that the data pertain to even-odd and odd-even target nuclides, whereas the calculation of Buck and Perey with which the data agree specifically pertains, to the case of even-even target nuclides.

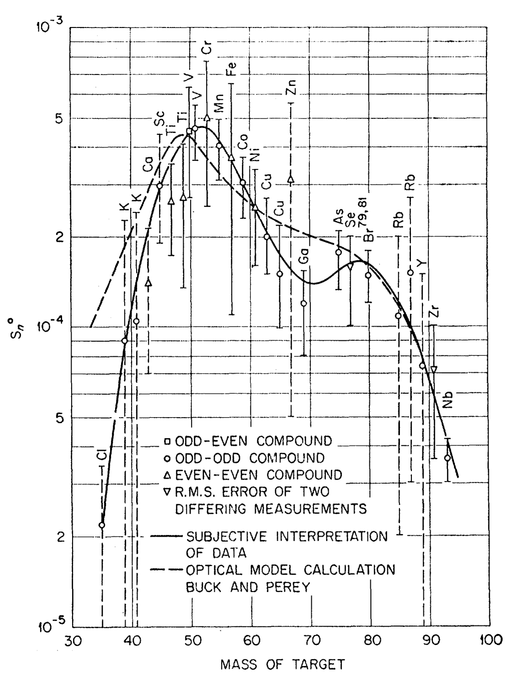


FIG. 2. Summary of *s*-wave strength functions observed for even-odd and odd-even target nuclides. A recent determination of the strength function of the odd-odd target nuclide $^{50}$V is also shown.

Indirect evidence from the strength function has just been given that resonances show local overlap or cluster tendencies, at least near neutron dissociation energy, for nuclides whose strength functions are near maximum, or alternatively, for nuclides near the 28-neutron shell. It is clear that the number of resonances observed in this present experiment is too small to make valid statistical tests of distribution functions. However, we suggest that, from casual inspection of Fig. 1, certain features appear which have some systematic basis for recognition: (a) The two titanium isotopes appear relatively deficient in small resonances, and (b) clustering tendencies appear in $^{54}$Cr, $^{57}$Fe, and $^{61}$Ni. To make these observations quantitative, we have plotted the corresponding distribution functions in Figs. 3 and 4, respectively, along with the Porter-Thomas and Wigner distributions for comparison. With the present instrument for measuring cross sections, care must be used in saying how many levels there are for which $\Gamma_n^0 < 0.25\bar{\Gamma}_n^0$. In the case of the titanium isotopes, no levels should be missed of such size, but below this, comparisons with theory whould not be attempted. If a $\chi^2$ test is performed to compare the observed distribution with the Porter-Thomas distribution, excluding those resonances with $(\Gamma_n^0)^{1/2} \lesssim 0.5(\bar{\Gamma}_n^0)^{1/2}$, we indeed find a poor fit. Let us emphasize, however, that the number of levels is too small for the test to be strictly valid and no rigorous conclusion can be drawn. Likewise, in the case of the nearest-neighbor spacings for $^{53}$Cr, $^{57}$Fe, and $^{61}$Ni in Fig. 4, the tendency for too many small spacings is evident and again a $\chi^2$ test, even excluding the large interval for which there is an exceedingly small chance for occurrence, gives poor agreement with the Wigner distribution. Again, however, we must be cautious about drawing conclusions, since the sample of resonant states is too small for the test to be valid. The presence of two-spin states also complicates the argument because, while the two independent populations will have the effect of introducing small spacings where none existed in the single-population case, two spin-state heavy nuclides actually seem to be rather well described by the single population Wigner distribution.[20]

By means of the Porter-Thomas and the Wigner distributions it would be possible to predict certain features of a sample of neutron resonances. It seems that these distribution functions, if they are valid, give little notion of the salient features of $^{47}$Ti, $^{49}$Ti, $^{53}$Cr, $^{57}$Fe, and $^{61}$Ni, and hence regarding widths and spacings, we can do little more based upon the present study than call for more information or other proof, before concluding that the Porter-Thomas and Wigner distributions on reduced widths and nearest-neighbor spacings are very useful models for at least these members of the 3*s* resonance region. A more tangible result of this study is the establishment of Fig. 2 as the 3*s* strength function.

The experiments in this paper were motivated partly by the notion that the resonant states of relatively light nuclei, especially if chosen in close proximity to a closed shell, should be of relatively simple composition, and being so, might exhibit features to differentiate them from resonances in heavy nuclei. Although on inspection the total cross sections of Fig. 1 do seem to show distinctive features, we have not been able to place them on any firm statistical basis.

We have pursued the question of the nature of the observed resonances a bit further by investigating what correlations might be found between the observed resonance structure and the states of allowed spin and parity that can be calculated in a theoretical manner. Such a calculation is relatively easy to make, starting with the shell model and considering the pair interaction in terms of the BCS theory.[21] Of course, the BCS treatment may be a rather poor approximation to be used here, because the nuclei of concern are light and lie in the neighborhood of closed shells, their neutrons and protons occupying similar orbits.

The energies of the shall-model orbits considered here are given in Table III. They are close to those given by Nilsson,[22] but some modifications were made by taking remarks of Cohen[23] into account. The pairing energies considered between protons and neutrons are assumed to have the strength parameter $G^{21}$ of the magnitude $G=24/A$ MeV. Using these parameters the BCS equations are solved and the spectra of quasiparticles obtained. The evaluation of the energies of the two- and four-quasiparticle states is then straightforward.

Since the incident neutrons that produce the resonances are with few, if any, exceptions *s*-wave, the spins of the resonances will be $I=J\pm\frac{1}{2}$, where $J$ is the spin

TABLE III. Energies of the shell-model orbits used in the quasiparticle calculation of excitation energy.

| | Energy (MeV) |
|---|---|
| $1d_{5/2}$ | −1.09 |
| $2s_{1/2}$ | 0.00 |
| $1d_{3/2}$ | 1.64 |
| $1f_{7/2}$ | 7.06 |
| $2p_{3/2}$ | 10.00 |
| $1f_{5/2}$ | 10.90 |
| $2p_{1/2}$ | 11.60 |
| $1g_{9/2}$ | 13.11 |

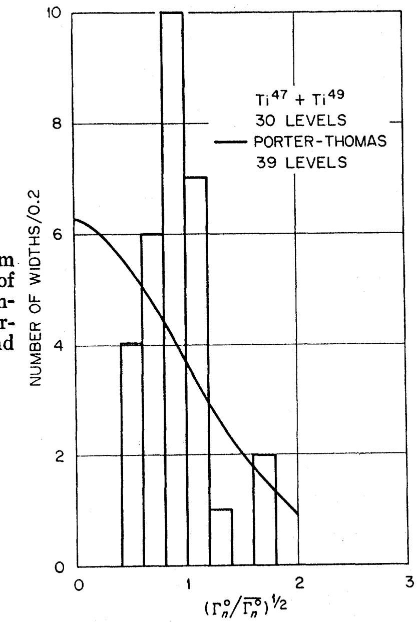


FIG. 3. Histogram of the frequency of reduced-width amplitudes for the target nuclides $^{47}$Ti and $^{49}$Ti.

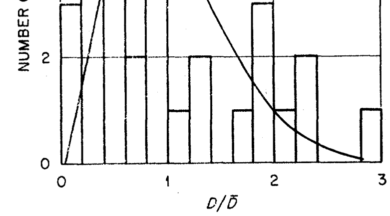


FIG. 4. Histogram of the frequency of nearest-neighbor level spacings for the nuclides $^{53}$Cr, $^{57}$Fe, and $^{61}$Ni.

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of the target. The $J$ (and parity) of the targets $^{43}$Ca, $^{47}$Ti, $^{49}$Ti, $^{53}$Cr, $^{57}$Fe, and $^{61}$Ni are, respectively, $\frac{7}{2}^-$, $\frac{5}{2}^-$, $\frac{7}{2}^-$, $\frac{3}{2}^-$, $\frac{3}{2}^-$, and $\frac{3}{2}^-$; correspondingly, $I^\pi$ for the compound systems $^{44}$Ca, $^{48}$Ti, $^{50}$Ti, $^{54}$Cr, $^{58}$Fe, and $^{62}$Ni are, respectively, $(3^-,4^-)$, $(2^-,3^-)$, $(3^-,4^-)$, $(1^-,2^-)$, $(1^-,2^-)$, and $(1^-,2^-)$. The energies around which we have evaluated the average level spacings are separation energies $S$, which, for the above six even-even nuclei, are, respectively, 11.14, 11.62, 10.94, 9.72, 10.05, and 10.59 MeV.[24] The energy interval over which $\bar{D}$, the average spacing, was obtained was arbitrarily taken as $S-2$ MeV $\leqslant E \leqslant S+2$ MeV.

In summary, we have calculated for the above compound nuclei the number of two- and four-quasiparticle states of allowed spin and parity, whose energies $E$ satisfy the relation $S-2$ MeV $\leqslant E \leqslant S+2$ MeV. By dividing 4 MeV by this number, the theoretical level spacing $\bar{D}_{\text{th}}$ is obtained and the result is given in the last column of Table II.

As is seen, $\bar{D}_{\text{th}}$ is in good agreement with the corresponding experimental spacing $\bar{D}$ for $^{47}$Ti, $^{49}$Ti, $^{57}$Fe, and $^{61}$Ni. In particular, the slight increase of $\bar{D}$ by going from $^{47}$Ti to $^{49}$Ti is reproduced theoretically. The sharp drop of $\bar{D}$ in going from $^{57}$Fe to $^{61}$Ni is also well accounted for. Thus, it seems that our theory, in spite of its crudeness, explains the experiment rather well.

Rather bad disagreement between $\bar{D}_{\text{th}}$ and $\bar{D}_{\text{obs}}$ occurred, however, for $^{43}$Ca and $^{53}$Cr. Since the BCS approach is expected to be the worst for $^{43}$Ca among the six nuclei considered here, the disagreement for this nucleus is not so surprising. In the other hand, the disagreement for $^{53}$Cr is a bit puzzling.

In order to carry the correlation between the calculation and the experiment further, a digression back to an introductory discussion is necessary. The earliest notations about the compound nucleus have evolved in the last several years into a concept of a state that is subject to decomposition into a hierarchy of states representing more and more complex excitations of the target nucleus. According to Feshbach[6] the simplest of such state, through which more complex states must first pass, is the two-particle one-hole (three-quasiparticle) state (when for simplicity an even-even target is considered), which is second in the hierarchy of states only to the initial single particle, or one-particle no-hole state. He shows that this so-called doorway state will impose a structure on the giant resonance in the strength function, characterized by the lifetime of the three-quasiparticle states, a point which had been considered implicitly in a somewhat different way by Bruckner *et al.* sometime ago.[25,26]

Based on the formalism of Fesbach, Shakin[27] performed some realistic calculations to describe the strength functions for eV neutrons on even-even targets and fairly good agreement with experiment was obtained. In principle we can extend Shakin's analysis to the present case, but the large excitation energy of the compound system and the fact that the dominating states are of four-quasiparticle character almost prohibit such an attempt to be made with any physical significance. We should thus be contented with, at least at this moment, the above estimate of $\bar{D}$ and its fair agreement with experiment.

In this connection, it is now illuminating to consider Fig. 5, in which we exhibit the accumulative number of levels of allowed spin and parity as a function of excitation energy. We note first of all, excepting for $^{53}$Cr$+n$, a suggestion that the observed states are predominantly four-quasiparticle excitation. Figure 5 also designates the binding energy (the site of the experiment) and the lowest energy at which six-quasiparticle excitation can occur. One or two interesting correlations now appear between Fig. 5 and properties of the resonances shown in Fig. 1. Features of the cross sections that appear in Fig. 1, which are also reflected in Table II, are the suggestions of a rise in strength function in $^{47}$Ti$+n$ at about 35 keV and a concentration in strength function of $^{53}$Cr$+n$ near zero neutron energy. The calculations, on which Fig. 5 is based, are crude, but we think that it may be more than coincidental that they predict that: (a) In the energy region studied, $^{54}$Cr should have two- and four-quasiparticle excitations *with equal probability* and (b) The "threshold" for six-quasiparticle excitation, and hence six-quasiparticle states generally, lie appreciably nearer the energy region studied in the case of $^{47}$Ti$+n$ than in the case of $^{49}$Ti$+n$, or any of the other nuclides studied.

## IV. CONCLUSIONS

The nuclides $^{47}$Ti, $^{49}$Ti, $^{53}$Cr, $^{57}$Fe, and $^{61}$Ni have masses that span the $3s$ strength-function resonance and also the major neutron number 28. Over an energy interval of about 70 keV the strength functions of these nuclides are in good agreement with the optical-model calculations of Buck and Perey. Superficially there appears to be some irregularities in certain of the nuclides in regards to the distributions of widths or spacings, but, because of insufficiently good resolution and too few resonances, firm statistical support for the existence of these irregularities is lacking. A quasiparticle calculation suggests that most of the states observed are of the four-quasiparticle type, excepting that in $^{54}$Cr there may be also two-quasiparticle states, and that the onset of six-quasiparticle states is suggested in the case of $^{47}$Ti$+n$.

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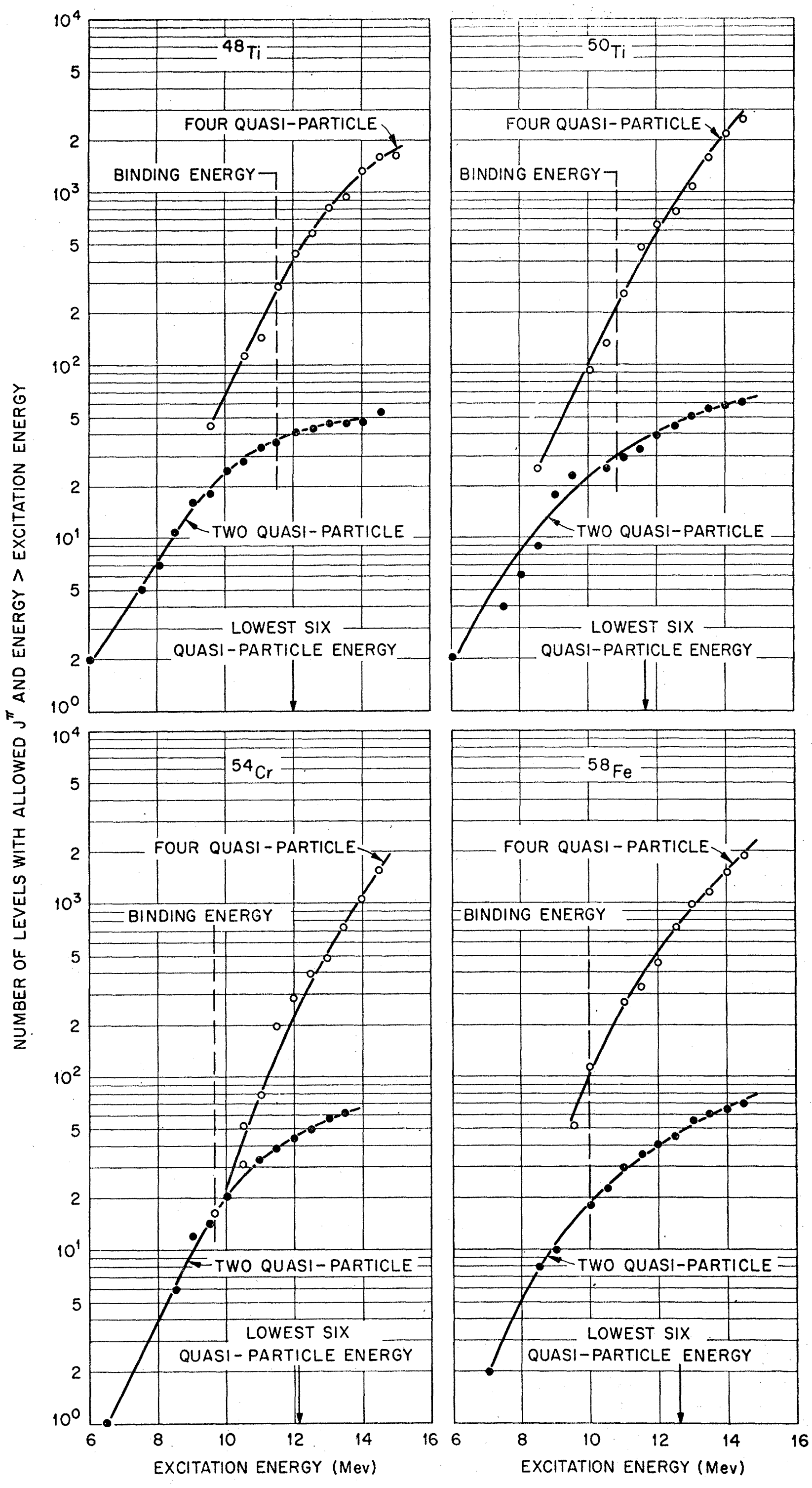


FIG. 5. Accumulative number of levels of allowed spin and parity as a function of excitation energy as calculated from the BCS theory (see text).

This experiment has thus contributed information about the $3s$ strength function and has suggested experiments which might reveal whether it is possible to discern states of increased complexity as they become energetically possible, and, if so, how the strength function behaves.