PHYSICAL REVIEW VOLUME 151, NUMBER 3 18 NOVEMBER 1966

Neutron Form Factors from Inelastic Electron Scattering in Tritium and Helium-3*

E. B. HUGHES,† M. R. YEARIAN, AND R. HOFSTADTER
High Energy Physics Laboratory, Stanford University, Stanford, California
(Received 31 May 1966)

Measurements are reported of the inelastic scattering of electrons from the mirror nuclei of tritium and helium-3. The ratio of the quasi-elastic electron-tritium and electron–helium-3 cross sections has been measured to a precision of about 3% for values of the square of the four-momentum transfer q^2 equal to 2.5 and 4.6 F^{-2}. Detailed mesaurements of the shape of the entire inelastic spectra and estimates of the total inelastic cross sections have also been made. While these data are primarily intended to furnish additional experimental information which will be of use in determining the structure of the three-body nuclei, it is also pointed out that such data are a possible source of information on the structure of the neutron.

I. INTRODUCTION

IN recent years an extensive series of experiments has been carried out on the scattering of electrons from the light nuclei. It was recognized by Collard and Hofstadter[1] that a comparative study of the structures of these nuclei, as revealed by their electromagnetic interactions, would provide an excellent opportunity to investigate the nuclear force between small numbers of nucleons. In particular, experiments on the nuclei of tritium and helium-3 might be expected to reveal the existence or absence of three-body nuclear forces.

The nuclei of tritium and helium-3 are generally believed to comprise an isotopic doublet and to have very similar ground-state wave functions. The structures of these two nuclei have been investigated by the method of elastic electron scattering[1,2] and by an inelastic electron-scattering experiment in which the scattered electron and an ejected proton were detected in coincidence.[3] The theoretical interpretation of these and other results, including the measurement of binding energies, the photodisintegration of helium-3,[4] muon capture by helium-3,[5] and slow-neutron capture by deuterium,[6] has led to a much improved knowledge of the wave function of the three-nucleon system and potentially provides a sensitive determination of this wave function. At the present time it is believed that the three-nucleon system has a probability of approximately 94% of being in a $^2S_{1/2}$ ground-state which is fully symmetric in the space coordinate of all three nucleons.[7] However, finite probabilities of other components of the ground-state wave function, notably a $^2S_{1/2}$ state of mixed spatial symmetry and three $^4D_{1/2}$ states, are suggested by the existence of an exchange magnetic moment and by the more rapid decrease with increasing momentum transfer of the charge form factor of helium-3 relative to the other three-nucleon form factors.

Inherent also in electron scattering experiments involving the three-body nuclei is a determination of the electromagnetic form factors of the neutron. However, theoretical difficulties have so far prevented the elastic-scattering measurements from being interpreted in this way. In the present experiment we have made measurements of the inelastic electron-tritium and electron–helium-3 cross sections, both at the maximum of the broad inelastic electron peak (the quasi-elastic region) and throughout the inelastic continuum. The ratio of the quasi-elastic electron-tritium and electron–helium-3 cross sections has been measured to a precision of about 3% for values of the four-momentum transfer q^2 equal to 2.5 and 4.6 F^{-2}. At $q^2=2.5$ F^{-2} the measurements are confined to a single electron scattering angle of 90°, but at $q^2=4.6$ F^{-2} measurements were made at angles of 40°, 75°, and 120°. Measurements of the entire inelastic continuum were obtained at $q^2=2.5$ F^{-2} for a scattering angle of 90°, and at $q^2=4.6$ F^{-2} for a scattering angle of 75°.

Previous measurements of inelastic cross sections from helium-3 have been reported by Buchanan *et al.*[8] The present results, however, are obtained with an improved experimental technique and also include measurements on tritium. The purpose of the present measurements is to increase the general wealth of experimental data available on the three-body nuclei and also to make a preliminary attempt to extract neutron form factors from these data. In particular we wish to investigate methods of obtaining form-factor information which are based on a comparison of data from both tritium and helium-3 and which therefore might be less dependent upon a detailed knowledge of the three-nucleon wave function.

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* Work supported in part by the U. S. Office of Naval Research, Contract Nonr 225(67).

† Present address: Princeton-Pennsylvania Accelerator, Princeton, New Jersey.

[1] H. Collard and R. Hofstadter, Phys. Rev. **131**, 416 (1963).

[2] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B57 (1965).

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[4] B. L. Berman, L. J. Koester, and J. H. Smith, Phys. Rev. **133**, B117 (1964).

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[6] N. T. Meister, T. K. Radha, and L. I. Schiff, Phys. Rev. Letters **12**, 509 (1964).

[7] B. F. Gibson and L. I. Schiff, Phys. Rev. **138**, B26 (1954).

[8] C. D. Buchanan *et al.*, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies* (Springer-Verlag, Berlin, 1966), Vol. I, p. 20.

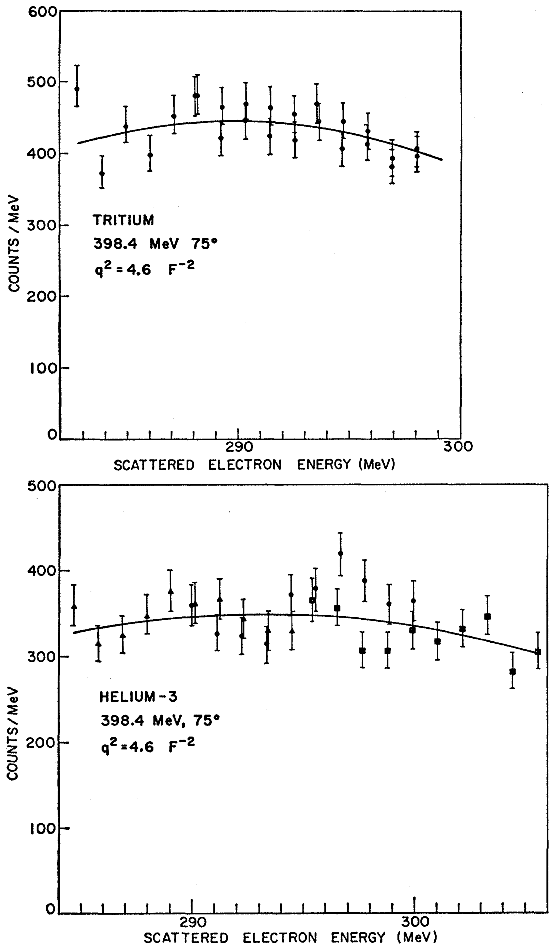

FIG. 1. An example of the measured electron-tritium and electron–helium-3 cross sections in the quasi-elastic region as a function of scattered electron momentum. The solid line represents a polynomial fit to the observed data points and was used to determine peak value of the cross section.

II. EXPERIMENTAL METHOD

The electron beam for this experiment was supplied by the Stanford Mark III linear accelerator. Thin-walled stainless-steel targets filled with tritium, helium-3, or hydrogen gas to a pressure of 3000 psi were used in the experiment and the scattered electrons were analyzed in momentum by a 72-in. double-focusing magnetic spectrometer. The scattered electrons were detected by an array of ten plastic scintillators located in the focal plane of the spectrometer and operated in coincidence with a single large Čerenkov counter. This experimental technique has been described in detail by Collard *et al.*[2]

III. RESULTS AND ANALYSIS

The Quasi-Elastic Data

Figure 1 shows typical data obtained at $q^2=4.6$ F^{-2} and an electron scattering angle of 75°. The experimental ratios of the quasi-elastic electron-tritium and electron–helium-3 cross section, uncorrected for radiative effects, are summarized in Table I.

TABLE I. The experimental ratios of the electron-tritium and electron–helium-3 quasi-elastic cross sections as a function of q^2 and scattering angle. Also shown are the ratios of the elastic electron-neutron and electron-proton cross sections which follow from the experimental ratios by means of Eq. (3). The errors attached are purely statistical.

q^2 (F^{-2})	E (MeV)	Angle (degrees)	$\dfrac{(d^2\sigma/dEd\Omega)_{\mathrm{H}^3}}{(d^2\sigma/dEd\Omega)_{\mathrm{He}^3}}$	$\dfrac{(d\sigma/d\Omega)_n}{(d\sigma/d\Omega)_p}$
2.5	248.8	90	0.619 ± 0.018	0.173 ± 0.028
4.6	667.5	40	0.579 ± 0.017	0.111 ± 0.025
			0.581 ± 0.017	0.114 ± 0.025
			0.582 ± 0.017	0.116 ± 0.025
4.6	398.4	75	0.655 ± 0.019	0.230 ± 0.032
			0.632 ± 0.018	0.193 ± 0.030
			0.604 ± 0.017	0.150 ± 0.027
4.6	296.4	120	0.675 ± 0.020	0.264 ± 0.035
			0.666 ± 0.017	0.249 ± 0.030

By analogy with the peak method of obtaining neutron form factors from measurements of the quasi-elastic electron-deuteron cross section, estimates of the neutron form factors can also in principle be made from the quasi-elastic electron-tritium and electron–helium-3 cross sections. In order to do this a theoretical description of the scattering process is necessary to allow for the scattering from the proton and the internal motion of the nucleons in the nucleus. At the present time such a theoretical treatment is not available and the peak method cannot be applied to either the electron-tritium or the electron–helium-3 cross sections. In this situation we can, however, make use of the expected similarity between the wave functions of tritium and helium-3 and use the comparison of the data from these two closely related nuclei in order to estimate the neutron form factors.

It is well known that the inelastic electron-deuteron cross section at the quasi-elastic peak, neglecting final-state interactions, can be related to the elastic electron-proton and electron-neutron cross sections by the

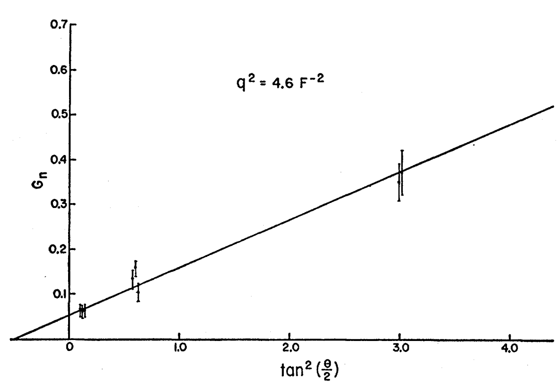

FIG. 2. The Rosenbluth plot for the neutron for $q^2=4.6$ F^{-2}.

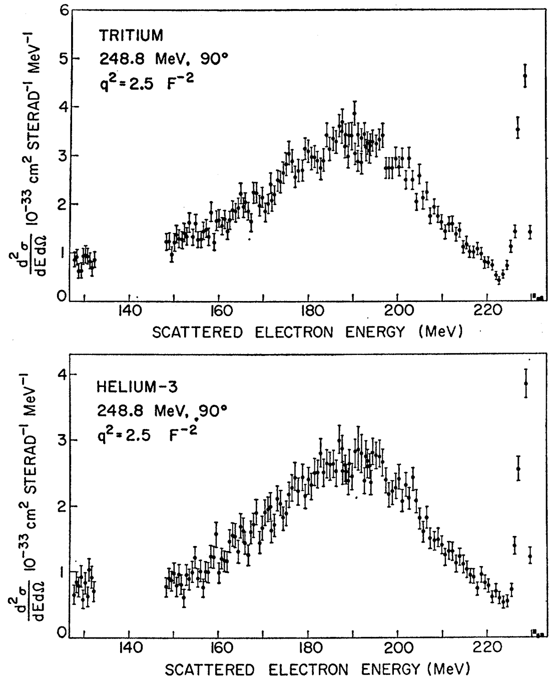

FIG. 3. The absolute inelastic electron-tritium and electron-helium-3 cross sections as a function of scattered electron momentum for $q^2=2.5$ F^{-2}.

following equation[9]:

$$\frac{d^2\sigma}{dEd\Omega}=M(p)\frac{m^2}{p(p^2+m^2)^{1/2}}\left\{\left(\frac{d\sigma}{d\Omega}\right)_n+\left(\frac{d\sigma}{d\Omega}\right)_p\right\}, \quad (1)$$

where $M(p)$ is a structure function dependent on the deuteron ground-state wave function, and essentially proportional to the probability of finding the neutron or proton at rest in the deuteron, p is the relative momentum of either of the nucleons in their final center-of-mass system, and m is the nucleon mass.

If we assume that the quasi-elastic electron-tritium and electron–helium-3 cross sections can be written in a form similar to Eq. (1), then the ratio R of these two cross sections can be written as follows:

$$R=\frac{(d^2\sigma/dEd\Omega)_{\mathrm{H}^3}}{(d^2\sigma/dEd\Omega)_{\mathrm{He}^3}}=\frac{2(d\sigma/d\Omega)_n+(d\sigma/d\Omega)_p}{(d\sigma/d\Omega)_n+2(d\sigma/d\Omega)_p}. \quad (2)$$

To obtain Eq. (2) we use the expected similarity between the wave functions of tritium and helium-3 and assume that the corresponding structure functions are sufficiently alike so that they cancel in forming the ratio. To the same approximation we assume that the radiative and final-state corrections to the quasi-elastic cross sections also cancel. Effectively, we are assuming that the three-nucleon ground-state wave function consists entirely of the symmetric $^2S_{1/2}$ state and that final-state interactions are negligible. Alternatively, we recognize the admixture of other components of the ground-state wave function and the existence of final-state interactions but expect that these effects will tend to cancel in forming the ratio.

[9] L. Durand, III, Phys. Rev. **123**, 1393 (1961).

TABLE II. The neutron form factors as a function of q^2. The expected form factors are taken from Eq. (14) of Ref. 5.

q^2 (F^{-2})	$(G_{En})^2$ Measured values	$-G_{Mn}/\mu_n$ Measured values	$(G_{En})^2$ Expected values	$-G_{Mn}/\mu_n$ Expected values
2.5	0.006±0.038		0.0	0.71
4.6	0.003±0.007	0.537±0.028	0.0	0.59

Using Eq. (2), the electron-neutron cross section can be expressed in terms of the electron-proton cross section and the experimental ratio R. The relation is as follows:

$$\frac{(d\sigma/d\Omega)_n}{(d\sigma/d\Omega)_p}=\frac{(1-2R)}{(R-2)}. \quad (3)$$

The values of the ratio of the electron-neutron and electron-proton cross section which follow from Eq. (3) are included in Table I. Absolute electron-neutron cross sections are obtained by normalization to the absolute electron-proton cross sections given by Janssens *et al.*,[10] and the Rosenbluth plot obtained for the neutron for q^2 equal to 4.6 F^{-2} is shown in Fig. 2.

The values we find for the charge and magnetic form factors of the neutron are given in Table II. These

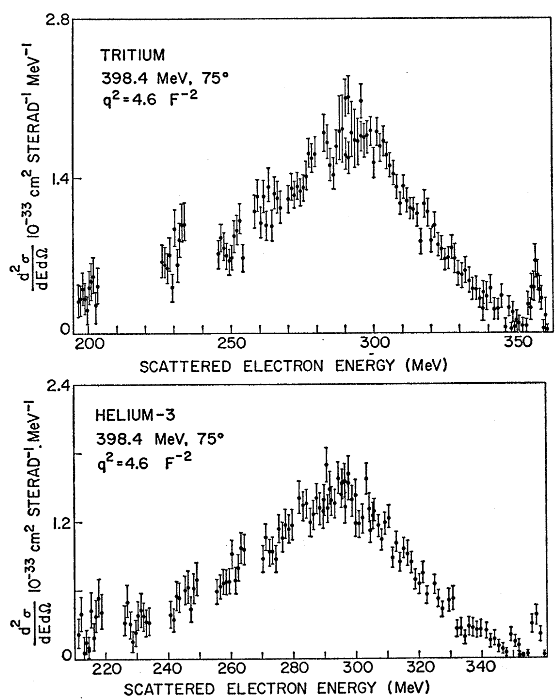

FIG. 4. The absolute inelastic electron-tritium and electron–helium-3 cross sections as a function of scattered electron momentum for $q^2=4.6$ F^{-2}.

[10] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, 922 (1966).

TABLE III. The total inelastic cross sections for tritium and helium-3 as a function of q^2 and scattering angle. Column 6 shows the correction factors applied to the measured cross sections, due to the experimental low-momentum cutoff.

q^2 (F^{-2})	Angle (degrees)	Target	σ_{observed} (cm^2/Sr)	% cutoff	Correction factor	$\sigma_{\text{corrected}}$ (cm^2/Sr)
2.5	90	H^3	$(1.64\pm0.05)\times10^{-31}$	30.0	1.043	$(1.71\pm0.09)\times10^{-31}$
		He^3	$(2.68\pm0.08)\times10^{-31}$	30.0	1.043	$(2.79\pm0.15)\times10^{-31}$
4.6	75	H^3	$(1.32\pm0.04)\times10^{-31}$	30.0	1.032	$(1.36\pm0.06)\times10^{-31}$
		He^3	$(1.82\pm0.05)\times10^{-31}$	25.0	1.058	$(1.92\pm0.13)\times10^{-31}$

values can be compared with the recent values given by inelastic scattering experiments in deuterium[11] which we also show in Table II. For q^2 equal to 2.5 F^{-2}, the ratio R was measured for only one value of the electron-scattering angle and separate determinations of the neutron form factors cannot be made. If the value of the magnetic form factor is taken to be that given by Hughes *et al.*,[11] then the electric form factor can be determined. The value obtained is given in Table II. These results show that values of the neutron form factors of the expected order of magnitude can be deduced from Eq. (2).

The Inelastic Continuum

Figures 3 and 4 show the inelastic electron spectra obtained from the tritium and helium-3 targets for $q^2=2.5$ F^{-2} and a scattering angle of 90°, and $q^2=4.6$ F^{-2} and a scattering angle of 75°. The momentum range includes both the elastic peak and the broad quasi-elastic peak and extends to a momentum approximately 30% below the position of the quasi-elastic peak.

The measurement of the inelastic continuum in the momentum region below the quasi-elastic peak was complicated by the production of large numbers of negative pions of the same momentum and at the same angle as the scattered electrons. The detection of such pions by the counter system was prevented by inserting a thickness of two radiation lengths of lead immediately in front of the plastic scintillation counters. Electrons initiated showers in the lead and gave rise to larger pulses in the scintillators, whereas the pulse height produced by the pions was unaffected and could be suppressed by suitable discrimination.

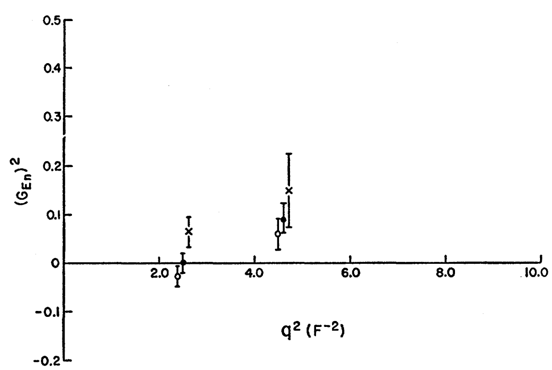

FIG. 5. The values of the square of the neutron electric form factor $(G_{En})^2$ as a function of q^2. The full circle points were obtained using Eq. (7), the crosses using Eq. (8) and the open circles using Eq. (9). The crosses and open circles have been shifted slightly in the q^2 direction for purposes of clarity.

In order to obtain the total inelastic cross sections, the experimental distributions shown in Figs. 3 and 4 were integrated numerically using an IBM 7090 computer. The integration was terminated at a momentum ranging from 25 to 30% below the position of the quasi-elastic peak and did not include the elastic peak. In the absence of a theoretical computation of the radiative corrections to the inelastic continua, an estimate of these corrections was obtained by using the radiative corrections appropriate to similar inelastic continua in electron-deuteron scattering. Likewise, an estimate of the correction necessary for that part of the experimental cross section which falls below the momentum cutoff even in the absence of radiative effects was also obtained through the use of the corrections found necessary in the analysis of the inelastic deuteron data. These corrections are summarized in Table III together with the total inelastic electron-tritium and electron–helium-3 cross sections. The errors shown on the cross sections include both the statistical error, which was always about 3%, and a systematic error due to the uncertainty in the two theoretical corrections mentioned above. The latter error was taken to be equal to the size of the corrections themselves.

In the past, neutron form factors have been obtained from measurements of total inelastic electron-deuteron cross sections by means of the so-called area method, in which the total inelastic cross section is related to the sum of the free electron-proton and electron-neutron cross sections. Such sum rules for inelastic electron-deuteron scattering have been given by Jankus[12] and Blankenbecler[13] and can be written in the form shown in Eq. (4).

$$\left(\frac{d\sigma}{d\Omega}\right)_d=(1+\Delta)\left\{\left(\frac{d\sigma}{d\Omega}\right)_n+\left(\frac{d\sigma}{d\Omega}\right)_p\right\}, \qquad (4)$$

where the parameter Δ is typically of the order of 0.02.

The use of Eq. (4) with $\Delta=0$ as a means of obtaining neutron form factors has recently been investigated in

[11] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965).

[12] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).

[13] R. Blankenbecler, Phys. Rev. **111**, 1684 (1958).

detail by Hughes *et al.*,[14] and it was concluded that the area method, when provided with a more precise theoretical calculation of the parameter Δ, is a potentially useful source of information on the neutron. Information on the neutron form factors is presumably also contained in measurements of the total inelastic electron-tritium and electron–helium-3 cross sections. No sum rules similar to Eq. (4), however, have yet been established for these nuclei. If, nevertheless, we assume that sum rules can be written for tritium and helium-3 as shown in Eqs. (5) and (6),

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{H}^3}=(1+\Delta_{\mathrm{H}^3})\left\{2\left(\frac{d\sigma}{d\Omega}\right)_n+\left(\frac{d\sigma}{d\Omega}\right)_p\right\}, \tag{5}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{He}^3}=(1+\Delta_{\mathrm{He}^3})\left\{\left(\frac{d\sigma}{d\Omega}\right)_n+2\left(\frac{d\sigma}{d\Omega}\right)_p\right\}, \tag{6}$$

then there are three ways in which the neutron form factors can be estimated. These are as follows:

(a) From Eq. (5), according to which

$$\frac{(d\sigma/d\Omega)_n}{(d\sigma/d\Omega)_p}=\frac{R_1-(1+\Delta_{\mathrm{H}^3})}{2}, \tag{7}$$

where

$$R_1=\frac{(d\sigma/d\Omega)_{\mathrm{H}^3}}{(d\sigma/d\Omega)_p},$$

(b) From Eq. (6), according to which

$$\frac{(d\sigma/d\Omega)_n}{(d\sigma/d\Omega)_p}=R_2-2(1+\Delta_{\mathrm{He}^3}), \tag{8}$$

where

$$R_2=\frac{(d\sigma/d\Omega)_{\mathrm{He}^3}}{(d\sigma/d\Omega)_p},$$

and

(c) From the ratio of Eqs. (5) and (6), according to which

$$\frac{(d\sigma/d\Omega)_n}{(d\sigma/d\Omega)_p}=\frac{(1+\Delta_{\mathrm{H}^3})-2R_3(1+\Delta_{\mathrm{He}^3})}{R_3(1+\Delta_{\mathrm{He}^3})-2(1+\Delta_{\mathrm{H}^3})}, \tag{9}$$

where

$$R_3=\frac{(d\sigma/d\Omega)_{\mathrm{H}^3}}{(d\sigma/d\Omega)_{\mathrm{He}^3}}.$$

If in the absence of any theoretical estimates we take Δ_{H^3} and Δ_{He^3} to be equal to zero, it might be expected that the reliability of methods (a) and (b) would be most seriously affected by this approximation. On the other hand, the effects of this approximation should tend to cancel as far as method (c) is concerned.

At each value of q^2 the total inelastic cross section has been measured for only one value of the scattering angle and therefore the form factors of the neutron cannot be determined independently. If the magnetic form factor is assigned the value given by Hughes *et al.*,[11] then the value of the square of the electric form factor found by each of the methods (a), (b), and (c) is shown in Fig. 5. These results indicate that all three methods yield values of the square of the electric form factor which are close to the expected value of this form factor. It is noticeable, however, that larger nonzero values of $(G_{En})^2$ are predicted at $q^2=4.6$ F^{-2} than at $q^2=2.5$ F^{-2}, which indicates perhaps that the approximations underlying these various methods become less correct as q^2 increases. There is also no obvious superiority of method (c), which involves the comparison of tritium and helium-3.

IV. CONCLUSIONS

In the present experiment precise measurements have been made of the ratio of the inelastic electron-tritium and electron–helium-3 cross sections at the quasi-elastic peak for low values of q^2. It is found that neutron form factors of the expected order of magnitude can be deduced from these ratios under the simplifying assumption that the quasi-elastic cross sections are proportional to the sum of the individual electron-nucleon cross sections. Precise measurements have also been made of the shape of the inelastic electron continuum in scattering from tritium and helium-3.

While these data are primarily intended to furnish additional experimental information which will be of use in determining the structure of the three-body nuclei, it is also pointed out that such data are a possible source of information on the structure of the neutron. Although too much weight is not attached to the estimates of the neutron form factors given in the present analysis, it is suggested that comparisons of electron scattering data from such closely related nuclei as tritium and helium-3 might provide the basis for determinations of the neutron form factors which are relatively insensitive to the details of nuclear structure.

ACKNOWLEDGMENTS

We gratefully acknowledge the assistance we have received from the staffs of the Los Alamos Scientific Laboratory and Stanford University in helping to overcome the special problems connected with the tritium and helium-3 targets. We also wish to thank R. Parks and M. Ryneveld for their invaluable assistance in setting up and operating the experimental equipment.

[14] E. B. Hughes, T. A. Griffy, R. Hofstadter, and M. R. Yearian, Phys. Rev. 146, 973 (1966).