The lack of detailed agreement between calculated and experimental values of $A(\theta)$ does indicate that the scattering matrix of this model is not exact. Two possible reasons for this are, (1) the neglected couplings in the $d+\mathrm{T}$ system as shown in Table I might drastically change the structure of the scattering matrix (as previously mentioned, we are in process of correcting this flaw), and (2) the coupling potentials that were used were central potentials that depended only upon $J$ and $P$, and this may be too simple to fit the data. It is interesting to note that the fact that $A(\theta)$ was too small at most angles seems to be a disease that this model shares with conventional direct-reaction theories. Tanifuji[27,29] has emphasized that since the $p+\mathrm{T}$ system exhibits strong spin polarization,[36] the neglect of the stripping process as shown in Fig. 11 is to neglect a spin-dependent potential. This neglect would not affect differential and total cross sections, but could make it impossible to simultaneously fit the spin polarizations and left-right asymmetry. Still, because of the complex energy behavior of the solutions to the coupled equations, it is proposed to first couple all the $d+\mathrm{T}$ states in Table I with central coupling potentials to get the best fit to experimental data before resorting to hard cores, spin-dependent stripping potentials, and nonlocal coupling potentials.

[36] T. A. Tombrello, Phys. Rev. **138**, B40 (1965).

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# Improved Solution to the Bethe-Faddeev Equations*

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An approximate analytic solution is derived for the Bethe-Faddeev three-body equations in nuclear matter. The solution is no more complicated than the original approximation proposed by Bethe, but it is more accurate and avoids the discontinuities that appear in the original solution. In a certain limiting case, the solution agrees with the one previously proposed by Moszkowski on the basis of a variational treatment.

## I. INTRODUCTION

RECENTLY, Bethe[1] has shown that the contribution of three-body correlations to the energy per particle of nuclear matter is given by

$$E^{(3)}/A=\rho^2\int g(r_{23})F(r_{23})d\tau_{23}. \tag{1}$$

In this formula, $\rho$ is the particle density, $g(r_{23})$ gives the radial dependence of the off-energy-shell effective interaction or $G$ matrix, and $F(r_{23})$ is defined by

$$F(r_{23})=\int\eta(r_{12})Z_1(r_{12},r_{13},r_{23})d\tau_1. \tag{2}$$

Here, $\eta(r_{12})$ is the on-energy-shell two-body "difference function," i.e., it is the difference between the unperturbed and the correlated wave functions for two particles. The three-body function $Z_1$, which is called $\Phi-\Psi^{(1)}$ by Bethe, satisfies the "Bethe-Faddeev equation"

$$Z_1(r_{12},r_{13},r_{23})=\eta(r_{12})+\eta(r_{13})-(1/e_{12})g_{12}Z_3(r_{12},r_{13},r_{23}) \\ -(1/e_{13})g_{13}Z_2(r_{12},r_{13},r_{23}), \tag{3}$$

with two similar equations for $Z_2$ and $Z_3$. The operator $g_{12}$, the off-energy-shell $G$ matrix for particles 1 and 2, obeys the equation

$$g_{12}=v_{12}-v_{12}(1/e_{12})g_{12}, \tag{4}$$

where $v_{12}$ is the nucleon-nucleon potential. The propagator $(1/e_{12})$ is given by

$$e_{12}=-\nabla_{12}{}^2+\gamma^2, \tag{5}$$

where $\gamma$ is a constant which is estimated by Bethe to be between 3.1 $\mathrm{F}^{-1}$ and 3.7 $\mathrm{F}^{-1}$, depending on the radius of the repulsive core in the two-body potential.

These equations were derived with the aid of three approximations, as discussed by Bethe.[1] (1) The initial momentum of each of the three interacting particles has been put equal to zero. (2) Reference-spectrum approximation: The energy spectrum for intermediate states is pure kinetic energy, and the exclusion principle is neglected for these states. (3) The dependence of $e_{12}$ on the momentum of particle 3 has been averaged out.

Simple and accurate methods are known for calculating the two-body functions $\eta(r)$ and $g(r)$.[2] The problem, therefore, is to solve (3) for the three-body function $Z_1$, and Bethe[1] has found a very simple approximate solu-

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] H. A. Bethe, Phys. Rev. **138**, B804 (1965).
[2] H. A. Bethe, B. H. Brandow, and A. G. Petschek, Phys. Rev. **129**, 225 (1963). This paper, and its authors, will be referred to as BBP.

tion. The purpose of this note is to derive an approximate solution which is more accurate, but no more complicated, then the one in Bethe's paper. The new solution for $Z_1$ also has the advantage of being a continuous function, while Bethe's original solution has discontinuities.

## II. APPROXIMATE EQUATIONS FOR $Z_1$

The basic idea of the present treatment is the same as that of Ref. 1, namely, to find a simple approximation for the effect of the operator $(1/e_{12})g_{12}$ on a three-body wave function. We consider first the two-body problem, restricting our attention to central two-body potentials which contain a hard core. Following Sec. 5 of BBP, we have

$$(1/e_{12})g_{12}\exp(i\mathbf{k}_0\cdot\mathbf{r}_{12})=\sum_L i^L(2L+1)(k_0r_{12})^{-1}\times\chi_L(r_{12})P_L(\hat{k}_0\cdot\hat{r}_{12}),\quad(6)$$

where $\chi_L=k_0r_{12}j_L(k_0r_{12})$ inside the core and $\chi_L\to 0$ as $r_{12}\to\infty$. According to Sec. 8 of BBP, if there is no attractive potential outside the core, it is a good approximation to write

$$\chi_L(r_{12})=k_0cj_L(k_0c)\exp[-\gamma(r_{12}-c)],\quad r_{12}>c,\quad(7)$$

where $c$ is the core radius. Putting these results for $\chi_L$ into Eq. (6) gives

$$\begin{aligned}1/e_{12})g_{12}\exp(i\mathbf{k}_0\cdot\mathbf{r}_{12})&=\zeta(r_{12})\exp(i\mathbf{k}_0\cdot\mathbf{r}_{12}),\quad r_{12}<c,\\&=\zeta(r_{12})\exp(i\mathbf{k}_0\cdot\hat{r}_{12}c),\quad r_{12}>c,\end{aligned}\quad(8)$$

where $\hat{r}_{12}$ is a unit vector in the direction of $\mathbf{r}_{12}$ and

$$\begin{aligned}\zeta(r_{12})&=1,\quad r_{12}<c\\&=(c/r_{12})\exp[-\gamma(r_{12}-c)],\quad r_{12}>c.\end{aligned}\quad(9)$$

Equation (8) is our basic approximation for $(1/e_{12})g_{12}$; it is exact for $r_{12}<c$. Since the solution for $\chi_L$ is dominated by the repulsive core and the large off-energy-shell value of $\gamma$ (as shown in Ref. 2), the presence of an attractive force outside the hard core can be taken into account by an appropriate change in the form of $\zeta(r_{12})$ for $r_{12}>c$.

The approximation made by Bethe[1] was to replace the right-hand side of Eq. (8) by $\zeta(r_{12})\exp(i\mathbf{k}_0\cdot\mathbf{r}_{12})$ for all $r_{12}$. This is exact inside the core but less accurate than (8) outside the core. According to Bethe's approximation, applying $(1/e_{12})g_{12}$ to any three-body function is the same as multiplying that function by $\zeta(r_{12})$. Thus the coupled integral equations (3) are reduced to linear algebraic equations for $Z_1$, $Z_2$, $Z_3$, and the solution is

$$Z_1=\frac{\eta_{12}(1-\zeta_{13})+\eta_{13}(1-\zeta_{12})-\eta_{23}(\zeta_{12}+\zeta_{13}-2\zeta_{12}\zeta_{13})}{1-\zeta_{12}\zeta_{13}-\zeta_{12}\zeta_{23}-\zeta_{13}\zeta_{23}+2\zeta_{12}\zeta_{13}\zeta_{23}},\quad(10)$$

where $\eta_{ij}$ and $\zeta_{ij}$ have been written for $\eta(r_{ij})$ and $\zeta(r_{ij})$. This solution, which is the one obtained by Bethe,[1] will be referred to as the "old solution." The old solution is exact when all three interparticle distances are less than $c$. In this case all $\eta$'s and $\zeta$'s are equal to unity, and the exact result is $Z_1=(1+\eta)/(1+2\zeta)=\frac{2}{3}$.

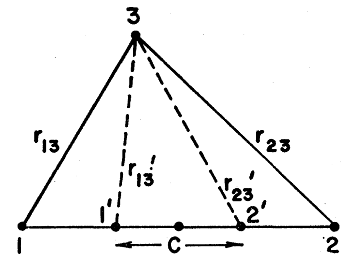


FIG. 1. Illustration of basic approximation for the operator $(1/e_{12})g_{12}$, as explained in the text.

To see what the improved approximation (8) implies for the effect of $(1/e_{12})g_{12}$ on a three-body function, we note that a typical Fourier component of this function can be written

$$\exp[i(\mathbf{k}_a\cdot\mathbf{r}_1+\mathbf{k}_b\cdot\mathbf{r}_2+\mathbf{k}_c\cdot\mathbf{r}_3)]=\exp(i\mathbf{k}_c\cdot\mathbf{r}_3+i\mathbf{K}_{ab}\cdot\mathbf{R}_{12})\exp(i\mathbf{k}_{ab}\cdot\mathbf{r}_{12}),\quad(11)$$

where $\mathbf{K}_{ab}$ and $\mathbf{k}_{ab}$ are total and relative momenta for particles 1 and 2, $\mathbf{R}_{12}$ and $\mathbf{r}_{12}$ are their center-of-mass and relative positions. Applying $(1/e_{12})g_{12}$ to this Fourier component gives, according to (8),

$$\begin{aligned}\exp(i\mathbf{k}_3\cdot\mathbf{r}_3+i\mathbf{K}_{ab}\cdot\mathbf{R}_{12})\zeta(r_{12})&\times\exp(i\mathbf{k}_{ab}\cdot\mathbf{r}_{12}),\quad r_{12}<c,\\&\times\exp(i\mathbf{k}_{ab}\cdot\hat{r}_{12}c),\quad r_{12}>c.\end{aligned}\quad(12)$$

So if $r_{12}<c$, we simply multiply the original Fourier component by $\zeta(r_{12})$, in agreement with Bethe's method. But if $r_{12}>c$, we obtain $\zeta(r_{12})$ times the original Fourier component evaluated at a new point in configuration space. At this new point, $\mathbf{r}_3$ and $\mathbf{R}_{12}$ are unchanged, but the magnitude of $\mathbf{r}_{12}$ has been reduced to $c$.

It is clear from these remarks that, for a three-body function $F(r_{12},r_{13},r_{23})$, Eq. (8) implies

$$\begin{aligned}(1/e_{12})g_{12}F(r_{12},r_{13},r_{23})&=\zeta(r_{12})F(r_{12},r_{13},r_{23}),\ r_{12}<c,\\&\quad\zeta(r_{12})F(c,r_{13}',r_{23}'),\ r_{12}>c,\end{aligned}\quad(13)$$

where

$$r_{13}'=[\tfrac{1}{2}(r_{13}^2+r_{23}^2)-\tfrac{1}{4}(r_{12}^2-c^2)+(c/2r_{12})(r_{13}^2-r_{23}^2)]^{1/2},\quad(14a)$$

$$r_{23}'=[\tfrac{1}{2}(r_{13}^2+r_{23}^2)-\tfrac{1}{4}(r_{12}^2-c^2)+(c/2r_{12})(r_{23}^2-r_{13}^2)]^{1/2}.\quad(14b)$$

The case $r_{12}>c$ is illustrated in Fig. 1. We start with the particles at the vertices of the large triangle and then apply $(1/e_{12})g_{12}$ to the function $F$. The result is $\zeta(r_{12})$ times the value taken by $F$ when particles 1 and 2 have moved to new positions 1' and 2', which are a distance $c$ apart. Particle 3 and the center of mass of 1 and 2 have been kept fixed. The distance from 1' to 3 is called $r_{13}'$ and is given by (14a), and similarly for $r_{23}'$.

TABLE I. Comparison of the exact numerical solution of Eq. (15), the analytic approximate solution (19), and the old solution (10)—evaluated for different values of $r_{12}$, $r_{13}$, and $r_{23}$.

| $r/c$ | Exact | Analytic approximation | Old | $r/c$ | Exact | Analytic approximation | Old |
|---|---|---|---|---|---|---|---|
| | $Z_1(r_{12}=r, r_{13}=c, r_{23}=c)$ | | | | $Z_1(r_{12}=c, r_{13}=c, r_{23}=r)$ | | |
| 1.0 | 0.667 | 0.667 | 0.625 | | 0.667 | 0.667 | 0.750 |
| 1.1 | 0.657 | 0.657 | 0.620 | | 0.686 | 0.686 | 0.761 |
| 1.2 | 0.646 | 0.646 | 0.613 | | 0.709 | 0.709 | 0.773 |
| 1.4 | 0.619 | 0.619 | 0.600 | | 0.761 | 0.761 | 0.804 |
| 1.6 | 0.588 | 0.588 | 0.578 | | 0.824 | 0.824 | 0.844 |
| 1.8 | 0.551 | 0.551 | 0.550 | | 0.897 | 0.897 | 0.900 |
| 2.0 | 0.514 | 0.514 | 0.514 | | 0.972 | 0.972 | 0.972 |
| | $Z_1(r_{12}=r, r_{13}=r, r_{23}=c)$ | | | | $Z_1(r_{12}=r, r_{13}=c, r_{23}=r)$ | | |
| 1.0 | 0.667 | 0.667 | 0.500 | | 0.667 | 0.667 | 0.750 |
| 1.1 | 0.638 | 0.643 | 0.478 | | 0.681 | 0.679 | 0.761 |
| 1.2 | 0.599 | 0.608 | 0.453 | | 0.700 | 0.696 | 0.773 |
| 1.4 | 0.495 | 0.507 | 0.401 | | 0.753 | 0.747 | 0.804 |
| 1.6 | 0.361 | 0.369 | 0.312 | | 0.819 | 0.815 | 0.844 |
| 1.8 | 0.207 | 0.209 | 0.200 | | 0.896 | 0.896 | 0.900 |
| 2.0 | 0.056 | 0.056 | 0.056 | | 0.972 | 0.972 | 0.972 |
| | $Z_1(r_{12}=r, r_{13}=r, r_{23}=r)$ | | | | $Z_1(r_{12}=1.4c, r_{13}=r, r_{23}=1.4c)$ | | |
| 1.0 | 0.667 | 0.667 | 0.667 | 0.0 | 0.755 | 0.747 | 0.804 |
| 1.1 | 0.660 | 0.665 | 0.651 | 0.5 | 0.754 | 0.747 | 0.804 |
| 1.2 | 0.641 | 0.654 | 0.628 | 1.0 | 0.753 | 0.747 | 0.804 |
| 1.4 | 0.552 | 0.576 | 0.550 | 1.18 | 0.650 | 0.667 | 0.655 |
| 1.6 | 0.400 | 0.418 | 0.409 | 1.36 | 0.569 | 0.592 | 0.566 |
| 1.8 | 0.214 | 0.217 | 0.217 | 1.72 | 0.443 | 0.459 | 0.443 |
| 2.0 | 0.056 | 0.056 | 0.056 | 2.08 | 0.358 | 0.362 | 0.347 |

Using the approximation (13) allows us to write Eqs. (3) in the form

$$Z_1(r_{12},r_{13},r_{23})=\eta_{12}+\eta_{13}-\zeta_{12}Z_3-\zeta_{13}Z_2\,, \tag{15}$$

where $Z_3=Z_3(r_{12},r_{13},r_{23})$ for $r_{12}<c$ and $Z_3(c,r_{13}',r_{23}')$ for $r_{12}>c$, $Z_2=Z_2(r_{12},r_{13},r_{23})$ for $r_{13}<c$ and $Z_2(r_{12}'',c,r_{23}'')$ for $r_{13}>c$, and $r_{12}''$ and $r_{23}''$ are defined in analogy with $r_{13}'$ and $r_{23}'$. Similar equations hold for $Z_2$ and $Z_3$. Equations (15) are our approximation to Eqs. (3). They have also been derived, in a similar way, by M. W. Kirson.[3] We will show that to a good approximation these equations can be solved analytically.

## III. SOLUTION OF THE EQUATIONS FOR $Z_1$

When all three interparticle distances are less than $c$, Eqs. (15) are exact and are identical to Bethe's equations. The solution in this region is known to be $Z_1=Z_2=Z_3=\frac{2}{3}$.

Next consider the case $r_{12}>c$, $r_{13}<c$, $r_{23}<c$. It is easy to see that $r_{13}'<c$ and $r_{23}'<c$. Hence $Z_3(c,r_{13}',r_{23}')$, which multiplies $\zeta_{12}$ in Eqs. (15), is equal to $\frac{2}{3}$ and these equations take the form

$$\begin{aligned} Z_1&=\eta_{12}+\eta-\tfrac{2}{3}\zeta_{12}-\zeta Z_2\,,\\ Z_2&=\eta_{12}+\eta-\tfrac{2}{3}\zeta_{12}-\zeta Z_1\,,\\ Z_3&=2\eta-\zeta Z_2-\zeta Z_1\,,\end{aligned} \tag{16}$$

[3] M. W. Kirson (private communication).

where we have put $\eta_{13}=\eta_{23}=\eta$ and $\zeta_{13}=\zeta_{23}=\zeta$ since $r_{13}$ and $r_{23}$ are less than $c$. Solving and then letting $\eta\to 1$ and $\zeta\to 1$, we obtain, for $r_{12}>c$, $r_{13}<c$, $r_{23}<c$,

$$Z_1=Z_2=(\eta+\eta_{12}-\tfrac{2}{3}\zeta_{12})/(1+\zeta)\to \tfrac{1}{2}(1+\eta_{12}-\tfrac{2}{3}\zeta_{12})\,, \tag{17a}$$

$$Z_3=2[\eta-\zeta(\eta_{12}-\tfrac{2}{3}\zeta_{12})]/(1+\zeta)\to 1-\eta_{12}+\tfrac{2}{3}\zeta_{12}\,. \tag{17b}$$

When two or more of $(r_{12},r_{13},r_{23})$ are larger than $c$, we are forced to solve the equations numerically. Formula (15) gives $Z_1(r_{12},r_{13},r_{23})$ in terms of $Z$'s at different points in space, and these $Z$'s in turn depend on $Z$'s at still different points. Thus one is led to a chain of equations; and this chain terminates when all the $Z$'s on the right-hand side of (15) can be evaluated from the analytic solutions, which are valid when two or more of $(r_{12},r_{13},r_{23})$ are less than $c$. Except for certain special cases, the chain of equations will terminate after a finite number of steps because, as can be seen from Fig. 1, in successive steps the particles tend to get closer together.

An example of a case in which the chain of equations does not terminate occurs when the three particles initially lie on a single straight line. Even after a large number of steps, the right-hand side of (15) will contain $Z$'s that are to be evaluated at points arbitrarily close to, but still outside, the region of applicability of the analytic solutions.

We can obtain an analytic approximation to the solution of (15), and thus avoid numerical computations, in the following way. We approximate Eqs. (15) by the simpler equations

$$Z_1(r_{12},r_{13},r_{23})=\eta_{12}+\eta_{13}-\zeta_{12}Z_3-\zeta_{13}Z_2\,, \tag{18}$$

where $Z_3=Z_3(r_{12},r_{13},r_{23})$ for $r_{12}<c$ and $Z_3(c,r_{13},r_{23})$ for $r_{12}>c$, and $Z_2=Z_2(r_{12},r_{13},r_{23})$ for $r_{13}<c$ and $Z_2(r_{12},c,r_{23})$ for $r_{13}>c$— and analogous equations hold for $Z_2$ and $Z_3$. The solution can be obtained analytically and is given by

$$\begin{aligned} Z_1(r_{12},r_{13},r_{23})=&\,\eta_{12}(1-\zeta_{13}+\tfrac{1}{2}\zeta_{13}\zeta_{23})\\ &+\eta_{13}(1-\zeta_{12}+\tfrac{1}{2}\zeta_{12}\zeta_{23})-\eta_{23}(\zeta_{12}+\zeta_{13}-\zeta_{12}\zeta_{13})\\ &+\zeta_{12}\zeta_{13}+\tfrac{1}{2}(\zeta_{12}+\zeta_{13})\zeta_{23}-\tfrac{4}{3}\zeta_{12}\zeta_{13}\zeta_{23}\,. \end{aligned} \tag{19}$$

That this is the correct solution can be seen by substituting it, and the two similar formulas for $Z_2$ and $Z_3$, into Eq. (18) and using the fact that $\eta(r)=\zeta(r)=1$ whenever $r\leqslant c$. Equation (19) is our main result. We will first show it to be a good approximation to the solution of (15) and then compare it with the old solution (10).

Expression (19) reduces to simpler forms when at least one of $r_{12}$, $r_{13}$, $r_{23}$ is less than $c$ because some of the $\eta_{ij}$ and $\zeta_{ij}$ can be put equal to unity. If two or more of the variables are less than $c$, then (19) agrees with the analytic solution of (15).

To see whether the analytic approximation (19) is an accurate solution of (15) in other regions of configuration space, the chain of equations arising from (15) has been solved numerically and this exact solution has been compared with Eq. (19). All the numerical work has been carried out for case B of Bethe's paper,[1] i.e., we take $\eta$ and $\zeta$ to be

$$\eta(r)=([2.2-r/c]/1.2)^2, \quad 1\leqslant r/c\leqslant 2.2, \tag{20a}$$

$$\zeta(r)=([1.9-r/c]/0.9)^2, \quad 1\leqslant r/c\leqslant 1.9. \tag{20b}$$

Inside the core, $\eta=\zeta=1$, while $\eta$ vanishes for $r>2.2c$ and $\zeta$ vanishes for $r>1.9c$.

Some of the results are shown in Table I. The analytic approximation usually differs by less than 2% from the numerical solution, although the difference sometimes gets as large as 4% when all three interparticle distances are greater than $c$. The old solution is usually much less accurate, especially at small distances.

The old solution also suffers from discontinuities. For example, it gives $Z_1(c,c,r_{23})=(1-\eta_{23})/(1-\zeta_{23})$ for $r_{23}>c$, and the limit of this as $r_{23}$ approaches $c$ from above is $\frac{3}{4}$ in case B, while we know that $Z_1(c,c,c)=\frac{2}{3}$. Our analytic approximation, on the other hand, is clearly a continuous function as long as $\eta$ and $\zeta$ are continuous.

Our approximate solution for $Z_1$ is related to Moszkowski's treatment of the three-body problem.[4] If one constructs the total three-body wave function $\Psi$ according to Moszkowski's equation,

$$2(1-\Psi)=Z_1+Z_2+Z_3, \tag{21}$$

the result, in the special case $\eta=\zeta$, is $\Psi=(1-\eta_{12})\times(1-\eta_{13})(1-\eta_{23})$. This is precisely the formula suggested by Moszkowski on the basis of a variational treatment.

TABLE II. Values of $F(r_{23})$ in units of $4\pi c^3$. The exact values were obtained by numerical solution of Eq. (15), the analytic approximation is from Eq. (19).

| $r_{23}/c$ | Exact solution | Analytic approximation |
|---|---|---|
| 0.25 | 0.552 | 0.558 |
| 0.50 | 0.541 | 0.547 |
| 0.75 | 0.525 | 0.530 |
| 1.0 | 0.509 | 0.509 |
| 1.2 | 0.543 | 0.543 |
| 1.4 | 0.583 | 0.584 |
| 1.6 | 0.629 | 0.630 |
| 1.8 | 0.675 | 0.678 |
| 2.0 | 0.717 | 0.719 |

[4] S. A. Moszkowski, Phys. Rev. **140**, B283 (1965).

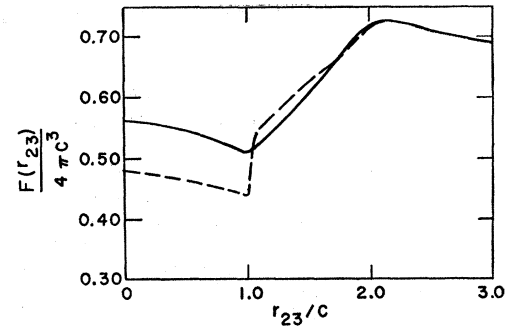


FIG. 2. The function $F(r_{23})$ plotted against $r_{23}/c$. The solid curve is obtained by using the analytic approximation (19) for $Z_1$. The dashed curve results when $Z_1$ is calculated from the "old solution" (10).

The three-body energy comes out somewhat more repulsive with our solution than with the old one. To illustrate this, the function $F(r_{23})$, which occurs in expression (1) for the energy, has been calculated and the results are plotted in Fig. 2. The dashed line was obtained by using the old solution for $Z_1$; the solid line results from our formula (19). Inside the core, where $g(r_{23})$ is repulsive, the solid curve lies about 15% higher than the dashed curve. The resulting change in three-body energy, however, will be less than 1 MeV per particle. Bethe's conclusion that the three-body energy is small of course remains unaltered.

One more indication that expression (19) is an adequate solution of (15) is provided by the values of $F(r_{23})$ shown in Table II. The exact numerical solution of (15) was used to obtain the first column; the second column was calculated by use of the analytic approximation (19). The excellent agreement shows that the small errors in the analytic approximation for $Z_1$ have been largely averaged out by the integration that occurs in calculating $F(r_{23})$ by use of Eq. (2).

## IV. CONCLUSION

The analytic approximation (19) is just as simple as the old formula (10), and it has the advantages of being more accurate and avoiding discontinuities. Also, Kirson[3] has found it to be superior to the old solution as a starting point for more detailed calculations in which the effects of exclusion and nonzero hole momenta are taken into account.

## ACKNOWLEDGMENTS

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