Mixing Ratios in the Ground-State Decays of the 3.68- and 3.85-MeV Levels of C^{13}†

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The mixing ratios for two gamma-ray transitions in C^{13} have been measured: For the transition $3.68 \rightarrow 0$, $x(E2/M1) = -(0.096_{-0.021}^{+0.030})$ while for the transition $3.85 \rightarrow 0$, $x(E3/M2) = +(0.12 \pm 0.03)$. The phase convention quoted by Poletti and Warburton is used. The phase of the mixing ratio for the $3.68 \rightarrow 0$ transition is compared with that for the mirror transition in N^{13} and with the phase of the mixing ratio for the transitions between $\frac{3}{2}^-$ and $\frac{1}{2}^-$ hole states in N^{15} and O^{15}. The $3.85 \rightarrow 0$, $\frac{5}{2}^+ \rightarrow \frac{1}{2}^-$ transition is also discussed and compared with the $\frac{5}{2}^+ \rightarrow \frac{1}{2}^-$ transitions in mass 15.

I. INTRODUCTION

THE mass-13 nuclei have provided a number of interesting tests for independent-particle-model (IPM) calculations both completely within the $1p$ shell[1] and with excitation of one particle to the $(2s,1d)$ shell.[2] In this respect the gamma-ray decay of the $\frac{5}{2}^+$ level at 3.85 MeV in C^{13} is an interesting one: There is an $E1$ decay to the $\frac{3}{2}^-$ level at 3.68 MeV (24%), an $E2$ decay to the $\frac{1}{2}^+$ level at 3.09 MeV (0.9%), and a decay which must be predominantly $M2$ to the $\frac{1}{2}^-$ ground state (75%).[3,4] (A recent investigation by Gorodetzky *et al.*[5] gives the first and last percentages as 37±4% and 62±4%.) The mean lifetime of the 3.85-MeV level has been measured by Simpson, Clark, and Litherland[6] as (7.5_{-2}^{+3}) psec. A simple single-particle estimate of the $E3$ transition rate to the ground state indicated that a careful measurement of the angular distribution of the 3.85-MeV gamma rays de-exciting the level would probably yield an experimental lower limit for the strength of the possible $E3$ component in this transition and might be able to give a value for this strength. A knowledge of this $E3$ rate is of some interest since to first order (neutron jumping) it is zero.

A knowledge of the phase of the mixing ratio for the $3.68 \rightarrow 0$ transition in C^{13} (and the mirror $3.51 \rightarrow 0$ transition in N^{13}) is of some importance since this $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transition is, in the LS-coupling limit,[1] between the two lowest ^{22}P states of the s^4p^9 configuration. This transition is therefore quite similar to the two lowest s^4p^{11} $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions in mass 15 since these also, in LS-coupling notation, are transitions between two ^{22}P states. The mass-15 transitions in question have usually been taken to be the N^{15} $6.32 \rightarrow 0$ and O^{15} $6.18 \rightarrow 0$ transitions. However, at the time this work was begun it had been stated[7] that the phases of the $E2/M1$ mixing ratios of these two transitions are both opposite to those expected for the s^4p^{11} $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions. This had prompted Rose and collaborators[7,8] to propose a model in which the O^{15} 6.18- and N^{15} 6.32-MeV states are taken to be mixtures of a $p_{1/2}$ hole coupled to the 2^+ 6.92-MeV state of O^{16} and of a $p_{3/2}$ hole coupled to the O^{16} closed core, with the former dominant. It was felt desirable to explore the consequences of this model. Since the above model was invoked to explain the absolute phases of the $E2/M1$ mixing ratios of the two $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ mass-15 transitions we can cast some light on its validity by a comparison of the relative phases of the $E2/M1$ mixing ratios of $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions in question: two in mass 13 and two in mass 15. In fact this comparison of relative phases has been extended to cover some transitions in mass 11 and mass 14 also.[9] Preliminary results of this comparison gave quite strong evidence that the phases of $E2/M1$ mixing ratios of the N^{15} $6.32 \rightarrow 0$ and O^{15} $6.18 \rightarrow 0$ transitions are those expected for the s^4p^{11} $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions. It was therefore quite pleasing when it was found[10] that, after all, the absolute phases for these two transitions were those expected for s^4p^{11} $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$. In the present paper we report on the measurement of the $E2/M1$ ratio of the C^{13} $3.68 \rightarrow 0$ transition. A full comparison of this result with theory will be given in a subsequent paper.[9]

II. EXPERIMENTAL METHOD

A. The Gamma-Gamma Correlation Experiment

We consider first the mixing ratio of the C^{13} $3.68 \rightarrow 0$ transition. The method used is that of measuring the angular correlations between the two gamma rays in the C^{13} $3.85 \rightarrow 3.68 \rightarrow 0$ cascade following population of the 3.85-MeV level via the $C^{12}(d,p)C^{13}$ reaction. Since

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[1] See, e.g., A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954); A. M. Lane, *ibid.* **A66**, 977 (1953); **A68**, 197 (1955). More recent calculations have been made by S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965) and D. Amit and A. Katz, Nucl. Phys. **58**, 388 (1964).
[2] E.g., T. Sebe, Prog. Theoret. Phys. (Kyoto) **30**, 290 (1963).
[3] R. J. Mackin, W. R. Mills, and J. Thirion, Phys. Rev. **102**, 802 (1956).
[4] R. E. Pixley, J. V. Kane, and D. H. Wilkinson, Phys. Rev. **120**, 943 (1960).
[5] S. Gorodetzky, R. M. Freeman, A. Gallmann, and F. Haas, Phys. Rev. **149**, 801 (1966).
[6] J. J. Simpson, M. A. Clark, and A. E. Litherland, Can. J. Phys. **40**, 769 (1962).
[7] H. J. Rose and J. S. Lopes, Phys. Letters **18**, 130 (1965).
[8] H. J. Rose, J. S. Lopes, and W. Greiner, Phys. Letters **19**, 686 (1966).
[9] A. R. Poletti, E. K. Warburton, and D. Kurath (to be published).
[10] H. J. Rose and D. Brink (to be published); H. J. Rose and J. S. Lopes, Phys. Letters **22**, 601 (1966).

this reaction results in alignment of the 3.85-MeV level the correlation is a triple one. We use the procedure and method of analysis designated as Method I by Litherland and Ferguson.[11] The measured[6] lifetime of the $\frac{5}{2}^+$, C^{13} 3.85-MeV level, together with the known branching ratio implies that for an enhancement of 100 times over the Weisskopf[12] estimate for a 170-keV $M2$ transition in C^{13} the $M2/E1$ mixing ratio of the $3.85 \rightarrow 3.68$ transition is limited by $|x| < 0.01$. For the purposes of the measurements to be described this means that we can take the 170-keV transition as being pure $E1$. This considerably simplifies the problem of measuring the $E2/M1$ mixing ratio for the $3.68 \rightarrow 0$ transition. By setting one gamma-ray detector at 90° with respect to the beam and recording coincidences with a second detector rotating about the target in the plane defined by the first detector and the beam, two of the seven geometries listed by Litherland and Ferguson[11] were determined. Two further geometries were determined simultaneously by recording coincidences between the movable detector and a third fixed detector placed at 90° to the beam and to the first fixed detector. The information obtained from these four geometries was sufficient to determine the mixing ratio of the $3.68 \rightarrow 0$ transition together with the population parameters of the 3.85-MeV state.

A deuteron beam of energy 2.51 MeV was chosen together with an unenriched C^{12} target 40-keV thick to the incident deuterons in order to take advantage of the high yield from the $C^{12}(d,p_3)C^{13}$ resonance[13] at 2.49 MeV, which has a center-of-mass resonance[13] width of 40 ± 3 keV. The target was deposited on a 0.001-in. thick molybdenum backing since this was found to give the least background under the 170-keV photopeak and negligible absorption of this gamma ray. Previous work by Chase, Johnson, and Warburton[13] had indicated that the alignment of the residual 3.85-MeV excited state of C^{13} at this bombarding energy was particularly strong. These workers have also shown that $W(0°)/W(90°)$ for the 170-keV line remained constant over the resonance so that any uncertainties in the bombarding energy or target thickness were unimportant. The coincidences between each of the fixed counters and the movable one were recorded simultaneously and routed into the first halves of two separate 400-channel analyzers. A sample of each of the random coincidence spectra was simultaneously routed into the second half of each analyzer. The reaction was monitored by, and the angular distributions normalized against, the 0.170–3.68 coincidences between the two fixed crystals. Because of variations in the beam, target inhomogeneities and buildup of carbon on the target this was found to be more reliable than monitoring on the beam or on the singles counting rate in one of the fixed crystals. A least-squares fit to each of the four angular correlations as the expansion $W(\theta°) = A_0 + A_2 P_2(\cos\theta)$ gave the coefficients A_2/A_0 listed in Table I. A_4/A_0 may differ only negligibly from zero since the intermediate state (3.68 MeV) is $J = \frac{3}{2}$ and the $3.85 \rightarrow 3.68$ transition is essentially pure dipole as discussed above. Further analysis was carried out by fitting the four correlations simultaneously and determining χ^2 as a function of the values of the best fitting population parameters of the 3.85-MeV level, while stepping values of the $3.68 \rightarrow 0$ mixing ratio from $-\infty$ to $+\infty$. This was accomplished with the aid of a computer program written by W. W. True and modified by us so that the input data for the different geometries were automatically normalized. For all four geometries which we considered, the end points of each correlation are common to another correlation, e.g., geometry I, $\theta_1 = 0°$ gives the same point as geometry VII, $\theta_7 = 0°$. There are four such common points for the four geometries used and three unknowns to find; namely, the normalization constants for three of the geometries. These three normalization constants were found by the method of least-squares. In order to utilize all the measured points and to take advantage of the known analytical form of the angular correlations, i.e., $W(\theta) = A_0 + A_2 P_2(\cos\theta)$ for all four geometries, the least-squares fit was performed in terms of the A_0 and A_2 coefficients and their uncertainties as follows: Let a_p, b_p, c_p, d_p, be the values of $W(\theta = p)$ generated by this fitting for the angles specified by p where a, b, c, and d refer to the geometries I, VII, VI, and II, respectively. Let $p = 0$ refer to $\theta = 0°$, $p = 1$, to $\theta = 90°$ and let α_0, α_1, β_0, etc., be the experimental errors for the points specified by $p = 0$, 1 so that to normalize the geometries it will be found necessary to minimize the following expression,

$$\frac{(a_0 - b_0 x)^2}{\alpha_0^2 + \beta_0^2 x^2} + \frac{(b_1 x - c_1 y)^2}{\beta_1^2 x^2 + \gamma_1^2 y^2} + \frac{(c_0 y - d_0 z)^2}{\gamma_0^2 y^2 + \delta_0^2 z^2} + \frac{(d_1 z - a_1)^2}{\delta_1^2 z^2 + \alpha_1^2},$$

where x, y, z are the normalization constants for geometries VII, VI, and II, respectively, with the normalization constant for geometry I fixed at unity. This is a nonlinear problem—however, the values of the denominators do not depend sensitively on the values of

TABLE I. Angular correlation coefficients determined from the triple gamma-gamma correlation measurement of the $3.85 \rightarrow 3.68 \rightarrow 0$ cascade gamma rays in C^{13}. The coefficients have not been corrected for the finite solid angle subtended by the detectors. In subsequent analysis these were, however, taken as $Q_2 = 0.955$ for the 170-keV transition and $Q_2 = 0.965$ for the 3.68-MeV transition.

Geometry	A_2/A_0
I	-0.318 ± 0.005
II	-0.214 ± 0.006
VI	-0.185 ± 0.006
VII	-0.283 ± 0.007

[11] A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).

[12] D. H. Wilkinson, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 862 ff.

[13] L. F. Chase, Jr., R. G. Johnson, and E. K. Warburton, Phys. Rev. **120**, 2103 (1960).

x, y, z near their values which minimize the above expression. Hence in the denominators we can use the values of x, y, z which are given by reference to geometry I, i.e., by an approximate (unweighted) normalization. The problem is now linear. Differentiation with respect to x, y, z in turn and equating each expression to zero gives three linear algebraic equations which are easily solved. It was found that the normalization of the data had a large effect on the value of χ^2 at the minimum, but did not change the position of the minimum as a function of $\tan^{-1}x$. A preliminary hand normalization gave $\chi^2 \approx 3$ at the minimum, while the computer normalized data yielded $\chi^2 = 1.2$ at the minimum.

A further modification of the program ensured that only solutions for which all the population parameters were positive or zero were considered. The results of this least-squares fitting program are illustrated in Fig. 1 which gives χ^2 versus arctan x for the $3.68 \rightarrow 0$ transition. This χ^2 curve has two solutions, the smaller being $x = -(0.096_{-0.021}^{+0.030})$ where the limits are those of the 1% confidence level. The larger value of x can be rejected since measurements[14] of the angular correlation of the internal pairs associated with the $3.68 \rightarrow 0$ transition have shown that the $3.68 \rightarrow 0$ transition must be mainly $M1$ ($x^2 < 0.25$ to two standard deviations). The simultaneous best fitting curves are plotted in Fig. 2 along with the normalized experimental data. It is seen

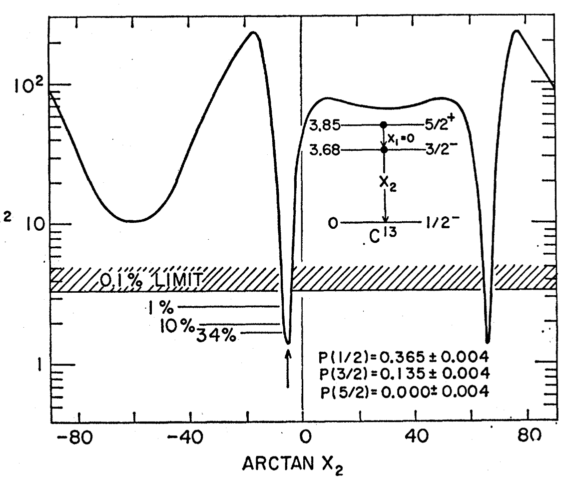

FIG. 1. Plot of χ^2 versus arctan x_2 for a simultaneous fit to the gamma-gamma triple-correlation data on the $3.85 \rightarrow 3.68 \rightarrow 0$ cascade transitions in C^{13}. As indicated in the insert, x_2 is the $E2/M1$ mixing ratio in the $3.68 \rightarrow 0$ transition, while x_1 refers to possible $M2/E1$ mixing in the $3.85 \rightarrow 3.68$ transition. Justification for the assumption $x_1 = 0$ is discussed in the text. χ^2 is normalized so that its expectation value is unity. The values of χ^2 corresponding to confidence limits of 34%, 10%, 1%, and 0.1% are marked. The solution for x_2 corresponding to arctan $x_2 \sim 65°$ is eliminated by the results of Ref. 14. Values of the population parameters $P(\alpha)$ determined from this fitting procedure are given for the allowed solution corresponding to arctan $x_2 = -5.5°$.

that the four distributions are well fitted. The population parameters for the 3.85-MeV state were also obtained. Their importance will be discussed in connection with the gamma-ray distributions discussed below.

B. The Gamma-Ray Distribution Experiment

It is not always necessary to detect the reaction product at 0° or 180° with respect to the beam in order to apply the Method II analysis of Litherland and Ferguson.[11] It is only necessary to have axial symmetry with respect to the z axis (e.g., the beam axis).[15] In the present case it was already known that even with the reaction product unobserved the 3.85-MeV state was strongly aligned.[13] It was decided, therefore, to use a three-crystal pair spectrometer to determine the angular distribution of the 3.85-MeV gamma ray since in this manner it could be reasonably well-resolved from the 3.68-MeV gamma ray and the accuracy of spectrum stripping is greater than for singles spectra. Figure 3 shows two typical spectra, one obtained at 0° and the other at 90°. A Gaussian least-squares fitting code[16] was used to determine the yield at each angle of the 3.09-, 3.68-, and 3.85-MeV gamma rays. The solid lines drawn in Fig. 3 are the best fitting curves obtained by this code. It will be noticed that the fit is not very good at the very top of each peak. However, this has previously been shown[15] to make an error of approximately 2.5% in the over-all area of each peak and since the error is in the same direction in all three cases the effect on the extracted angular distributions will be much less. Since the 3.09-MeV gamma ray is isotropic, the yields of the 3.68- and 3.85-MeV gamma rays were normalized to the yield of the 3.09-MeV gamma ray at each angle. The target was geometrically centered over the center of rotation of the three-crystal spectrometer beforehand. The accuracy of this centering was investigated by taking points at 0°, ±15°, ±30°, 37.5°, 45°, 52.5°, ±60°, ±75°, ±90°. It was found that there was no clear evidence for any mis-centering or for a deviation of the angular scale from true zero. Evidence for the latter statement was obtained by determining the value of χ^2 for least-squares fits of the angular distributions of the 3.68- and 3.85-MeV gamma rays to $A_0 + A_2P_2[\cos(\theta + \Delta\theta)] + \cdots$ as a function of $\Delta\theta$ where $\Delta\theta$ is the deviation of the zero of the angular scale from true zero. For the 3.85-MeV gamma-ray angular distribution χ^2 was a minimum at $\Delta\theta = 0°$ ($\chi^2 = 0.75$) and crossed the 10% confidence limits ($\chi^2 = 1.60$) at $\Delta\theta = -2.2°$ and 1.7°. For the 3.68-MeV angular distribution χ^2 varied slowly over this range increasing monotonically from 1.18 to 1.67 as $\Delta\theta$ increased from $-2.2°$ to $+1.7°$. We conclude that $\Delta\theta = (0_{-2.2}^{+1.7})°$. Furthermore, it was found that $\Delta\theta$ could have been as great as ±3° without

[14] E. K. Warburton, D. E. Alburger, A. Gallmann, P. Wagner, and L. F. Chase, Jr., Phys. Rev. 133, B42 (1964); and unpublished data.

[15] E. K. Warburton, J. W. Olness, D. E. Alburger, D. J. Bredin, and L. F. Chase, Jr., Phys. Rev. 134, B338 (1964).

[16] P. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. 33, 70 (1962). The use of this code is discussed in Ref. 15.

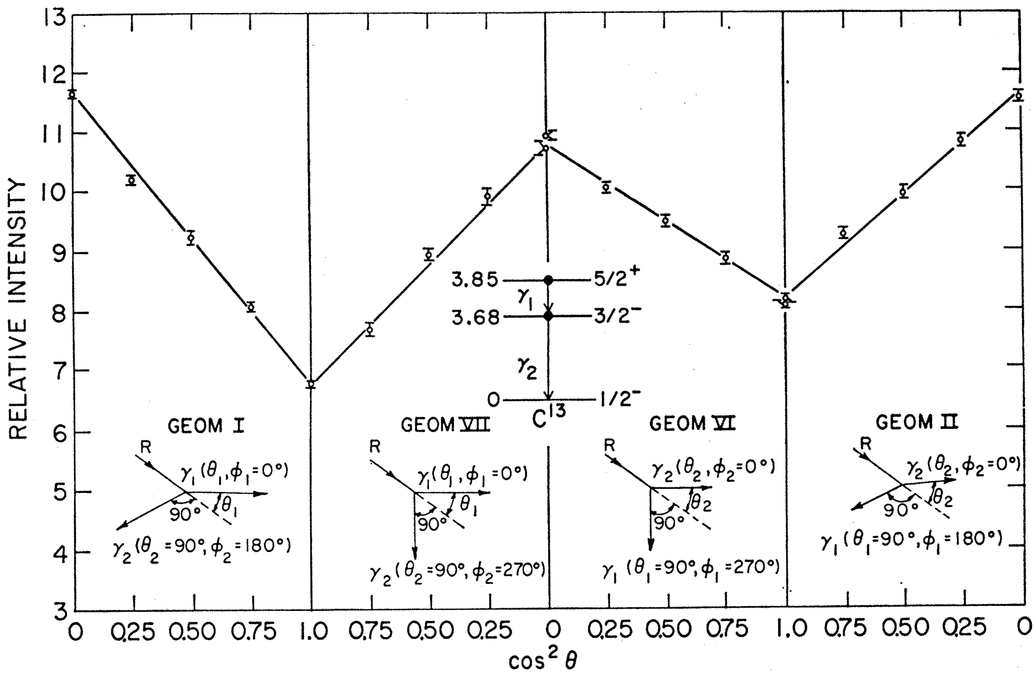

FIG. 2. Results of the gamma-gamma triple correlation measurement for the 3.85 → 3.86 → 0 cascade gamma rays in C^{13}. The open circles give the experimental points as measured for the four indicated geometries. Each correlation was obtained separately and they have been normalized using a least-squares method so that the sum of the squares of the differences between common experimental points measured in two different geometries is a when measured in terms of the statistical errors on these points. The particular configuration corresponding to each geometry is shown schematically below each angular distribution. The distributions are plotted against $\cos^2\theta$ in order to display the fact that only terms in $P_2(\cos\theta)$ are present. The solid curves show the fit to these data, as computed in the fitting procedure illustrated in Fig. 1, corresponding to the minimum in χ^2 at arctan $x_2 = -5.5°$.

significantly affecting the values of the Legendre coefficients A_2/A_0 and A_4/A_0.

The problem in determining the angular distribution of the 170-keV gamma ray lay in reducing as much as possible the background under the 170-keV photopeak. This was achieved firstly by using a chopped beam and gating the analyzer recording the 170-keV spectrum only during the beam bursts in order to minimize the background from the 10-minute annihilation activity due to N^{13}; secondly by carefully shielding the crystal from as much room background as possible; and thirdly by mounting the target on a 0.001-in. thick molybdenum foil which was found to give less background beneath the 170-keV photopeak than tantalum, nickel, or stainless steel. The molybdenum backing mounted at 45° with respect to the beam direction was thin enough so that any absorption effects for the 170-keV gamma ray could be neglected. The yield at each angle was normalized to the measured beam current, dead-time corrections being taken into account for the analyzer. The yield at each point was determined by the computer (Gaussian plus background) least-squares fitting program.[16] The angular distributions which were obtained for the 3.85-MeV and 170-keV radiations are shown in Fig. 4, while the Legendre polynomial coefficients are summarized in Table II. The χ^2 curve obtained by the simultaneous least-squares fitting of the two distributions as a function of the mixing ratio for the 3.85-MeV transition is shown in Fig. 5. The smaller of the two solutions for x is $x = +(0.12\pm0.05)$ where the error limits correspond to those at the 1% confidence limit; the solution to one standard deviation is $x = +(0.12\pm0.03)$. The larger value of x obtained from Fig. 5 can be rejected on two counts, the known[6] lifetime of the level and the results of internal-pair angular-correlation[14] measurements. In obtaining the errors on x we have limited the possible substate populations to $P(\frac{1}{2})$ and $P(\frac{3}{2})$ since for the gamma-gamma correlation the solution obtained showed that $P(\frac{5}{2})=0$ while at the minimum of the curve in Fig. 4, $P(\frac{5}{2})$ is also zero. We note that the value of 0 obtained for $P(\frac{5}{2})$ means that a lower (or equal) value of χ^2 would have been obtained for negative (unphysical) values of $P(\frac{5}{2})$.

TABLE II. The measured angular distributions of the 3.85-MeV and 170-keV radiations at a deuteron bombarding energy of 2.51 MeV. The coefficients have been corrected for the finite solid angle subtended by the detectors. The attenuation coefficients were taken as $Q_2=1.000$ and $Q_4=0.9900$ for the 3.85-MeV radiation and $Q_2=0.9800$ for the 170-keV radiation.

Gamma ray	A_2/A_0	A_4/A_0
170 keV	−0.332±0.008	···
3.85 MeV	0.380±0.009	−0.252±0.012

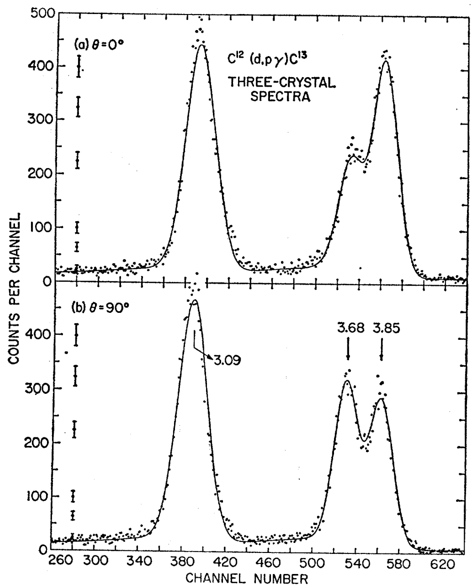

FIG. 3. Two typical pulse-height spectra obtained using the three-crystal pair spectrometer to observe the gamma rays from the reaction $C^{12}(d,p\gamma)C^{13}$ at an incident deuteron energy of 2.51 MeV. Figure 3(a) shows the spectrum obtained at $\theta=0°$, Fig. 3(b) that at 90°. The three peaks correspond to the ground-state transitions from the three bound levels of C^{13}. Each spectrum was obtained for the same integrated beam current. It will be noticed that there are substantial anisotropies in the angular distributions of the two higher energy gamma rays.

The populations $P(\frac{1}{2})$, $P(\frac{3}{2})$, $P(\frac{5}{2})$ of the 3.85-MeV level obtained in the two experiments are given in Table III and shown on Figs. 1 and 5. The theoretical estimates are discussed in the next section. It is seen that there is very good agreement between the two completely independent determinations of these quantities for a beam energy of 2.51 MeV and target thickness of

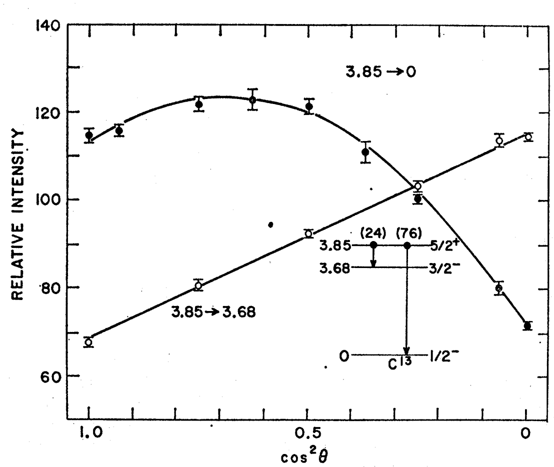

FIG. 4. Results of angular-distribution measurements on the primary gamma rays de-exciting the C^{13} 3.85-MeV level populated in the $C^{12}(d,p)C^{13}$ reaction. The experimental data were obtained at a deuteron bombarding energy of 2.51 MeV. The solid curves show the results of an even-order Legendre polynomial fit to these data to determine the expansion coefficients A_k. Solutions thus obtained for the ratios A_2/A_0 and A_4/A_0 are given in Table II.

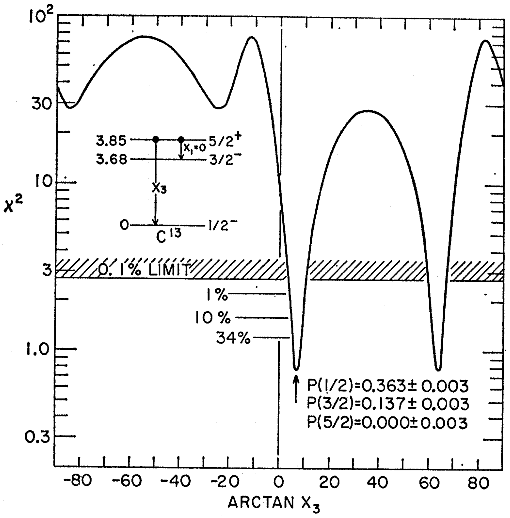

FIG. 5. Chi-squared versus arctan x_3 where x_3 is the $E3/M2$ mixing ratio for the 3.85-MeV transition in C^{13}. The solid curve was obtained by simultaneously fitting the angular distributions corresponding to the two transitions shown in the figure. It was assumed that $x_1=0$. The smaller of the two solutions for x_3 is significantly different from zero, the larger solution is ruled out by a consideration of the lifetime of the level and by results obtained from Ref. 14. The populations of the substates of the 3.85-MeV level obtained at the minimum of the χ^2 curve are also shown on the figure. They agree very well with those determined from the gamma-gamma correlation experiment—see Fig. 1 and Table III.

40 keV; i.e., the values determined by the triple gamma-gamma correlation agree completely with those determined by the angular distribution experiment. This gives us a high degree of confidence in the limits we have set on the mixing ratios obtained from the two experiments. It also insures that the measured angular distribution of the $E1$ $3.85 \rightarrow 3.68$ transition is consistent with that predicted by the angular-correlation experiment.

III. DISCUSSION

A. The $C^{12}(d,p_3)C^{13}$ Reaction Mechanism

In the present work we have gained information on gamma-ray transitions in C^{13} without assuming any model for the nuclear reaction forming this nucleus. The fact that the analysis is independent of the reaction mechanisms is a very important advantage of the method used.[11,17] However, in this type of experiment

TABLE III. The population parameters for the 3.85-MeV level of C^{13} excited by the (d,p) reaction at 2.51 MeV. The first line lists the populations obtained from the gamma-gamma triple correlation experiment, while the second line lists those obtained from the angular-distribution experiment.

Experiment	$P(\frac{1}{2})$	$P(\frac{3}{2})$	$P(\frac{5}{2})$
Gamma-gamma	0.365±0.004	0.135±0.004	$0.000_{-0.000}^{+0.004}$
Distribution	0.363±0.003	0.137±0.003	$0.000_{-0.000}^{+0.003}$
Theory (see text)	0.383	0.110	0.007

[17] E. K. Warburton and H. J. Rose, Phys. Rev. **109**, 1199 (1958).

a nuclear model can be used as a guide in the choice of experimental conditions and at the completion of the experiment the values obtained for the alignment parameters (which contain all the information pertaining to the mechanism of formation) can be compared to the predictions of various nuclear models. This procedure was followed in the present work.

The C^{13} 3.85-MeV level was formed primarily via the 2.49-MeV resonance in $C^{12}+d$. This N^{14} resonance has $J^{\pi}=4^{-}$ and the $C^{12}(d,p_3)C^{13}$ reaction appears to proceed via capture of $l=3$ deuterons and emission of $l=1$ protons.[18] These resonance parameters correspond[19] to population parameters for the C^{13} 3.85-MeV level of

$$P(\tfrac{1}{2})=0.389\,,\quad P(\tfrac{3}{2})=0.106\,,\quad P(\tfrac{5}{2})=0.005\,.$$

There is a background underlying the $C^{12}+d$ resonance which a stripping mechanism appears to describe quite well.[13,20] An estimate[21] of the population parameters corresponding to this background gives

$$P(\tfrac{1}{2})=0.364\,,\quad P(\tfrac{3}{2})=0.124\,,\quad P(\tfrac{5}{2})=0.012\,.$$

These are, coincidently, quite close to those predicted for the resonance. The constancy of the angular distribution of the 170-keV line as a function of the bombarding energy near $E_d=2.5$ MeV indicates that there is no appreciable interference between the resonance and the stripping background. We thus obtain the third line of Table III for the weighted average corresponding to the beam energy and target thickness used. We see, by comparison with the experimentally determined populations that the assumed reaction mechanism is quite consistent with our experimental results.

B. Comparison With Other Measurements

Previous measurements of the two mixing ratios measured by us in the present work have been made by Fletcher, Tilley, and Williamson[20] who on the basis of distorted-wave stripping analysis of the $C^{12}(d,p\gamma)C^{13}$ reaction concluded that $\Gamma(E2)/\Gamma(\gamma)\lesssim 5\%$ ($|x|\lesssim 0.22$) for the $3.68\rightarrow 0$ transition and $\Gamma(E3)/\Gamma(\gamma)\lesssim 2\%$ ($|x|\lesssim 0.14$) for the $3.85\rightarrow 0$ transition. Gorodetzky *et al.*[5] who detected the protons leading to the 3.85-MeV level in an annular counter placed at 180° and determined the angular distribution of the coincident gamma rays, obtained four solutions for the value of the $3.85\rightarrow 0$ mixing ratio each having rather a large uncertainty associated with it. Three of the four values can be rejected because of the known lifetime[6] of the level and the results of internal pair angular correlation[14] measurements. The fourth gives $x=+0.1$ (no error specified). The results of the present work are consistent with these previous results but have much smaller errors. Furthermore the present results are obtained in a model-independent manner whereas the results of Ref. 20 depend rather critically on the details of a nuclear-reaction model.

C. The C^{13} $3.85\rightarrow 0$ Transition

The known lifetime[6] of this level together with the measured branchings[3–5] and the mixing ratio measured in the present work allow the width for the $E3$ transtion to the ground state to be calculated. The average of the branchings obtained by Mackin, Mills and Thirion[3] and by Gorodetzky *et al.*[5] is $(30\pm4)\%$ and $(69\pm4)\%$ to the 3.68-MeV and ground state, respectively. Using these values we obtain,

$$\Gamma(E3)=(8.6_{-6.6}^{+16.2})\times10^{-7}\text{ eV (to 1\% confidence limit)}$$

or

$$\Gamma(E3)=(8.6_{-5.1}^{+11})\times10^{-7}\text{ eV (to one standard deviation)},$$

where the contributions to the errors come in approximately equal measure from the uncertanties in the mixing ratio of the 3.85-MeV transition and the lifetime of the 3.85-MeV level. Measuring the transition strength in terms of the Weisskopf single-particle (s.p.) estimate as given by Wilkinson[12] we obtain,

$$|M(E3)|^2=17.5_{-13.5}^{+33.5}\text{ Weisskopf units (1\% confidence limit)}$$

or

$$|M(E3)|^2=17.5_{-10.4}^{+22.5}\text{ Weisskopf units (one standard deviation).}$$

To a first approximation the C^{13} $3.85\rightarrow 0$ transition is a single-neutron $d_{5/2}\rightarrow p_{1/2}$ transition with the C^{12} core considered as an inert 0^{+} core.[1,2] Perhaps a more meaningful unit of comparison for the $E3$ strength in question is that calculated for a $d_{5/2}\rightarrow p_{1/2}$ transition with an effective charge of $\beta_3 e$ for the neutron. The effective charge β_L in an EL transition characterizes, in the weak-surface-coupling approximation, the collective contribution of higher shells. Evidence for $\beta_3\cong 0.5$ comes from consideration of $E3$ transitions in neighboring nuclei,[22,23] namely, C^{14} and O^{16}, and this value we shall

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[19] The population parameters can be calculated from the formula for the gamma-ray angular distribution from such a resonance. For instance, the gamma-ray angular-distribution formula given by S. Devons and L. J. B. Goldfarb, in *Handbuch der Physik*, edited by S. Flugge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 362.

[20] N. R. Fletcher, D. R. Tilley, and R. M. Williamson, Nucl. Phys. **38**, 18 (1962).

[21] E. K. Warburton and L. F. Chase, Jr., Phys. Rev. **120**, 2095 (1960).

[22] T. K. Alexander and K. W. Allen, Can. J. Phys. **43**, 1563 (1965).

[23] D. E. Alburger, A. Gallmann, J. B. Nelson, J. T. Sample, and E. K. Warburton, Phys. Rev. **148**, 1050 (1966).

TABLE IV. Experimental determinations of the $E3/M2$ mixing ratio of the $\frac{5}{2}^+ \rightarrow \frac{1}{2}^-$ transitions in the mass 13 and 15 nuclei. The phase convention used is that of Refs. 11 or 24.

Nucleus	Transition	Mixing ratio, x	Reference
C^{13}	$3.85 \rightarrow 0$	$+(0.12 \pm 0.03)$	Present work
O^{15}	$5.24 \rightarrow 0$	$+(0.035_{-0.04}^{+0.11})$	5
N^{15}	$5.27 \rightarrow 0$	$-(0.15 \pm 0.06)$	a
		$-(0.09 \pm 0.02)$	b
		$-(0.16 \pm 0.02)$	c

[a] E. K. Warburton, J. S. Lopes, R. W. Ollerhead, A. R. Poletti, and M. F. Thomas, Phys. Rev. **138**, B104 (1965).
[b] D. Pelte, B. Povh, and W. Scholtz, Nucl. Phys. **78**, 241 (1966).
[c] O. Häusser, R. D. Gill, J. S. Lopes and, H. J. Rose, Nucl. Phys. (to be published).

use. We then obtain

$$\Gamma(E3,\text{s.p.}) = 2.6 \times 10^{-7} \text{ eV}.$$

The C^{13} $3.85 \rightarrow 0E3$ transition is 3.3 of these single-particle units, i.e.,

$$|M(E3)|^2 = 3.3_{-2.0}^{+4.2} \text{ single-particle units (one standard deviation).}$$

We conclude that the $E3$ strength is significantly enhanced over that expected on the basis of any single-particle estimates. It would be interesting to compare the experimental figure with a more realistic IPM calculation including collective enhancement in some way. Sebe[2] has obtained excellent agreement for the $M2$ strength of this transition.

While not as amenable to a theoretical treatment as the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions in mass 13 and 15 it is of some interest to compare the phases of the measured $E3/M2$ mixing ratios for the $\frac{5}{2}^+ \rightarrow \frac{1}{2}^-$ transitions in these nuclei. These have been measured for C^{13}, O^{15}, and N^{15} (for the unbound N^{13} $\frac{5}{2}^+$ level $\Gamma_p \gg \Gamma_\gamma$ and the gamma decay of this level has not been observed). The various measurements are collected and compared in Table IV. The phase convention used is that of Litherland and Ferguson[11] as quoted by Poletti and Warburton.[24] It will be noticed that the phase for the C^{13} transition is opposite to that for the N^{15} transition and is the same as that for the O^{15} transition. This directly parallels the pattern which occurs for the phases of the $\frac{3}{2} \rightarrow \frac{1}{2}^-$ transitions which will be discussed below.

D. The $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ Transitions in the $A = 13$ or 15 Nuclei

It is now possible to compare the phases and magnitudes of the $E2/M1$ mixing ratios for all four $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions listed in Table V which gives a summary of the various measurements of these quantities. The phase convention used in this comparison is the same as that[11,24] used in Table IV. It will be seen that the phases of the transitions in C^{13} and O^{15} are opposite to those of the N^{13} and N^{15} transitions. It has already been shown[25,26] that a calculation of the relative phases of the mixing ratios in various related transitions is of interest: In the N or $Z = 11$ systems it lead to a further understanding of the experimentally determined phases and also enabled predictions[26] to be made for the phases of mixing ratios as yet unmeasured. In that case the nuclei were known to be deformed and the Nilsson[27] model was employed. In the present case, as mentioned in the Introduction, the appropriate model is the independent-particle model.[1] A preliminary comparison of the relative phases of the mixing ratios for the four $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions in the mass 13 or 15 systems was thus made in the LS-coupling limit with the following result: The relative phase of all four experimentally determined mixing ratios was found to be as given by experiment if the transitions were taken to be the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions between the two lowest ^{22}P states in both the s^4p^{11} (mass 15) case and the s^4p^9 (mass 13) case. The calculation of Lane and Radicati[1] showed that there is no phase change for the C^{13} $M1$ transition case in going from the LS-limit into intermediate coupling. Preliminary calculations indicate that this is also the case for the C^{13} $E2$ transition and also for the $M1$ and $E2$ transitions in N^{13}. Because of this it seems very likely that all four transitions are adequately described by the independent-particle model within the $1p$ shell. A more thorough comparison of these phases in mass 13 and 15 along with a number of others in the $1p$ shell will be made as mentioned in a further paper.,

[24] A. R. Poletti and E. K. Warburton, Phys. Rev. **137**, B595 (1965).

TABLE V. Experimental determinations of the $E2/M1$ mixing ratio of the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions in the mass 13 and 15 nuclei. The phase convention used is that of Refs. 11 and 24.

Nucleus	Transition	x	Reference
C^{13}	$3.68 \rightarrow 0$	$-(0.096_{-0.021}^{+0.030})$	Present work
N^{13}	$3.51 \rightarrow 0$	$+(0.092 \pm 0.02)$	a
O^{15}	$6.18 \rightarrow 0$	$-(0.12 \pm 0.03)$	b
		$-(0.17 \pm 0.01)$	c
		$-(0.19_{-0.07}^{+0.06})$	d
N^{15}	$6.32 \rightarrow 0$	$+(0.09_{-0.03}^{+0.06}$	e
		$+(0.13 \pm 0.02)$	c

[a] F. C. Young, J. C. Armstrong, and J. B. Marion, Nucl. Phys. **44**, 486 (1963); and private communication from F. C. Young.
[b] B. Povh and D. F. Hebbard, Phys. Rev. **115**, 608 (1959).
[c] J. Lopes, O. Häuser, H. Rose, A. Poletti, and M. Thomas, Nucl. Phys. **76**, 223 (1966).
[d] Reference 5.
[e] E. Warburton, J. Lopes, R. Ollerhead, A. Poletti, and M. Thomas, Phys. Rev. **138**, B104 (1965).

[25] A. J. Howard, J. P. Allen, and D. A. Bromley, Phys. Rev. **139**, B1135 (1965).
[26] A. R. Poletti and D. F. H. Start, Phys. Rev. **147**, 800 (1966).
[27] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No 16 (1955).