# Competition between $E2$ and $M1$ Gamma Transitions in Nuclei of High Spin*

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(Received 6 July 1966)

The effect of the competition between $M1$ and $E2$ electromagnetic transitions on the angular distribution of gamma rays from nuclei with high spin was studied. The angular distribution of such gamma rays was calculated as a function of their energy. This energy dependence of the angular distribution can be used to test the assumption that nuclei with high spin rotate as a whole.

## I. INTRODUCTION

THE parity selection rules for $E2$ and $M1$ electromagnetic transitions are identical. Therefore for states in which the spin difference $\Delta J$ is $\pm 1$ or 0, the two emissions complete with one another. Such competition in transitions between low-spin states has been investigated extensively.[1–6] The emphasis in the present study was on the effect of this competition on the angular distribution of the emitted gamma rays.

High-spin states are produced in heavy-ion reactions, in which the direction of the spin of the emitting system is in a plane perpendicular to the direction of the heavy-ion beam or is close to this plane. Therefore the gamma rays are emitted anisotropically. However, the angular distribution of $E2$ gamma rays differs from that of $M1$ rays. Also, the intensity of the two types of radiation depends strongly on the energy of the emitted gamma rays. Therefore the angular distribution of the emitted gamma rays depends on their energy.

In the present paper the angular distribution of gamma rays from high-spin states was calculated as a function of their energy. The evaluation takes into consideration the competition between $E2$ and $M1$ transitions. First, the angular distribution for a single gamma transition was calculated. The angular distribution for a single gamma-ray transition is less isotropic than that for a cascade, and the fact that the spin of the compound nucleus is not exactly in the plane perpendicular to the heavy-ion beam also reduces the anisotropy. However, both effects do not reduce the anisotropy significantly.[7] Therefore a calculation based on a single transition is sufficient to obtain a first-order approximation of the angular distribution.

For high-spin values the angular momentum may be entirely associated with the rotation of the nucleus as a whole.[8,9] A comparison between results of the proposed theory and experiments offers a test for this proposition. If indeed the effect of the pairing interaction disappears for high-spin values, the nucleus can be treated as a deformable liquid drop with a moment of inertia equal to the moment of inertia of a rigid body with the same shape. This is the view adopted in the present paper, and all the discussion is based on this assumption. The discussion is limited to nuclei that are spherical in their ground state, so that all deformations are due to rotation.

According to the liquid-drop model, the equilibrium shapes of rotating liquid-drop nuclei are closely approximated by spheroids.[10–18] Although these spheroids are only approximate equilibrium shapes, they are excellent approximations even for angular momenta much higher than those considered here.

Spheroidal rotating nuclei have vanishing electric dipole moments, nonvanishing magnetic dipole moments, and nonvanishing electric quadrupole moments. Therefore the leading radiations are $M1$ and $E2$, both of which were considered in this study; the competition between them as exhibited by the angular distribution was given special emphasis. Radiations of higher multipolarities can be neglected in comparison with these two leading radiations.

To calculate the components of the respective multipole tensors, the shapes of equilibrium calculated by Beringer and Knox[18] were used. These shapes are sufficiently accurate for the purpose. The magnetic dipole

* Work supported by the U. S. Atomic Energy Commission.

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moments of the Beringer and Knox shapes differ from the corresponding magnetic moments of the more rigorously calculated Carlson and Pou Lu[11] shapes by 0.5% at most. The respective difference in the value of the quadrupole moments is 1% at most.

According to Beringer and Knox,[18] for nuclei with spins smaller than a critical value the equilibrium shape is an oblate spheroid whose axis of rotation coincides with the axis of cylindrical symmetry. However, for nuclei with spins higher than this critical value the equilibrium shape is a prolate spheroid whose rotating axis coincides with one of the minor axes. The proposed method for evaluating angular distribution is applicable to both types of equilibrium shapes. For completeness, all necessary equations are included to enable calculations of the angular distributions from high-spin states having oblate or prolate equilibrium shapes. However, the validity of the method is demonstrated explicitly by applying it to one particular case, that in which the equilibrium shape is an oblate spheroid.

Since the calculation is based on the liquid-drop model, the components of the contribution multipole tensors were calculated classically. In other words, the nuclear matrix elements were replaced by their classical counterparts. However, since the transitions are in the statistical region, the proportionality of the transition probability to the density of final states is retained, as required.

First, the angular distribution of gamma rays from a high-spin state was calculated with respect to a system of coordinates in which the $z$ axis points along the spin of the emitting system. The angular distribution was then rewritten with respect to a system of coordinates in which the $z$ axis is in the direction of the heavy-ion beam. This angular distribution is isotropic as long as the direction of the spin of the emitting nucleus is random. However, since the preferential direction of the spin of the emitting system is perpendicular to the direction of the heavy-ion beam, an anisotropic distribution results. Next the first angular distribution calculated was rewritten with respect to a system of coordinates in which the $z$ axis is in the direction of the heavy-ion beam. To obtain the angular distribution with respect to the direction of the heavy-ion beam, the contributions from all possible spin orientations in the direction perpendicular to the heavy-ion beam were summed. This last value is the one that is to be compared with experimental results.

## II. THEORY

First, the angular distribution in a system of coordinates in which the $z$ axis coincides with the axis of rotation is calculated. The probability per unit time $P(\theta,\phi)$ of emitting a photon into a unit of solid angle characterized by the angles $\theta$ and $\phi$ is[19,20]:

$$P(\theta,\phi)\alpha\frac{1}{4hk^3}|\{\sum_{l,m}(-)^{l+1}a_E(l,m)[\mathbf{Y}_m{}^{l(l,1)}\times\mathbf{n}]+a_M(l,m)\mathbf{Y}_m{}^{l(l,1)}\}|^2\rho(E,J). \quad (1a)$$

The angular distribution is proportional to $P(\theta,\phi)$.

For $E2$ and $M1$ radiation, Eq. (1a) can be rewritten as:

$$P(\theta,\phi)\alpha\frac{1}{4hk^3}\{\sum_{m,m',J'}a_E{}^*(2,m)a_E(2,m')([\mathbf{Y}_m{}^{2(2,1)}\times\mathbf{n}]^*\cdot[\mathbf{Y}_{m'}{}^{2(2,1)}\times\mathbf{n}])\rho(E,J')+i\sum_{m,m',J'}a_E{}^*(2,m')a_M(1,m)$$
$$\times([\mathbf{Y}_{m'}{}^{2(2,1)}\times\mathbf{n}]^*\cdot\mathbf{Y}_m{}^{1(1,1)})\rho(E,J')+\sum_{m,m',J'}a_M{}^*(1,m)a_M(1,m')(\mathbf{Y}_m{}^{1(1,1)*}\cdot\mathbf{Y}_m{}^{1(1,1)})\rho(E,J')\}. \quad (1b)$$

The first sum on the right-hand side of Eq. (1b) contains the contributions due to $E2$ transitions, the second sum contains the contributions due to interference between $E2$ and $M1$ radiations, and the third sum contains the contributions due to $M1$ radiations. In the first sum the summation over $m$ and $m'$ extends from $-2$ to $+2$; in the last two sums the summation extends from $-1$ to $+1$. For a given $J$, the summation of $J'$ in the first sum extends from $J-2$ to $J+2$ and that in the last two sums extends from $J-1$ to $J+1$. The first and the last sums also contain contributions from interference terms between different components of radiation of the same multipolarity. As will be seen later, all interference terms vanish in the present case.

Here the classical emission probability per unit time per unit solid angle is multiplied by the density of final states, $\rho(E,J)$. In Eq. (1) $\mathbf{Y}_m{}^{l(l,1)}$ is the vector spherical harmonic, which is a function of $\theta$ and $\phi$; $k$ is the wave number; $\mathbf{n}$ is the unit vector in the direction of radiation; and[19,20]

$$a_E(l,m)=\frac{4\pi k^{l+2}}{i(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}Q_{l,m}, \quad (2a)$$

$$a_M(l,m)=\frac{4\pi ik^{l+2}}{(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}M_{l,m}, \quad (2b)$$

where

$$Q_{l,m}=\int r^lY_{l,m}{}^*(\mathbf{r})\rho_e d\mathbf{r}, \quad (3a)$$

$$M_{l,m}=-\frac{1}{(l+1)c}\int r^lY_{l,m}{}^*(\mathbf{r})\,\mathrm{div}(\mathbf{r}\times\mathbf{j})d\mathbf{r}. \quad (3b)$$

[19] J. M. Blatt and W. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 584.

[20] J. A. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 1962), p. 550.

In Eqs. (3a) and (3b) $Y_{l,m}^*(\mathbf{r})$ is the scalar solid spherical harmonic, $\rho_z$ is the nuclear charge density, and $\mathbf{j}$ is the nuclear current density. Values for $Q_{l,m}$ and $M_{l,m}$ have to be calculated separately for the two types of equilibrium shapes discussed by Beringer and Knox.[18]

For the electric quadrupole moment for oblate equilibrium shapes, a simple integration yields

$$Q_{2,0}=\frac{1}{4}\left(\frac{5}{\pi}\right)^{1/2}\tfrac{1}{5}ZeR_0^2\left(\frac{2}{\eta^{2/3}}\right)(\eta^2-1)\,, \tag{4a}$$

$$Q_{2,\pm1}=Q_{2,\pm2}=0\,, \tag{4b}$$

where $Ze$ is the nuclear charge, $R_0$ is the radius of the undeformed nucleus, and $\eta$ is the ratio of the minor to the major axis. The dependence of $\eta$ on spin has been discussed by Beringer and Knox.[18]

Similarly, for the electric quadrupole moment for prolate equilibrium shapes, a simple integration yields

$$Q_{2,0}=\frac{1}{4}\left(\frac{5}{\pi}\right)^{1/2}\tfrac{1}{5}ZeR_0\frac{1}{\eta^{4/3}}(\eta^2-1)\,, \tag{5a}$$

$$Q_{2,\pm1}=0\,, \tag{5b}$$

$$Q_{2,\pm2}=\frac{1}{4}\left(\frac{15}{2\pi}\right)^{1/2}\tfrac{1}{5}ZeR_0^2\frac{1}{\eta^{4/3}}(\eta^2-1)\,. \tag{5c}$$

The nuclear current density is related to the nuclear charge density by

$$\mathbf{j}=\rho_z v=\rho_z\boldsymbol{\omega}\times\mathbf{r}\,, \tag{6}$$

where $v$ is the local velocity, and $\boldsymbol{\omega}$ is the angular velocity of rotation.

Using Eq. (6) to carry out the integration of Eq. (3b) yields the magnetic moment for oblate equilibrium shapes

$$M_{1,0}=\left(\frac{3}{4\pi}\right)^{1/2}\tfrac{2}{5}\eta^2 ZeR_0^2\omega=\left(\frac{3}{4\pi}\right)^{1/2}\tfrac{2}{5}\eta^2\frac{ZeR_0^2L}{\mathcal{I}}$$
$$=\left(\frac{3}{4\pi}\right)^{1/2}\eta^2\frac{e\hbar}{2Mc}\frac{Z}{A}J\,, \tag{7}$$
$$M_{1,\pm1}=0\,,$$

where $\mathcal{I}$ is the nuclear moment of inertia, which is taken as rigid for nuclei with high spin; $L$ equals $J\hbar$; and $J$ is the nuclear spin. The result of Eq. (7) is consistent with the fact that the gyromagnetic ratio for nuclei rotating as a whole is $Z/A$.[21,22]

Similarly, the magnetic moment for prolate equilibrium shapes is

$$M_{1,0}=\frac{1}{2}\left(\frac{3}{4\pi}\right)^{1/2}\frac{\eta^2}{1+\eta^2}\frac{e\hbar}{2Mc}\frac{Z}{A}J\,, \tag{8a}$$

$$M_{1,\pm1}=0\,. \tag{8b}$$

For the density of levels appearing in Eq. (1), the following form is used[23–27]:

$$\rho(E,J)=\rho_E(E)(2J+1)\exp(-J^2/2\sigma^2)\,. \tag{9}$$

The parameter $\sigma^2$ is related to the nuclear moment of inertia $\mathcal{I}$ and to the nuclear temperature $T$ by

$$\sigma^2=\mathcal{I}T/\hbar^2\,. \tag{10}$$

The vector product $\mathbf{Y}_m^{l(l,1)}\times\mathbf{n}$ can be easily expressed in terms of $\mathbf{Y}_m^{l+1(l,1)}$ and $\mathbf{Y}_m^{l-1(l,1)}$ as[28]

$$\mathbf{Y}_m^{l(l,1)}\times\mathbf{n}=-\left(\frac{l}{2l+1}\right)^{1/2}\mathbf{Y}_m^{l+1(l,1)}+\left(\frac{l+1}{2l+1}\right)^{1/2}\mathbf{Y}^{l-1(l,1)}\,. \tag{11}$$

The scalar products of the vector spherical harmonics appearing in Eq. (1b) can be easily evaluated by using Eq. (11). From the property of these scalar products and the symmetry properties of the components of the multipole tensors as given in Eqs. (4b), (5b), (5c), and (8b), it follows that all interference terms vanish.

Now the angular distribution $P(\theta,\phi)$ is calculated with respect to a system of coordinates in which the $z$ axis coincides with the axis of rotation. For oblate shapes of equilibrium, Eqs. (1b), (2a), (2b), (4a), (4b), (7b), and (11) are used, and the appropriate scalar products of the vector spherical harmonics are evaluated.

$$P(\theta,\phi)\propto\frac{\rho(E')}{4h}\left\{\left(\frac{e\hbar}{2Mc}\right)^2\left(\frac{Z}{A}\right)^2(J\eta)^2\left(\frac{E}{\hbar c}\right)^3\left[\sum_{J'}(2J'+1)\exp\left(-\frac{J'^2}{2\sigma^2}\right)\right]\sin^2\theta\right.$$
$$\left.+\frac{1}{50}\frac{Z^2e^2R_0^2(\eta^2-1)^2}{\eta^{4/3}}\left(\frac{E}{\hbar c}\right)^5\left[\sum_{J'}(2J'+1)\exp\left(-\frac{J'^2}{2\sigma^2}\right)\right]\cos^2\theta\sin^2\theta\right\}=A\sin^2\theta+B\cos^2\theta\sin^2\theta\,, \tag{12}$$

where $\theta$ is the polar angle with respect to the direction of rotation. Note that the angular distribution does not depend on the final energy $E'$ but does depend on the gamma energy $E$.

A similar equation is obtained for prolate shapes of equilibrium by using Eqs. (1b), (2a), (5a), (5b), (5c), (8a), (8b), and (11). This equation is not written here because of its length.

Since prolate shapes of equilibrium are reached only for very high spin values,[18] this discussion is limited to oblate shapes of equilibrium. This case is sufficient to

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demonstrate the dependence of angular distribution on energy. Obviously for low-energy gamma rays the angular distribution is dominated by the first term in Eq. (12), or the magnetic dipole radiation; whereas for high-energy gamma rays the angular distribution is dominated by the second term, or the electric quadrupole radiation.

After the angular distribution in Eq. (12) is rewritten with respect to a system of coordinates in which the $z$ axis points along the direction of the heavy-ion beam and the over-all spin distribution is averaged, the angular distribution with respect to the heavy-ion beam is obtained. This is the value that is to be compared with the experimental value.

$$P(\Theta)=A(1-\tfrac{1}{2}\cos^2\Theta)+\tfrac{1}{8}B(1+2\cos^2\Theta-3\cos^2\Theta), \quad (13)$$

where $\Theta$ is the polar axis with respect to the direction of the heavy-ion beam.

## III. RESULTS

Equations (12) and (13) were used to calculate the angular distribution for various gamma-ray energies from $Cu^{66}$ with a spin of $J=30$. A rigid moment of inertia was used, and the nuclear temperature was 1.5 MeV. The value of $\eta$ for this case, taken from Beringer and Knox,[18] is 1.5.

The angular distribution was calculated for energies of 0, 0.5, 1, 2, and 5 MeV and also for the limiting case of infinite energy. The results are summarized below.

For $E=0$ MeV,

$$P(\Theta)\propto 1-0.5\cos^2\Theta. \quad (14a)$$

For $E=0.5$ MeV,

$$P(\Theta)\propto 1-0.41\cos^2\Theta-0.108\cos^4\Theta. \quad (14b)$$

For $E=1$ MeV,

$$P(\Theta)\propto 1-0.174\cos^2\Theta-0.391\cos^4\Theta. \quad (14c)$$

For $E=2$ MeV,

$$P(\Theta)\propto 1+0.438\cos^2\Theta-1.125\cos^4\Theta. \quad (14d)$$

For $E=5$ MeV,

$$P(\Theta)\propto 1+1.473\cos^2\Theta-2.368\cos^4\Theta. \quad (14e)$$

For $E=10$ MeV,

$$P(\Theta)\propto 1+1.843\cos^2\Theta-2.812\cos^4\Theta. \quad (14f)$$

For $E=\infty$ MeV,

$$P(\Theta)\propto 1+2\cos^2\Theta-3\cos^4\Theta. \quad (14g)$$

Equations (14) show that the angular distribution varies as a function of the gamma-ray energy. For vanishing energy (the hypothetical low-limit case) the angular distribution reaches the asymptotic value typical of $M1$ transitions. For infinite energy (the hypothetical upper-limit case) the angular distribution reaches the asymptotic value typical of $E2$ transition. For realistic energies the angular distribution varies between the two limits.

## IV. DISCUSSION

The $Cu^{66}$ nucleus was chosen for demonstration purposes because for this case Beringer and Knox[18] calculated $\eta$ explicitly as a function of the nuclear spin. The demonstrated effect is even more pronounced in heavy nuclei, which are of particular interest. In heavy nuclei the angular distribution varies faster as a function of energy between the limiting cases.

A comparison between the angular distribution of gamma rays emitted by high-spin nuclei as predicted by the present theory and an experimentally measured angular distribution is of utmost interest. An agreement between theory and experiment would indicate the validity of the assumption on which the theory is based, namely, that for high-spin states the nucleus can be considered as a deformed charge liquid-drop rotating as a whole. On the other hand, a marked disagreement between theory and experiment would force the conclusion that even for high-spin states such a simplified picture is not valid.

Studies[9,29–31] of nuclear moments of inertia indicate that for high-spin values, for which the effect of the pairing interaction disappears, the nuclei behave as regular fluids with rigid-body values for the moments of inertia. The use of the present theory offers an additional test to prove or disprove this assumption.

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