Comparison of Low-Energy Total and Momentum-Transfer Scattering Cross Sections for Electrons on Helium and Argon*

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Modified effective-range theory has allowed an unambiguous extrapolation of recent total electron-atom and -molecule scattering cross sections to zero energy. This extrapolation completely determines the differential scattering cross section and therefore the momentum transfer or diffusion cross section for the range of applicability of the effective-range theory. Calculations of momentum-transfer cross sections derived from the total cross-section data of Golden and Bandel for the cases of He and Ar are presented and discussed. It is shown that diffusion cross sections obtained by unfolding swarm experiment data are inconsistent with diffusion cross sections obtained from total cross-section data using effective-range theory.

INTRODUCTION

IN general, electron-molecule cross-section determinations have been divided into two categories[1]:

1. Direct total cross-section determinations from electron-beam attenuation measurements in the desired gas as a function of gas pressure using electrons of sharply defined energy.

2. Indirect diffusion or momentum-transfer cross-section determinations from electron swarm experiments or afterglow microwave attenuation experiments.

These two types of cross section are, of course, related directly through the differential scattering cross section. However, for very low electron energies, differential scattering cross-section determinations are indeed difficult to perform. Method 2 (which is the newer of the two methods) has become quite useful since it allows cross-section determinations for extremely low values of electron energy (a regime inaccessible to method 1). Furthermore, many problems for which low-energy cross sections are necessary require momentum-transfer cross sections. However, the unfolding of the momentum-transfer cross section from the actual measurement is not a straightforward one, since it involves assumptions concerning the electron-energy distribution function which might be subject to question. Furthermore, since the electron-energy distribution function is necessarily broad, method 2 cannot discern rapid variations of cross section with energy. Hence, it would be convenient to be able to derive low-energy momentum-transfer cross sections from precise, direct total-cross-section measurements.

O'Malley[2] has recently applied atomic effective-range theory[3] to approximate determinations of electron-rare-gas scattering lengths from the old direct total-cross-section measurements of Ramsauer and Kollath,[4,5] as well as to the more indirect measurements of momentum-transfer cross sections by a number of authors.[6–13]

Since the work of O'Malley,[2] more precise direct measurements have been made of the total cross section for electrons on helium[14] and for electrons on argon.[15] This new information,[14,15] and the revised momentum-transfer cross-section determinations in He,[16] warrant a more precise comparison of the electron total- and momentum-transfer cross sections in He and Ar.

METHOD

Following O'Malley,[2] the partial-wave expansions for the total scattering cross section σ_t and for the momentum-transfer cross section σ_m are written as follows (in square angstroms):

$$\sigma_t = 3.517 \sum_{L=0}^{\infty} (2L+1)\frac{\sin^2(\eta_L)}{k^2}, \quad (1)$$

and

$$\sigma_m = 3.517 \sum_{L=0}^{\infty} (L+1)\frac{\sin^2(\eta_L - \eta_{L+1})}{k^2}, \quad (2)$$

where $k^2 = (2m/\hbar^2)E$ and

$$E(\text{eV}) = 13.6(ka_0)^2,$$

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where a_0 is the electron radius of the first Bohr orbit in Å and E is the electron energy in (eV).

For a scattering system composed of a charged particle plus a neutral polarizable particle, the atomic effective-range formulas may be written[2,3]

$$\frac{\tan\eta_0}{k} = -A - 0.2840\alpha\sqrt{E} - 0.04902 A\alpha E\ln E + BE, \quad (3)$$

$$\frac{\tan\eta_1}{k} = 0.05679\alpha\sqrt{E} - 0.07353 A_1 E, \quad (4)$$

$$\frac{\tan\eta_L}{k} = \frac{0.8518\alpha\sqrt{E}}{(2L+3)(2L+1)(2L-1)}, \quad (L>1) \quad (5)$$

where A is the scattering length in units of a_0, α is the atomic electric polarizability in units of a_0^3, and A_1 and B are measured in units of a_0^3. Since Eqs. (3)–(5) are valid for small E, the $\tan\eta_L$ terms may be replaced by $\sin\eta_L$ or even $\eta_L(\mathrm{mod}\,\pi)$ with the expansions remaining valid. These expansions should be valid up to electron energies of about 3 eV for He and 0.6 eV for Ar, which follows from the work of O'Malley.[2] Equations (1)–(5) may be used to determine momentum-transfer cross sections from total-cross-section data or vice versa.[17] In this work, both were actually done. The parameters A, B, and A_1 may be evaluated by replacing $\tan\eta_L$ by $\sin\eta_L$ in Eqs. (3)–(5), substituting Eqs. (3)–(5) into Eq. (1), and fitting the resulting equation to total-cross-section measurements.[17,18] Following the evaluation of the effective-range parameters from total-cross-section data, σ_m may be evaluated from Eq. (2).[19] Alternatively, the parameters A, B, and A_1 may be evaluated by replacing $\tan\eta_L$ by η_L in Eqs. (3)–(5), substituting Eqs. (3)–(5) into Eq. (2) by replacing $\sin(\eta_L - \eta_{L+1})$ by $(\eta_L - \eta_{L+1})$, and fitting the resulting equation to momentum-transfer cross-section determinations.[17,18] Then σ_t may be evaluated from Eq. (1).[19] For this work, cross sections were calculated using the $L=0$, 1, 2 partial waves for σ_t and using the $L=0$, 1, 2, 3 partial waves for σ_m.

HELIUM

Using the method outlined above, the values of the parameters A, B, and A_1 were determined from the data of Golden and Bandel for σ_t.[14] The values so obtained are given in Table I,[20,21] together with the values previously obtained by O'Malley[2] from the data of Ramsauer and Kollath,[4,5] and the values obtained in this work from the data of Frost and Phelps for σ_m.[16,22] It should be noted that the value of A_1 determined by O'Malley[2] from the differential cross-section measurements of Ramsauer and Kollath[5] in helium is in very good agreement with that determined in this work.

Figure 1 shows a plot of the total-scattering cross section versus electron energy using the parameters

TABLE I. Values of the parameters for the modified effective-range formulas.

Atom	Source	α (a_0^3)	A (a_0)	B (a_0^3)	A_1 (a_0^3)
He	σ_t-Golden and Bandel, this work	1.36	1.15	0.273	0.264
	σ_t-Golden and Bandel,[a]	1.36	1.15	0.348	...
	σ_t and $\sigma(\theta)$-Ramsauer and Kollath,[b]	1.36	1.19	0.266	0.273
	σ_m-Frost and Phelps[c] this work	1.36	1.21	0.292	0.359
Ar	σ_t-Golden and Bandel, this work	11.0	−1.65	1.11	11.6
	σ_t-Ramsauer and Kollath,[b]	11.0	−1.70	1.23	8.0

[a] Reference 14.
[b] Reference 2, 4, and 5.
[c] See Refs. 16 and 22.

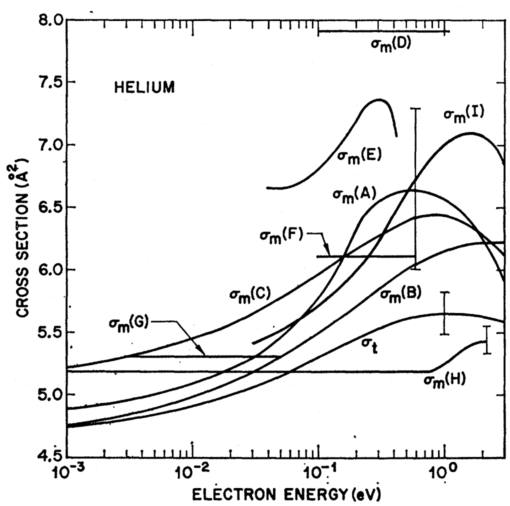

FIG. 1. Low-energy e^--He scattering cross sections versus electron energy. σ_t—best fit to the data of Golden and Bandel between 0.3 and 3.0 eV as described in text; $\sigma_m(A)$—data of Frost and Phelps; $\sigma_m(B)$—calculated from σ_t as described in text; $\sigma_m(C)$—best fit to the data of Frost and Phelps between 0.01 and 3.0 eV as described in text; $\sigma_m(D)$—data of Bowe; $\sigma_m(E)$—data of Anderson and Goldstein; $\sigma_m(F)$—data of Phelps, Pack, and Frost; $\sigma_m(G)$—data of Pack and Phelps; $\sigma_m(H)$—data of Gould and Brown; and $\sigma_m(I)$—data of Crompton and Jory.

[17] The sum of the squares of the percentage differences between measured and calculated values of σ was minimized by variation of the parameters A, B, and A_1.

[18] For this work, the values of α were assumed to be known and were taken from J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Clarendon Press, Oxford, England, 1932), p. 214.

[19] Actually, the differential scattering cross section $\sigma(\theta)$ could also be evaluated. See Ref. 2.

[20] The value of B for helium obtained in this work is slightly different than that obtained in Ref. 14, where a cruder representation of the energy dependence of the cross section was used, and a value for A_1 was not determined. However, the value of A obtained in this work is the same as that obtained in Ref. 14.

[21] In this work, when α was allowed to vary as a fourth parameter in fitting the helium data of Ref. 14, the value of α obtained as the best fit to the data was also $1.36a_0^3$.

[22] The actual values of σ_m obtained by Frost and Phelps in Ref. 16 for energies between 0.01 and 3.0 eV were furnished by Dr. Phelps.

determined in this work from the data of Golden and Bandel for helium.[14] The error bar was determined in a way described in Ref. 14. Also shown in Fig. 1 is a plot of the momentum-transfer cross section versus electron energy as determined from the data of Golden and Bandel[14] as described above. This calculated curve may be compared with the various other momentum-transfer cross-section determinations shown on the plot.[7,8,10–12,23] Over much of the energy range the agreement between the results of Frost and Phelps[16] and calculated values of σ_m from the data of Golden and Bandel[14] is excellent ($\sim$2%). The maximum difference ($\sim$11%) between these two curves is seen to be in the neighborhood of the cross-section maximum. The agreement between the results of Crompton and Jory[23] and calculated values of σ_m from the data of Golden and Bandel[14] is somewhat poorer ($<$15%). This curve, $\sigma_m(C)$, does not fit the data given by Frost and Phelps,[16] $\sigma_m(A)$, any better than that given by Golden and Bandel,[14] $\sigma_m(B)$. However, the scattering length as obtained in this work by fitting the data of Frost and Phelps[16] ($1.21a_0$) does not seriously disagree with that obtained by Frost and Phelps ($1.18a_0$).[22] In contrast to this failure of the effective-range formula to fit the data of Frost and Phelps[16] well, the effective-range fit to the data of Golden and Bandel[14] yielded a curve from which no datum deviates by more than a few percent. Finally, it should be noted that the error bar shown on the plot as being associated with the data of Frost and Phelps[16] was determined by the present author as being the size of the variation in the cross section which Frost and Phelps[16] could not distinguish over a voltage interval of about 1.0 eV.[24] With this in mind, the measurements of Frost and Phelps[16] and those of Golden and Bandel[14] are consistent with each other. If effective-range theory is correct, a better measure of the momentum-transfer cross section is obtained from precise measurements of the total cross section than from the unfolding of swarm experiment data, even when the unfolding is performed with care to a relatively simple problem such as He in the case of Frost and Phelps,[16] or Crompton and Jory.[23]

Since the total cross section is a very slowly varying function over the energy range used by Golden and Bandel for helium,[14] the change in cross section from the best fit to the total-cross-section data due to a change in one parameter from its best value may easily be evaluated. A change in one parameter sufficient to give a factor of 4 increase in the sum of the squares of the percentage differences between measured and calculated values of σ_t could certainly be detected. Therefore, such a change was taken as a measure of the precision of each of the parameters. The deviations of the experimental points from the best-fit line were assumed to be random and on the average $\pm$1% from the best-fit line. It then follows that a change in either A or B of about $\pm$1% will be detected and a change of about $\pm$8% in A_1 will be detected. Computations made by varying the parameters from their best values yielded agreement with the above calculations.

ARGON

Using the method discussed above the values of the parameters A, B, and A_1 were determined from the data of Golden and Bandel for σ_t in argon.[15] The values so obtained are given in Table I together with the values previously obtained by O'Malley[2] from the data of Ramsauer and Kollath.[4,5]

Figure 2 shows a plot of the total cross section versus electron energy using the data of Golden and Bandel in argon.[15] As in the case of helium, no datum deviates by more than a few percent from this curve for σ_t. Figure 2 also shows a plot of the momentum-transfer cross section versus electron energy as determined from the data of Golden and Bandel,[15] as described above. The data of Frost and Phelps[16] (the only data available which cover completely the range of electron energies encompassing the minimum) are also shown. The calculated Ramsauer-Townsend minimum in the momentum-transfer cross section is seen to lie about a factor of 3.7 deeper than the corresponding minimum in the total cross section and about a factor of 4.4 deeper than that given by Frost and Phelps.[16] The S-wave phase shift is zero at the Ramsauer-Townsend minimum for the total cross section, whereas the difference between the S- and P-wave phase shifts is approximately 0 at

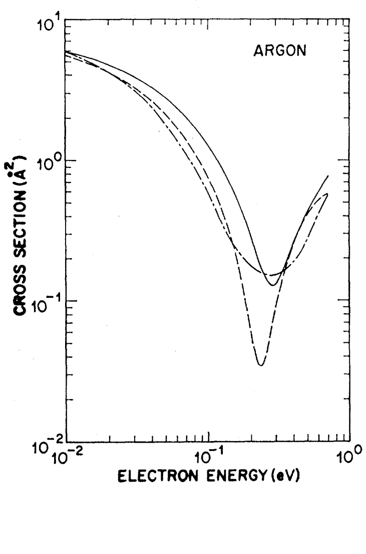

FIG. 2. Low-energy e^--Ar scattering cross sections versus electron energy. The solid line is the best fit to the data of Golden and Bandel for σ_t between 0.1 and 0.5 eV as described in text; the curve with alternating long and short dashes is that given by Frost and Phelps for σ_m; and the curve with long dashes has been calculated for σ_m from the σ_t curve as described in text.

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[24] Frost and Phelps state in Ref. 16 that their analysis could not distinguish the momentum-transfer cross-section curve presented here from one constructed so as to oscillate about that one with amplitude and frequency found by Ramsauer and Kollath in Ref. 4 and by C. E. Normand, Phys. Rev. **35**, 1217 (1930). According to the present author, the largest of these fluctuations is a change in cross section of about 20% with a frequency of more than 1 eV. This is inconsistent with the claim of Frost and Phelps in Ref. 16 that the momentum-transfer cross section in He at the maximum is correct to within about 4.5%.

the Ramsauer-Townsend minimum in the momentum-transfer cross section. Hence, the total cross section in the neighborhood of the minimum is given by the P-wave phase shift while the momentum-transfer cross section is approximately given by the difference between the P- and D-wave phase shifts. Therefore, one would expect the minimum momentum-transfer cross section to lie deeper than the minimum total cross section. The fact that the results of Frost and Phelps[16] did not find such a deep Ramsauer-Townsend minimum must be attributed to their necessarily broad distribution of electron energies.

In the case of argon, where there is a strong energy dependence of the cross section, it is quite clear that a much better measure of the momentum-transfer cross section is obtained from precise measurements of total cross sections than from the unfolding of swarm-experiment data.

In the case of argon, the cross section is a rapidly varying function of electron energy and hence it is more difficult to evaluate the precision in the determination of the effective-range parameters. It can be stated that A and B are less precise, and A_1 more precise than is the case in helium. Computations made by varying the parameters using the same criterion as used for helium showed that a change of $\pm 3\%$ in A and $\pm 5\%$ in either B or A_1 will be detected.[25]

[25] O'Malley in Ref. 2 has estimated a precision of about 2% in the determination of A for argon from the data of Ramsauer and Kollath.

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Scattering Cross Sections of Argon and Atomic Oxygen to Thermal Electrons*

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The electron-scattering cross section for momentum transfer of several neutral species has been experimentally determined. An S-band microwave interferometer was employed to investigate the plasma produced by the incident shock wave in a shock tube. This plasma was in the state of equilibrium, so the relative species concentrations and the electron velocity distribution were known. For electron-collision frequencies small compared with the signal frequency, the microwave data could be intepreted to yield the scattering cross sections. The cross section of the argon atom was measured from 1800 to 5500°K. The results agree with previous drift measurements when those cross sections are averaged over the Maxwellian distribution of electron velocities. Above 3000°K the oxygen molecule becomes dissociated and the oxygen-atom cross section can be measured. From 3000 to 4000°K, this atomic cross section is $(1.2\pm0.2)\times10^{-15}$ cm^2. For molecular nitrogen and oxygen the small-collision-frequency requirement could not be fully satisfied. The cross sections for these gases were approximately 10^{-15} cm^2 from 2000 to 3500°K, with no observable dependency on gas density.

INTRODUCTION

MEASUREMENTS have been made in the past of the electron-scattering cross section in argon[1] and molecular nitrogen[2,3] using a swarm technique. The molecular oxygen-scattering cross section has been measured up to 900°K by Mentzoni[4] using a microwave interferometer. The atomic-oxygen cross section was measured at 4000°K by Lin,[5] using a combination of microwave-attenuation and conductivity probe data. To extend the data range of the atomic-oxygen species and to obtain data for argon in a similar environment, the present shock-tube experiments were performed.

The plasmas are produced by a shock wave in a shock tube. This plasma generator was selected because the species distribution can be calculated from the equilibrium normal shock-wave relations. The operating conditions were selected so that the electrons in the shock tube would be thermally equilibrated with the neutrals; therefore, their temperature can be taken to be the same as the temperature of the neutrals which was varied from 2000 to 5000°K. The cross sections which are obtained are then averages over a Maxwellian velocity distribution. By taking the ratio of the measured attenuation and phase shift of an S-band microwave signal propagating through the plasma, the electron collision frequency can be determined. From this collision frequency the scattering cross sections to electrons could be inferred because the species distribution and electron temperature were known.

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