Phonon and Electron Drag Coefficients in Single-Crystal Aluminum

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Measurements have been made of the attenuation of the fast and slow shear waves along the ⟨110⟩ direction in an aluminum single crystal in the frequency range from 30 MHz to 170 MHz and in the temperature range 66°K to 300°K. The attenuation can be divided into a square-law frequency term and a term consistent with the theoretical form of the dislocation attenuation at high frequencies. On account of the fact that aluminum has three free electrons per atom and a low resistivity, the attenuation due to electron viscosity is large enough to measure at room temperature, and at low temperatures contributes the principal square-law component. The difference between the measured value and the electron component can be used to evaluate the phonon-viscosity square-law component. The asymptotic value of the dislocation attenuation can be used to evaluate the drag coefficient, and for the first time a definite electron drag coefficient is demonstrated. The measurements can be used to establish a ratio of 3·4 between the limiting nonlinearity radii for electrons and phonons. While the absolute value cannot be obtained with accuracy by this method—on account of a lack of knowledge of the number $\bar{N}$ of dislocations per cc—theory indicates that the value should be between 0.9 and 1.6×10^{-3} dyn sec/cm², which is a relatively large damping coefficient.

I. INTRODUCTION

IN the linear, small-amplitude range of motion, it is well known[1,2] that dislocations are damped by the conversion of dislocation energy to phonons. Recently it has been demonstrated[3,4] that dislocations are damped by the conversion of dislocation energy to electron motion. A theory[5] of this effect was given which accounted for the difference in the nonlinear-dislocation breakaway loss, first observed by Love and Shaw,[3] and the divergence between the measured attenuation in the normal and in the superconducting regions and that calculated from the Bardeen-Cooper-Schrieffer (BCS) theory. This evidence is somewhat indirect since it requires some assumptions about dislocation-loop lengths and cutoff radii. Hence a direct measurement of this drag coefficient is desirable, and it is the purpose of this paper to present measurements in aluminum single crystals which allow such in evaluation.

Aluminum is a particularly advantageous material for this purpose since the direct electron damping of acoustic waves is so large that a measurable component exists even for room temperature. This is partly because there are three free electrons per atom. Also, according to the measurements of Lax[6] and Filson,[7] the divergence from a spherical Fermi surface is large enough so that the measured attenuation is 1.45 times the calculated attenuation for the free-electron model. Their measurements for a polycrystalline aluminum show that the attenuation is consistent with an electron viscosity determined by the equation

$$\eta_e=\frac{9\times10^{11}\hbar^2}{5e^2R}(9\pi^2N)^{2/3}\times1.45=\frac{3.88\times10^{-8}}{R}\ \text{(aluminum)}. \tag{1}$$

In this equation $\hbar$ is Planck's constant h divided by 2π, N is the number of atoms per cc (6.11×10^{22} for aluminum), e the electronic charge (4.8×10^{-10} esu) and R, the resistivity in Ω cm. The resistivity as given in Refs. 6 and 7—low temperatures—and the International Critical Tables—high temperatures—is shown by Fig. 1.

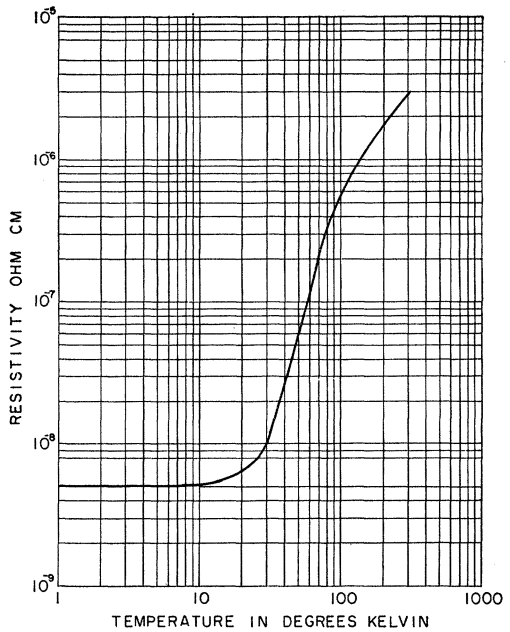

FIG. 1. Resistivity of a 99.995% aluminum sample as a function of the temperature.

* This research has been performed under contract Nonr 266-91, Institute of Fatigue and Reliability, Columbia University.

[1] A. Granato and K. Lucke, in *Physical Acoustics*, edited by W. P. Mason (Academic Press Inc., New York, 1966), Vol. IV A, Chap. VI.

[2] W. P. Mason, in *Physical Acoustics*, edited by W. P. Mason (Academic Press Inc., New York, 1965), Vol. III B, Chap. VI.

[3] R. E. Love and R. W. Shaw, Rev. Mod. Phys. 34, 260 (1964); R. E. Love, R. W. Shaw, and W. A. Fate, Phys. Rev. 138, A1453 (1965).

[4] B. R. Tittmann and H. E. Bömmel, Phys. Rev. Letters 14, 246 (1965).

[5] W. P. Mason, Appl. Phys. Letters 6, 111 (1965); Phys. Rev. 143, 229 (1966).

[6] E. Lax, Phys. Rev. 115, 1591 (1959).

[7] D. H. Filson, Phys. Rev. 115, 1516 (1959).

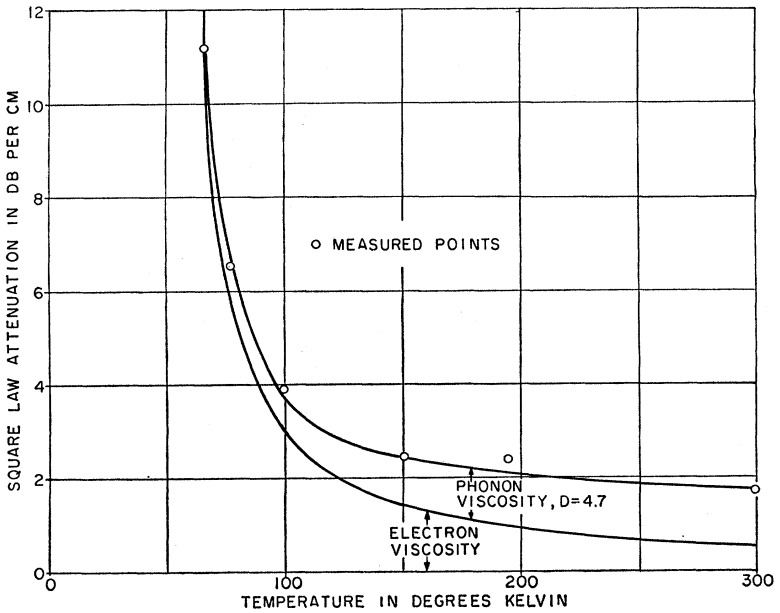

FIG. 2. Square-law attenuation along a ⟨110⟩ direction for a shear wave with a polarization along ⟨1$\bar{1}$0⟩ direction. Circles represent measured points. Electron viscosity loss was calculated from Eq. (1). The difference between measured loss and electron viscosity loss evaluates the phonon viscosity with a nonlinearity factor $D=4.7$.

Dislocations are damped by interactions with electrons and with phonons. These interactions produce a drag on the dislocation per unit length which is proportional to the velocity of the dislocation through the medium. The drag coefficient B, which determines the force on the dislocation due to interaction with the electronic shear viscosity of Eq. (1), and with the shear phonon viscosity discussed in Sec. III, has the dimensions of dyn sec/cm^2. Besides phonon viscosity another source of dislocation damping proposed by Leibfried[8,9]—due to the difference in radiation pressure of phonons on the front and back sides of the dislocation—takes the form

$$B=aE_0/10V_s, \qquad (2)$$

where a is the lattice constant, V_s the shear velocity in the glide plane and E_0 the thermal energy density. For aluminum this damping has a value 0.55×10^{-4} which is much smaller than the phonon viscosity drag term and is here neglected.

The attenuation for electrons, given for the slow shear wave, is

$$A_{\mathrm{Np/cm}}=\omega^2\eta_e/2\rho V^3=2.12\times10^{-7}/R \qquad (3)$$

at 150 MHz using the value of $\rho=2.73$; $V=3.11\times10^5$ cm^2/sec which result from the constants measured by Kamm and Alers[10] given in Eq. (4). By employing the resistivity given by Fig. 1, the calculated electronic attenuation in dB per cm—1 Np equals 8.68 dB—is

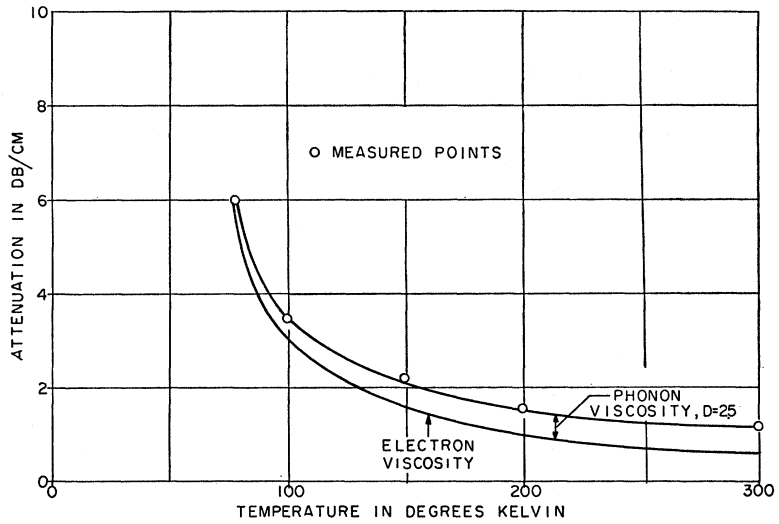

FIG. 3. Square-law attenuation along a ⟨110⟩ direction for a shear wave with a polarization along ⟨100⟩ direction. Circles represent measured points. Solid lines represent division into electron and phonon viscosity terms as in Fig. 2.

[8] W. P. Mason, J. Appl. Phys. **35**, 2779 (1964).
[9] G. Leibfried, Z. Physik **127**, 344 (1950).
[10] G. A. Alers, in *Physical Acoustics*, edited by W. P. Mason (Academic Press Inc., New York, 1965), Vol. III B, Chap. I, p. 34, Table X.

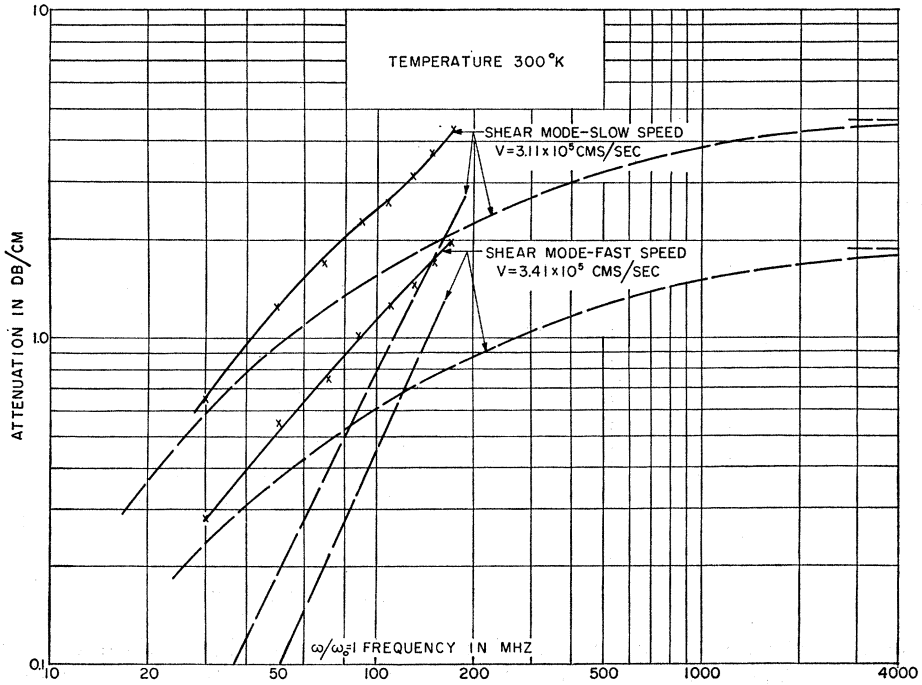

FIG. 4. Measured fast and slow shear-wave loss at 300°K as a function of the frequency. Dashed lines represent a breakdown of the loss into square-law loss and dislocation loss. Asymptotic value indicated by dashed lines at right.

shown by the curve of Fig. 2 marked electron viscosity. This attenuation is proportional to the square of the frequency and as discussed in the next section, the square-law term can be separated from the dislocation term by employing a sum of two standard forms. The actual measured square-law terms are shown by the circles. At very low temperatures the square-law term is given mostly by electron damping, but at higher temperatures a damping due to phonon viscosity predominates. As discussed in the Sec. III, this attenuation measurement allows an evaluation of the nonlinearity constant D appearing in the phonon-viscosity term. As discussed in Eq. (12) this nonlinearity term D is related to the deviation of the ordinary second order elastic constants from linearity. For the slow speed shear mode, the constant $D=4.7$. A similar measurement was made for the fast shear mode controlled by the c_{44} constant—velocity is 3.41×10^5 cm/sec—and the attenuation has the same electronic component but a smaller phonon-viscosity term with a D value of 2.5 as shown by Fig. 3.

II. EVALUATION OF FREQUENCY-SQUARE LOSS AND DISLOCATION COMPONENT

The actual measurements from which the square-law and dislocation components can be obtained were made

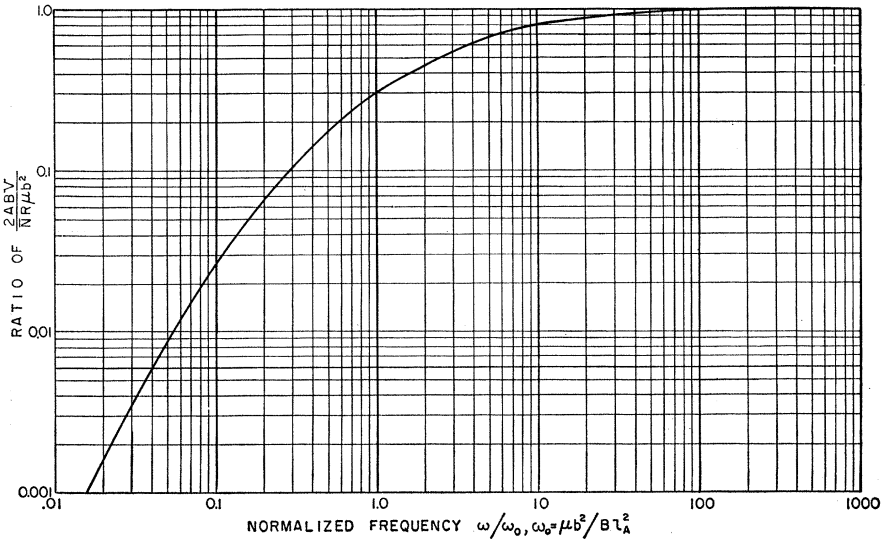

FIG. 5. Normalized curve for a dislocation loss (after Oen, Holmes and Robinson).

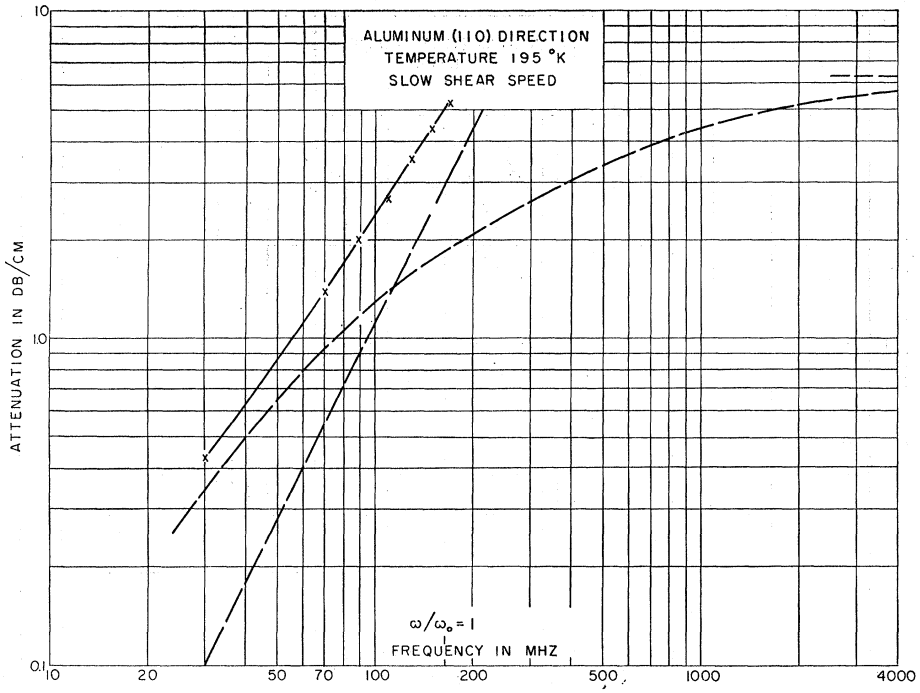

FIG. 6. Measured loss for slow shear wave at 195°K. Dashed curves show breakdown of loss into a frequency square-law term and a dislocation term.

by measuring the shear-wave attentuation along the ⟨110⟩ direction with the polarization along a cube axis (fast wave) and along the ⟨1$\bar{1}$0⟩ direction (slow wave). The measured values are shown by the crosses of Fig. 4. The slow wave is controlled by the elastic constant $\frac{1}{2}(c_{11}-c_{12})$, while the fast wave is controlled by the constant c_{44}. Using the elastic moduli and density determined by Kamm and Alers[10], i.e., $c_{11}=11.428$, $c_{12}=6.204$, $c_{44}=3.171\times10^{11}$ dyne/cm^2,

$$\rho=2.733 \tag{4}$$

the velocities are 3.11×10^5 and 3.41×10^5 cm/sec.

As discussed in detail in a previous paper,[11] the attenuation of an unstrained single crystal, at high frequencies, can be represented by an attenuation proportional to the square of the frequency, due to electron viscosity, phonon viscosity and thermoelastic effects, plus a dislocation term of standard form which has the shape shown by Fig. 5. This curve represents the solution of the Granato-Lücke theory of dislocation damping for an exponential distribution of pinning points as obtained by Oen, Holmes, and Robinson,[12] This solution neglects the mass term since for all measurements made the dislocation loops are overdamped and no resonances are observable. This form of the dislocation loss has been well verified for copper[13,14] by separating out the dislocation contribution from the other losses by neutron-irradiating the sample. The success of separating out the dislocation component from the square-law term by direct addition of two standard terms was demonstrated in a previous paper.[11]

As shown by Fig. 4, the measured attenuation for both the fast and slow shear waves can be fitted by square-law curves shown by the dashed straight lines and dislocation curves of the type shown by Fig. 5 with the disposable constant $\omega/\omega_0=1$ for a frequency of 80 MHz. The limiting value for an infinite frequency is shown by the dashed straight line above each curve. From Fig. 5 the drag coefficient can be evaluated from the limiting attenuation from the formula

$$B=\bar{N}\Omega\mu b^2/2AV. \tag{5}$$

In this Eq. A is the attenuation in Np/cm (1 Np=8.68 dB), V is the velocity of the wave, $\bar{N}$ the number of dislocations, Ω the orientation factor which relates the strain in the acoustic wave to the average strain in the glide plane, μ is the shear modulus in the glide plane $[\frac{1}{3}(c_{11}-c_{12}+c_{44})=2.798\times10^{11}$ dyn/cm$^2]$ and b the Burger's vector equal to 2.86×10^{-8} cm for aluminum. The orientation factor Ω has been calculated[13] for the two shear waves along the ⟨110⟩ direction to be

$$\Omega c_{44}=\tfrac{1}{3}[c_{44}/(c_{11}-c_{12}+c_{44})]=0.126, \tag{6}$$
$$\Omega((c_{11}-c_{12})/2)=(c_{11}-c_{12})/2(c_{11}-c_{12}+c_{44})=0.311.$$

Using the values given above, the two limiting values determine the drag coefficient to be for the two cases

$$B=2.08\times10^{-10}\,\bar{N}\ \text{(slow speed)},$$
$$B=1.95\times10^{-10}\,\bar{N}\ \text{(high speed)}, \tag{7}$$

[11] W. P. Mason and A. Rosenberg (to be published).
[12] O. S. Oen, D. K. Holmes, and M. T. Robinson, Solid State Division Annual Report, Oak Ridge Laboratory, 1960 (unpublished).
[13] G. A. Alers and D. O. Thompson, J. Appl. Phys. **32**, 283 (1961).
[14] T. Suzuki, A. Ikushima, and N. Aoki, Acta Met. **12**, 1231 (1964).

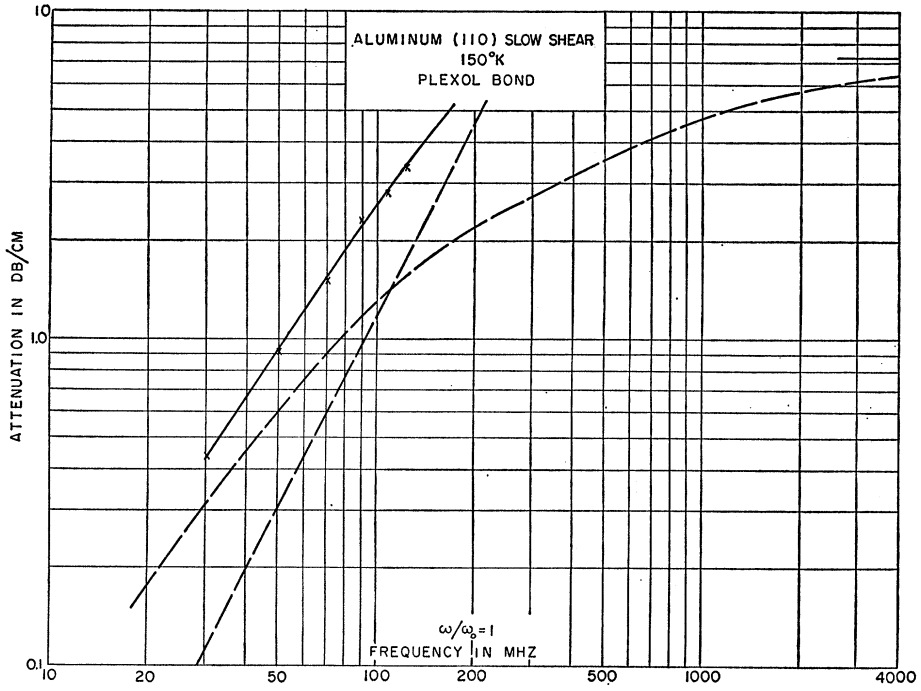

FIG. 7. Measured loss for slow shear wave at 150°K. Dashed lines show breakdown of loss into a square-law term and a dislocation term.

which is a reasonable agreement. Since the slow-speed wave has the higher attenuation and hence can be more accurately measured, all the dislocation measurements are limited to this wave.

To determine the drag coefficient one has to know or approximate the number of dislocations. In aluminum this is a difficult matter since an etch-pit technique has not been developed. X-ray techniques have been used, but these cannot be applied to a crystal $\frac{1}{2}$-in. diam. Measurements[15] for small-size crystals grown at slow rates show values which increase with the diameter and are in excess of 10^5 per cc for a crystal 0.1 in. in

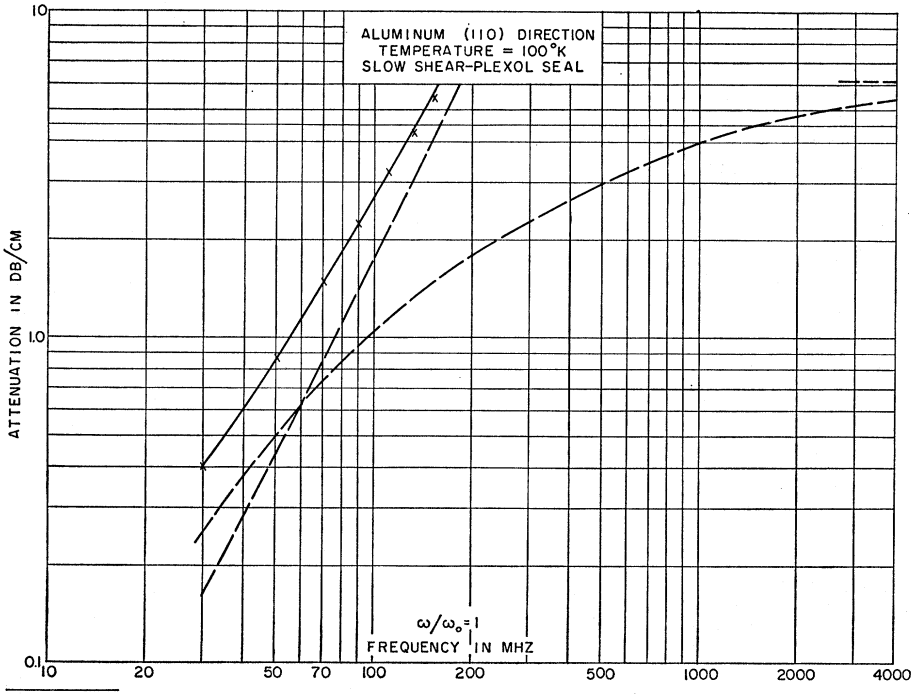

FIG. 8. Measured loss for a slow shear wave at 100°K. Dashed lines show breakdown of loss into a square-law term and a dislocation term.

[15] S. Howe and C. Elbaum, Phil. Mag. 6, 1227 (1961).

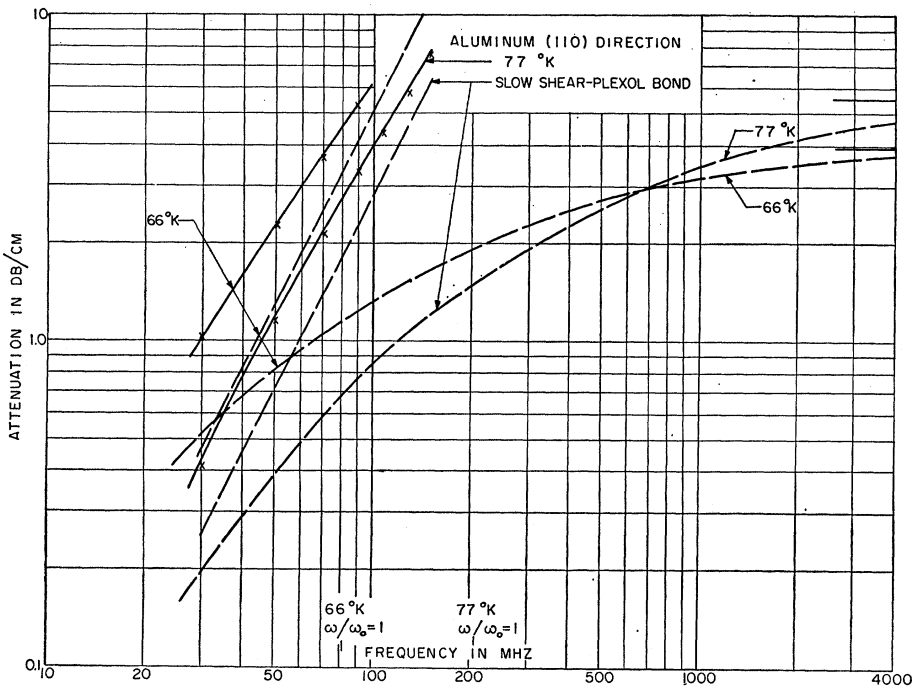

FIG. 9. Measured loss for a slow shear wave at 66°K and at 77°K. Dashed lines show breakdown of losses into a square-law term and a dislocation term.

diam. Perhaps the best estimate for the present purpose is obtained from an electron microscope examination[16] of a thin section of an annealed crystal which showed a value of 5×10^6 dislocation length per cc. With this value the drag coefficient becomes 1.04×10^{-3} dyn sec/cm^2, which is close to the theoretical values discussed in Sec. III. This is a rather large value of the damping constant B but it confirms the large value found by Dorn, Mitchell and Hauser[17] who used a stress pulse technique. Hence, although an accurate absolute value of B cannot be obtained by this method, it is possible to demonstrate the existence of an electron drag coefficient and obtain an estimate of the ratio of the electron nonlinear radius to the phonon nonlinear radius as discussed in Sec. III.

Since both the square-law term and the dislocation-drag term depend critically on the temperature, measurements were made from 66 to 300°K and are shown plotted on Figs. 6–9 inclusive. The measured values are shown by the solid lines with crosses, while the separation into square-law and dislocation terms are shown by the dashed lines. The position of the arbitrary constant $\omega/\omega_0=1$ is plotted against the frequency. All of these measurements are for the slow shear wave. The fast shear wave was also measured but since it showed substantially the same results only the square law values at 150 MHz are shown plotted by Fig. 3.

Using the limiting value of attenuation shown by the straight line at the right of each figure, the relative drag coefficients are shown plotted by the circles of Fig. 10. In order to agree with theoretical values, the effective number of dislocations $\bar{N}$ is taken as 8×10^6 per cc. It should be emphasized, however, that direct measurements do not establish the absolute value within a region which probably is within values from $\frac{1}{3}$ to 3 times this value.

III. EVALUATION OF ELECTRON AND PHONON DISLOCATION-DRAG COMPONENTS

The square-law attenuation of Figs. 2 and 3 is accounted for partly by electron viscosity—Eq. (1)—and partly by direct conversion of acoustic energy into phonon energy. For shear waves the only process that has been suggested for this conversion is the Akheiser effect which results from a separation of the temperatures of the various phonon modes by a suddenly applied stress followed by a relaxation of these temperatures to an average value ΔT above the ambient. ΔT is zero for a shear wave since there is no increase in temperature associated with a shearing strain. It was shown by the writer[18] that the suddenly applied stress causes an increase in elastic modulus equal to

$$\Delta c=3\sum_i E_i(\gamma_i{}^j)^2, \tag{8}$$

where E_i is the thermal energy associated with mode i and $\gamma_i{}^j$ is the Grüneisen number associated with the particular mode and strain. For the Debye approximation which was used in deriving this equation, all the modes have the same variation with temperature as

[16] R. L. Segall and P. G. Partridge, Phil. Mag. 4. 912 (1959).
[17] J. E. Dorn, J. Mitchell, and F. Hauser, Exptl. Mech. 5, 353 (1965).
[18] W. P. Mason and T. B. Bateman, J. Acoust. Soc. Am. 36, 646 (1964).

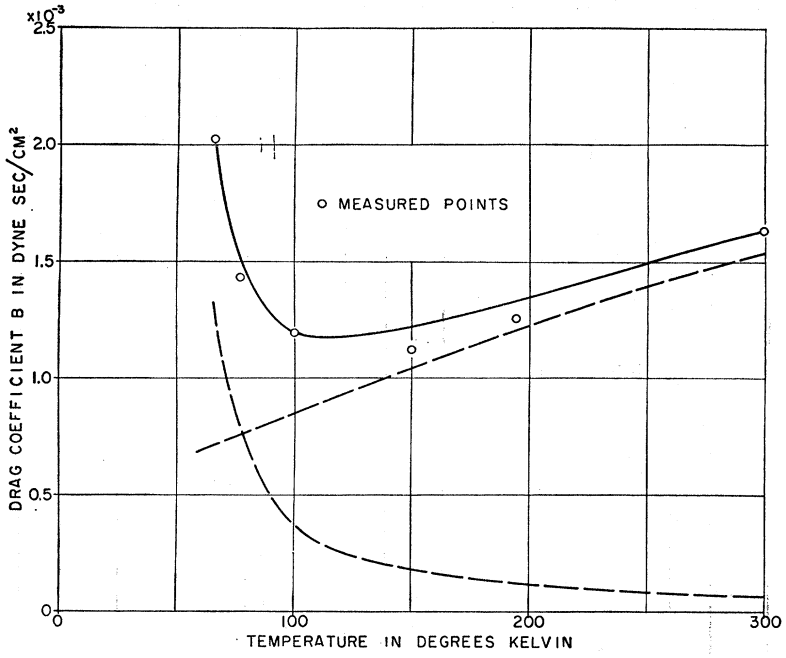

FIG. 10. Measured dislocation drag terms as a function of the temperature. Dashed curves show phonon and electron drag terms on the assumption that $a_0=3b/4$ for the phonon term. Electron term is evaluated to give best fit for the total measured drag coefficient.

does the total thermal energy E_0 and hence Eq. (8) reduces to

$$\Delta c=3E_0[\textstyle\sum_i(\gamma_i{}^j)^2]/n, \tag{9}$$

where n is the number of modes used to evaluate the summation. In comparisons with experiment the 39 pure modes propagated along the ⟨100⟩, ⟨110⟩, and ⟨111⟩ axes have been used to evaluate this expression.

Maxwell first showed that the product of an increase Δc of an elastic modulus, produced by a suddenly applied shearing strain, by the relaxation time required to equilibrate the motions of the gas molecules to their average value was equal to the viscosity η of the gas. In the present case the relaxation time to equilibrate the various phonon temperatures to the average value is the thermal relaxation time

$$\tau=3K/C\bar{V}^2, \tag{10}$$

where K is the thermal conductivity, C the specific heat per unit volume and $\bar{V}$ the Debye average velocity which for an isotropic material is

$$\bar{V}=[\tfrac{1}{3}(2/V_s{}^3+1/V_l{}^3)]^{-1/3}. \tag{11}$$

Here V_s and V_l are the shear and longitudinal velocities of the medium. For crystalline materials various methods[19] have been used to calculate this velocity and for aluminum the value is 3.5×10^5 cm/sec. Hence the "phonon" viscosity associated with a shear wave is

$$\eta_p=9\frac{E_0K}{C\bar{V}^2}\sum_i\frac{(\gamma_i{}^j)^2}{n}=D\left(\frac{E_0K}{C\bar{V}^2}\right);\quad D=\frac{9\sum_i(\gamma_i{}^j)}{n}, \tag{12}$$

where the nonlinearity constant D is determined by the summation of the square of the Grüneisen numbers multiplied by $9/n$ where n is the number of modes used to determine D.

As discussed in several publications[18] the Grüneisen number can be calculated when the second-order–ordinary–elastic constants and the third-order elastic constants are known. This process has recently been applied[20] for six crystals for which the third-order moduli have been measured and the results are in good agreement with the measured attenuations. For the present case the third-order elastic moduli of aluminum have not been measured partly because the elastic nonlinearity is combined with a dislocation nonlinearity. However, the measured shear-wave attenuation of Figs. 2 and 3 allow an evaluation of the nonlinearity constant D.

For a nonconducting crystal all the thermal energy is carried by phonons and the conductivity K used is the total thermal conductivity. For a metal, however, electrons also carry thermal energy. In equilibrating the phonon temperatures, however, it is only the thermal energy carried by phonons that comes into play. Hence it is necessary to separate the lattice conductivity from the electronic. This has been done for monovalent metals by direct measurement[21] but not for aluminum. A theoretical formula due to Leibfried and Schlömann,[22] given by the Eq. (13), approximates the lattice thermal conductivity. This takes the form

$$K_{(\mathrm{W/cm^\circ K})}=3.6aA\Theta^3/\gamma^2T, \tag{13}$$

[19] G. Alers and O. L. Anderson, in *Physical Acoustics*, edited by W. P. Mason (Academic Press Inc., New York, 1965), Vol. III B, Chaps. I and II.

[20] W. P. Mason and T. B. Bateman, J. Acous. Soc. Am. **40**, 852 (1966).

[21] G. K. White and S. B. Woods, Phil. Mag. **45**, 1343 (1954).

[22] G. Leibfried and E. Schlömann, Nachr. Akad. Wiss. Goettingen, Math.-Physik. KI., IIa **11**, 71 (1954).

where a is the lattice spacing, A a constant equal to 92.9, Θ the Debye temperature, γ the Grüneisen constant and T the absolute temperature. For aluminum this formula gives

$$K=214/T. \tag{14}$$

According to the measurements of White and Woods[21] this overestimates the measured values by a factor of 2. Hence for aluminum the lattice thermal conductivity is taken to be

$$K=107/T=0.36 \text{ W/cm°K at } 300°\text{K}. \tag{15}$$

By using this thermal conductivity, the ratio of E_0/C taken from tables, and $\bar{V}$ equal to 3.5×10^5 cm/sec, one can derive a value of D which best fits the difference between the measured square law loss and the electronic square-law loss. The top solid lines of Figs. 2 and 3 represent the sum of the two types of losses with a D value equal to 4.7 for the slow wave and 2.5 for the fast wave. The measured points are in good agreement with the sum of terms. At low temperatures the electron viscosity predominates but it also makes a measurable contribution at room temperatures. For a shear wave in the glide plane with an elastic constant $(c_{11}-c_{12}+c_{44})/3$ the effective value of D is

$$(2\times4.7+2.5)/3=3.95. \tag{16}$$

Since a dislocation is surrounded by a strain field, there is an energy loss due to phonon viscosity as discussed in previous publications.[8,2] For a screw dislocation the expression is particularly simple since only one shearing strain is involved. Summing all the squares of the strain rate times the phonon viscosity over space surrounding the dislocation it was shown[8] that the drag coefficient due to this source is

$$B=(b^2/8\pi a_0^2)\eta=b^2D/8\pi a_0^2(E_0K/C\bar{V}^2), \tag{17}$$

where a_0 is a nonlinearity limiting radius determined by the condition that the nonlinear third-order elastic moduli are sufficiently large to cause the concept of a phonon as an acoustic wave to lose meaning. As discussed in a previous publication[11] the value $a_0=3b/4$ is reasonable. This value may vary some between crystals, and for aluminum, which has large nonlinear terms, a value of $a_0=b$ may be a better approximation. For an edge dislocation a similar expression occurs with an added term due to compressional phonon viscosity. This term turns out to be small compared to the effect of shear phonon viscosity and the value for an edge dislocation is taken to be

$$B=\tfrac{3}{4}[b^2\eta/8\pi(1-\sigma)^2a_0^2] \tag{18}$$

which is very close in value to (17).

Recently it has been shown[5] that electron viscosity will produce a similar drag effect and in fact the same equation—(12)—will account for the effect if we replace the phonon viscosity of (12) by the electron viscosity of (1). A different nonlinearity radius a_0 may result since this is determined by the nonlinearity value of the Fermi surface rather than the elastic constants. This was estimated to be 10^{-7} cm.

Since both the phonon and electron viscosities are evaluated from the square-law frequency losses given by Figs. 2 and 3 one can calculate the drag coefficients to be expected for these two effects and determine the best ratio of the nonlinear radii to agree with experiment. The values of Fig. 10 are drawn on the assumption that the phonon radius of $3b/4$ is correct and the ratio of electron to phonon radii is

$$\text{ratio } a_{0e}/a_{0p}=3.4$$

which follows from the equality of the electron and phonon drag coefficients at 77°K. On the other hand if we take the assumed nonlinear electron radius of 10^{-7} cm as more nearly correct, the phonon radius becomes 2.9×10^{-8} cm which is very close to the Burger vector $b=2.86\times10^{-8}$ cm. For this case the drag coefficient at room temperature becomes 0.9×10^{-3} dyn sec/cm^2. Either value falls within the probable dislocation range and hence direct measurement cannot discriminate between the two values.