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Critical-Current Behavior in Narrow Thin-Film Superconductors

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Critical currents in planar thin films of tin and lead have been measured as a function of temperature using very narrow strip samples. There is a critical sample width of the order of one micron above which the critical-current behavior is dominated by flux-flow processes. For narrower samples the temperature dependence of the average critical current density is in excellent agreement with the usual relation deduced from free-energy considerations, and the low-temperature values are in good agreement with the simple depairing criterion. At higher temperatures the observed critical current densities are significantly larger than the Bardeen-Rogers predictions based on calculations of the BCS energy-gap variation with current. It is proposed that in these narrow films flux vortices due to the self-field of the current at the film edges are too large to enter the film and by their subsequent motion degrade the critical current. The approximate size of such vortices is calculated and good agreement with the experimental critical widths is obtained. The observation, in wide films only, of "training" phenomena similar to those seen in bulk type-II superconductors also tends to support this hypothesis.

CRITICAL currents have been measured in planar thin films of tin and lead using very narrow samples. For sufficiently narrow strips, the temperature dependence of the *average* critical current density was found to be in excellent agreement with the usual relation deduced from free-energy considerations. The low-temperature average critical current density (j_{co}) for such specimens is in good agreement with that predicted by the simple "depairing" criterion which should be valid at $T=0$. At higher temperatures the observed critical current densities are significantly larger than would be predicted from calculations of the variation of the BCS energy gap with current,[1–3] which reduce to essentially the depairing result at $T=0$. These large values of the average j_{co} and their temperature variation in accord with the free-energy prediction are observed only when the thin film strip has a width less than a certain critical value which depends on the film material and thickness.

For a uniform current distribution in films of thickness δ, much less than the penetration depth λ, the usual free-energy calculation of the critical current density leads to the simple relation

$$j_c(t)=H_c(t)/\lambda(t)=(H_0/\lambda_0)(1-t^2)^{3/2}(1+t^2)^{1/2}. \quad (1)$$

Here $t\equiv T/T_c$, H_c is the bulk critical field, and H_c and λ depend on temperature in the way suggested by the Gorter-Casimir two-fluid theory. At $T=0$ the depairing argument[1,4] predicts the somewhat lower critical current density, $j_{co}=0.82\ H_0/\lambda_0$.

The films investigated in this work were produced by vacuum evaporation at pressures of about 5×10^{-7} Torr onto microscope slide substrates held at 77°K and had thicknesses from 170 to 910 Å, widths down to 0.4 μ, and lengths of order 500 μ. The evaporation masks were formed by scribing with a razor blade a film of clear plastic lacquer, previously applied to the substrate and subsequently removed with acetone following the evaporation. Film thickness was measured using a standard quartz crystal-thickness monitor. Strip widths were measured using an optical microscope and carry an uncertainty of about 0.1μ. At this resolution no tapering of the film edges could be observed. Critical currents were measured by using a dc four-terminal method capable of detecting potential differences of 5×10^{-9} V. A low-resistance (0.1-Ω) shunt connected in parallel with the sample prevented thermal runaway and consequent destruction of the film when the critical current of the strip was exceeded. With normal-state sample resistances of order 50Ω the system was in principle capable of detecting the resistance of a normal cross section of the strip about 1.0 Å in length, even at current levels appropriate to temperatures quite near T_c. With this sensitivity the critical-current criterion of first voltage onset appears to be meaningful.

Experimental values of the average critical current density for a tin film 1.9 μ wide are shown as a function of temperature in Fig. 1. The solid curve is a plot of Eq. (1) with the value of j_{co} chosen to fit the data, while the dashed curve gives for the same value of j_{co}, the critical current density from Rogers' calculation.[2] The agreement between the experimental data and the temperature dependence predicted by Eq. (1) is typical of that obtained for very narrow specimens of tin and lead in this work. In Table I experimental values of j_{co} determined by this curve-fitting procedure are given for a variety of films along with the corresponding values predicted by the simple depairing theory. The predicted value of j_{co} depends on the film thickness and purity primarily through the factor λ^{-1}. In this case the values of λ_0 appropriate for a given film thickness were taken from the calculations of Douglass[5] based on the Ginzburg-Landau theory and assuming the impurity-limited mean free path l to be infinite. The finite value of l that

[1] J. Bardeen, Rev. Mod. Phys. 34, 667 (1962).
[2] K. T. Rogers, Ph.D. thesis, University of Illinois, 1960 (unpublished).
[3] R. H. Paramenter, RCA Rev. 23, 323 (1962).
[4] M. Tinkham, in *Low Temperature Physics*, edited by C. Dewitt *et al.* (Gordon and Breach, Science Publishers, Inc., New York, 1962), p. 149.
[5] D. H. Douglass, Jr., Phys. Rev. 124, 735 (1961).

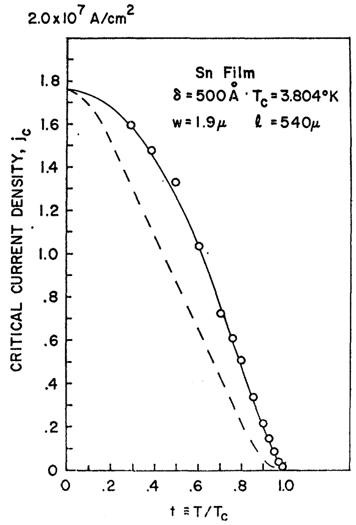

FIG. 1. Variation of average critical current density with reduced temperature for a narrow thin film of tin. The solid curve gives the temperature dependence predicted by the free-energy theory and the dashed curve that predicted by the Rogers-Bardeen calculation.

would need to be used in these calculations in order to give exact agreement is also given.

Values of j_{co}[6] as a function of film width for a typical series of lead films are shown in Fig. 2. For very narrow films, j_{co} is independent of the width within the limits of experimental uncertainty. As the strip width is increased beyond a certain value, j_{co} begins to decrease and the average critical current density shows marked deviations from the behavior predicted by Eq. (1). In this latter regime one expects that the actual current density will be highest at the film edges and that a determination of the true critical current density from a total critical-current measurement must rely heavily on a prediction of the current distribution in the film.[7] In wide films the average current density is much smaller but again is independent of the strip width.

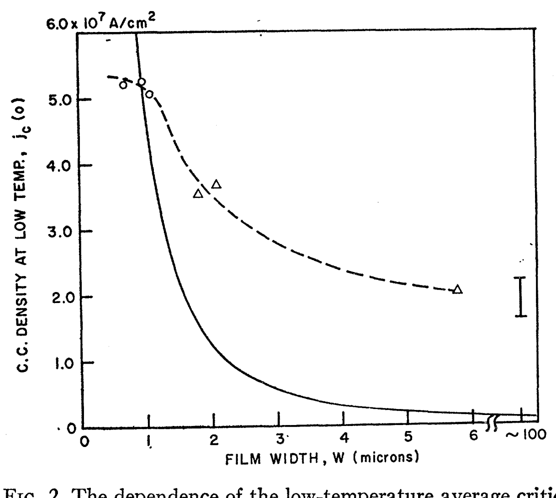

FIG. 2. The dependence of the low-temperature average critical current density on strip width for thin lead films with $\delta \approx 500$ Å. The solid curve indicates the estimated width of a flux vortex pair due to the self-field of the current at the film edges. The bar indicates the range of values observed for very wide films.

The basis for identifying a critical width for flat thin film strips may be understood semiquantitatively in terms of the flux-vortex picture for thin films in perpendicular magnetic fields.[8,9] As the current is increased in a wide flat film, flux vortices due to the self-field of the current may be expected to enter the film at the edges since the entry of such vortices tends to produce a more uniform current distribution in the film and thus to lower the energy of the system. In the present work dc voltages of about 10^{-7} volts have been observed in wide films at current levels such that the effective "resistance" would be characteristic of a normal section of the strip only 1 Å long in the direction of current flow. It thus seems highly probable that in wide films the initial breakdown of the superconducting state with increasing current occurs as the result of the dissipative motion of vortices across the film. According to this hypothesis, films which are too narrow to contain a pair of vortices (oppositely directed at opposite edges of the film) should not suffer this critical-current degradation due to dissipative flux motion. One can also infer that the critical current in wide films gives an indication of the pinning forces on vortices, and that the small voltage which appears in the absence of thermal runaway is a measure of the "viscosity" which the vortices encounter.

TABLE I. A comparison of observed values of the low-temperature average critical current density j_{co} with the values predicted by the simple depairing theory assuming the impurity-limited mean free path l to be infinite.

Material	δ (Å)	w (μ)	j_{co}(obs.) (A/cm²)	j_{co}(theor., $l=\infty$) (A/cm²)	l (Å)
Sn	500	1.9	1.76×10^7	1.92×10^7	1970
Sn	295	1.0	1.54×10^7	1.57×10^7	1940
Sn	170	2.9	1.11×10^7	1.28×10^7	324
Pb	475	1.0	5.26×10^7	7.46×10^7	180
Al	500	3.0	$2.9\ \times10^6$	3.16×10^6	2000

If the vortices are assumed to be circular and to contain a single fluxoid quantum, $\varphi_0 = h/2e$, then their effective size is given approximately by the relation $D_v = 2(\varphi_0/\pi B)^{1/2}$ where B is the average magnetic field in the vortex. If for simplicity the field lines are taken to be circles then the field at the film edges would lead to vortices of approximate diameter

$$D_v = 2(\varphi_0/\mu_0 j\delta)^{1/2}, \qquad (2)$$

where μ_0 is the permeability of vacuum. In Fig. 2 the

[6] Note that for Pb films wider than 1 μ the temperature dependence of Eq. (1) was not followed and approximate j_{co} values were obtained from the lowest temperature data in these cases. These values of j_{co} are indicated by the triangles in Fig. 2.

[7] R. E. Glover and H. T. Coffey, Rev. Mod. Phys. **36**, 299 (1964).

[8] M. Tinkham, Phys. Rev. **129**, 2413 (1963).

[9] R. D. Parks, in *Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio, 1964*, edited by J. G. Daunt *et al.* (Plenum Press, Inc., New York, 1965), p. 34.

width of a pair of such vortices is shown by the solid curve. In spite of the oversimplification inherent in the proposed model, the intersection of the two curves in Fig. 2 at the experimental critical width indicates very good agreement and similar agreement is obtained for tin films at their much lower critical current density for which the corresponding width is about 2μ.

The suggested relation between the vortex size and critical film width can also account for the observation of the "training" phenomenon in films of width greater than the critical value. The number of quenching trials needed to achieve the maximum critical current in wide films tended to increase with film width, as might be expected in view of the larger number of possible vortex arrangements in the wider films. On the other hand, "training" was never observed in films of less than the critical width. The use of very small samples, therefore, may offer special advantages for thin-film experiments in which the presence of flux vortices severely complicates the interpretation of results.

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Energy Exchanges Attending Field Electron Emission

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The energy exchange attending field electron emission (Nottingham effect) is shown to be localized to the emitting area of the cathode. It is further shown that the magnitude and direction (i.e., heating or cooling) depend strongly on cathode temperature, work function, and applied electric field. The temperature boundary separating emission cooling and heating is considerably below theoretical expectations for clean and for zirconium-oxygen-coated tungsten. The existing theory of the Nottingham effect, examined in the light of these and other results, must be modified to include the variation of average energy of the conducting carriers with temperature and field.

I. INTRODUCTION

ELECTRON emission is accompanied by energy exchanges between the conduction electrons and lattice, which become particularly important at the very high emission densities feasible with field and thermal-field (T-F) emission cathodes. Their study is of basic interest since it provides a complementary check, through a direct measurement of the average energy of emitted electrons, of the theory of field and T-F emission; it is also of practical importance because these energy exchanges control the cathode-emitter-tip temperature and set an upper limit on the feasible emission density. The work reported herein is an attempt to confirm, by direct measurement of the energy exchange, the theoretically predicted temperature dependence of the energy exchange and the reversal of its direction (from cathode heating to cooling) at high emitter temperatures.

There are two main emission-induced energy-exchange phenomena. The familiar resistive Joule heating effect was studied in the case of field emission by Dyke *et al.*[1] and Dolan, Dyke, and Trolan.[2] In the usual case where resistivity increases rapidly with temperature, resistive heating by itself leads to an inherently unstable situation at high emission densities. Since stable high-density emission is observed,[3] there must exist another factor having a strong and stabilizing influence on the cathode-tip temperature.

Such a stabilizing factor is provided by the energy exchange resulting from the difference between the average energy of the emitted electrons $\langle E\rangle$ and that of the replacement electron supplied from the Fermi sea, $\langle E'\rangle$. In the case of thermionic emission this phenomenon, discussed by Richardson[4] and later by Nottingham,[5] is well known and produces cooling of a cathode with a work function ϕ by an average energy amount $e\phi+2kT$ per emitted electron. The corresponding effect in field and T-F emission was first discussed by Fleming and Henderson,[6] who were unable to detect it experimentally, and has been a subject of controversy[5,6] with respect to the correct value of $\langle E'\rangle$ and hence the direction of the effect (cathode cooling occurs when $\langle E\rangle>\langle E'\rangle$, and heating when $\langle E\rangle<\langle E'\rangle$). Preliminary data reported earlier[7] tended to support the view of Nottingham who took $\langle E'\rangle$ to be the Fermi energy E_f and, on that basis, predicted heating of the cathode in the case of field emission. Thus, the energy exchange

[1] W. P. Dyke, J. K. Trolan, E. E. Martin, and J. P. Barbour, Phys. Rev. **91**, 1043 (1953).
[2] W. W. Dolan, W. P. Dyke, and J. K. Trolan, Phys. Rev. **91**, 1054 (1953).
[3] E. E. Martin, J. K. Trolan, and W. P. Dyke, J. Appl. Phys. **31**, 782 (1960).
[4] O. A. Richardson, Phil. Trans. Roy. Soc. London **A201**, 497 (1903).
[5] W. B. Nottingham, Phys. Rev. **59**, 907 (1941).
[6] J. E. Henderson and G. M. Fleming, Phys. Rev. **48**, 486 (1935); **54**, 241 (1938); **58**, 908 (1941).
[7] F. M. Charbonnier, R. W. Strayer, L. W. Swanson, and E. E. Martin, Phys. Rev. Letters **13**, 397 (1964).