# Bloch Equation for Conduction-Electron Spin Resonance*

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An effective Bloch equation describing the spin magnetization of the electrons in a metal is derived starting from the usual spin transport equation. No assumption is made as to the direction of the static field, so the effect of the orientation of the field on the diffusion constant is taken into account. The diffusion constant, at resonance, is unchanged when the magnetic field is normal to the surface of the metal, but is altered by a factor $(1+\omega_c^2\tau^2)^{-1}$ for the parallel direction, where $\omega_c$ is the cyclotron frequency and $\tau$ the relaxation time. The effect of these results on the transmitted magnetization is discussed.

## I. INTRODUCTION

SINCE the pioneering work of Dyson[1] there have been several different approaches to the theory of electron spin resonance in metals.[2,3] Azbel, Gerasimenko, and Lifshitz,[2,4] modifying the usual transport equation to include the interaction of the electron spins with the magnetic field, correctly predicted the transmission of magnetization through the metal at resonance. This transmitted radiation has recently been observed by a number of investigators.[5–7] It was also pointed out by Azbel *et al.* that for long relaxation times $\tau$ the transmitted magnetization depended on the orientation of the static magnetic field $\mathbf{H}_0$ to the surface of the metal. Their analysis becomes extremely complicated and a simplified approach, based on Torrey's[8] modified Bloch[9] equation, to account for the diffusion of nuclei in nuclear magnetic resonance studies, has been suggested by Kaplan.[3] In this treatment, a term $D_0\nabla^2\mathbf{M}$ is added to the phenomenological Bloch equation in order to describe the diffusion of electrons in and out of the skin depth. Kaplan considered the case when $\mathbf{H}_0$ was normal to the surface and $D_0$ was taken to be $\frac{1}{3}v_F^2\tau$; $v_F$ is the Fermi velocity. The case when $\mathbf{H}_0$ was parallel to the surface was not treated by Kaplan, but one intuitively expects that the diffusion constant will no longer be given simply by $D_0$ since cyclotron effects would alter the diffusion of the electrons into the metal if $\omega_c\tau\gg1$, where $\omega_c$ is the cyclotron frequency.

In this paper an attempt is made to present a more rigorous justification for the use of the modified Bloch equation to describe conduction-electron spin resonance in metals and in so doing we hope to generalize this equation for arbitrary direction of the static field $\mathbf{H}_0$. This would give the simplified approach proposed by Kaplan a wider range of applicability and allow one to semiquantitatively understand many of the recent transmission experiments.

We find that an effective Bloch equation can be derived starting with the proposed kinetic equation of Azbel *et al.* with the simplifying assumption of spherical energy surfaces. Explicit expressions for the diffusion constant are obtained for an arbitrary direction of the field. As one would expect, the diffusion constant for the field normal to the metal is simply given by $\frac{1}{3}v_F^2\tau$ at resonance, whereas this is only true for the parallel case when $\omega_c\tau\ll1$. The general expression for the diffusion constant in a parallel field is

$$D_{11}=v_F^2\tau/3(1+\omega_c^2\tau^2) \quad \text{(at resonance)}.$$

## II. DERIVATION OF THE MODIFIED BLOCH EQUATION

The kinetic equation[2,4] describing the change in the distribution function of noninteracting electrons in a metal, including the interaction of their spins with the magnetic field, is given by

$$\frac{\partial\mathfrak{f}}{\partial t}+\mathbf{v}\cdot\nabla_r\mathfrak{f}-\frac{e}{c\hbar}(\mathbf{E}+\mathbf{v}\times\mathbf{H})\cdot\nabla_k\mathfrak{f}-\frac{i\mu}{\hbar}[\boldsymbol{\sigma}\cdot\mathbf{H},\,\mathfrak{f}]+\left(\frac{\partial\mathfrak{f}}{\partial t}\right)_{\text{coll}}=0, \quad (1)$$

where $\mu$ is the magnetic moment of the electron and the components of $\boldsymbol{\sigma}$ are the Pauli spin matrices $\sigma_x$, $\sigma_y$, and $\sigma_z$. The distribution function $\mathfrak{f}$ is a $2\times2$ matrix whose diagonal elements give the probability that an electron is at the point $\mathbf{r}$, in state $\mathbf{k}$ with spin in the direction of or opposite to the magnetic field $\mathbf{H}$. The magnetic field $\mathbf{H}$ consists of two fields, a static field $\mathbf{H}_0$ and a rotating field $\mathbf{H}_1$ orthogonal to $\mathbf{H}_0$. To a good approximation, we can assume that the Lorentz force acting on the electron is governed solely by $\mathbf{H}_0$ and that the small electric field generated by the penetration of $\mathbf{H}_1$ into the metal is negligible. We therefore omit the electric

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[1] F. J. Dyson, Phys. Rev. **98**, 349 (1955).

[2] M. Ia. Azbel, V. I. Gerasimenko, and I. M. Lifshitz, Zh. Eksperim. i Teor. Fiz. **32**, 1212 (1957) [English transl.: Soviet Phys.—JETP **5**, 986 (1957)]; the spin transport equation had also been used in a paper by V. P. Silin, Zh. Eksperim. i Teor. Fiz. **30**, 421 (1956) [English transl.: Soviet Phys.—JETP **3**, 305 (1956)], where an attempt is made to derive a Bloch equation.

[3] J. I. Kaplan, Phys. Rev. **115**, 575 (1959).

[4] M. Ia Azbel and I. M. Lifshitz, Progr. Low Temp. Phys. **3**, 288 (1961).

[5] R. B. Lewis and T. R. Carver, Phys. Rev. Letters **12**, 693 (1964).

[6] N. S. Vander Ven and R. T. Schumacher, Phys. Rev. Letters **12**, 695 (1964).

[7] S. Schultz and C. Latham, Phys. Rev. Letters **15**, 148 (1965); S. Schultz and M. R. Shananbarger, *ibid.* **16**, 178 (1966).

[8] H. C. Torrey, Phys. Rev. **104**, 563 (1956).

[9] F. Bloch, Phys. Rev. **70**, 460 (1946).

field term and replace $\mathbf{H}$ by $\mathbf{H}_0$ in the Lorentz term in Eq. (1).

The general expression for the magnetization is given by[10]

$$\mathbf{M}(\mathbf{r},t)=\frac{\mu}{8\pi^3}\int d^3k\,\mathrm{Tr}(\sigma\mathfrak{f}). \tag{2}$$

As in Ref. 2, it proves convenient to separate the density operator into a part which describes the magnetization of the metal and a part which describes the other transport properties. This is accomplished by the substitution

$$\mathfrak{f}=\boldsymbol{\sigma}\cdot\mathfrak{F}+FI, \tag{3}$$

where $I$ is the unit matrix. This can always be done since $\boldsymbol{\sigma}$ and $I$ form a complete set of $2\times2$ matrices. With this substitution and using the usual commutation rules[11] for the components of $\boldsymbol{\sigma}$, Eq. (2) reduces to

$$\mathbf{M}(\mathbf{r},t)=\frac{\mu}{4\pi^3}\int\mathfrak{F}d^3k. \tag{4}$$

With the aid of Eq. (3) one can separate the kinetic equation into two equations which describe the change of $\mathfrak{F}$ and $F$, namely,

$$\frac{\partial\mathfrak{F}}{\partial t}+\mathbf{v}\cdot\nabla_r\mathfrak{F}-\frac{e}{c\hbar}(\mathbf{v}\times\mathbf{H}_0)\cdot\nabla_k\mathfrak{F}-\frac{2\mu}{\hbar}(\mathfrak{F}\times\mathbf{H})+\left(\frac{\partial\mathfrak{F}}{\partial t}\right)_{\mathrm{coll}}=0 \tag{5}$$

and a similar equation for $F$ except that the fourth term in Eq. (5) is absent. In obtaining (5) we have used the following relation:

$$(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{B})=\mathbf{A}\cdot\mathbf{B}+i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B}), \tag{6}$$

where $\mathbf{A}$ and $\mathbf{B}$ are ordinary vectors.

Since the spin-flip relaxation time $T_1$ is much longer than the collision relaxation time $\tau$, we can separate the collision term into two parts,

$$\left(\frac{\partial\mathfrak{f}}{\partial t}\right)_{\mathrm{coll}}=\left(\frac{\partial\mathfrak{f}}{\partial t}\right)_{\tau}+\left(\frac{\partial\mathfrak{f}}{\partial t}\right)_{T_1}, \tag{7}$$

where $(\partial\mathfrak{f}/\partial t)_\tau$ represent the change in $\mathfrak{f}$ due to momentum-changing collisions but with no spin reversal and $(\partial\mathfrak{f}/\partial t)_{T_1}$ represents only spin-flip collisions. The momentum-changing collisions are inelastic collisions due to the electron-phonon interaction but at sufficiently low temperatures and, since only electrons around the Fermi surface are affected ($E_F\gg kT$), we can neglect the small energy change that the electron suffers and treat these collisions as elastic. If one assumes that $(\partial\mathfrak{f}/\partial t)_{T_1}$ can be represented by a relaxation time[12] and uses the fact that the longitudinal and transverse relaxation times are equal (i.e., $T_2=T_1$) for electrons in a metal,[13–15] he can write[2]

$$\left(\frac{\partial\mathfrak{F}}{\partial t}\right)_{T_1}=\frac{\mathfrak{F}-\chi(E)\mathbf{H}_0}{T_1}\quad\text{for}\quad\mu H_0\ll kT, \tag{8}$$

where

$$\chi(E)=\mu\frac{\partial f_0}{\partial E}\quad\text{and}\quad f_0=\left[\exp\frac{(E-E_F)}{kT}+1\right]^{-1}.$$

Our coordinate system is such that the direction of $\mathbf{H}_0$ is along the $z$ axis. Substituting Eq. (8) into Eq. (5) yields

$$\frac{\partial\mathfrak{F}}{\partial t}+\mathbf{v}\cdot\nabla_r\mathfrak{F}-\frac{e}{c\hbar}(\mathbf{v}\times\mathbf{H}_0)\cdot\nabla_k\mathfrak{F}-\frac{2\mu}{\hbar}(\mathfrak{F}\times\mathbf{H})+\frac{\mathfrak{F}-\chi\mathbf{H}_0}{T_1}+\left(\frac{\partial\mathfrak{F}}{\partial t}\right)_\tau=0. \tag{9}$$

The transverse components of the magnetization are determined by the $x$ and $y$ components of $\mathfrak{F}$, which can be written as

$$\frac{\partial\mathfrak{F}_t}{\partial t}+\mathbf{v}\cdot\nabla_r\mathfrak{F}_t-\frac{e}{c\hbar}(\mathbf{v}\times\mathbf{H}_0)\cdot\nabla_k\mathfrak{F}_t-\frac{2\mu}{\hbar}(\mathfrak{F}_t\times\mathbf{H}_0)+\left(\frac{\partial\mathfrak{F}_t}{\partial t}\right)_\tau-\frac{2\mu}{\hbar}(\mathfrak{F}_z\times\mathbf{H}_1)+\frac{\mathfrak{F}_t}{T_1}=0, \tag{10}$$

where $\mathfrak{F}_t=\hat{\mathbf{i}}\mathfrak{F}_x+\hat{\mathbf{j}}\mathfrak{F}_y$ and $\mathfrak{F}_z=\hat{\mathbf{k}}\mathfrak{F}_z$, or in scalar form

$$\frac{\partial\mathfrak{F}^+}{\partial t}+\mathbf{v}\cdot\nabla_r\mathfrak{F}^+-\frac{e}{c\hbar}(\mathbf{v}\times\mathbf{H}_0)\cdot\nabla_k\mathfrak{F}^++i\omega_0\mathfrak{F}^+-\gamma_0 i\mathfrak{F}_zH^++\left(\frac{\partial\mathfrak{F}^+}{\partial t}\right)_\tau+\frac{\mathfrak{F}^+}{T_1}=0, \tag{11}$$

where

$$\mathfrak{F}^+=\mathfrak{F}_x+i\mathfrak{F}_y,\quad\omega_0=\gamma_0H_0,\quad\text{and}\quad H^+=H_{1x}+iH_{1y}.$$

Of course, there is a similar equation for $\mathfrak{F}_z$ and, in fact, the two equations are coupled. However, if we are far from saturation, $M_z$ will, to a good approximation, be equal to its equilibrium value $M_0$ or, similarly,

$$\mathfrak{F}_z=\chi(E)H_0=\mu(\partial f_0/\partial E)H_0.$$

This choice of $\mathfrak{F}_z$ yields for $M_z$

$$M_z=\frac{\mu}{4\pi^3}\int\mathfrak{F}_zd^3k=\mu\int\mathfrak{F}_zdE\frac{1}{4\pi^3}\int\frac{dS}{|\nabla_kE|}. \tag{12}$$

[10] L. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1958).

[11] L. Landau and E. M. Lifshitz, *Quantum Mechanics* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1958).

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[15] Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1963), Vol. 14.

Substituting for $\mathfrak{F}(z)$ and approximating $\partial f_0/\partial E$ to be a delta function yields

$$M_z=2\mu^2 g(E_F)H_0, \tag{13}$$

where $g(E_F)$ is the density of states at the Fermi surface. This is the usual result for the temperature-independent Pauli paramagnetism. Thus the usual linearization procedure is accomplished by substituting $\chi(E)H_0$ for $\mathfrak{F}_z$ in Eq. (11) and this is the case which shall concern us here.

Our primary interest is deriving an effective Bloch equation and no attempt will be made to solve the differential equation for $\mathfrak{F}^+$. This can easily be done for the simplifying case of spherical energy surfaces or if the energy is a quadratic function of $\mathbf{k}$[16] and to see this we make the following substitution:

$$\mathfrak{F}^+=g_0(\mathbf{r},k,t)+\mathbf{v}\cdot\mathbf{g}_1(\mathbf{r},k,t), \tag{14}$$

where $g_0$ and $\mathbf{g}_1$ only depend on the modulus of $k$. Upon substitution into (11) and using the fact that $\mathbf{v}$ is equal to $\hbar\mathbf{k}/m^*$, (11) separates into an even and an odd equation in $\mathbf{k}$. Thus

$$\frac{\partial g_0}{\partial t}+\mathbf{v}\cdot\nabla_r(\mathbf{v}\cdot\mathbf{g}_1)+i\omega_0 g_0-i\gamma_0\mu\frac{\partial f_0}{\partial E}H_0H^+ +\frac{g_0}{T_1}=0, \tag{15a}$$

$$\mathbf{v}\cdot\frac{\partial\mathbf{g}_1}{\partial t}+\mathbf{v}\cdot\nabla_r g_0+i\omega_0\mathbf{v}\cdot\mathbf{g}_1+\frac{\mathbf{v}\cdot\mathbf{g}_1}{\tau}+\frac{\mathbf{v}\cdot\mathbf{g}_1}{T_1} -\frac{e}{m^*c}(\mathbf{v}\times\mathbf{H}_0)\cdot\mathbf{g}_1=0. \tag{15b}$$

We have used the fact that $(\partial g_0/\partial t)_\tau=0$ and $(\partial\mathbf{v}\cdot\mathbf{g}_1/\partial t)_\tau=\mathbf{v}\cdot\mathbf{g}_1/\tau$ for spherical energy bands and elastic collisions. We can further simplify (15b) to

$$\mathbf{v}\cdot\nabla_r g_0-\frac{e}{m^*c}\mathbf{v}\cdot(\mathbf{H}_0\times\mathbf{g}_1)+\left(i\omega_0+\frac{1}{\tau^*}\right)\mathbf{v}\cdot\mathbf{g}_1=0, \tag{16}$$

where

$$\tau^*=\frac{\tau_r}{1-i\omega\tau_r} \quad\text{and}\quad \tau_r=\frac{\tau T_1}{\tau+T_1}\simeq\tau \quad\text{for}\quad T_1\gg\tau.$$

In obtaining (16) we assumed the applied field is circularly polarized and rotates with a single frequency $\omega$. One can solve Eq. (16) by factoring the $\mathbf{v}$ and taking a cross and dot product with $\mathbf{H}_0$. The result is

$$\mathbf{g}_1=-\left\{\frac{\gamma_c^2\tau^{*3}(\nabla_r g_0\cdot\mathbf{H}_0)\mathbf{H}_0-\gamma_c\tau^{*2}(1+i\omega_0\tau^*)(\nabla_r g_0\times\mathbf{H}_0)+\tau^*(1+i\omega_0\tau^*)^2\nabla_r g_0}{(1+i\omega_0\tau^*)[(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}]}\right\}. \tag{17}$$

The magnetization is given by

$$M^+=M_x+iM_y=\frac{\mu}{4\pi^3}\int\mathfrak{F}^+d^3k=\frac{\mu}{4\pi^3}\int g_0 d^3k, \tag{18}$$

and the rate of change of magnetization is

$$\frac{\partial M^+}{\partial t}=\frac{\mu}{4\pi^3}\int\frac{\partial g_0}{\partial t}d^3k, \tag{19}$$

where

$$\frac{\partial g_0}{\partial t}=-\mathbf{v}\cdot\nabla_r(\mathbf{v}\cdot\mathbf{g}_1)-\left(i\omega_0+\frac{1}{T_1}\right)g_0+i\gamma_0\mu\frac{\partial f_0}{\partial E}H_0H_0^+. \tag{20}$$

Upon integration we have

$$\frac{\partial M^+}{\partial t}=\frac{\mu}{4\pi^3}\int-\mathbf{v}\cdot\nabla_r(\mathbf{v}\cdot\mathbf{g}_1)d^3k -\left(i\omega_0+\frac{1}{T_1}\right)M^+ +i\gamma_0 M_0H^+. \tag{21}$$

If one were to neglect the term $\int\mathbf{v}\cdot\nabla_r(\mathbf{v}\cdot\mathbf{g}_1)d^3k$ the resulting equation would reduce to the usual Bloch equation for an homogeneous system. This term describes the fact that the rotating field $H_1$ can only penetrate into the skin depth and that the electrons can now diffuse in and out of the skin depth. Since we made no assumption about the direction of the field $H_0$ we will be able to see how the Bloch equation depends on the orientation of $H_0$ to the surface, and how the orientation affects the diffusion process.

The first term on the right-hand side (r.h.s.) of Eq. (21) can be written as

$$\sum_{i,j}\int v_i v_j\nabla_i g_{1j}d^3k. \tag{22}$$

For a spherical energy surface and assuming an isotropic relaxation time, the only contribution to the integral comes from the $i=j$ terms and, since only electrons at the Fermi surface contribute, we can replace $v_i^2$ by $\frac{1}{3}v_F^2$. This would also be true for an arbitrary dispersion law if the crystal had cubic symmetry or was a polycrystalline sample. Then $v_F^2$ would be the square of the Fermi velocity averaged over the Fermi surface. Therefore we could write Eq. (22) as $\frac{1}{3}v_F^2\int\nabla_r\cdot\mathbf{g}_1 d^3k$.

The effective Bloch equation is obtained by substituting our expression for $\mathbf{g}_1$ and carrying out the integration.

[16] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, London, 1954).

The result is

$$\frac{\partial M^+}{\partial t}=\frac{v_F^2\tau^*}{3(1+i\omega_0\tau^*)} \times\left\{\frac{\gamma_e^2\tau^{*2}\mathbf{H}_0\cdot\nabla_r(\nabla_r M^+\cdot\mathbf{H}_0)+(1+i\omega_0\tau^*)^2\nabla^2M^+}{[(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}]}\right\} -\left(i\omega_0+\frac{1}{T_1}\right)M^+ +i\gamma_0M_0H^+. \quad (23)$$

Our coordinate system was chosen such that the $z$ axis was along the direction of $\mathbf{H}_0$. Let us define the $z'$ axis to be normal to the metal and for the particular case of an infinite slab $M^+$ will only be a function of $z'$. Equation (23) can then be written as

$$\frac{\partial M^+}{\partial t}=\frac{v_F^2\tau^*}{3(1+i\omega_0\tau^*)}\frac{[(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}\cos^2\alpha]}{[(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}]}\frac{d^2M^+}{dz'^2} -\left(i\omega_0+\frac{1}{T_1}\right)M^+ +i\gamma_0M_0H^+, \quad (24)$$

where $\alpha$ is the angle between the field $\mathbf{H}_0$ and the normal to the metal, i.e., the angle between $z$ and $z'$. If we define the diffusion constant $D$ to be

$$D=\frac{v_F^2\tau^*}{3(1+i\omega_0\tau^*)}\left\{\frac{(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}\cos^2\alpha}{(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}}\right\}, \quad (25)$$

Eq. (24) becomes

$$\frac{\partial M^+}{\partial t}=D\frac{d^2M^+}{dz'^2}-\left(i\omega_0+\frac{1}{T_1}\right)M^+ +i\gamma_0M_0H^+. \quad (26)$$

We see that (25) is the equation used by Kaplan but the diffusion constant is no longer given by $\frac{1}{3}v_F^2\tau$. For the two cases of a perpendicular or a parallel field $\cos\alpha$ will be 1 and 0, respectively, and the diffusion constant becomes

$$D_\perp=\frac{v_F^2\tau^*}{3(1+i\omega_0\tau^*)}$$

and (27)

$$D_\parallel=\frac{v_F^2\tau^*}{3}\frac{(1+i\omega_0\tau^*)}{[(1+i\omega_0\tau^*)^2+\omega_c^2\tau^{*2}]}.$$

Remembering that $\tau^*=\tau/(1-i\omega\tau)$ we have at resonance,

$$D_\perp=\tfrac{1}{3}v_F^2\tau \quad \text{and} \quad D_\parallel=\frac{v_F^2\tau}{3(1+\omega_c^2\tau^2)}. \quad (28)$$

This result demonstrates that, for the normal case, $D$ is simply given by $\frac{1}{3}v_F^2\tau$, as one would expect, but for a parallel field, $D$ becomes $v_F^2\tau/3\omega_c^2\tau^2$ when $\omega_c\tau\gg1$. In all cases $D$ is equal to $\frac{1}{3}v_F^2\tau$ when $\omega_c\tau\ll1$.

## III. TRANSMISSION

In order to find the magnetization in the metal one must solve Maxwell's equations together with Eq. (23). For the case of an infinite slab one finds that the space part of the field has the form $\exp(-Kz')^3$ where there are two values for the decay constant $K$. In general, one finds a large but short-range field which decays in the skin depth $\delta$ and a small but slowly decaying field with decay constant given by

$$K=(1/\delta_{\text{eff}})[1+i(\omega-\omega_0)T_1]+O(\delta^2/\delta_{\text{eff}}^2) \quad (29)$$

and at resonance

$$K=1/\delta_{\text{eff}}, \quad (30)$$

where $\delta_{\text{eff}}=(DT_1)^{1/2}$. When the field is perpendicular to the metal $D_\perp=\frac{1}{3}v_F^2\tau$ while $D_\parallel=v_F^2\tau/3(1+\omega_c^2\tau^2)$ for a parallel field. Therefore the decay of the magnetization into the metal depends on the orientation of the static field $H_0$ when $\omega_c\tau\gg1$. Substituting for $D_\perp$ and $D_\parallel$ we find that the decay constants for the two field directions are

$$K_\parallel=\frac{\omega_c\tau}{l}\left(\frac{3\tau}{T_1}\right)^{1/2} \quad \text{and} \quad K_\perp=\frac{1}{l}\left(\frac{3\tau}{T_1}\right)^{1/2} \quad \text{for } \omega_c\tau\gg1, \quad (31)$$

where $l$ is the mean free path of the electrons. Of course the amplitude of the field will also be dependent on the decay constants and this dependence can be obtained by solving the boundary-value problem. This will be done in a forthcoming paper[17] and will not be discussed here. The important point is that the signal transmitted will depend on the orientation of the field for $\omega_c\tau\gg1$, and if one is trying to determine the decay constant by measuring the amplitude of the transmitted signal as a function of sample thickness[6] this dependence has to be taken into account.

## CONCLUDING REMARKS

We have seen that it is possible to derive an effective Bloch equation for conduction-electron spin resonance assuming spherical energy surfaces. The equation takes the form proposed by Kaplan with a modified diffusion constant given by Eq. (25) which is valid for all orientations of the static field if $\omega_c\tau\ll1$, $D$ is simply given by $\frac{1}{3}v_F^2\tau$ and there will be no dependence on the direction

[17] T. G. Castner (to be published).

of $H_0$ whereas this no longer remains true if $\omega_c\tau \gg 1$. Cyclotron effects alter the diffusion of electrons into the metal which manifests itself in a more complex diffusion constant. For the extreme cases of parallel and perpendicular fields we find that $D_\perp$ is equal to $D_0$ and $D_\parallel$ equals $D_0/(1+\omega_c^2\tau^2)$ at resonance. It was also pointed out that these results will affect the transmitted magnetization in a way that agrees with the conclusions in Ref. (2). The effective Bloch equation is valid for all orientations of the static field and should be helpful in a semiquantitative understanding of recent experiments in conduction-electron spin resonance.

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# Magnetic Irreversible Solution of the Ginzburg-Landau Equations*

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We present the exact magnetic irreversible solution of the Ginzburg-Landau equations for a cylinder of infinite length (whose Ginzburg-Landau parameter $\kappa$ is unity and whose radius $R$ is three coherence lengths $\xi$) in an axial magnetic field $H_0$ for all values of $H_0$. Solutions for other values of $\kappa$ (0.3 to 3) and $R/\xi$ (2 to 12) are also discussed. We have determined, as a function of $H_0$ and as a function of position, the order parameter, the vector potential, the internal magnetic field, and the current density; and also as a function of $H_0$, the total number of superconducting electrons per unit length of the cylinder and the magnetization per unit volume. This solution is magnetically irreversible and hysteretic because of persistent currents which flow in the sample perpendicular to the applied magnetic field. The magnetization is reversible only over intervals of $H_0$ over which the number of fluxoids is conserved; otherwise it is irreversible. This solution does not depend on defects and is the counterpart to Abrikosov's magnetic reversible mixed-state solution. It is dominant in thin specimens.

## I. INTRODUCTION

IT was predicted by Abrikosov[1] that the magnetization per unit volume $4\pi M$ of a type-II superconductor of infinite extent is magnetic reversible when in the mixed state. $4\pi M$ depends only on the value of the applied magnetic field $H_0$ and not on the previous history of the sample. Experimentally, however, a certain degree of irreversibility is always found, and it is large for Ginzburg-Landau[2] $\kappa$ values of order unity[3] and small when $\kappa \gg 1$. Thin samples appear to be always irreversible[4] regardless of the quality of the sample preparation. For magnetic fields $H_0$ larger than the bulk critical field $H_{c2}$ the surface remains superconducting up to[5] $H_{c3}$. The superconducting surface is able to carry persistent circulating currents[6] around the circumference of the sample. The direction of circulation of these currents depends on the direction of the change of the external magnetic field.[7] A long macroscopic cylinder, therefore, appears as a "giant vortex" ($H_{c2} \lesssim H_0 \leq H_{c3}$) whose physical size is determined by the sample dimensions. Such a surface state in the form of a giant vortex exists also for $H_0 < H_{c2}$ as has been shown experimentally[8] just below $H_{c2}$. This state is quite different from the mixed state since it can carry a total current whereas the ideal mixed state (without pinning centers) cannot.[9,10] Over a finite interval of the external magnetic field this current can be changed such that it conserves the number of fluxoids enclosed in the sample.[6-8,11] When a total current is flowing in a macroscopic specimen in order to conserve the number

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