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Energy and Angular Distributions of Electrons Ejected from Hydrogen and Helium by 100- to 300-keV Protons*

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Measurements have been made of the differential cross sections for ejection of electrons of various energies at various angles from hydrogen and helium gases bombarded by 100- to 300- keV protons. After electrostatic analysis, individual electrons were counted using an electron-multiplier detector. Measurements were made at nine angles from 10° to 160° and at electron energies down to 2 eV. Comparison is made with Born-approximation calculations using hydrogen wave functions but with the results scaled to molecular hydrogen and to helium. The Gryziński classical theory is also used to calculate cross sections and these are compared with the Born approximation and with the experimental values. A simple semi-empirical equation has been found which fits the energy distributions fairly well at low and intermediate energies.

I. INTRODUCTION

ONE of the important processes that takes place when fast charged particles traverse a gas is ionization. Theoretical calculations of the cross section for ionization can be made for simple systems using the Born approximation and these results can be scaled to other systems. The Gryziński classical theory can also be used and offers the advantage of ease of computation. Total ionization cross sections calculated on the basis of these methods yield satisfactory agreement with experiment at high enough energies but become less satisfactory at lower energies. For protons bombarding hydrogen gas, for example, the departure from agreement begins to become serious below about 80 keV. The corresponding figure for helium gas is about 250 keV. It would be desirable to have an experimental basis for checking the accuracy of these and other theoretical treatments of ionization. However, a more intimate comparison of experiment with theory is desirable to better ascertain the applicability of a given theoretical treatment. Such a comparison can be made using differential cross sections. Kuyatt and Jorgensen[1] (hereafter referred to as KJ) pointed out the need for a more extensive comparison between the Born predictions and experiment especially at the higher energies where the Born approximation is expected to be valid. KJ presented some calculations using hydrogen wave functions and Rudd and Jorgensen[2] (hereafter referred to as RJ) presented further Born computations. Oldham[3] has recently made improved calculations of cross sections differential in both energy and angle of ejected electrons.

Previous experimental work of this kind was done by KJ who measured the angular and energy dependence of the cross section for ejection of electrons from hydrogen by protons in the energy range 50 to 100 keV and by RJ who obtained data for protons on helium from 50 to 150 keV. The present work extends the data for hydrogen and helium to 300 keV and thus enters the energy range where the theoretical treatments provide good total ionization cross sections.

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[1] C. E. Kuyatt and T. Jorgensen, Jr., Phys. Rev. **130**, 1444 (1963).
[2] M. E. Rudd and T. Jorgensen, Jr., Phys. Rev. **131**, 666 (1963).
[3] W. J. B. Oldham, Phys. Rev. **140**, A1477 (1965).

II. EXPERIMENTAL METHOD

The apparatus was similar to that used previously[2] so only a brief description will be given here. The proton beam traverses the gas in the collision chamber and is collected by a shielded, biased Faraday cup. The beam current incident on the cup is integrated by a polystyrene capacitor and operational amplifier. Proton beam currents were typically 0.1 to 0.2 μA. Generally a total proton charge of around 1 μC was collected for each reading. An automatic shutoff stopped the current integration at a predetermined point and simultaneously stopped the scaler which counted electrons.

Electrons could be extracted through any of nine ports arranged from 10° to 160° from the beam direction. An arrangement of baffles was placed inside the collision chamber so that the electron detection system "looked" into a blackbody cavity at each of the ports. This was done to prevent reflected electrons from appreciably affecting the results.

The energy analysis in this work was done by a parallel-plate electrostatic analyzer such as that described by Harrower[4] and others. The relation between the plate spacing (3.24 cm) and the separation between entrance and exit slits was such that the energy of the electrons passed by the analyzer was equal to the potential on the back plate and was independent of the preacceleration voltage placed on the front plate. This latter voltage was kept at +10 V. With the slit widths used the energy resolution of the analyzer was about 4%. The electron detector was a 14-stage Dumont SPM-03-301 electron multiplier operated at a potential of 4000 V. Output pulses of about 10-mV amplitude were amplified and counted by the scaler.

The steady and 60-cycle components of the magnetic field were annulled by three pairs of 4-ft-diam Helmholtz

[4] G. A. Harrower, Rev. Sci. Instr. **26**, 850 (1965).

coils. The steady component was read with a Rawson Model 727 rotating-coil gaussmeter connected to a high-gain preamplifier and oscilloscope. Magnetic fields in the scattering and analyzer regions were reduced below 5 mG. The vacuum was maintained by an oil diffusion pump used with a water-cooled baffle and a zeolite sorbent trap. The base pressure was about 2×10^{-6} Torr.

Pressure measurements were made with a GIC-017 ionization gauge calibrated to within 5% by a liquid-nitrogen-trapped GM-110 McLeod gauge. No account was taken of the error due to the pumping action of the cold trap in the McLeod-gauge line. However, since only hydrogen and helium were used the error due to this effect should be only a few percent.

III. MEASUREMENTS

Account was taken of absorption of electrons by the target gas as in the previous work. The fraction transmitted was calculated by the relation $t=e^{-\alpha p x}$, where α is the absorption coefficient at the electron energy in question, p is the target gas pressure, and x is the effective path length at the pressure p. The value of x here was 6.1 ± 0.6 cm. Absorption coefficients given by Normand[5] and by Golden and Bandel[6] were used. Hydrogen was used at pressures of 0.5 to 0.9 μ and helium at 1.4 to 1.8 μ. At these pressures the absorption correction ranged from about 25% at the lowest energies down to less than 1% above about 300–400 eV.

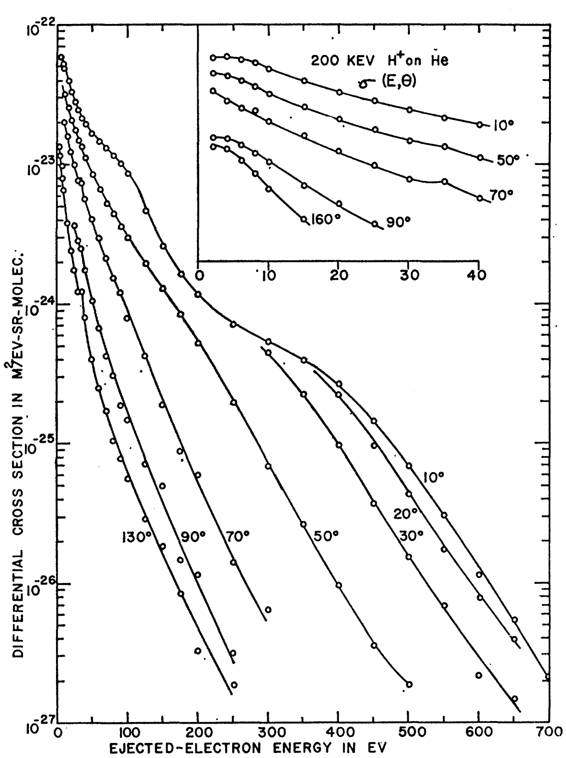

FIG. 1. Doubly differential cross section for ejection of electrons by 200-keV protons in helium.

[5] C. E. Normand, Phys. Rev. 35, 1217 (1930).
[6] D. E. Golden and H. W. Bandel, Phys. Rev. 138, A14 (1965).

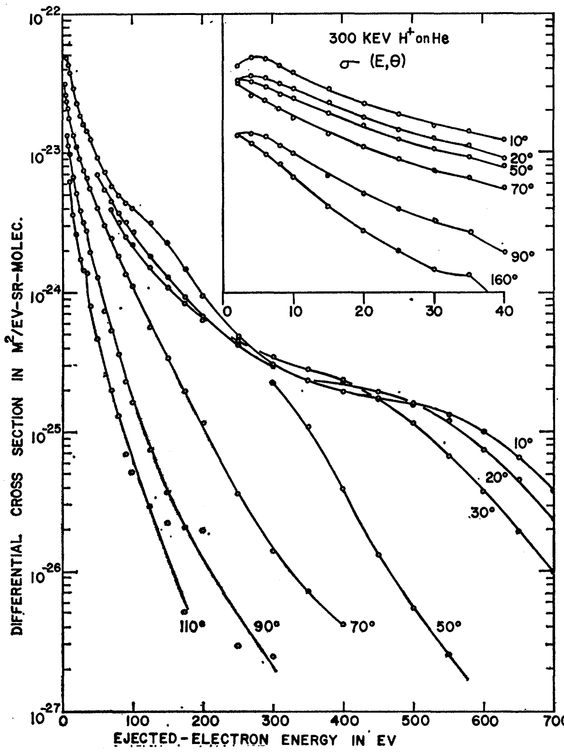

FIG. 2. Doubly differential cross section for ejection of electrons by 300-keV protons in helium.

It was found that there was an appreciable number of counts which depended on the beam and the gas pressure but which were not due to electrons or other charged particles since their number was unaffected by any electric or magnetic fields in the region of the analyzer. This effect which was more pronounced with hydrogen was attributed to photons from the collisional excitation of the gas by the proton beam. Photons from the scattering center could go directly into the first dynode after one reflection from the shiny back surface of the analyzer. For some of the runs the back plate was coated with a colloidal solution of carbon and this reduced the effect considerably. Nevertheless, these counts were subtracted in the calculation of cross sections as were the counts from the residual background gas.

The dependence of the cross sections on the magnitude of the beam current, the target gas pressure, and the preacceleration voltage was not appreciable. Furthermore, data taken near the beginning of the experimental period agreed well with reruns taken near the end of the period.

Numbers of counts, pressure, and other data were recorded directly on IBM cards and calculation of the cross sections was done by a computer using Eq. (2) of KJ. The efficiency of the multiplier detector was taken as 0.78, the value measured by RJ. An attempt was made to measure this quantity in the same manner as

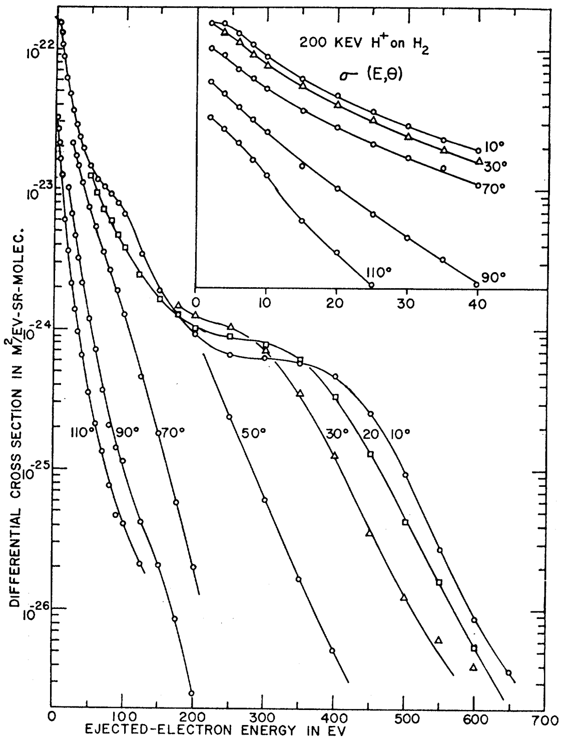

FIG. 3. Doubly differential cross section for ejection of electrons by 200-keV protons in hydrogen.

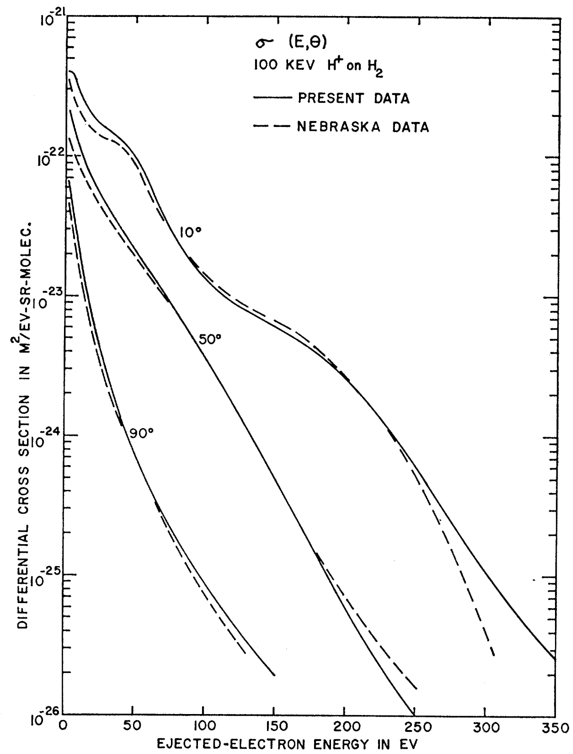

FIG. 5. Comparison of present cross-section data with that of Rudd and Jorgensen (see Ref. 2).

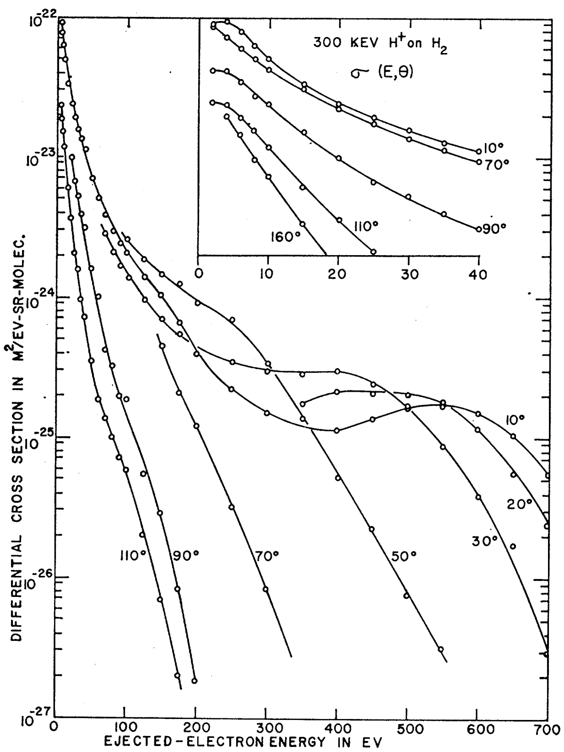

FIG. 4. Doubly-differential cross section for ejection of electrons by 300-keV protons in hydrogen.

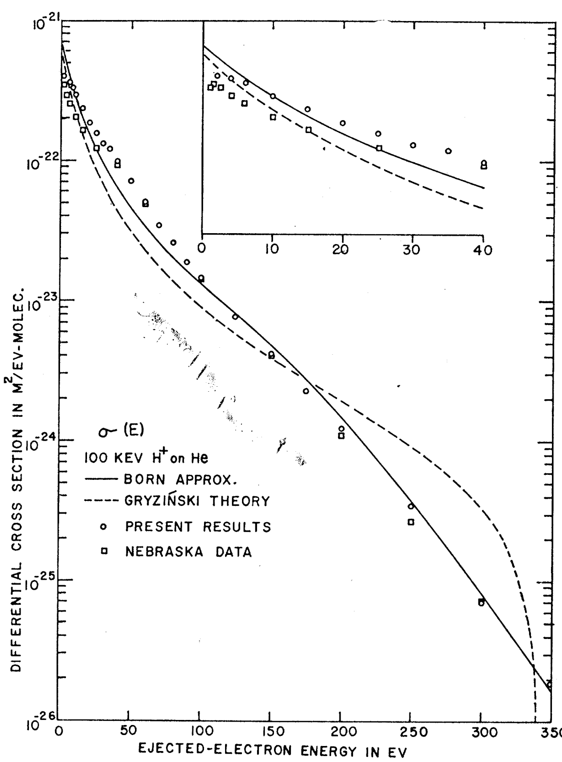

FIG. 6. Differential cross section for ejection of electrons by 100-keV protons in helium. Integrated over all angles.

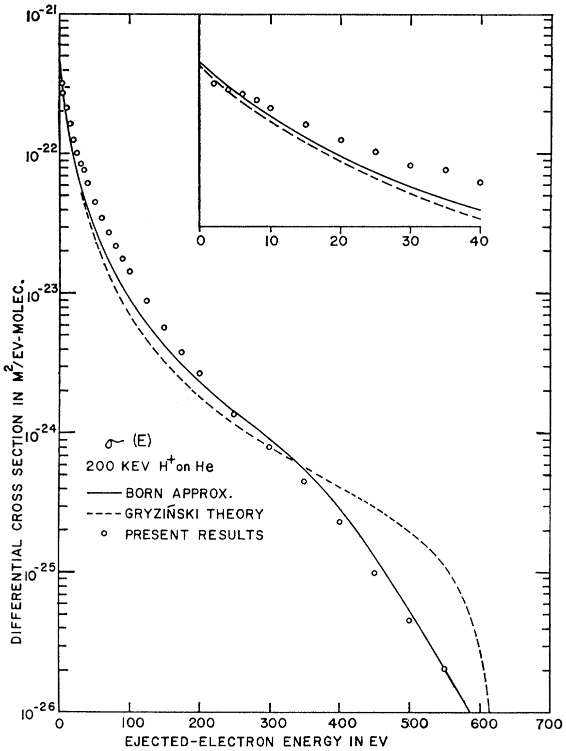

FIG. 7. Differential cross section for ejection of electrons by 200-keV protons in helium. Integrated over all angles.

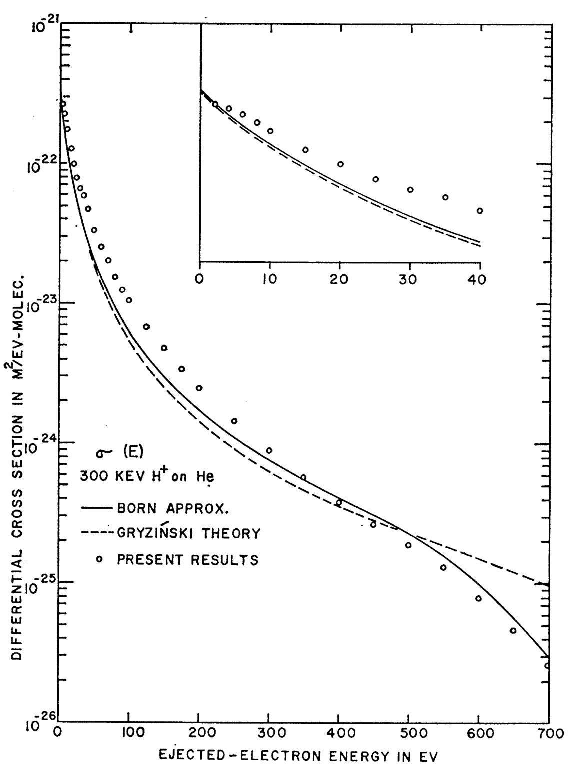

FIG. 8. Differential cross section for ejection of electrons by 300-keV protons in helium. Integrated over all angles.

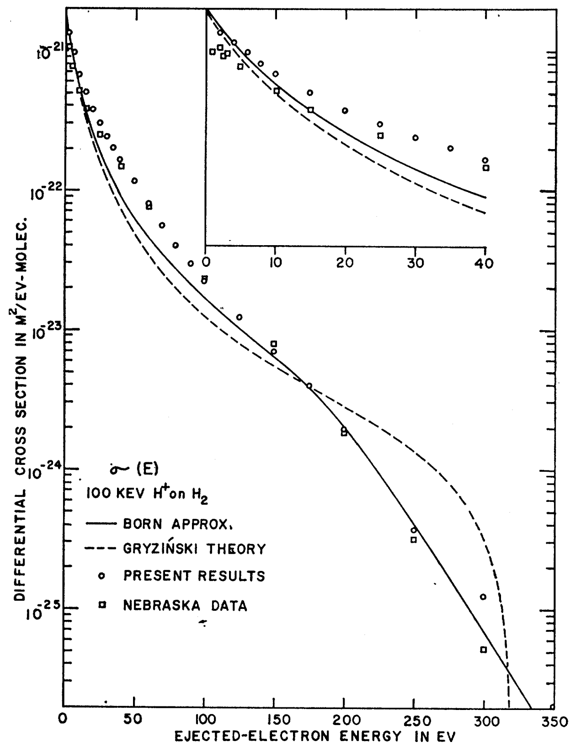

FIG. 9. Differential cross section for ejection of electrons by 100-keV protons in hydrogen. Integrated over all angles.

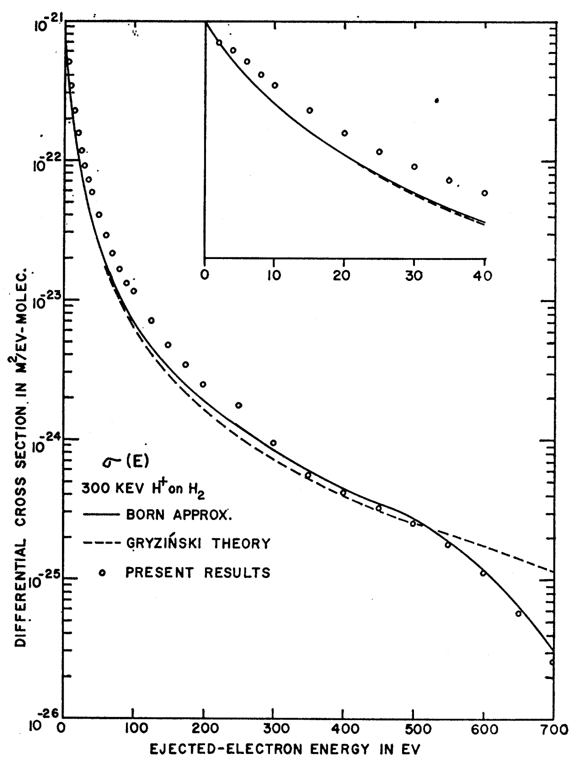

FIG. 10. Differential cross section for ejection of electrons by 300-keV protons in hydrogen. Integrated over all angles.

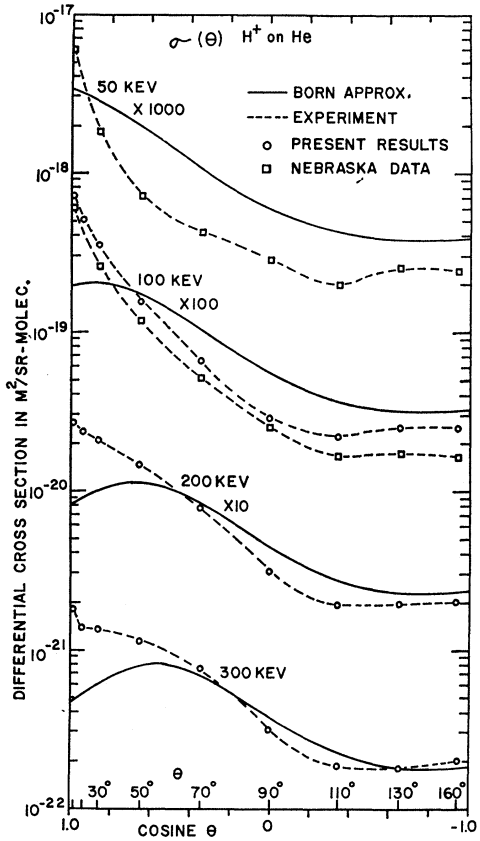

FIG. 11. Angular distribution of electrons of all energies ejected by protons in helium.

RJ, i.e., by reading current to a cup which replaced the electrostatic analyzer. This yielded cross sections differential in angle which could be compared to the integral of the doubly differential cross sections over all electron energies. This was done at the 10° and 20° ports and yielded an average efficiency of 0.85±0.08. This just overlaps the actual value used. For reasons discussed later it is believed that the 0.78 figure is closer to the correct efficiency.

IV. EXPERIMENTAL RESULTS

Doubly differential cross sections were measured at 10°, 20°, 30°, 50°, 70°, 90°, 110°, 130°, and 160° with 100-, 150-, 200-, and 300-keV protons on hydrogen gas and at the same angles with 100-, 200-, and 300-keV protons on helium gas. Figures 1–4 show some of the cross sections plotted versus ejected electron energy with the angle of ejection as a parameter. The small irregularity at 35 eV in some of the helium curves is due to the auto-ionization peaks reported previously.[7]

On Fig. 5 a comparison with the previous work at Nebraska for 100-keV protons on hydrogen is plotted. The agreement is very good at intermediate and high electron energies but below about 50 eV the present cross sections are higher than the previous results. The discrepancy is as much as 30% at energies around 10 or 20 eV. The cause of this difference is not known and this points up the diffiuclty of making this kind of measurement at low electron energies.

The doubly differential cross-section data may be integrated once to obtain singly differential cross sections (differential in either energy or angle) or twice to obtain the total cross section for production of electrons. Thus,

$$\sigma(\theta)=\int_0^\infty \sigma(E,\theta)dE\,,$$

$$\sigma(E)=2\pi\int_0^\pi \sigma(E,\theta)\,\sin\theta d\theta\,,$$

$$\sigma_i=2\pi\int_0^\infty\int_0^\pi \sigma(E,\theta)\,\sin\theta d\theta dE\,.$$

These integrations have been done by the computer and values of $\sigma(E)$ are shown in Figs. 6–10 and later in Fig. 17. Graphs of $\sigma(\theta)$ are given in Figs. 11 and 12. The squares are the Nebraska data. As might be expected, higher energy protons eject more high-energy electrons. It is evident from the angular distributions that higher

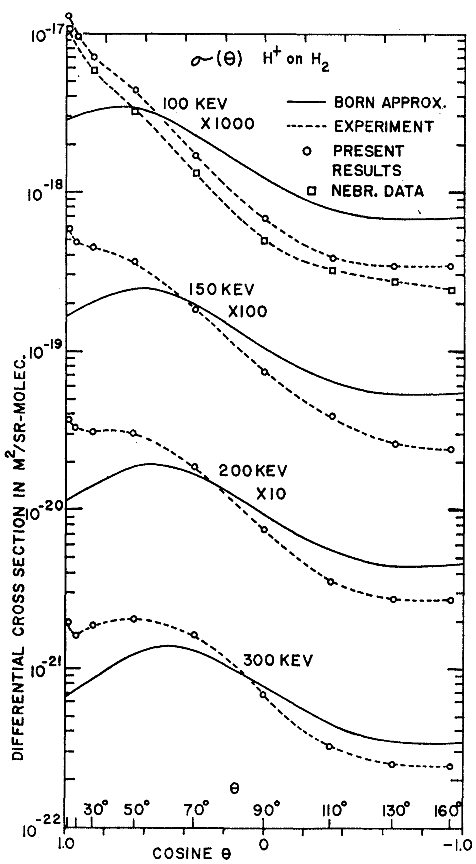

FIG. 12. Angular distribution of electrons of all energies ejected by protons in hydrogen.

[7] M. E. Rudd, Phys. Rev. Letters 13, 503 (1964); 15, 580 (1965).

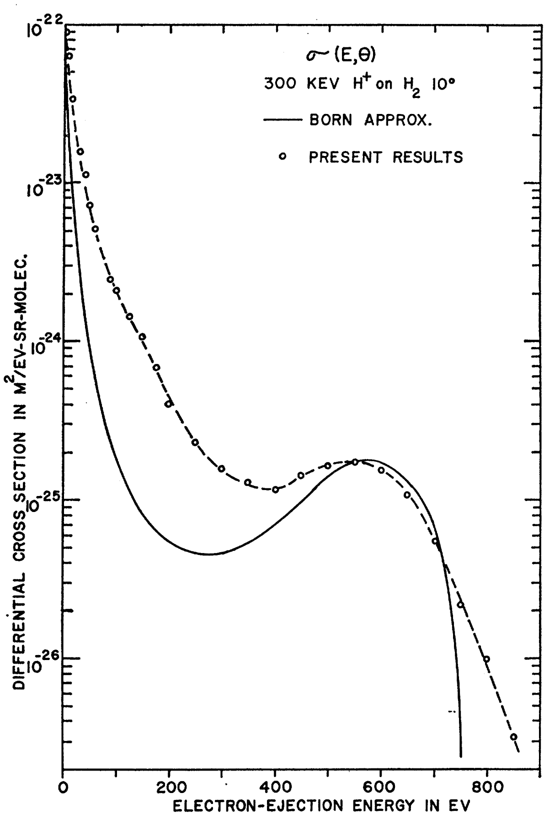

FIG. 13. Comparison of Born calculation and experiment for the energy distribution of electrons ejected at 10° by 300-keV protons in hydrogen.

energy protons eject electrons more nearly isotropically whereas low-energy protons tend to eject them in a nearly forward direction.

When integrated over both energy and angle, total cross sections for ionization are obtained which are uniformly 28% higher than the generally accepted values of other experimenters. The values of these cross sections are, of course, directly influenced by the choice of the value of efficiency of the detector. Because of the uncertainty in this number and because of the discrepancy in the energy distributions with previous work, the results of this experiment must be considered to be uncertain by about 30%.

V. COMPARISON WITH QUANTUM THEORY

Massey and Mohr[8] and others have applied the Born approximation to the ionization of hydrogen atoms. Bates and Griffing[9] have made some computations for heavy-particle collisions of the distributions in electron energy but not in angle. KJ give an equation derived from that of Massey and Mohr which, after numerical integration over one variable, yields the doubly differential cross sections we are measuring. Bates and Griffing give a procedure by which the total cross sections can be scaled from atomic hydrogen to other targets. However, the procedure for scaling cross sections differential in electron-ejection energy are somewhat more involved. One must scale not only the cross section and the projectile energy, but also the energy of the ejected electron. The requirement to scale the electron energies was not understood when the previous computations using the KJ equation were made[2] and consequently the published values are not correct. The correct scaling equation is

$$\sigma(E,E_p,U,n)=n(U_{\mathrm{H}}/U)^3\sigma(EU_{\mathrm{H}}/U,E_pU_{\mathrm{H}}/U,U_{\mathrm{H}},1),$$

where E is the energy of the ejected electron, E_p is the incident proton energy, n is the number of electrons per molecule, and U and U_{H} are the ionization potentials of the target molecule and of the hydrogen atom, respectively. We have scaled only to molecular hydrogen and to helium. Presumably, the same scaling procedure would also work for more complicated systems if one treated each electronic shell separately and then added the partial cross sections.

Using this scaling equation and the Born approximation equation given by KJ, computations of doubly and singly differential cross sections and total cross sections

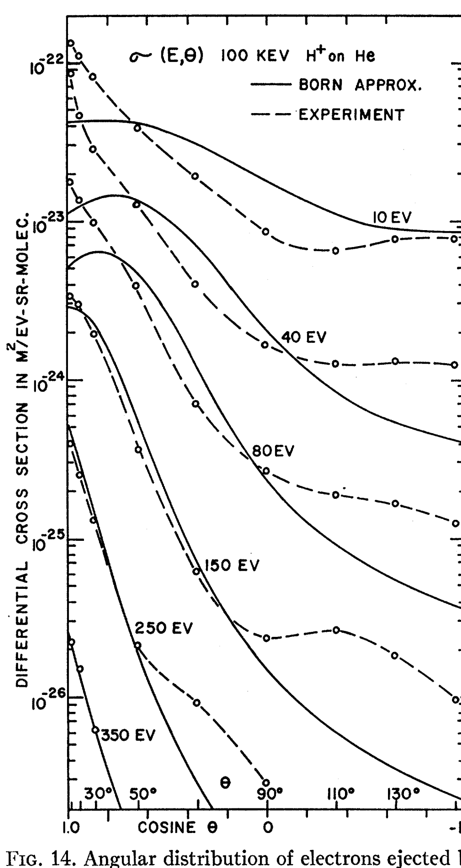

FIG. 14. Angular distribution of electrons ejected by 100-keV protons in helium.

[8] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. (London) **A140**, 613 (1933).

[9] D. R. Bates and G. Griffing, Proc. Phys. Soc. (London) **A66**, 961 (1953).

were made. As was expected the integrated cross sections were identical to those given by Bates and Griffing. Comparison of the calculated doubly differential cross sections with experiment are given in Figs. 13–16. In Fig. 13 the rather pronounced peak in the energy distribution of about 550 eV is reproduced fairly well by the theory although around 150 eV the two curves differ by more than a factor of 10.

The angular distributions as shown in Figs. 14–16 exhibit rather good agreement over intermediate angles especially at the higher electron energies, but discrepancies appear in the forward and backward directions. Agreement does not improve greatly upon integration over electron energies as is shown in Figs. 11 and 12. However, integration over angles does produce fair agreement as shown in Figs. 6–10 where Born curves and experiment never disagree by more than a factor of 1.6 for either hydrogen or helium.

The good agreement of the present results with the Nebraska work and with the Born approximation at high electron energies is the justification for using the value 0.78 for the detector efficiency.

VI. COMPARISON WITH CLASSICAL THEORY

Recently interest in the use of a classical theory of atomic collisions has arisen partly because the computations are so much easier than those using quantum-mechanical methods. The approach of Gryziński[10] is probably best known and has been used by a number of investigators[11] to calculate ionization cross sections. Previous comparisons[2] with the Gryziński theory showed fair agreement but some of his earlier work has been found to be in error.[12] We have now made computations of cross sections using Eq. (53) of his more recent work[13] assuming a delta-function distribution of orbital electron velocities with his $\mathcal{E}_1$ equal to the ionization potential U and his ΔE equal to $E+U$. E is the ejected electron energy. U was taken as 15.4 eV for hydrogen and 24.6 eV for helium. Since Gryziński considers only a binary collision, i.e., a collision between the incoming particle and one electron, the effect of the nucleus is neglected except that it provides the binding energy. Therefore angular distributions, being strongly affected by the nucleus, would not be expected to be predicted accurately in this approximation. However, energy distributions do not have this limitation and the theory can be expected to provide these as well as the total cross sections.

Results are plotted in Figs. 6–10 where it is seen that while the agreement is not as good in general as that given by the Born approximation, nevertheless the theory agrees with experiment within a factor of 2 over nearly all of the energy range. The use of the delta-function velocity distribution results in the cross section dropping to zero at a certain value of electron energy while the experimental data and the Born approximation continue to have finite values. If the delta function were replaced by a more realistic velocity distribution the theoretical values at very high electron energies would be considerably improved.

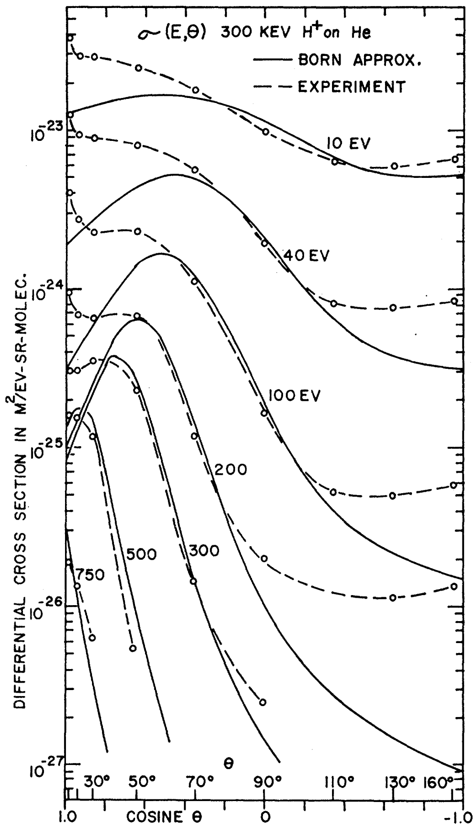

FIG. 15. Angular distribution of electrons ejected by 300-keV protons in helium.

According to the Gryziński theory the dependence of the cross section on ejected-electron energy E is $\sigma(E) \propto (E+U)^{-3}$ for low electron energies. If this is correct a plot of log(cross section) versus $\log(E+U)$ would be a straight line with a slope of -3. To check this the plot shown in Fig. 17 was drawn. The graphs are indeed straight lines but the slopes are somewhat less than 3. The straight-line behavior extends farther the higher the incident proton energy. The dropping away of the very lowest energy points from the straight-line dependence is probably instrumental and it is likely that more careful low-energy measurements would find the cross sections following the straight line down to 0 eV ejection energy.

The straight lines were used to obtain values for the constants A and B in the equation $\sigma(E)=A(E+U)^{-B}$. The results are given in Table I. We have also made calculations of total cross sections for ionization of

[10] M. Gryziński, Phys. Rev. **115**, 374 (1959).
[11] See, e.g., R. H. McFarland and J. D. Kinney, Phys. Rev. **137**, A1058 (1965) and A. E. Kingston, *ibid.* **135**, A1529 (1964).
[12] R. G. Alsmiller, Jr., Oak Ridge National Laboratory Report No. ORNL-3232, 1962 (unpublished).
[13] M. Gryziński, Phys. Rev. **138**, A322 (1965).

TABLE I. Values of constants in $\sigma(E)=A(E+U)^{-B}$. (E and U are in eV.)

Gas	Proton energy (keV)	A in (10^{-19} m^2/eV molecule)	B	Eq. valid for $E\lesssim$
He	100	1.75	1.80	50
He	200	2.86	2.03	100
He	300	3.43	2.14	175
H_2	100	2.45	1.80	50
H_2	150	3.34	2.00	80
H_2	200	4.52	2.13	125
H_2	300	5.42	2.28	300

hydrogen and helium by protons using Gryziński's Eq. (54). Agreement with experimental values of Hooper *et al.*[14] and with the Born approximation is very good at high proton energies (above about 300 keV for He and above 200 keV for H_2).

VII. CONCLUSIONS

It is evident from this work that while the Born approximation yields good values for total ionization cross sections at high energies and reproduces certain features of the experimental energy and angular distributions of ejected electrons, close quantitative agreement is lacking. Especially poor agreement occurs in the angular distributions. Use of more accurate wave functions might give better argeement but would increase the numerical work. The disagreement may be due to the treatment of the hydrogen molecule as two atoms, because of the scaling procedure, or to something of a more fundamental nature. If measurements such as these could be made at fairly high energies with atomic hydrogen as the target, this question might be resolved.

These measurements reinforce the previous conclusion[2] that the Gryziński classical theory can be very useful for rapid calculations of cross sections. The accuracy, however, is not as good as that given by the Born approximation.

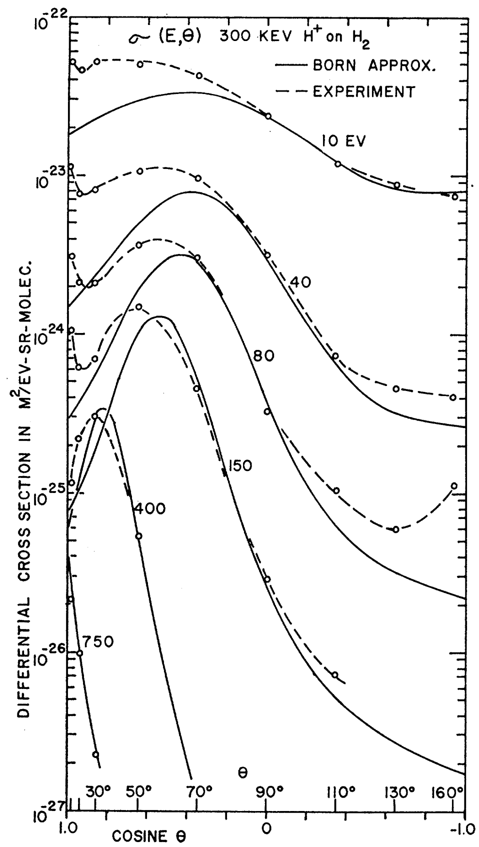

FIG. 16. Angular distribution of electrons ejected by 300-keV protons in hydrogen.

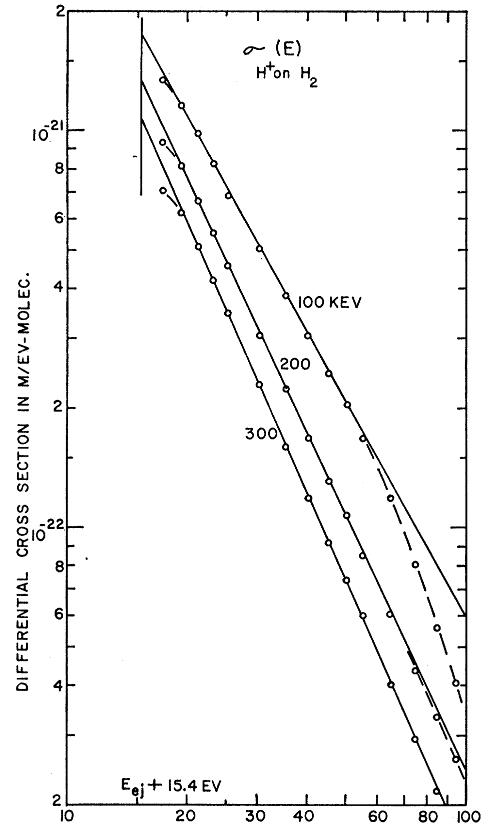

FIG. 17. Differential cross section for electrons ejected at all angles plotted against ejection energy plus ionization potential for protons in hydrogen.

ACKNOWLEDGMENTS

We wish to thank C. E. Kuyatt for his assistance in formulating the scaling procedure in the Born approximation calculations. We are grateful to G. Sandness, C. Jacobs, Jr., D. Lang, and D. Gregoire who materially assisted in constructing the apparatus and in taking the data. We also wish to thank B. Erickson for doing the computer work on the Born approximation.

[14] J. W. Hooper *et al.*, Phys. Rev. **128**, 2000 (1962).