Strong Vertex According to $SL(6,C)$

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The baryon-meson vertex is calculated to lowest nonvanishing order in momentum transfer, in a relativistic $SU(6)$ theory in which the particles are assigned to unitary (infinite-dimensional) representations of $SL(6,C)$. The coupling 56,56,35 contains no arbitrary parameters except for the over-all universal coupling strength. All the results are in excellent agreement with experiment.

1. INTRODUCTION

As soon as Gürsey and Radicati, and Sakita,[1] presented their $SU(6)$ proposal, it was realized that "pure $SU(6)$" was a theory that needed to be generalized. To some the need to reconcile the theory with relativity was paramount; others insisted on its essentially nonrelativistic nature. But all agreed that the field of application of $SU(6)$ must be extended beyond the purely static domain. The first attempt was made by Gürsey and Radicati.[1] They noticed that a strong Yukawa vertex, invariant under static $SU(6)$ and under space reflections, does not exist. Any satisfactory vertex is therefore a step beyond static $SU(6)$. The proposal of Gürsey and Radicati was[1]

$$V\sim\psi^{*ABC}\psi_{A'BC}(\mathbf{q}\pi^-+\pi^-\mathbf{q})_A{}^{A'}. \qquad (1.1)$$

Here ψ_{ABC} is the tensor basis for the 56-dimensional representation of $SU(6)$ associated with baryons, $\pi^-{}_A{}^B$ is the tensor basis associated with the 35-dimensional meson representation, and $\mathbf{q}_A{}^B$ is the "kinetic spurion"

$$\mathbf{q}_A{}^B=\delta_a{}^b\mathbf{q}\cdot\boldsymbol{\sigma}_\alpha{}^\beta.$$

[We use indices $A, B=1,\cdots,6$ for $SU(6)$; $a, b=1,\cdots,3$ for $SU(3)$; α, $\beta=1$, 2, for spin.] The three-vector $\mathbf{q}$ is the momentum of the meson.

The appearance of the kinetic spurion in (1.1) constitutes, of course, breaking of $SU(6)$ invariance. Once we admit this kind of vertex, we might as well write down the most general "invariant" involving one kinetic spurion, namely,

$$V\sim C_1\psi^{*ABC}\psi_{A'BC}(\mathbf{q}\pi^-+\pi^-\mathbf{q})_A{}^{A'} +C_2\psi^{*ABC}\psi_{A'B'C}\mathbf{q}_A{}^{A'}\pi^-{}_B{}^{B'} +C_3\psi^{*ABC}\psi_{ABC}(\mathbf{q}\pi^-)_D{}^D \qquad (1.2)$$

with three arbitrary coefficients.

Although the ansatz (1.2) can be made to fit experiments by a suitable choice of the three undetermined parameters, it does not have aesthetic appeal; nor does it have a theoretical justification. It was natural, therefore, to attempt to justify it, and if possible to determine the three parameters, by regarding it as invariant under a larger symmetry group. Since momenta are involved, and since the kinematics are typically relativistic, it is tempting to assume that relativity is relevant. Thus was born an era of intense activity directed towards a reconciliation of $SU(6)$ with invariance under the transformations of the Poincaré group. Many early attempts were less than completely successful, which led to a widespread disillusionment and a feeling that "relativistic $SU(6)$" was an idle dream. Nevertheless, the desirability of enlarging the field of application of $SU(6)$ remains, and the impossibility of a meaningful relativistic $SU(6)$ remains unproven.

We present here the first calculation of experimental consequences of the symmetry $SL(6,C)$, described previously by one of the authors.[2-6] In Sec. 2 we give a review of the basic features of the theory. Sections 3 and 4 consist of the calculation of the strong baryon-baryon-meson vertex to lowest order in the momenta of the particles involved, and its expression in terms of the usual $SU(6)$ states. In Sec. 5 we present the numerical results of the calculation. Finally, Sec. 6 is concerned with the problem of crossing symmetry.

Briefly, the numerical results are: (1) $D/F=1.8$ for pseudoscalar mesons. (2) $D/F=1.5$ for magnetic interaction of vector mesons. (3) Almost pure F for electric interaction of vector mesons. (4) An improved value for the width of the N^*. (5) Almost pure $M1$ for the $N^*N\rho$ interaction.

2. REVIEW OF RELATIVISTIC $SU(6)$

A model world, in which both $SU(6)$ and the Poincaré group are exact symmetries, has a very large invariance group, namely,[3]

$$G=P\times S \quad \text{(semidirect product)}, \qquad (2.1)$$

where P is the Poincaré group and S is a noncompact

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[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); B. Sakita, Phys. Rev. 137, B1756 (1964).

[2] C. Fronsdal, J. Math. Phys. (to be published.)

[3] P. Budini and C. Fronsdal, Phys. Rev. Letters 14, 968 (1965).

[4] For a very detailed application of this technique to $SL(2,C)$, see G. Bisiacchi and C. Fronsdal, Nuovo Cimento 41, 35 (1966). A more complete discussion of the representations of $SL(n,C)$, including a comparison of our methods with those of Gel'fand and Naimark and Harish-Chandra, may be found in Ref. 2.

[5] For more details, see C. Fronsdal, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), p. 585.

[6] For more details, see C. Fronsdal (Ref. 5), p. 665. The parity operator P must satisfy $PM_A{}^B=N_A{}^BP$; when $N=-\frac{3}{2}$ we have $P=\pm(-)^t$, so that the successive multiplets are 56^-, 700^+, 4592^-, $\cdots$.

group that contains $SL(6,C)$ as a subgroup. We consider the simplest case, when

$$S=SL(6,C).$$

This group contains a subgroup $SU(3)\otimes SL(2,C)$. The second factor is isomorphic to the (homogeneous) Lorentz group; let $s_{\mu\nu}$ be the generators of this subgroup and let $L_{\mu\nu}$ be the generators of Lorentz transformations. Then the precise meaning of (2.1) may be expressed by the statement that

$$L^x{}_{\mu\nu}\equiv L_{\mu\nu}-s_{\mu\nu}$$

commute with all the generators of S. Thus

$$P\times S=P^x\otimes S,$$

where P^x is isomorphic, but not identical, to P. The generators of P^x are the same as those of P, except that $L_{\mu\nu}$ are replaced by $L^x{}_{\mu\nu}$.

Representations of the group (1.1) may be constructed most easily from a representation of P^x and a representation of S. A theory of the $SU(6)$ type can result only if we choose a representation of P^x with real mass and zero spin.[3] A basis for a representation of this type will have the form

$$\psi_\sigma(x),\quad \sigma=1,2,\cdots,$$

where the transformations of P^x act on the argument x and those of S act on the index σ only.

It is necessary to take a unitary representation for S, and since S is noncompact this implies that the index σ runs over an infinite set. Nevertheless, it is convenient to start with a family of finite representations, and pass to a unitary representation by analytic continuation.[4] The simplest finite representations of $SL(6,C)$ have the tensor basis

$$\psi_{A_1\cdots A_{n+k}}{}^{\bar{B}_1\cdots\bar{B}_N} \tag{2.2}$$

symmetric in upper indices and symmetric in lower indices. The generators of $SL(6,C)$ are a set $M_A{}^B$ of operators that act on the undotted indices[6a] and satisfy the commutation relations of $SU(6)$, and a set $N_{\bar{A}}{}^{\bar{B}}$ of operators that act on the dotted indices and satisfy the same commutation relations.[5] A maximal compact subgroup is generated by the subset

$$2\lambda_A{}^B=M_A{}^B+N_{\bar{A}}{}^{\bar{B}}; \tag{2.3}$$

this subgroup is the static $SU(6)$ introduced by Gürsey and Radicati and Sakita.

The reduction of (2.2) into irreducible representations of $SU(6)$ is, for positive integral N and k[6]

$$\psi_{A_1\cdots A_{N+k}}{}^{\bar{B}_1\cdots\bar{B}_N}=S\sum_{t=0}\frac{(2t+k+n-1)!N!(N+k)!}{t!(t+k)!(N-t)!(N+t+k+n-1)!}\times\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}\delta_{A_{t+k+1}}{}^{B_{t+1}}\cdots\delta_{A_{N+k}}{}^{B_N}, \tag{2.4}$$

where S stands for symmetrization with respect to upper indices and symmetrization of the lower indices, $n=6$, and

$$\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}=(\text{traceless projection of})\times\delta_{B_{t+1}}{}^{A_{t+k+1}}\cdots\delta_{B_N}{}^{A_{N+k}}\psi_{A_1\cdots A_{N+k}}{}^{B_1\cdots B_N}. \tag{2.5}$$

The sum over t in (2.4) is finite, because the coefficient vanishes when $N-t$ is a negative integer. The smallest $SU(6)$ representation that appears in (II,4) is the symmetric $\tilde{\psi}_{A_1\cdots A_k}$; application to physical baryons requires that one take $k=3$.

The matrix elements of $M_A{}^B$ and $N_{\bar{A}}{}^{\bar{B}}$ between the basis vectors (2.5) can easily be calculated[7,2] (we give the result below for the relevant value of N), and these are simple analytic functions of the parameter N. The requirement that

$$(M_A{}^B)^\dagger=N_{\bar{B}}{}^{\bar{A}},$$

as must be true in a unitary representation,[5] yields the condition $N=-\frac{9}{2}+i\rho$, ρ real. In addition one has to require that there exist an operator that can be interpreted as intrinsic parity,[6] and this fixes N at $N=-\frac{9}{2}$. For this value of N the reduction (2.4) simplifies to

$$\psi_{A_1\cdots A_{N+k}}{}^{\bar{B}_1\cdots\bar{B}_N}=S\sum_{t=0}^{\infty}(-)^t\frac{(2t+k+n-1)!}{t!(t+k)!}\times\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}\delta_{A_{t+k+1}}{}^{B_{t+1}}\cdots\delta_{A_{N+k}}{}^{B_N}. \tag{2.6}$$

Unlike the case of positive integer N, the reduction now contains an infinite number of terms. In spite of their formal nature, (2.5) and (2.6) provide useful shortcuts to correct results in practical calculations.

The main results that are needed to evaluate the invariant vertex are the following[2,7]:

$$\left(2\lambda_i{}^j+\frac{k}{n}\delta_i{}^j\right)\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}=\sum_{s=1}^{t+k}\delta_{A_1}{}^j\tilde{\psi}_{A_1\cdots i\cdots A_{t+k}}{}^{B_1\cdots B_t}-\sum_{s=1}^{t}\delta_i{}^{B_s}\psi_{A_1\cdots A_{t+k}}{}^{B_1\cdots j\cdots B_t}, \tag{2.7}$$

$$\begin{aligned}2i\lambda'{}_i{}^j\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}&=(M_i{}^j-N_i{}^j)\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}\\&=-(2t+k+n)\tilde{\psi}_{iA_1\cdots A_{t+k}}{}^{jB_1\cdots B_t}-\frac{t(t+k)}{(2t+k+n-1)(2t+k+n-2)}\\&\quad\times S\Big[\delta_i{}^j\delta_{A_1}{}^{B_1}\tilde{\psi}_{A_2\cdots A_{t+k}}{}^{B_2\cdots B_t}+(t-1)\delta_i{}^{B_1}\delta_{A_1}{}^{B_2}\tilde{\psi}_{A_2\cdots A_{t+k}}{}^{B_3\cdots B_t}+(t+k-1)\delta_{A_1}{}^j\delta_{A_2}{}^{B_1}\tilde{\psi}_{iA_3\cdots A_{t+k}}{}^{B_2\cdots B_t}\\&\quad-\frac{(t-1)(t+k-1)}{2t+k+n-3}\delta_{A_1}{}^{B_1}\delta_{A_2}{}^{B_2}\tilde{\psi}_{iA_3\cdots A_{t+k}}{}^{jB_3\cdots B_t}-(2t+k+n-2)\delta_i{}^{B_1}\delta_{A_1}{}^j\tilde{\psi}_{A_2\cdots A_{t+k}}{}^{B_2\cdots B_t}\Big].\end{aligned} \tag{2.8}$$

[6a] For typographical reasons "dotted" indices have bars instead of the more conventional dots.
[7] A. Salam and J. Strathdee, Trieste, Report IC/65/79, 1965 (unpublished).

The operator representing intrinsic parity may be defined for this representation by[6]

$$P\psi_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}=(-)^t\psi_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}.$$

It commutes with $\lambda_A{}^B$ and anticommutes with $\lambda'_A{}^B$.

We could develop the meson representation in a similar fashion, but we prefer the more direct approach of the following section.

3. THE VERTEX IN TERMS OF STATIC $SU(6)$ MULTIPLETS

About the meson representation we know that its reduction according to $SU(6)$ must contain a 35-dimensional $SU(6)$ multiplet with negative parity. In addition, the meson representation must be self-conjugate, because the π^0 is the same as its antiparticle; at first sight this seems to require that an $SU(6)$ singlet meson exist, but in fact it does not.[8] For the present we assume that the reduction according to $SU(6)$ gives a singlet (κ), a **35** with negative parity ($\pi^-{}_A{}^B$), possibly a **35** with positive parity ($\pi^+{}_A{}^B$), and a number of others that we need not name. We can prove that no more than two 35-dimensional representations can occur, and that these have opposite parities.[9]

An invariant baryon-baryon-meson vertex will then have the form

$$V=J\kappa+J^-{}_A{}^B\pi^-{}_B{}^A+J^+{}_A{}^B\pi^+{}_B{}^A+\cdots, \qquad (3.1)$$

where J and $J^\pm{}_A{}^B$ are baryon-baryon currents, linear in ψ and linear in ψ^*. Since J is an $SU(6)$ scalar, it must necessarily couple $SU(6)$ multiplets with the same dimension and therefore with the same parity; hence J and κ have positive parity. The set of all currents in (3.1) must transform contragrediently to the mesons, hence according to a unitary, irreducible representation of $SL(6,C)$. In particular this requires that

$$M_A{}^BM_B{}^AJ=N_{\bar{A}}{}^{\bar{B}}N_{\bar{B}}{}^{\bar{A}}J=CJ, \qquad (3.2)$$

where C is a pure number that characterizes the current representation, and thus also the meson representation. Equation (3.2) has been used to calculate J;[2] the currents $J^\pm{}_A{}^B$ were calculated as follows. First we note that $\lambda'_A{}^BJ$ has the same transformation properties with respect to $SU(6)$ and parity as $J^-{}_A{}^B$, and that the traceless part of $\lambda'_A{}^CJ^-{}_C{}^B$ has the same transformation properties as $J^+{}_A{}^B$. There follows that

$$J^-{}_A{}^B\sim\lambda'_A{}^BJ,$$
$$J^+{}_A{}^B\sim(\text{traceless part of})\ \lambda'_A{}^CJ^-{}_C{}^B.$$

The constants of proportionality are next determined up to arbitrary phases by the requirement that the current representation be unitary. This turns out to be possible only if

$$C<-12.$$

Finally, the phases are determined by requiring that the current representation be self-conjugate.[10]

The final result for the static currents J and $J^\pm{}_A{}^B$, to the extent that it is needed here, is[11]

$$\begin{aligned}J&=-[-\tfrac{7}{5}C]^{-1/2}[\bar{\psi}^{*ABC}\psi_{ABC}+\cdots],\\ J^-{}_A{}^B&=\bar{\psi}^{*CDE}\bar{\psi}_{ACDE}{}^B+\bar{\psi}_A{}^{*BCDE}\bar{\psi}_{CDE}+\cdots,\\ J^+{}_A{}^B&=-[-\tfrac{3}{4}(C+12)]^{-1/2}[\bar{\psi}^{*BCD}\bar{\psi}_{ACD}+\cdots].\end{aligned} \qquad (3.3)$$

With this normalization the currents transform exactly as the mesons:

$$i\lambda'_A{}^B\kappa=\left[-\frac{C}{35}\right]^{1/2}\pi^-{}_A{}^B,$$

$$\begin{aligned}i\lambda'_A{}^B\pi^-{}_C{}^D=&-\left[-\frac{C}{35}\right]^{1/2}(\delta_C{}^B\delta_A{}^D-\tfrac{1}{6}\delta_C{}^D\delta_A{}^B)\kappa\\ &-\frac{3}{8}\left[-1-\frac{C}{12}\right]^{1/2}[\pi^+{}_A{}^D\delta_C{}^B+\pi^+{}_C{}^B\delta_A{}^D\\ &-\tfrac{1}{3}\pi^+{}_A{}^B\delta_C{}^D-\tfrac{1}{3}\pi^+{}_C{}^D\delta_A{}^B]+\cdots,\end{aligned}$$

$$\begin{aligned}i\lambda'_A{}^B\pi^+{}_C{}^D=&\frac{3}{8}\left[-1-\frac{C}{12}\right]^{1/2}[\pi^-{}_A{}^D\delta_C{}^B+\pi^-{}_C{}^B\delta_A{}^D\\ &-\tfrac{1}{3}\pi^-{}_A{}^B\delta_C{}^D-\tfrac{1}{3}\pi^-{}_C{}^D\delta_A{}^B]+\cdots.\end{aligned}$$

The unwritten terms indicated by $+\cdots$, involve higher $SU(6)$ multiplets represented by tensors with four indices (189^+, 189^-, 405^+, and 405^-).

Our results for the vertex function are given by (3.1) and (3.3). Now we have to express this in terms of physical states, i.e., states that transform according to irreducible representations of the Poincaré group P.

4. THE PHYSICAL BASIS

So far we have used a basis adapted to the decomposition of the group $G=P\times S$ as the direct product $P^x\otimes S$. However, before confronting the results with real physics we have to take account of the fact that P, and not P^x, is the physical Poincaré group. It is interesting to note that the distinguished role of P is entirely due to symmetry-breaking effects. Because physical masses are nondegenerate we cannot prepare arbitrary one-particle states, but only eigenstates of mass; and with the help of weak and electromagnetic

[8] The implication is clear for finite representations. For infinite unitary representations self-conjugacy of a representation means that it is equivalent to its contragredient. In fact, W. Rühl, CERN report TH 649 (unpublished), assigns the mesons to a self-conjugate representation that has no singlet and only one 35.

[9] In fact it turns out that the meson representation is precisely as was conjectured in Ref. 6.

[10] In Ref. 6 the meson representation was described by means of the tensor $\varphi_{C_1\cdots C_M\bar{D}_1\cdots\bar{D}_M}{}^{A_1\cdots A_M\bar{B}_1\cdots\bar{B}_M}$. The relationship between C and M is $C=2M(M+5)$. Previously we have thought that self-conjugacy requires that M be real (and hence $-3<M<-2$), but that is not correct.

[11] See Ref. 2 for the complete results.

interactions we find out that eigenstates of mass transform irreducibly under P rather than under P^x.

The generators $s_{\mu\nu}$, which generate a subgroup of $SL(6,C)$ isomorphic to the Lorentz group, are linear combinations of the $SU(3)$-invariant operators $M_{\alpha\alpha}{}^{\alpha\beta}$ and $N_{\alpha\dot{\alpha}}{}^{\alpha\dot{\beta}}$, namely,[12]

$$s_{\mu\nu}=\frac{1}{2i}[(\sigma_{\mu\nu})_A{}^B M_B{}^A+(\sigma_{\mu\nu})_{\bar{A}}{}^{\bar{B}}N_{\bar{B}}{}^{\bar{A}}]. \tag{4.1}$$

In particular, a pure Lorentz transformation L_{i0} is the sum of $L^x{}_{i0}$, which in momentum space transforms the momentum only, and s_{i0}; thus

$$L_{i0}=i\left(p_i\frac{\partial}{\partial p_0}+p_0\frac{\partial}{\partial p_i}\right)+(\sigma_i)_B{}^A\lambda'{}_A{}^B. \tag{4.2}$$

The finite Lorentz transformation which connects the rest system to the system in which the momentum is p, without rotation, is

$$L(p)=\exp[i\theta^i L_{i0}],\quad \theta^i=\frac{p_i}{|p|}\tanh^{-1}\frac{|p|}{p_0}.$$

Consider first the quark representation ξ_A. In this case

$$2iL_{i0}\xi_A=2is_{i0}\xi_A=(\sigma_i)_B{}^C M_C{}^B\xi_A=(\sigma_i)_A{}^C\xi_C$$

and thus

$$L(p)=e^{\frac{1}{2}\theta\cdot\sigma}=[2m(p_0+m)]^{-1/2}(m+p_0+\mathbf{p}\cdot\boldsymbol{\sigma}).$$

Similar expressions are obtained for the action of $L(p)$ on $\xi_{\bar{A}}$, ξ^A, and $\xi^{\bar{A}}$. The result may be written

$$\begin{aligned}L(p)\xi_A&=[2m(p_0+m)]^{-1/2}(m\delta_A{}^B+p_{\bar{A}}{}^B)\xi_B,\\ L(p)\xi_{\bar{A}}&=[2m(p_0+m)]^{-1/2}(m\delta_A{}^B+p_A{}^{\bar{B}})\xi_{\bar{B}},\\ L(p)\xi^A&=[2m(p_0+m)]^{-1/2}(m\delta_B{}^A+p_B{}^{\bar{A}})\xi^B,\\ L(p)\xi^{\bar{A}}&=[2m(p_0+m)]^{-1/2}(m\delta_B{}^A+p_{\bar{B}}{}^A)\xi^{\bar{B}},\end{aligned} \tag{4.3}$$

where

$$p_A{}^{\bar{B}}=p^\mu(\sigma_\mu)_A{}^{\bar{B}},\ p_{\bar{A}}{}^B=p^\mu(\sigma_\mu)_{\bar{A}}{}^B.$$

From this we see that the "covariant trace"

$$(1/m)p_{\bar{A}}{}^B\xi_B\xi^{\bar{A}}$$

of the tensor $\xi_B\xi^{\bar{A}}$ is Lorentz-invariant. Let us replace

$$\delta_A{}^{\bar{B}}\to(1/m)p_A{}^{\bar{B}},\quad \delta_{\bar{A}}{}^B\to(1/m)p_{\bar{A}}{}^B \tag{4.4}$$

in the reduction formula (2.6) writing

$$\psi_{A_1\cdots A_{N+k}}{}^{\bar{B}_1\cdots\bar{B}_N}=S\sum_{t=0}^{\infty}(-)^t\frac{(2t+k+n-1)!}{t!(t+k)!}m^{t-N}\times\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p)p_{A_{t+k+1}}{}^{\bar{B}_{t+1}}\cdots p_{A_{N+k}}{}^{\bar{B}_N}, \tag{4.5}$$

[12] Here $(\sigma_{\mu\nu})_A{}^B=\frac{1}{2}(\sigma_\mu)_A{}^{\bar{C}}(\sigma_\mu)_{\bar{C}}{}^B-\frac{1}{2}(\sigma_\nu)_A{}^{\bar{C}}(\sigma_\mu)_{\bar{C}}{}^B$, and $(\sigma_{\mu\nu})_{\bar{A}}{}^{\bar{B}}$ is defined similarly, with $(\sigma_\mu)_A{}^{\bar{B}}=(1,\,-\boldsymbol{\sigma})$ and $(\sigma_\mu)_{\bar{A}}{}^B=(1,\,+\boldsymbol{\sigma})$.

where

$$\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p)=(\text{covariant traceless part of})\times M^{t-N}p_{\bar{B}_{t+1}}{}^{A_{t+k+1}}\cdots p_{\bar{B}_N}{}^{A_{N+k}}\psi_{A_1\cdots A_{N+k}}{}^{\bar{B}_1\cdots\bar{B}_N} \tag{4.6}$$

and "covariant traceless" means that

$$p_{\bar{B}_1}{}^{A_1}\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p)=0.$$

The terms of the expansion (4.5) are not mixed by the Lorentz generators $L_{\mu\nu}=L^x{}_{\mu\nu}+s_{\mu\nu}$, although they are mixed by $L^x{}_{\mu\nu}$ and by $s_{\mu\nu}$. Under Lorentz transformations the tensor $\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p)$ transforms exactly like $\xi_{A_1}\cdots\xi_{A_{t+k}}\xi^{\bar{B}_1}\cdots\xi^{\bar{B}_t}$.

The formula (4.5) is not a reduction according to static $SU(6)$, but a reduction according to another subgroup of $SL(6,C)$. In fact, define

$$\lambda_A{}^{\bar{B}}(p)=(1/2m)(M_A{}^C p_C{}^{\bar{B}}+p_A{}^{\bar{C}}N_{\bar{C}}{}^{\bar{B}}). \tag{4.7}$$

Comparing with (2.3) we see that $\lambda_A{}^{\bar{B}}(p)$ agrees with the $SU(6)$ generator $\lambda_A{}^B$ when $\mathbf{p}=0$. But the set (4.7) is covariant and satisfies covariant commutation relations; hence it spans an algebra that is isomorphic to $SU(6)$. We shall use the symbol $SU(6)_p$ to denote either the algebra or the group defined by (4.7). The tensor $\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p)$ transforms irreducibly under $SU(6)_p$, in exactly the same way as $\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}$ transforms under $SU(6)$. The physical meaning of $SU(6)_p$ becomes clear when we notice that the subset

$$(\sigma_\mu)_{\bar{B}}{}^A\lambda_A{}^{\bar{B}}(p)=\frac{1}{2m}p^\nu[(\sigma_{\nu\mu})_A{}^C M_C{}^A+(\sigma_{\mu\nu})_{\bar{B}}{}^{\bar{C}}N_{\bar{C}}{}^{\bar{B}}]$$

are the generators of Wigner's little group. Hence $SU(6)_p$ is the Gürsey-Radicati-Sakita synthesis of the little group with $SU(3)$. Reducing each $SU(6)_p$ multiplet according to the little group completes the reduction of the baryon representation into irreducible representations of the Poincaré group.

Finally we have to express $\tilde{\psi}\cdots{}^{\cdots}$ in terms of $\tilde{\psi}\cdots{}^{\cdots}(p)$. Both sets of tensors refer to states with momentum p; only the bases are different. Since they agree at $\mathbf{p}=0$, one has

$$e^{-i\theta^i s_{i0}}\tilde{\psi}\cdots{}^{\cdots}=L(p)^{-1}\tilde{\psi}\cdots{}^{\cdots}(p),$$

or

$$\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{B_1\cdots B_t}=e^{i\theta^i s_{i0}}L(p)^{-1}\tilde{\psi}_{A_1\cdots A_{t+k}}{}^{\bar{B}_1\cdots\bar{B}_t}(p). \tag{4.8}$$

For the purpose of evaluating the matrix elements of the current $J^-{}_A{}^B$ between the 56 physical baryon states, to the lowest nonvanishing order of momentum transfer, we put $t=1$ in (4.8) and retain terms of order $\boldsymbol{\theta}\approx\mathbf{p}$ only:

$$\tilde{\psi}_{A_1\cdots A_{k+1}}{}^{B_1}=\tilde{\psi}_{A_1\cdots A_{k+1}}{}^{\bar{B}_1}(p)+\frac{i}{m}(\mathbf{p}\cdot\boldsymbol{\sigma})_A{}^B\lambda'{}_B{}^A\psi_{A_1\cdots A_{t+k}}{}^{\bar{B}_1}(p).$$

We now use (2.8) and retain the terms involving the 56-

dimensional multiplet only, thus

$$\tilde{\psi}_{A_1\cdots A_{k+1}}{}^{B_1}\rightarrow -\frac{1}{2m}\frac{k+1}{(k+n+1)(k+n)}(\mathbf{p}\cdot\boldsymbol{\sigma})_B{}^A$$
$$\times S[k\delta_{A_1}{}^B\delta_{A_2}{}^{B_1}\tilde{\psi}_{AA_3\cdots A_{k+1}}(p)$$
$$-(k+n)\delta_A{}^{B_1}\delta_{A_1}{}^B\psi_{A_2\cdots A_{k+1}}(p)]. \quad (4.9)$$

There is another, and more direct way of obtaining this result, by substitution of (4.5) into (2.5).

Substituting (4.9) into (3.3) we obtain

$$J^-{}_A{}^B=(2/15)[\tfrac{1}{2}(\psi^{*\cdots}\mathbf{v}.{}^B\psi_{A\cdot\cdot}+\psi^{*B\cdot\cdot}\mathbf{v}_A\psi\cdots)$$
$$-\tfrac{1}{4}\psi^{*B\cdot\cdot}\mathbf{v}.{}^{\cdot}\psi_{A\cdot\cdot}+\tfrac{3}{8}\psi^{*\cdots}\psi\cdots\mathbf{v}_A{}^B$$
$$+\tfrac{5}{8}(\psi^{*\cdots}\mathbf{w}.{}^B\psi_{A\cdot\cdot}-\psi^{*B\cdot\cdot}\mathbf{w}_A{}^{\cdot}\psi\cdots)]+\cdots, \quad (4.10)$$

where dummy indices are abbreviated by dots, and

$$\mathbf{v}=-\mathbf{p}'/m'-\mathbf{p}/m,\quad \mathbf{w}=\mathbf{p}'/m'-\mathbf{p}/m. \quad (4.11)$$

In (4.10) the unwritten terms indicated by $+\cdots$ contain no 56, 56 matrix elements in the lowest order of $\mathbf{p}$ or $\mathbf{p}'$.

5. NUMERICAL RESULTS, BARYON-BARYON-MESON VERTEX

For mesons at rest, the vertex is obtained by substituting (3.3) and (4.10) into (3.1). The result is

$$V\sim-\left[-\frac{5}{7C}\right]^{1/2}(\psi^{*\cdots}\psi\ldots)\kappa$$
$$+(2/15)\{\tfrac{1}{2}\psi^{*B\cdot\cdot}(\mathbf{v}\pi^-+\pi^-\mathbf{v})_B{}^A\psi_{A\cdot\cdot}$$
$$-\tfrac{1}{4}(\psi^{*AB}\cdot\mathbf{v}_A{}^C\pi^-{}_B{}^D\psi_{CD\cdot})+\tfrac{3}{8}(\psi^{*\cdots}\psi\ldots)\mathrm{tr}(\mathbf{v}\pi^-)$$
$$+\tfrac{5}{8}\psi^{*B\cdot\cdot}(\mathbf{w}\pi^--\pi^-\mathbf{w})_B{}^A\psi_{A\cdot\cdot}\}$$
$$-\frac{1}{3}\left[-1-\frac{C}{12}\right]^{-1/2}(\psi^{*A\cdot\cdot}\pi^+{}_A{}^B\psi_{B\cdot\cdot})+\cdots. \quad (5.1)$$

Lorentz invariance (or Galilei invariance, to be precise) allows us to generalize this to any slow frame by replacing the expressions (4.11) for $\mathbf{v}$ and $\mathbf{w}$ by the following:

$$\mathbf{v}=2\mathbf{q}/\mu-\mathbf{p}'/m'-\mathbf{p}/m,\quad \mathbf{w}=\mathbf{p}'/m'-\mathbf{p}/m, \quad (5.2)$$

where $\mathbf{q}$ and μ are the meson momentum and mass.

The result (5.1), (5.2) is immediately applicable to the decay of a baryon into a baryon plus a meson. We shall also apply it to the perturbation theory vertex in which the meson momentum is spacelike. In this case it is not possible to transform the meson to rest, and it is not clear what meson mass should enter the formulae; for this reason we hestitate to compare the strength of $N^*N\pi$ and $NN\pi$. The annihilation vertex, the baryon-antibaryon-meson coupling, is discussed in the next section.

The pseudoscalar octet $P^-{}_a{}^b$, the vector octet $V^-{}_{\alpha a}{}^{\beta b}$ and the vector singlet $V^-{}_\alpha{}^\beta$ are defined by

$$\pi^-{}_A{}^B=\frac{1}{\sqrt{2}}\left[\mathbf{V}^-{}_a{}^b\cdot\boldsymbol{\sigma}_\alpha{}^\beta+\delta_\alpha{}^\beta P^-{}_a{}^b+\frac{1}{\sqrt{3}}\delta_a{}^b\mathbf{V}^-\cdot\boldsymbol{\sigma}_\alpha{}^\beta\right].$$

The baryon octet $\psi_{\alpha a}{}^b$ and the resonance decuplet $\psi_{\alpha\beta\gamma abc}$ are defined by

$$\psi_{ABC}=\psi_{abc\alpha\beta\gamma}+\frac{1}{3\sqrt{2}}[\epsilon_{abd}\epsilon_{\alpha\beta}\psi_{\gamma c}{}^d$$
$$+\epsilon_{acd}\epsilon_{\alpha\gamma}\psi_{\beta b}{}^d+\epsilon_{bcd}\epsilon_{\beta\gamma}\psi_{\alpha a}{}^d].$$

We normalize the vertex such that the pion-nucleon interaction occurs with the normalization $p\mathbf{v}\cdot\boldsymbol{\sigma}n\pi^+$. The result is (to lowest order in the momenta):

$(\bar{8},8,8^-{}_1)$. The baryon-octet coupling to the pseudoscalar octet is

$$\mathrm{tr}[\bar{\psi}\mathbf{v}\cdot\boldsymbol{\sigma}P^-\psi]+(2/7)\,\mathrm{tr}[\bar{\psi}\mathbf{v}\cdot\boldsymbol{\sigma}\psi P^-].$$

This means that the D/F ratio is 1.8, which is consistent with the experimental data on strong interactions. According to the partially conserved axial-vector current hypothesis, this should agree with the D/F ratio of weak interactions. From that point of view the value 1.8 is an excellent one. In certain models[13] this allows one to calculate $g_A/g_V=-\frac{1}{3}(D+F)/(D-F)=-1.17$, also in agreement with experiment.

$(\overline{10},8,8^-{}_1)$. The decay of the decuplet of resonances is represented by

$$(9/7)P^-{}_a{}^c\epsilon_{cdb}\bar{\psi}_i{}^{ade}\psi_e{}^b v^i.$$

As we mentioned above, there is a considerable theoretical uncertainty in comparing the strength of this coupling with that of $(\bar{8},8,8^-{}_1)$. If we follow the procedure of Gürsey and Radicati,[1] then the result is an N^* width of 80 MeV, a small improvement of their value of 60 MeV.

$(\bar{8},8,8^-{}_3)$. The baryon-octet coupling to the vector octet is

$$(11/21)\,\mathrm{tr}[\bar{\psi}\mathbf{v}\cdot\mathbf{V}_8{}^-\psi]-\tfrac{2}{3}\,\mathrm{tr}[\bar{\psi}\psi\mathbf{v}\cdot\mathbf{V}_8{}^-]$$
$$-(25/42)\,\mathrm{tr}[\bar{\psi}i\mathbf{w}\times\mathbf{V}_8{}^-\cdot\boldsymbol{\sigma}\psi]-(5/42)\,\mathrm{tr}[\bar{\psi}\boldsymbol{\sigma}\psi\cdot i\mathbf{w}\times\mathbf{V}_8{}^-].$$

The electric part (first line) has an F/D ratio of 25/3, not very much unlike models that take this coupling to be pure F. The magnetic part (second line) has $D/F=\frac{3}{2}$; if the ρ-meson mediates the electromagnetic interactions, then this value gives $\mu_p/\mu_n=-\frac{3}{2}$.

$(\bar{8},8,1^-{}_3)\cdots$ and to the vector singlet:

$$(40/7\sqrt{3})\mathbf{v}\cdot\mathbf{V}_1{}^-\,\mathrm{tr}[\bar{\psi}\psi]-(15/14\sqrt{3})i\mathbf{w}\times\mathbf{V}_1{}^-\,\mathrm{tr}[\bar{\psi}\boldsymbol{\sigma}\psi].$$

These results should be confronted with baryon-baryon scattering data.

[13] R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. Letters **16**, 377 (1966).

$(\overline{10},8,8^{-}{}_3)$. The production of resonances by the exchange of a vector meson in nucleon-nucleon or meson-nucleon scattering depends on the terms

$$-\frac{1}{7}V^{-}{}_{ia}{}^{c}\epsilon_{cdb}(\bar{\psi}_i{}^{ade}\mathbf{v}\cdot\boldsymbol{\sigma}\psi_e{}^b)-\frac{i}{7}(\mathbf{v}\times\mathbf{V}^{-}{}_a{}^c)_i\epsilon_{cdb}(\bar{\psi}_i{}^{ade}\psi_e{}^b)+\frac{10i}{7}(\mathbf{w}\times\mathbf{V}^{-}{}_a{}^c)_i\epsilon_{cdb}(\bar{\psi}_i{}^{ade}\psi_e{}^b).$$

The process of N^* decay into $N\gamma$ can be calculated by assuming dominance of an intermediate vector meson. The nature of this interaction has been measured indirectly through photoproduction of π^0 mesons using a polarized bremsstrahlung source.[14] The production is dominated by the exchange of the N^* at the resonant energy, 1236 MeV. The physically measured quantity R is the asymmetry of the differential cross section perpendicular and parallel to the electric field vector

$$R=\frac{\sigma_\perp-\sigma_{\parallel}}{\sigma_\perp+\sigma_{\parallel}}.$$

The asymmetry has been measured at energies of $E\gamma=235$, 285, and 335 MeV. Interpolating to the resonant energy, 300 MeV, we find $R=0.5\pm0.1$. We find that the $N^*N\gamma$ vertex is predominantly $M1$, and that the $E2$ contribution occurs with the correct phase.[15] In terms of the asymmetry parameter R, the value we predict is $R_{\mathrm{TH}}=0.7$. This value may be modified by the inclusion of higher order terms in the vertex, and by taking account of the large N^* and ρ-meson widths.

SU(6) singlet meson coupling. The coupling of the κ-meson in S wave:

$$(-90/7)[-10/7C]^{1/2}(\bar{\psi}\psi)\kappa.$$

Positive parity 35. This is also S wave; the octet-octet part is

$$(10/21)\left[-1-\frac{C}{12}\right]^{-1/2}[3\,\mathrm{tr}(\bar{\psi}S^{+}\psi-\bar{\psi}\psi S)+\mathrm{tr}(\bar{\psi}\boldsymbol{\sigma}\psi\cdot\mathbf{V}_8{}^{+}+5\bar{\psi}\boldsymbol{\sigma}\cdot\mathbf{V}_8{}^{+}\psi)+\sqrt{3}\mathbf{V}_1{}^{+}\cdot\mathrm{tr}(\bar{\psi}\boldsymbol{\sigma}\psi)]$$

and the $(\overline{10},8)$ part is

$$(20/7)\left[-1-\frac{C}{12}\right]^{-1/2}V_{ia}{}^{-c}\epsilon_{cdb}(\bar{\psi}_i{}^{ade}\psi_e{}^b).$$

The most important conclusion that can be drawn from these expressions is that the value C of the meson Casimir operator cannot be very close to the upper limit -12 set by unitarity.[10]

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[14] D. J. Drickey and R. F. Mozley, Phys. Rev. Letters 8, 291 (1962).

[15] For the nature of the assumptions made in this model calculation, see, for example, M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963).